Simple formula for coherent cooling

Revision 1

G. Stupakov

October 8, 2019

I. INTRODUCTION

We will try to derive the following formula for the coherent cooling posted on the board by S. Nagaitsev

$$t_{cool} = \frac{N_p}{\Delta\omega}\frac{c}{\sigma_{ze}}N_\sigma^2, \tag{1}$$

valid for short electron bunches, $\sigma_{ze} \ll \sigma_{zh}$.

II. DERIVATION

We will assume that the effective interaction wake generated by one hadron in a CeC system looksr like this

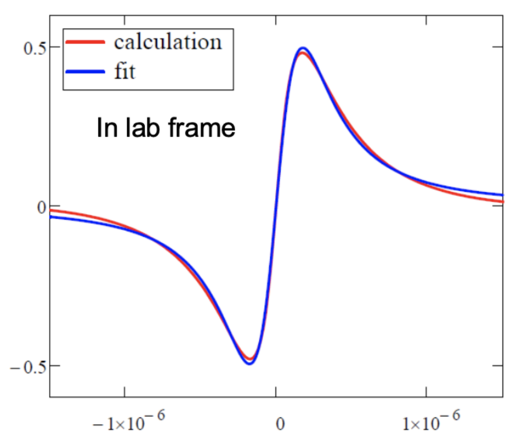

FIG. 1. Cartoon of a characteristic wake (from G. Wang presentation). The wake maximum is located at $z = \Delta z$ and is equal to V (has dimension of energy).

The wake maximum is located at $z = \Delta z$ and is equal to V (has dimension of energy).

We associate the bandwidth of this system is

$$\Delta\omega \sim \frac{c}{\Delta z} \tag{2}$$

Let us assume that the electron bunch length $\sigma_{ze} \sim \sigma_{zh}$.

Ions pass through a chicane and are shifted longitudinally by

$$\Delta z = R_{56}\eta, \tag{3}$$

where $\eta = \Delta E/E$. If we want to cool the bulk of the beam (that is roughly $1\sigma_\eta$) where σ_η is the rms energy spread, we select the R_{56} in such a way that it spreads the bulk hadron beam after the chicane to $\pm\Delta z$, which means

$$R_{56} = \frac{\Delta z}{\sigma_\eta}. \tag{4}$$

Particles with large energy deviation will be cooled by less efficiently than the bulk of the beam. However, if we want to effectively cool within the energy range $N_\sigma \sigma_h$, we need to select a smaller R_{56},

$$R_{56} = \frac{\Delta z}{N_\sigma \sigma_\eta}. \tag{5}$$

With this choice, the kick on the bulk of ions (within $1\sigma_\eta$) become smaller with the effective wake

$$V_{eff} = \frac{V}{N_\sigma} \tag{6}$$

With the kick V on each turn the cooling time is

$$N_{cool} \sim \frac{\sigma_\eta}{V_{eff}/E} \sim \frac{\sigma_\eta N_\sigma}{V/E}, \tag{7}$$

where E is the hadron energy, N_{cool} is the cooling time in revolution periods, and at this point we neglect the diffusion.

We now calculate the diffusion coefficient as

$$D \sim \left(\frac{V}{E}\right)^2 \frac{N_p}{\sigma_{zh}} \Delta z \tag{8}$$

where we multiply the square of the kick $(V/E)^2$ by the number of hadrons within the interaction length Δz. Note that the diffusion coefficient involves the characteristic wake V and not V_{eff}.

In the optimal regime, we crank up the amplification increasing the amplitude of V to the level when the diffusion rate is of the order of the cooling rate, $D/\sigma_\eta^2 \sim (N_{cool})^{-1}$,

$$\frac{1}{\sigma_\eta^2}\left(\frac{V}{E}\right)^2 \frac{N_p}{\sigma_{zh}}\Delta z \sim \frac{V/E}{\sigma_\eta N_\sigma} \tag{9}$$

from which we find

$$\frac{V}{E} = \frac{\sigma_\eta}{N_\sigma}\left(\frac{N_p}{\sigma_{zh}}\Delta z\right)^{-1} \tag{10}$$

Substituting this formula to Eq. (7) we obtain the cooling rate

$$N_{cool} \sim N_\sigma^2 \frac{N_p}{\sigma_{zh}}\Delta z. \tag{11}$$

This is the same as Eq. (1) in the limit $\sigma_{ze} \sim \sigma_{zh}$.