Theoretical Studies of Microbunched Electron Cooling (MBEC) for Future Electron-Ion Colliders

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Theorist's view on coherent cooling

Electron cooling is based on the friction force

$$\boldsymbol{F}(\boldsymbol{v}) = -\frac{4\pi n_e e^4 Z^2}{m_e}\Lambda \int d^3u\, f_e(\boldsymbol{u}) \frac{\boldsymbol{v}-\boldsymbol{u}}{|\boldsymbol{v}-\boldsymbol{u}|^3}$$

It is actually can be derived from a more general formula[1] valid for arbitrary medium with the dielectric tensor $\epsilon_{\alpha\beta}(\omega, \boldsymbol{k})$

$$F_\nu(\boldsymbol{v}) = -\frac{Z^2 e^2}{2\pi^2} \int d\omega d^3k\, \delta(\omega - \boldsymbol{k}\cdot\boldsymbol{v}) \frac{k_\nu k_\alpha k_\beta}{k^4} \mathrm{Im}\, \epsilon_{\alpha\beta}(\omega, \boldsymbol{k})$$

(the magnetized cooling force is also derivable from this).

The idea of coherent electron cooling[2] can be understood that we want to create the cooling medium with desired properties of the dielectric tensor $\epsilon_{\alpha\beta}(\omega, \boldsymbol{k})$.

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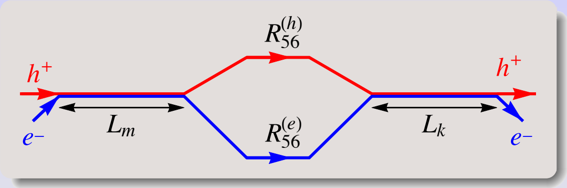
[1] J. Hubbard, Proceedings of the Royal Society of London. Series A. **260**, 114 (1961).

[2] Derbenev, AIP Conf. Proc. **253**, 103 (1992); Litvinenko, Derbenev. PRL, **102**, 114801 (2009).

Concept of generic coherent electron cooling

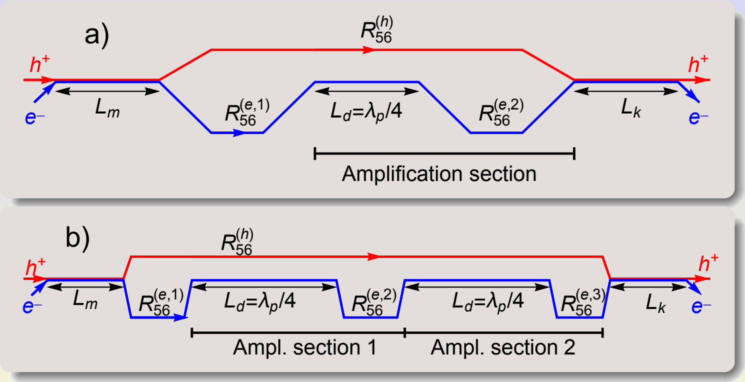


Electrons of the cooler beam with $\gamma_e = \gamma_h$ first interact with the hadron beam in a short modulator where their energy is perturbed by hadrons. The energy perturbations in the electron beam are then converted to density modulation in the chicane $R_{56}^{(e)}$. The longitudinal electric field of these density perturbations acts back on hadrons in the kicker. High-energy hadrons passing through $R_{56}^{(h)}$ move ahead and get a negative kick, low-energy move back and get a positive kick. Over many passages, this decreases the energy spread of the hadron beam.

This scheme is typically too weak to provide an adequate cooling and should be supplemented by an amplification of the signal in the electron beam.

Microbunched electron cooling (MBEC)^3.

In MBEC the amplification is provided by a sequence of drifts of $\lambda_p/4$ long and chicanes.



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[3] D. Ratner, PRL, **111**, 084802 (2013).

Theoretical studies of MBEC over the last 2 years

1. G. Stupakov, *Cooling rate for microbunched electron cooling without amplification*, PRAB, **21**, 114402 (2018)
2. G. Stupakov, P. Baxevanis, *Microbunched electron cooling with amplification cascades*, PRAB, **22**, 034401, 2019
3. P. Baxevanis and G. Stupakov, *Transverse dynamics considerations for microbunched electron cooling*, PRAB **22**, 081003 (2019).
4. G. Stupakov, *Microbunched Electron Cooling (MBEC) for Future Electron-Ion Colliders*, in HB2018, page WEA2WA02, 2018.
5. G. Stupakov, P. Baxevanis, *3D Theory of Microbunched Electron Cooling for Electron-Ion Colliders*, IPAC19, page 814, 2019.
6. P. Baxevanis and G. Stupakov, *Tolerances on energy deviation in microbunched electron cooling*, NAPAC19, WEPLH16, 2019.
7. P. Baxevanis and G. Stupakov, *Diffusion and nonlinear plasma effects in microbunched electron cooling*, NAPAC19, WEPLH17, 2019.

The model

We used the Vlasov equation to track the dynamics of microscopic 1D fluctuations in the electron and hadron beams during their interaction and propagation through the system.

Assumptions:

- 1D model: hadrons and electrons are treated as infinitely thin slices of charge Ze ($-e$ for electrons) with a Gaussian transverse charge distribution with rms sizes $\Sigma_x (\equiv \Sigma)$ and Σ_y.
- Perfect overlap of the electron and hadron beams in the modulator and the kicker.
- Particles (slices) do not shift relative to each longitudinally during the interaction in the modulator and the kicker.
- Chicanes shift particles in the longitudinal direction by $R_{56}\eta$ ($\eta = \Delta E/E$).
- There is a perfect mixing in the hadron beam on the scale Δz_{int} during one revolution in the ring.

Representative set of parameters for eRHIC MBEC

In numerical estimates I assume the following set of parameters for the hadron and electron cooler beams:

Proton energy [GeV]	275
Proton relative energy spread, $\sigma_{\eta h}$	4.6×10^{-4}
Electron energy [MeV]	150
Electron relative energy spread, $\sigma_{\eta e}$	1×10^{-4}
Electron beam charge, Q_e [nC]	1
Electron beam peak current [A]	30
Repetition rate [MHz]	112
RMS beam size in mod. and kicker, Σ_x, [mm]	0.7
L_m, L_k [m]	40

The electron bunch length, $\sigma_{ze} \approx 4$ mm, is much shorter than the proton bunch length, $\sigma_{ze} \lesssim \sigma_{zh} = 5$ cm.

The cooler-beam current is ~ 100 mA.

Longitudinal cooling time, no amplification[4]

The rate of energy spread change is (here $\eta = \Delta E/E$)

$$\frac{d\sigma_{\eta h}^2}{dt} = -\frac{\sigma_{\eta h}^2}{t_\mathrm{c}} + 2D$$

The cooling time t_c depends on $R_{56}^{(e)}$ and $R_{56}^{(h)}$. The optimal values are: $R_{56}^{(e)} = 0.4\Sigma_x/\sigma_{\eta e}\gamma = 1$ cm, $R_{56}^{(h)} = 0.4\Sigma_x/\sigma_{\eta h}\gamma = 0.2$ cm, with

$$N_\mathrm{c}^{-1} \equiv \frac{T}{t_\mathrm{c}} \approx \frac{0.3}{\sigma_{\eta h}\sigma_{\eta e}} \frac{1}{\gamma^3} \frac{Q_e c/\sigma_{zh}}{\sqrt{2\pi} I_A} \frac{r_h L_m L_k}{\Sigma_x^3} = 13.6\,\mathrm{h}$$

Here, $I_A \approx 17$ kA is the Alfven current and $r_h = (Ze)^2/m_h c^2$ is the classical radius for hadrons and T is the revolution period.
The cooling rate increases for smaller Σ_x, but we cannot focus both (hadron and electron) beams in the modulator and the kicker.

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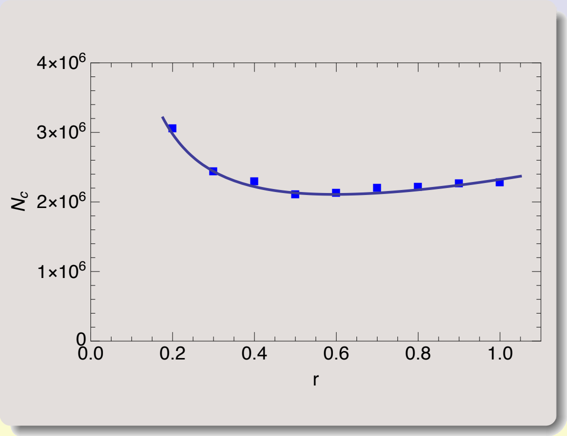
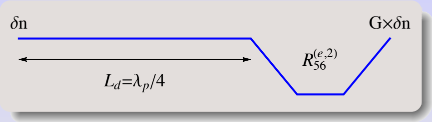
[4] G. Stupakov. PRAB, **21**, 114402 (2018)

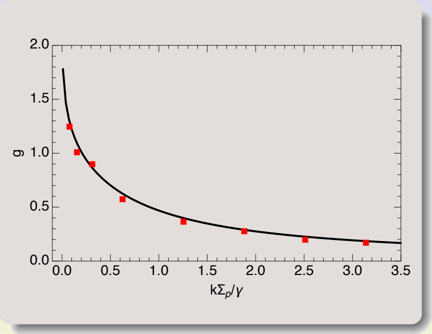
Numerical simulations

We used $N_e = 10^4$ electron macroparticles and the length of the "electron bunch" $\Delta z = 20\Sigma/\gamma$. The averaging was done over $M = 5 \times 10^6$ runs. The plot of the simulated cooling times as a function of the dimensionless chicane strength $r = R_{56}^{(h)}\sigma_{\eta h}\gamma/\Sigma = R_{56}^{(e)}\sigma_{\eta e}\gamma/\Sigma$.



Amplification of microbunching in the electron beam[5]





In 1D model, the amplification factor $G(k)$ is derived theoretically. For the optimized chicane strength (note the minus sign in G—this is for $R_{56}^{(e,2)} > 0$),

$$G(k) = -\frac{1}{\sigma_{\eta e}}\sqrt{\frac{I_e}{I_A\gamma}}\, g\left(\frac{k\Sigma_p}{\gamma}\right)$$

where Σ_p is the beam radius.
We also simulated g solving equations of motion for electrons in the drift with account of the Coulomb interactions. Red dots—the result of simulations.

This is a broadband amplifier. Unfortunately, small k (long period) plasma oscillations have small plasma frequency.

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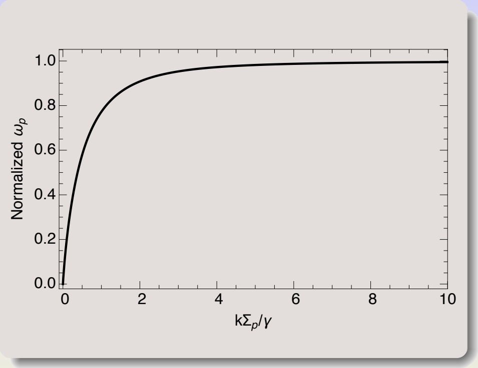
[5] Schneidmiller and Yurkov, PRSTAB **13**, 110701 (2010); Dohlus, Schneidmiller and Yurkov, PRSTAB **14** 090702 (2011); Marinelli et al., PRL **110**, 264802 (2013).

Dispersion of plasma oscillations

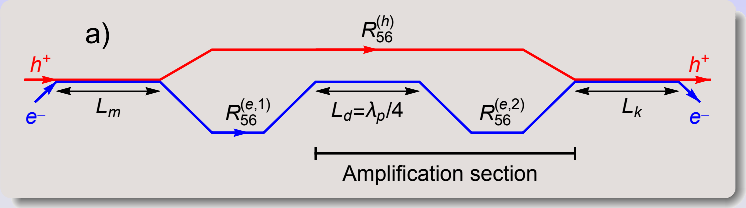


Plasma frequency is normalized by

$$\omega_{p0} = \frac{c}{\Sigma_p}\left(\frac{I_e}{I_A\gamma^3}\right)^{1/2}$$

This makes the optimal length of the amplification section longer than follows from simple estimates.

MBEC amplification using plasma oscillations in e-beam



Analytic theory predicts for the *peak* amplification factor for the beam current I_e (assuming $\Sigma_p = 0.1$ mm)

$$G \sim \frac{1}{\sigma_{\eta e}} \sqrt{\frac{I_e}{\gamma I_A}} \approx 24$$

For a quarter of plasma period at the peak current we have

$$\frac{1}{4}\lambda_p \approx 14.5 \text{ m}$$

Two stages of plasma amplification should be enough for eRHIC.

MBEC transverse cooling[7]

- The transverse cooling is achieved through introduction of the dispersion in the modulator and the kicker. We followed approach developed in[6].
- For the hadron transport line between the modulator and the kicker, the four-dimensional transfer matrix is given by

$$R = \begin{pmatrix} R_{33} & R_{34} & 0 & R_{36} \\ R_{43} & R_{44} & 0 & R_{46} \\ R_{53} & R_{54} & 1 & R_{56} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

  which is supposed to act on the combined vector $\mathbf{y} = (y, \theta_y, z, \eta)$.
- We adopt a simplified model in which $\beta_1 = \beta_2 = \beta$, $\alpha_1 = \alpha_2 = 0$, $D_1 = D_2 = D$ and $D_1' = D_2' = 0$ (1 and 2 refer to the modulator and the kicker).
- Neglecting diffusion effects, the cooling equations for energy spread σ_η and emittance ϵ are

$$\frac{d(\sigma_{\eta h})^2}{dt} = -\frac{(\sigma_{\eta h})^2}{N_c^\eta T}, \qquad \frac{d\epsilon}{dt} = -\frac{\epsilon}{N_c^\epsilon T}.$$

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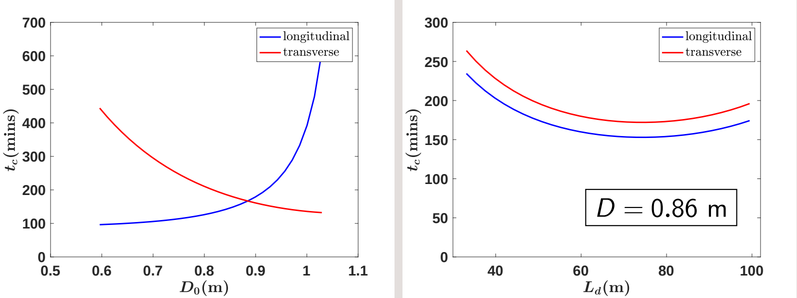
[6] V. Lebedev, Optical stochastic cooling, ICFA Beam Dyn. Newslett. 65, 100 (2014).

[7] P. Baxevanis and G. Stupakov. PRAB **22**, 081003 (2019).

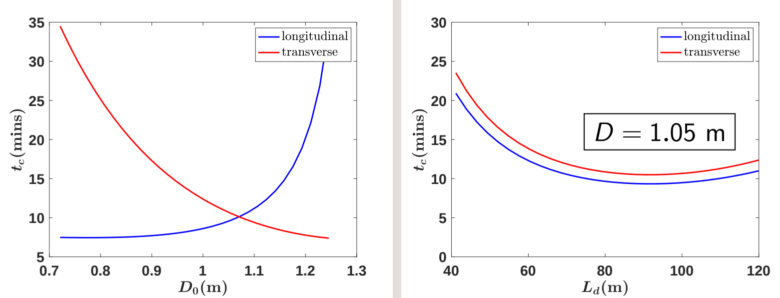
Optimization of the cooling rate, one ampl. stage

Our analytical formulas are encoded in a Matlab script and we can run various optimizations of the cooling rate. We maximized the transverse cooling rate, and then varied parameters relative to this optimal point. The optimized values: $D = 1.08$ m, phase advance from the modulator to the kicker $= 0.33$ rad (modulo 2π), plasma stage length $= 66.3$ m, $R_{56}^{h} = 0.75$ cm, $R_{56}^{(e,1)} = R_{56}^{(e,2)} = 1.83$ cm.



Optimization of the cooling rate, two ampl. stages

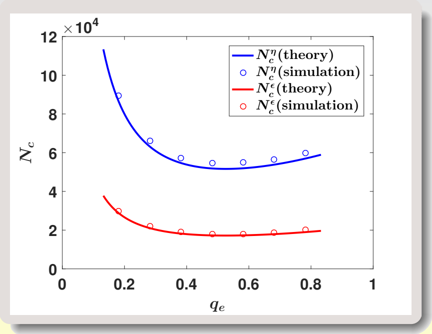
Optimized parameters for two amplification stages: $D = 1.31$ m, phase advance $= 0.38$ rad (modulo 2π), plasma stage length $= 82.2$ m, $R_{56}^{h} = 1.26$ cm, $R_{56}^{(e,1)} = R_{56}^{(e,2)} = R_{56}^{(e,3)} = 2.54$ cm.



Note relatively weak dependence of t_c on the length of the amplification section.

Comparison with simulation

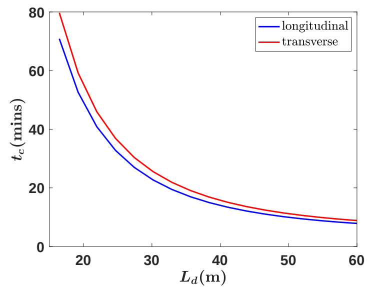
- To benchmark our theory, we compare with 1D simulation.
- The actual machine parameters would lead to a very high number of simulation macroparticles ($\sim 10^6$).
- Instead, we use an alternative parameter set which allows us to use fewer macroparticles ($\sim 10^4$ for electrons and $\sim 10^3$ for hadrons).

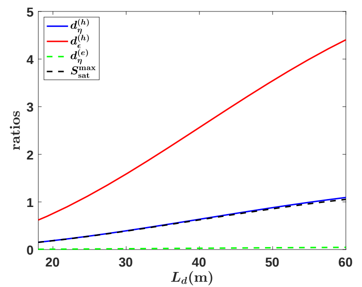


- Good agreement is observed between theory and simulation for one amplification section (q_e is the dimensionless chicane strength).

Effect of noise amplification and the saturation effects

This is the case of two amplification stages. In the right plot: $d_\eta^{(h)}$ is the amplified hadron noise/cooling ratio for longitudinal cooling, $d_\eta^{(e)}$ is the amplified electron noise/cooling ratio for longitudinal cooling, $d_\epsilon^{(h)}$ is the amplified hadron noise/cooling ratio for transverse cooling, $S_{\mathrm{sat}}^{\max}$ the saturation parameter. We need $d_\eta^{(h)} + d_\eta^{(e)}, d_\epsilon^{(h)} < 1$ and $S_{\mathrm{sat}}^{\max} \ll 1$.





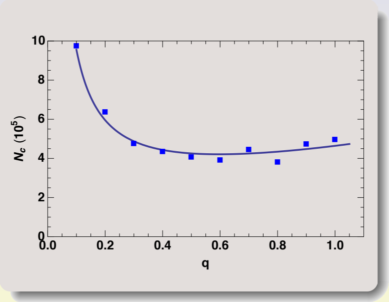
Note relatively weak dependence on the length of the amplification section.

3D effects in cooling without amplification[8]

In 3D theory particles are point charges and interact through the Coulomb field

$$f_z = -Ze^2 \frac{z\gamma}{(x^2+y^2+\gamma^2 z^2)^{3/2}}$$

The cooling rate is sensitive to the phase advance between the modulator and the kicker.



For the relative phase advance π ($x_e^K = -x_e^M$ $y_e^K = -y_e^M$ and $x_h^K = x_h^M$ and $y_h^K = y_h^M$) we found that the 3D theory gives the same cooling rate as 1D (q is dimensionless R_{56}). The plot shows the result of 3D simulations (dots) and 1D theory (solid line).

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[8] G. Stupakov, P. Baxevanis, *3D Theory of Microbunched Electron Cooling for Electron-Ion Colliders*, IPAC19, page 814, 2019.

Summary

- We have developed a theoretical model that describes the MBEC process for both the energy spread and the transverse emittance of the hadron beam. The model includes amplification stages that use 1/4 plasma oscillation drifts and chicanes.
- Our derivation is based on a one-dimensional (1D) Vlasov technique that tracks the evolution of the beam fluctuations through the MBEC setup.
- Simple formulas are obtained for the cooling times, allowing for fast optimization studies. Our analysis is benchmarked via comparison with 1D simulations.
- Noise effects and nonlinearity of amplification are now included in the analysis.
- Preliminary study of 3D effects confirms results of 1D model.
- From a practical point of view, cooling times below 1h appear to be feasible for the eRHIC parameters (both for energy spread and emittance) by making use of two plasma amplification stages.