RESONANCE AXIAL–VECTOR MASS EXTRACTED FROM EXPERIMENTS ON NEUTRINO–HYDROGEN AND NEUTRINO–DEUTERIUM SCATTERING

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Plan of the presentation

- Motivation for the study of the resonance reactions
- Development of the Rein-Sehgal model
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  - Normalization factors of Breit-Wigner distributions
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- Fine tune of non-interfering non-resonance background
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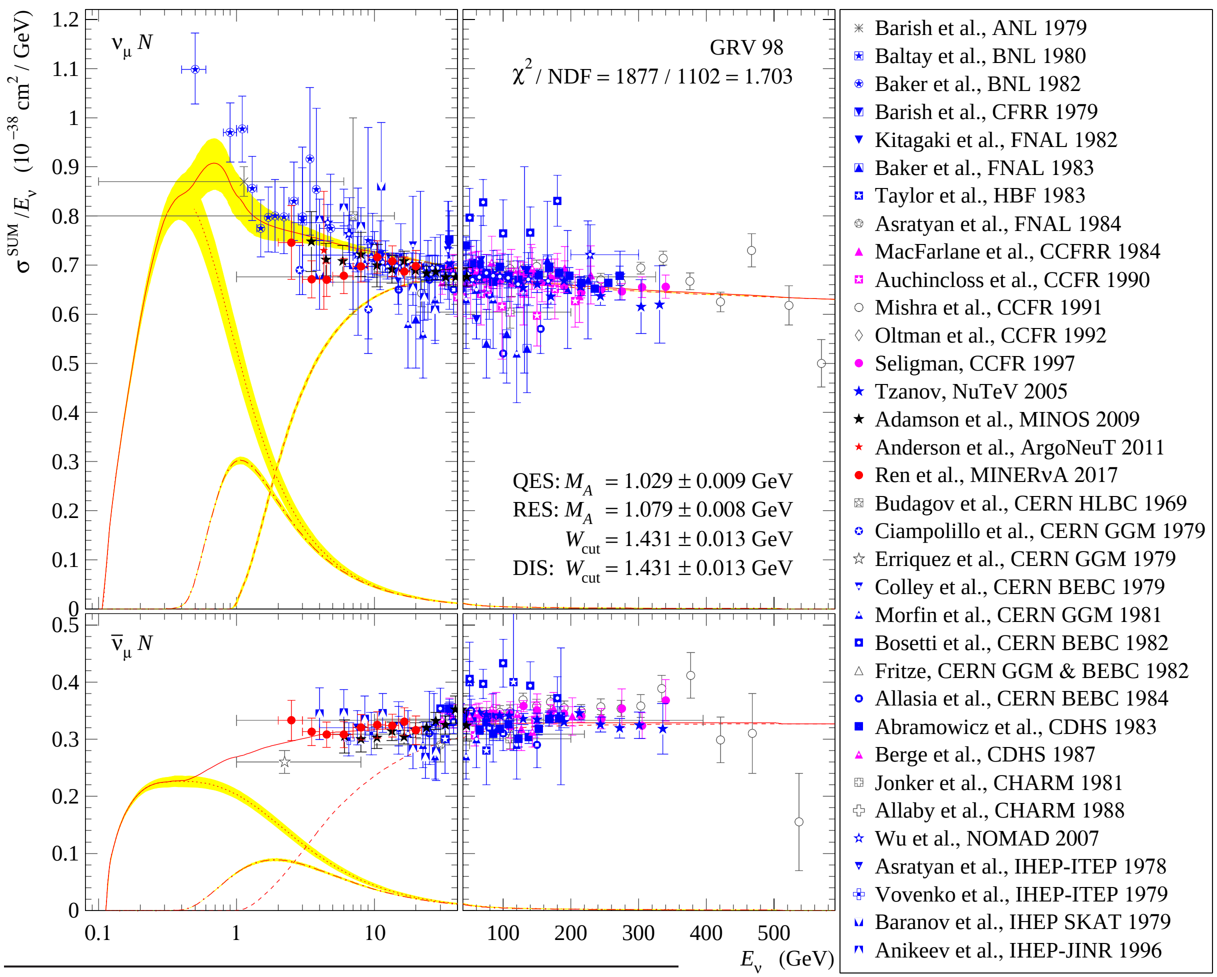
MOTIVATION FOR THE STUDY OF THE RESONANCE REACTIONS



Total CC ν_μ and $\overline{\nu}_\mu$ cross sections as functions of neutrino energy and normalized to energy for scattering off isoscalar nucleons in comparison with experimental data [a].

The curves and bands of theoretical uncertainties due to phenomenological parameters show the quasielastic (QES with contribution from Λ, Σ^-, and Σ^0 productions in the case of $\overline{\nu}_\mu$ reactions), resonance (RES), and deep inelastic (DIS) contributions and their sums.

[a] See for example K. S. Kuzmin, V. V. Lyubushkin, and V. A. Naumov, hep-ph/0511308; A. V. Akindinov *et al.*, Eur. Phys. J. C **79** (2019), 758, and references therein.

DEVELOPMENT OF THE REIN–SEHGAL MODEL

- For the phenomenological description of the CC and NC RES reactions we use the model proposed by D. Rein and L. M. Sehgal in 1981 (RS model) [a] and modified in recent years by other authors. The model is based on the formulation of the charged hadronic current in terms of the relativistic quark model by R. P. Feynman, M. Kislinger, and F. Ravndal (FKR) [b] taking into account contributions from 18'th interfering baryon and nucleon resonances below 2 GeV on the invariant mass of final system of hadrons W, and non-interfering non-resonance contributions as the background (NRB) of the reactions.
  The original version of the RS model neglects mass of the final charged lepton.
- In 2004 the final lepton mass correction and lepton polarization have been properly included into the leptonic current [c]. It was shown that the dynamic mass correction is very important for the ν_τ and $\overline{\nu}_\tau$ induced 1π production but, for the ν_μ and $\overline{\nu}_\mu$ induced reactions it is typically at the few percent level or less that is almost negligible in comparison with the kinematic mass corrections and with the intrinsic uncertainties of the RS model.
  The numerical algorithm for calculation of the cross-sections has been improved.
- In 2006 C. Berger and L. M. Sehgal improved the KLN model by taking into account the pion-pole contribution to the hadronic axial current, arising in the case of nonzero mass of final charged lepton (KLN-BS = KLN-BRS model) [d].

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[a] D. Rein, L. M. Sehgal, Annals Phys. **133**, 79 (1981).
[b] R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D **3**(1971) 2706.
[c] K. S. Kuzmin, V. V. Lyubushkin, and V. A. Naumov, Mod. Phys. Lett. A **19** (2004) 2815, Nucl. Phys. B (Proc. Suppl.) **139** (2005) 158.
[d] C. Berger and L. M. Sehgal, Phys. Rev. D **76** (2007) 113004.

- In 2008 K. M. Graczyk and J. T. Sobczyk investigated the different approaches to take into account nonzero lepton mass effect and modification of the axial current due to a pion pole term. It has been shows that their result is equivalent to KLN-BS. The alternative vector and axial form factors are proposed to improve the RS approach in the $\Delta(1232)$ resonance region [a].
- In 2018 M. Kabirnezhad (under supervision of J. T. Sobczyk and P. Przewlocki [b]) following the original paper by D. Rein [c] and ideas by E. Hernández, J. Nieves, and M. Valverde [d] suggested the new approach to calculation of the interference NRB with resonance contributions for the KLN-BS model [e]. This approach contains the 17'th interfering resonances below 2 GeV (resonance $F_{17}(1900)$ is excluded).
- Currently, the new modifications for the KLN-BS model are proposed [f]:
  - All previous versions of the model include the normalization factors of W-dependent Breit-Wigner distributions of resonances. The method of calculation of the normalization factors is ambiguous. We explain the problem and suggest easiest solution.
  - The KLN-BS model represents the non-interfering NRB by a resonance amplitude of P_{11} character with the Breit-Wigner factor replaced by an adjustable constant f_{NRB}. We define the f_{NRB} from a global fit of the experimental data.

[a] K. M. Graczyk and J. T. Sobczyk, Phys. Rev. D **77** (2008) 053001, Phys. Rev. D **77** (2008) 053003.

[b] M. Kabirnezhad, Ph.D. Thesis, Wroclaw U., 2017.

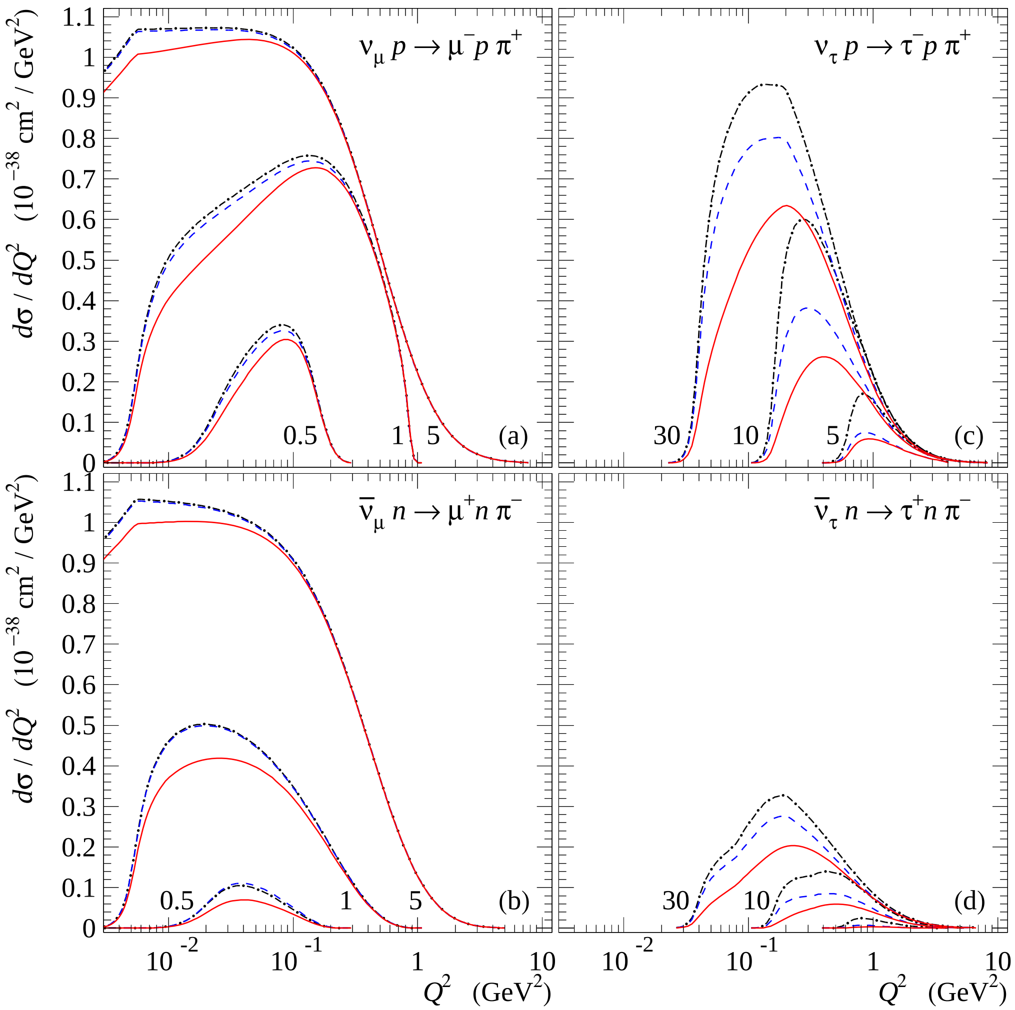
[c] D. Rein, Z. Phys. C **35** (1987) 43.

[d] E. Hernández, J. Nieves and M. Valverde, Phys. Rev. D **76** (2007) 033005.

[e] M. Kabirnezhad, Phys. Rev. D **97** (2018) 013002; JPS Conf. Proc. **12** (2016) 010043; J. Phys. Conf. Ser. **888** (2017) 012122.

[f] I. D. Kakorin, K. S. Kuzmin, and V.A. Naumov, paper in preparation.

NONZERO LEPTON MASS IN LEPTONIC CURRENT AND PION–POLE CONTRIBUTION TO THE HADRONIC AXIAL CURRENT



Differential cross sections as functions of $Q^2 = -q^2$ for different values of the neutrino energy (denoted near the groups of curves) for two neutrino (a), (c) and antineutrino (b), (d) RES reactions.

Dashed-dotted lines correspond to the cross sections predicted by the RS model with nonzero lepton mass included only into kinematics.

Dashed lines correspond to the cross sections calculated for the KLN model included the lepton mass into kinematics and definition of the leptonic current but not included the BS correction into hadronic current.

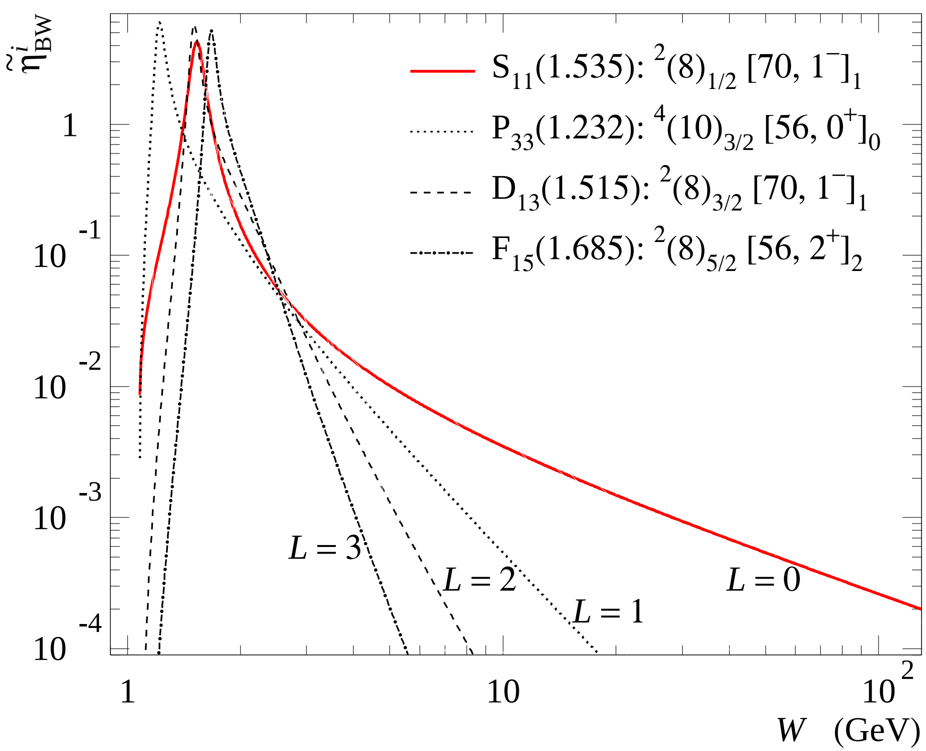
Solid lines show cross sections calculated for KLN-BS model included corrections for leptonic and hadronic currents and kinematics.

NORMALIZATION FACTORS OF BREIT-WIGNER DISTRIBUTIONS

Normalization factors of Breit-Wigner distributions of resonances are defined by

$$N_i = \int_{W_{\min}=m_{N'}+m_\pi}^{\infty} \tilde{\eta}^i_{BW}(W)\,dW, \quad \tilde{\eta}^i_{BW}(W) = \frac{1}{2\pi}\frac{\Gamma_i(W)}{(W-M_i)^2+\Gamma_i^2(W)/4},$$

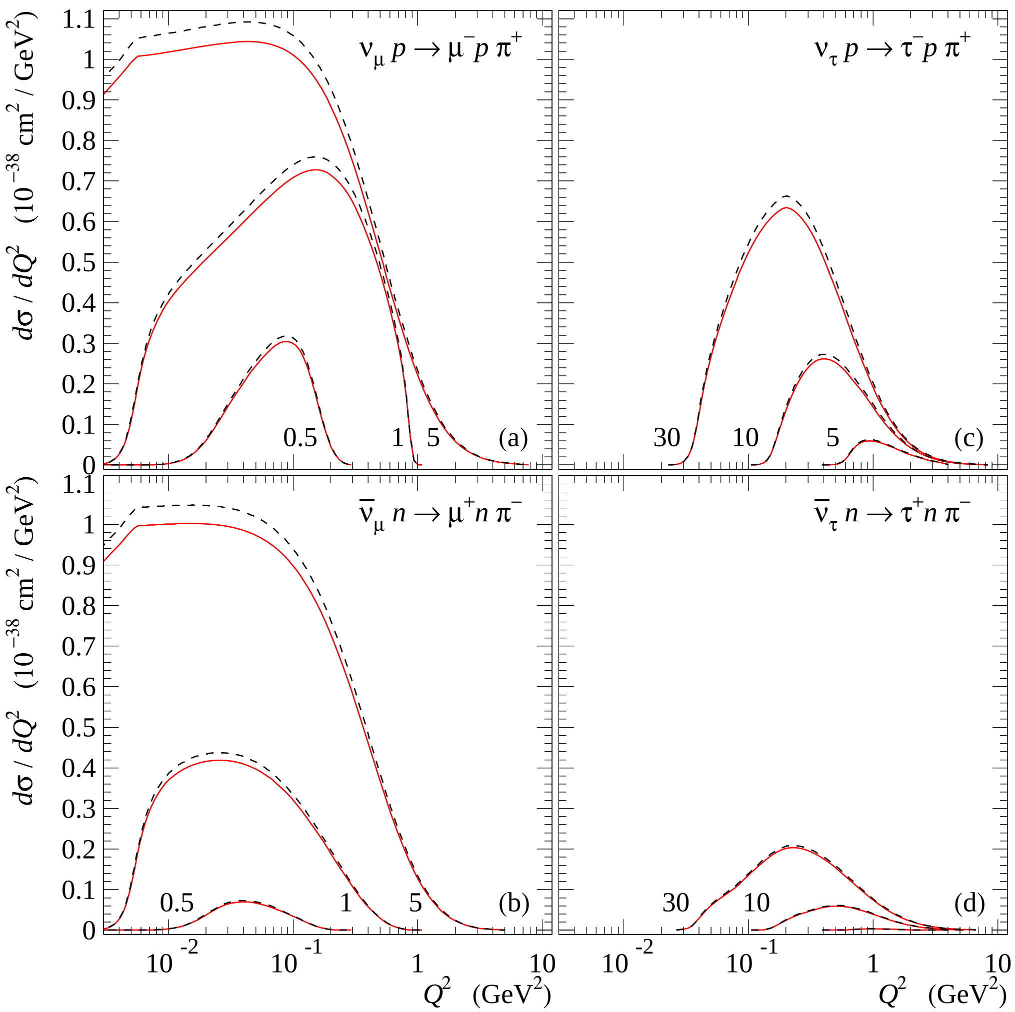
$$\Gamma_i(W) = \Gamma_i^0\left[q_\pi(W)/q_\pi(M_i)\right]^{2L+1}, \quad q_\pi(W) = 1/(2W)\sqrt{\left(W^2-m_{N'}^2-m_\pi^2\right)^2-4m_{N'}^2m_\pi^2},$$



where $m_{N'}$, m_π, L, M_i, and Γ_i^0 are masses of final nucleon and pion, total orbital angular momentum of resonance, resonance mass range, and Breit-Wigner width, accordingly. Index i indicates the resonance stares S $(L=0)$, P $(L=1)$, D $(L=2)$, and F $(L=3)$.

For numerical integration it is necessary to determine manually the upper limit of W but there are no any physical reasons to choice. As shown if the figure the asymptotic behavior of the Breit-Wigner distribution for S resonances ($\sim 1/W$) leads to unphysical $N_S=\infty$.

The condition of $N_i = 1$ avoids the ambiguity of calculation.



Differential Q^2-dependent cross sections of the same reactions and for the same neutrino energies as at the previous figure.

Dashed lines correspond to the cross sections predicted for the KLN-BS model with the normalization factors of Breit-Wigner distributions of resonances determined by Rein and Sehgal (values are listed in the table an the next page).

Solid lines correspond to cross sections for the KLN-BS model without the normalization factors.

The cross sections calculated without normalization factors of Breit-Wigner distributions of resonances are a few percent lower in comparison with the cross sections used controversial values of normalization factors.

NOTE! The GENIE generator (version 3.0.0 and earlier) utilizes somewhat different values for the N_i inherited from the NEUGEN neutrino event generator [a].

[a] H. Gallagher, Nucl. Phys. **B** (Proc. Suppl.) **112** (2002) 188.

PARAMETERS OF NUCLEON RESONANCES WITH MASSES BELOW 2 GEV INCLUDED INTO THE REIN–SEHGAL MODEL [a]

1	2	3	4	5	6	7	8
$P_{11}(1440)$	$[56, 0^+]_2$	****	1410–1470 (1440) (1430) (1450)	250–450 (350) (350) (370)	55–75 (0.65) (?) (0.65)	$+$	0.784
$D_{13}(1520)$	$[70, 1^-]_1$	****	1510–1520 (1515) (1515) (1525)	100–120 (110) (115) (125)	55–65 (0.60) 55–65 (0.56)	$-$	1
$S_{11}(1535)$	$[70, 1^-]_1$	****	1515–1545 (1530) (1535) (1540)	125–175 (150) (150) (270)	32–52 (0.42) (?) (0.45)	$-$	1.067
$S_{11}(1650)$	$[70, 1^-]_1$	****	1640–1680 (1650) (1655) (1640)	145–190 (150) (140) (140)	55–90 (0.60) 50–70 (0.60)	$+$	1.051
$D_{15}(1675)$	$[70, 1^-]_1$	****	1665–1680 (1675) (1675) (1680)	130–160 (150) (150) (180)	38–42 (0.40) 35–45 (0.35)	$+$	1.024
$F_{15}(1685)$	$[56, 2^+]_2$	****	1680–1690 (1685) (1685) (1680)	115–130 (120) (130) (120)	60–70 (0.65) 65–70 (0.62)	$+$	0.912
$D_{13}(1700)$	$[70, 1^-]_1$	***	1650–1800 (1720) (1700) (1670)	100–300 (200) (150) (80)	7–17 (0.12) (0.12) (0.10)	$-$	1.165
$P_{11}(1710)$	$[70, 0^+]_2$	****	1680–1740 (1710) (1710) (1710)	80–200 (140) (100) (100)	5–20 (0.10) (?) (0.19)	$+$	1.349
$P_{13}(1720)$	$[56, 2^+]_2$	****	1680–1750 (1720) (1720) (1740)	150–400 (250) (250) (210)	8–14 (0.11) (0.11) (0.19)	$+$	1.301
$F_{17}(1900)$	$[70, 2^+]_2$	**	1950–2100 (2020) (?) (1970)	200–400 (300) (300) (325)	2– 6 (0.04) (0.04) (0.06)	$+$	0.619
$P_{33}(1232)$	$[56, 0^+]_0$	****	1230–1234 (1232) (1232) (1234)	114–120 (117) (117) (124)	(99.4) (99.4) (1)	$+$	0.957
$P_{33}(1600)$	$[56, 0^+]_2$	****	1500–1640 (1570) (1600) (1640)	200–300 (250) (320) (370)	8–24 (0.16) 10–25 (0.20)	$+$	0.935
$S_{31}(1620)$	$[70, 1^-]_1$	****	1590–1630 (1610) (1630) (1620)	110–150 (130) (140) (140)	25–35 (0.30) 20–30 (0.25)	$+$	1.055
$D_{33}(1700)$	$[70, 1^-]_1$	****	1690–1730 (1710) (1700) (1730)	220–380 (300) (300) (300)	10–20 (0.15) 10–20 (0.12)	$+$	0.751
$F_{35}(1905)$	$[56, 2^+]_2$	****	1855–1910 (1880) (1880) (1920)	270–400 (330) (330) (340)	9–15 (0.12) 9–15 (0.15)	$-$	0.635
$P_{31}(1910)$	$[56, 2^+]_2$	****	1850–1950 (1900) (1890) (1920)	200–400 (300) (280) (300)	15–30 (0.20) 15–30 (0.19)	$-$	1.229
$P_{33}(1920)$	$[56, 2^+]_2$	***	1870–1970 (1920) (1920) (1960)	240–360 (300) (260) (300)	5–20 (0.12) 5–20 (0.17)	$+$	1.285
$F_{37}(1950)$	$[56, 2^+]_2$	****	1915–1950 (1930) (1930) (1950)	235–335 (285) (285) (340)	35–45 (0.40) 35–45 (0.40)	$+$	0.710

1 Resonance symbol $L_{2I,2J}(M_i)$, where $L = S, D, F, P$, the labels I and J indicate the isospin and spin, respectively, and M_i is the central mass.

2 FKR relativistic quark model assignment in terms of the flavor-spin $SU(6)$ basis $[D, L^P]_N$, where D is the dimensionality of the $SU(6)$ representation, L is the total quark orbital angular momentum, P is the total parity and N is the number of quanta of excitation.

3 Resonance status according to PDG: ****existence is certain, and properties are at least fairly well explored; ***existence ranges from very likely to certain, but further confirmation is desirable and/or quantum numbers, branching fractions, etc. are not well determined **evidence of existence is only fair.

4 Resonance mass M_i range (in MeV) according to PDG 2018 and (the central mass according to Rein and Sehgal, 1981).

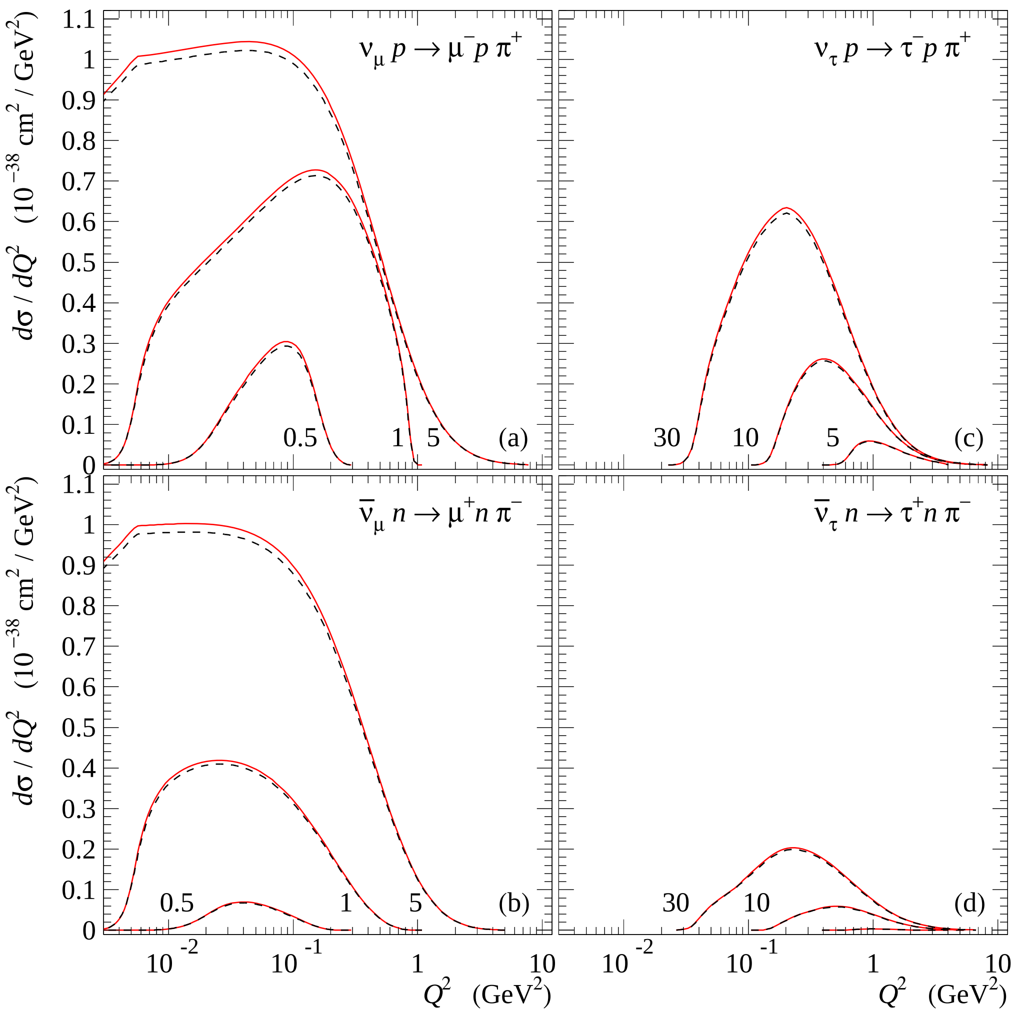
5 Breit-Wigner width Γ_i^0 range and, in parentheses, its mean value (in MeV) (the mean value according to Rein and Sehgal, 1981).

6 Branching ratio of the resonance decay into the $N\pi$ state (in %) and, in parentheses, the selected elasticity, χ_i.

7 The pure decay sign, $\text{sign}(N_i^*)$.

8 Normalization factor of Breit-Wigner distribution according to Rein and Sehgal, 1981.

[a] M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98** (2018) 030001, C. Patrignani *et al.* (Particle Data Group), Chin. Phys. **C 40** (2016) 100001, and R. E. Cutkosky *et al.*, Phys. Rev. D **20** (1979) 2839 cited by Rein and Sehgal, 1981.



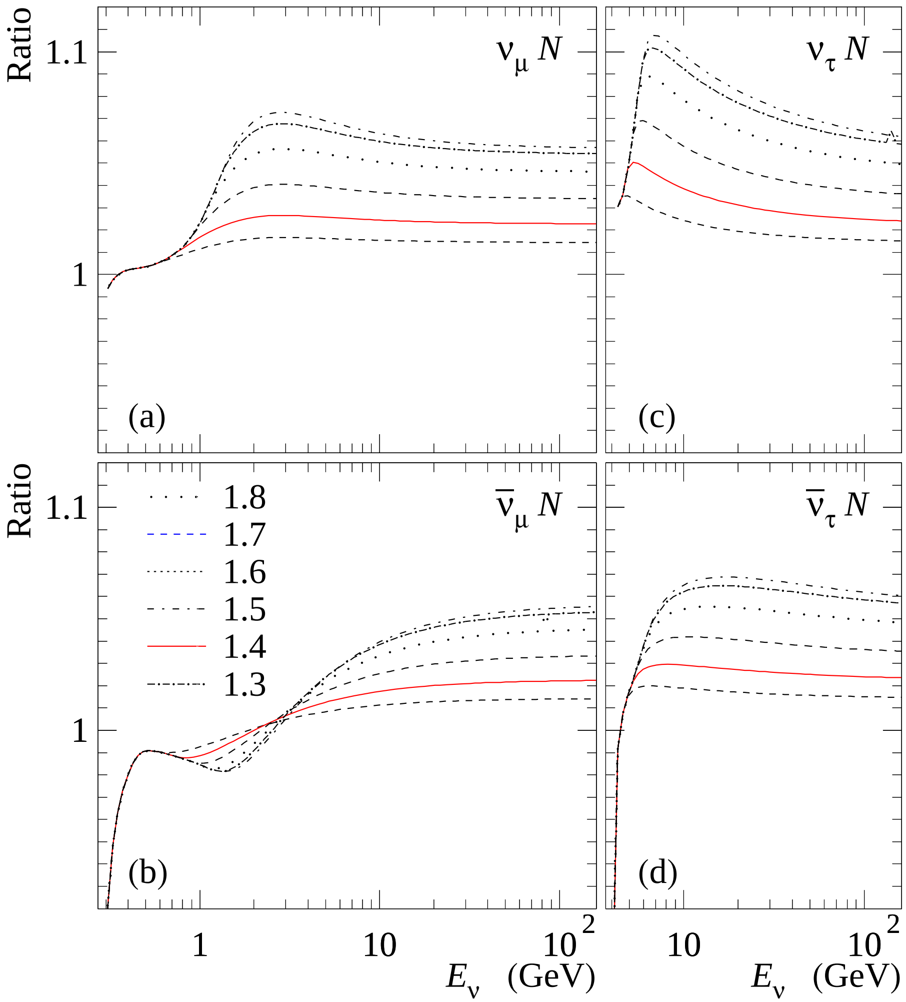
Differential Q^2-dependent cross sections (calculated for the KLN-BS model with with new definitions of $N_i = 1$) of the same reactions and for the same neutrino energies as at the previous figure.

Dashed and Solid lines correspond to cross sections calculated with outdated parameters of the resonances used in the original version of RS model and up-to-date parameters according to PDG 2018, respectively.

There is obviously a need to keep physical parameters up-to-date for precise determination of the RES cross sections.

NOTE! The official physics tunes of the all versions of the GENIE generator are obtained with earlier values of the parameters according to PDG 2016.

INTERFERENCE OF RESONANCE AMPLITUDES



Neutrino interactions with nuclei lead to generation of the hadron resonance with different quantum numbers. The amplitudes of different resonance states interfere to produce the calculated final state of hadron system. Each of the interfering resonances by simultaneous decay produce the same final system with one or several pions at a fixed invariant final mass W.

Figure shows the ratio of the total cross sections calculated without taking into account interference of the amplitudes of resonances to the default cross sections predicted by the KLN-BS model including the effects of interference. Cross sections are calculated without NRB contributions. Different lines show the ratio for different values of W.

NOTE! Currently the GENIE generator does not take into account the interference effect.

RESONANCE AXIAL MASS

The FKR model adopted for the RS model assume the standard dipole parametrization for the vector and axial-vector transition form factors

$$G^{V,A}\left(Q^2\right) \propto \left(1+\frac{Q^2}{4M_N^2}\right)^{1/2-n}\left(1+\frac{Q^2}{M_{V,A}^2}\right)^{-2}.$$

Here the vector mass is fixed on the "standard" value of $M_V = 0.84$ GeV [a], n is the integer number of oscillator quanta presented in the final resonance.

In the previous study the M_A is determined from the global likelihood analysis of the all available experimental data known at that time measured in experiments with different nuclear targets [b]. The world average value of $M_A = 1.12 \pm 0.03$ GeV has been used in the early versions of GENIE [c] and neutrino generators of MINERνA [d], MINOS [e], and T2K [f] experiments.

In the latest versions of GENIE (since 3.0.0) the value of M_A is an adjustable parameter in different physics tunes.

[a] In the original version of the RS model the axial mass was $M_A = 0.95$ GeV.

[b] K. S. Kuzmin, V. V. Lyubushkin, V. A. Naumov, Acta Phys. Polon. B **37** (2006) 2337.

[c] C. Andreopoulos *et al.*, Nucl. Instrum. Meth. A **614** (2010) 87. C. Andreopoulos *et al.*, arXiv:1510.05494 [hep-ph].

[d] C. L. McGivern *et al.* (MINERvA Collaboration), Phys. Rev. D **94** (2016) 052005.

[e] P. Adamson *et al.* (MINOS Collaboration), Phys. Rev. D **91** (2015) 012005.

[f] K. Abe *et al.* (T2K Collaboration), Nucl. Instrum. Meth. A **659** (2011) 106.

In the recent study T2K Collaboration uses the value of $M_A = 1.21$ GeV (arXiv:1909.03936 [hep-ex]).

METHOD OF DATA FITTING

We use the following least-square statistical model:

$$\chi^2 = \sum_i \left\{ \sum_{j\in G_i} \frac{\left[N_i T_{ij} - E_{ij}\right]^2}{\sigma_{ij}^2} + \frac{(N_i-1)^2}{\sigma_i^2} \right\}.$$

Here the index i enumerates the experimental data groups G_i, index $j \in G_i$ enumerates the bin-averaged experimental data E_{ij} from the group G_i, and σ_{ij} is the error of E_{ij} without the uncertainty due to the $\nu/\overline{\nu}$ flux normalization. The individual for each data group G_i flux normalization N_i is treated as free fitting parameter and included into the ordinary penalty term, $\left(N_i-1\right)^2/\sigma_i^2$, where σ_i is the flux normalization error. The T_{ij} represents the bin-averaged theoretical prediction, dependent on the set of fitting parameters $\boldsymbol{\lambda}$.

The procedure of minimization can be simplified by substituting into χ^2 equation $N_i = \mathcal{N}_i$, where $\mathcal{N}_i$ are obtained from the analytic solution to the equations $\partial\chi^2/\partial N_i = 0$,

$$\mathcal{N}_i(\boldsymbol{\lambda}) = \frac{1+\sigma_i^2\sum_{j\in G_i}\sigma_{ij}^{-2}T_{ij}E_{ij}}{1+\sigma_i^2\sum_{j\in G_i}\sigma_{ij}^{-2}T_{ij}^2}.$$

As follows from the analysis, the deviation of the normalization factors N_i from unity for each data group G_i does not exceed the doubled experimental uncertainty of the corresponding $\nu_\mu/\overline{\nu}_\mu$ flux normalization.

EXPERIMENTAL DATA, GLOBAL FIT, AND COMPARISON WITH THE DATA

In the present study the value of axial-vector mass can be obtained from the fit for the cross sections of the $\nu_\mu p \to \mu^- \Delta^{++}$, $\nu_\mu p \to \mu^- p\pi^+$, and $\overline{\nu}_\mu n \to \mu^+ n\pi^-$ reactions not requiring the NRB in the Rein-Sehgal approach, and measured with experiments with H_2 and D_2 targets.

The data set includes the following experimental data: Barish *et al.*, ANL 1979 [a] ($\langle d\sigma_\nu/dQ^2\rangle$), Radecky *et al.*, ANL 1982 [b] (σ_ν, $\langle d\sigma_\nu/dQ^2\rangle$), Furuno *et al.*, ANL 2003 ($\sigma_\nu/\sigma_\nu^{\text{QES}}$), Furuno *et al.*, BNL 2003 ($\sigma_\nu/\sigma_\nu^{\text{QES}}$) [c], Kitagaki *et al.*, BNL 1986 [d] (σ_ν), Bell *et al.*, FNAL 1978 [e] (σ_ν), Allen *et al.*, CERN BEBC 1980 [f] (σ_ν), Allasia *et al.*, CERN BEBC 1983 [g] ($\sigma_{\overline{\nu}}$), Barlag *et al.*, CERN BEBC 1984 [h] (σ_ν), Allen *et al.*, CERN BEBC 1986 [i] (σ_ν), Allasia *et al.*, CERN BEBC 1990 [j] (σ_ν, $\langle d\sigma_\nu/dQ^2\rangle$, $\sigma_{\overline{\nu}}$, $\langle d\sigma_{\overline{\nu}}/dQ^2\rangle$).

Thus, the fitted data set consists of the 105 data points with the 93 data points of ν_μ cross sections (88.6% of the total number of data points) and the 12 of $\overline{\nu}_\mu$ cross sections (11.4%).

The axial mass is $M_A = 1.176^{+0.071\,(0.087)}_{-0.067\,(0.083)}$ GeV ($\chi^2/\text{ndf} = 1.99$).

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[a] S. J. Barish *et al.*, Phys. Rev. D **19** (1979) 2521.

[b] G. M. Radecky *et al.*, Phys. Rev. D **25**, 1161 (1982); erratum – *ibid.* D **26** (1982) 3297.

[c] K. Furuno *et al.*, KEK Preprint 2003-48 (unpublished); M. Sakuda and E. F. Paschos, a talk at the NuInt 2002; M. Sakuda, a talk at the NOON 2003.

[d] T. Kitagaki *et al.*, Phys. Rev. D **34** (1986) 2554

[e] J. Bell *et al.*, Phys. Rev. Lett. **41** (1978) 1012, J. Bell *et al.*, Phys. Rev. Lett. **41** (1978) 1008.

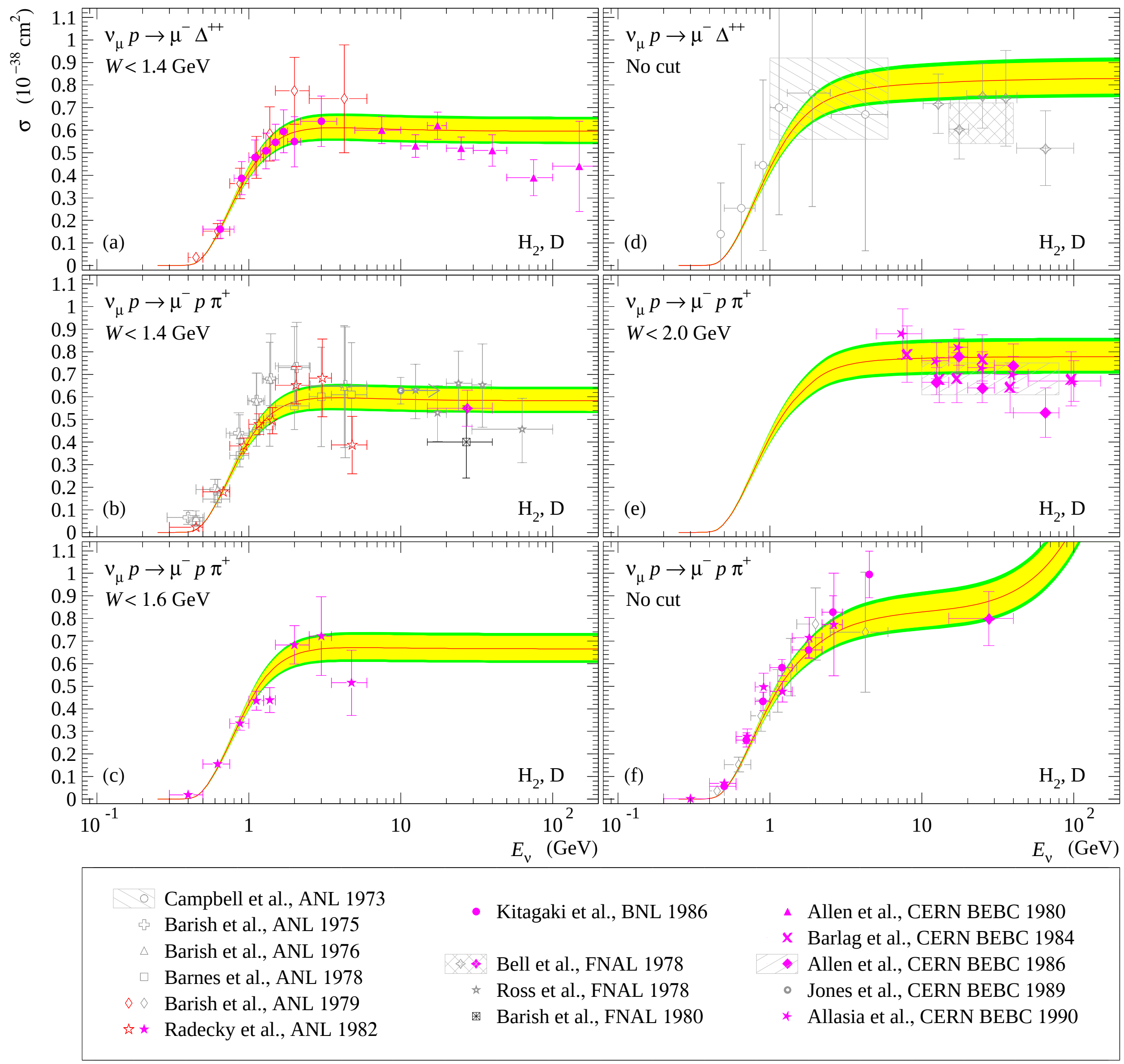
[f] P. Allen *et al.*, Nucl. Phys. B **176** (1980) 269.

[g] D. Allasia *et al.*, Z. Phys. C **20** (1983) 95.

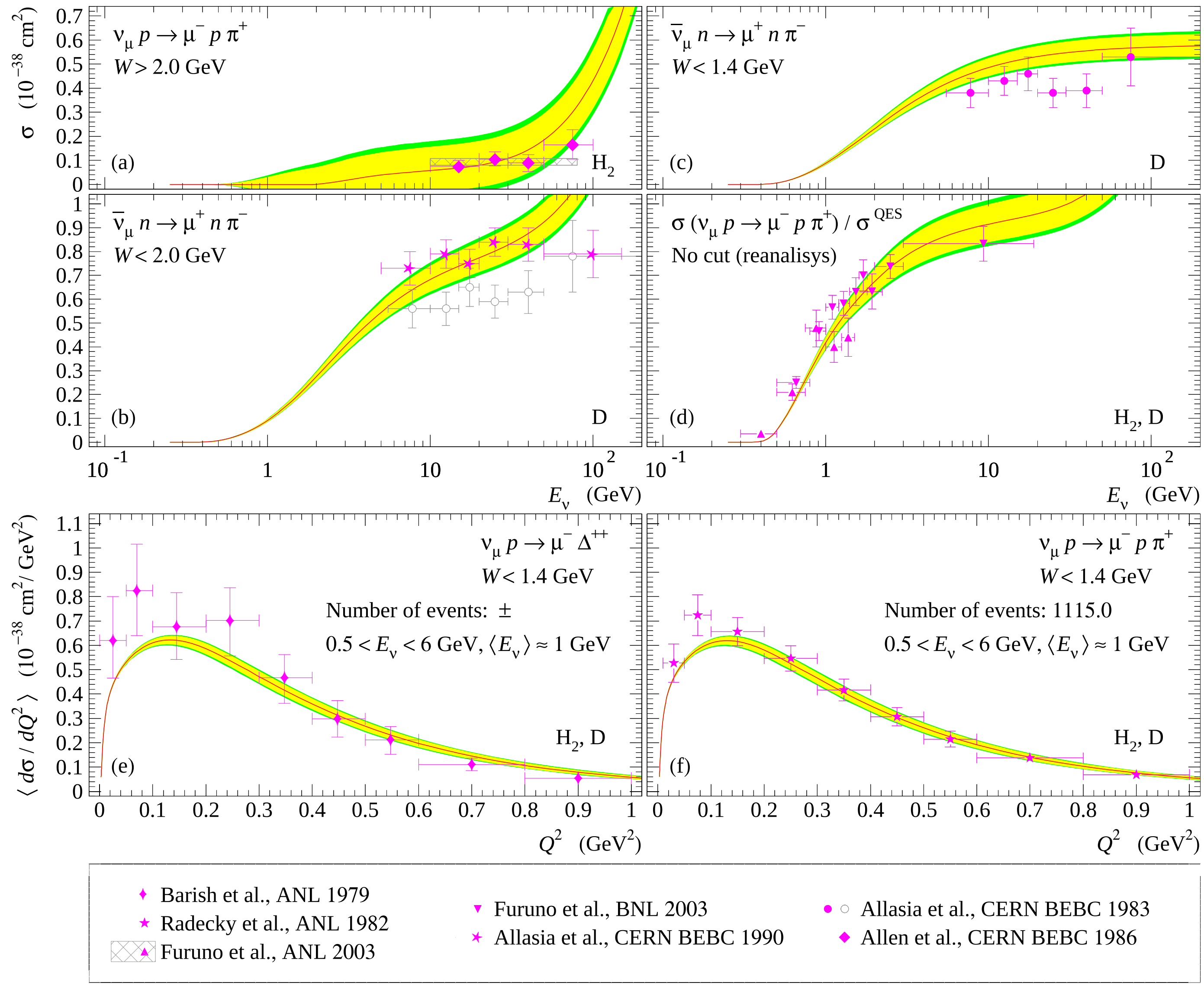
[h] S. Barlag, Preprint INIS-MF-9455, Ph. D. Thesis, Amsterdam University, 1984 (unpublished).

[i] P. Allen *et al.*, Nucl. Phys. B **264** (1986) 221.

[j] D. Allasia *et al.*, Nucl. Phys. B **343** (1990) 285.



σ (10^−38 cm^2)
ν_μ p → μ^− Δ^++
W < 1.4 GeV
(a)
H_2, D
ν_μ p → μ^− Δ^++
No cut
(d)
H_2, D
ν_μ p → μ^− p π^+
W < 1.4 GeV
(b)
H_2, D
ν_μ p → μ^− p π^+
W < 2.0 GeV
(e)
H_2, D
ν_μ p → μ^− p π^+
W < 1.6 GeV
(c)
H_2, D
ν_μ p → μ^− p π^+
No cut
(f)
H_2, D
E_ν (GeV)
Campbell et al., ANL 1973
Barish et al., ANL 1975
Barish et al., ANL 1976
Barnes et al., ANL 1978
Barish et al., ANL 1979
Radecky et al., ANL 1982
Kitagaki et al., BNL 1986
Bell et al., FNAL 1978
Ross et al., FNAL 1978
Barish et al., FNAL 1980
Allen et al., CERN BEBC 1980
Barlag et al., CERN BEBC 1984
Allen et al., CERN BEBC 1986
Jones et al., CERN BEBC 1989
Allasia et al., CERN BEBC 1990



$\nu_\mu\, p \to \mu^-\, p\, \pi^+$
$W > 2.0$ GeV
(a) H_2
$\bar{\nu}_\mu\, n \to \mu^+\, n\, \pi^-$
$W < 1.4$ GeV
(c) D
$\bar{\nu}_\mu\, n \to \mu^+\, n\, \pi^-$
$W < 2.0$ GeV
(b) D
$\sigma\,(\nu_\mu\, p \to \mu^-\, p\, \pi^+) / \sigma^{QES}$
No cut (reanalisys)
(d) H_2, D
$\sigma\ (10^{-38}\ \mathrm{cm}^2)$
E_ν (GeV)
$\nu_\mu\, p \to \mu^-\, \Delta^{++}$
$W < 1.4$ GeV
Number of events: ±
$0.5 < E_\nu < 6$ GeV, $\langle E_\nu \rangle \approx 1$ GeV
(e) H_2, D
$\nu_\mu\, p \to \mu^-\, p\, \pi^+$
$W < 1.4$ GeV
Number of events: 1115.0
$0.5 < E_\nu < 6$ GeV, $\langle E_\nu \rangle \approx 1$ GeV
(f) H_2, D
$\langle d\sigma / dQ^2 \rangle\ (10^{-38}\ \mathrm{cm}^2/\mathrm{GeV}^2)$
Q^2 (GeV^2)
Barish et al., ANL 1979
Radecky et al., ANL 1982
Furuno et al., ANL 2003
Furuno et al., BNL 2003
Allasia et al., CERN BEBC 1990
Allasia et al., CERN BEBC 1983
Allen et al., CERN BEBC 1986

FINE TUNE OF NON–INTERFERING NON–RESONANCE BACKGROUND

EXPERIMENTAL DATA, GLOBAL FIT, AND COMPARISON WITH THE DATA

The adjustable parameter f_{NRB} can be obtained from the global fit for the cross sections of the $\nu_\mu n \to \mu^- p\pi^0$, $\nu_\mu n \to \mu^- n\pi^+$, and $\overline{\nu}_\mu p \to \mu^+ p\pi^-$ reactions requiring the consideration of the NRB in the Rein-Sehgal approach and measured with experiments with H_2 and D_2 targets.

The axial mass M_A is fixed on the value obtained from the global fit of axial mass.

The corresponding data set includes the following experimental data: ANL 1982 [a] (σ_ν), Furuno *et al.*, ANL 2003 ($\sigma_\nu/\sigma_\nu^{\text{QES}}$), Furuno *et al.*, BNL 2003 [b] ($\sigma_\nu/\sigma_\nu^{\text{QES}}$), Kitagaki *et al.*, BNL 1986 [c] (σ_ν), Allasia *et al.*, CERN BEBC 1983 [d] (σ_ν), and Allen *et al.*, CERN BEBC 1986 [e] ($\sigma_{\overline{\nu}}$).

The data sets $\sigma_\nu\left(p\pi^0\right)$ and $\sigma_\nu\left(n\pi^+\right)$ obtained with Radecky *et al.*, ANL 1982 for $W < 1.4$ GeV and with no W-cut are recalculated by P. Rodrigues *et al.* [f] The ratios of the cross sections $\sigma_\nu\left(p\pi^0\right)/\sigma_\nu^{\text{QES}}$ and $\sigma_\nu\left(n\pi^+\right)/\sigma_\nu^{\text{QES}}$ are recalculated by K. Furuno *et al.*

Therefore, the fitted data set consists of the 86 data points with the 80 data points of ν_μ cross sections (93% of the total number of data points) and the 6 of $\overline{\nu}_\mu$ cross sections (7%).

The adjustable parameter is $f_{\text{NRB}} = 1.162^{+0.078\,(0.101)}_{-0.083\,(0.120)}$ ($\chi^2/\text{ndf} = 4.07$).

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[a] G. M. Radecky *et al.*, Phys. Rev. D **25** (1982) 1161; erratum – *ibid.* D **26** (1982) 3297.

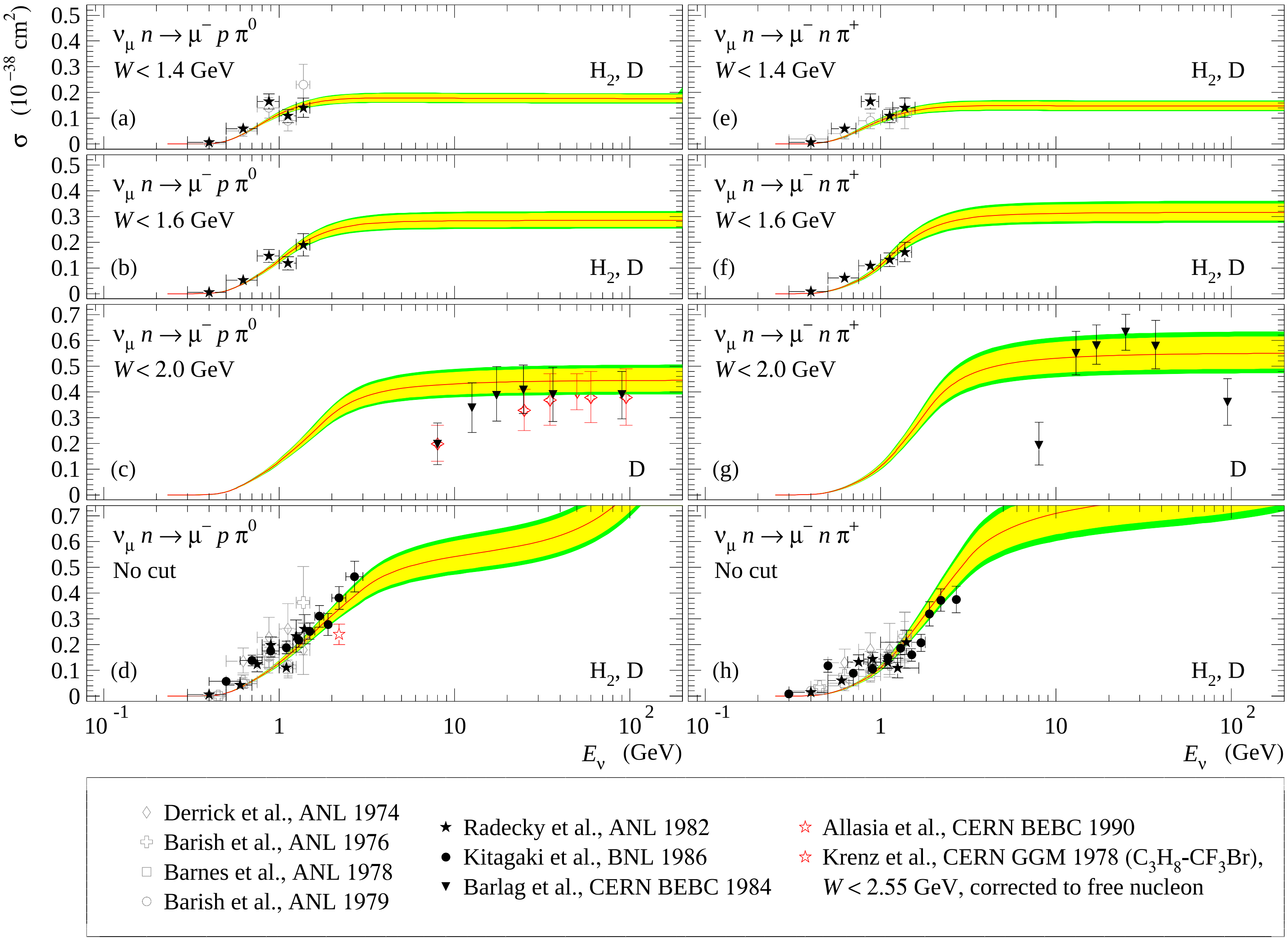
[b] K. Furuno *et al.*, KEK Preprint 2003-48 (unpublished); M. Sakuda and E. F. Paschos, a talk at the NuInt 2002; M. Sakuda, a talk at the NOON 2003.

[c] T. Kitagaki *et al.*, Phys. Rev. D **34** (1986) 2554.

[d] D. Allasia *et al.*, Z. Phys. C **20** (1983) 95.

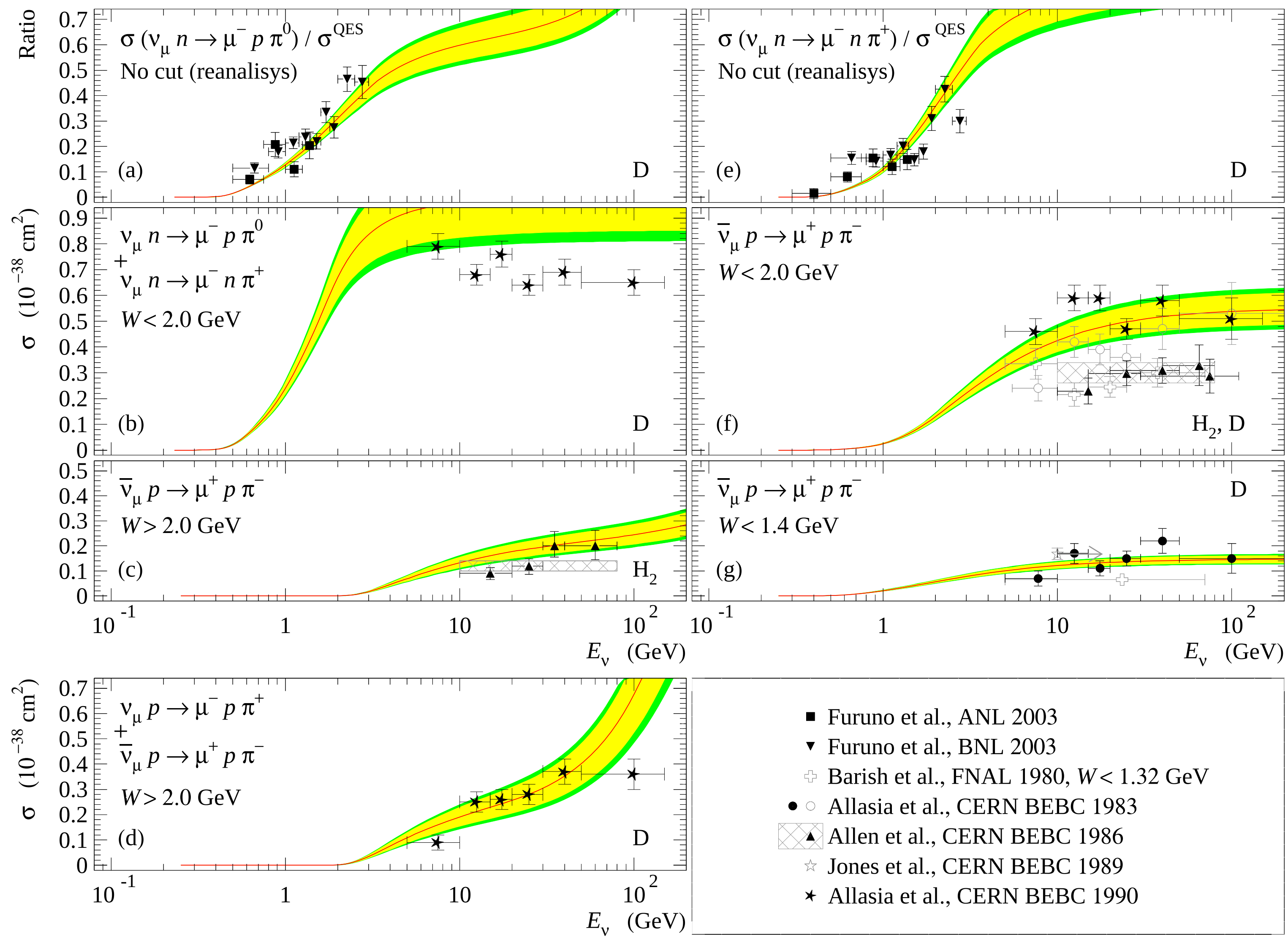
[e] P. Allen *et al.*, Nucl. Phys. B **264** (1986) 221.

[f] P. Rodrigues, C. Wilkinson and K. McFarland, Eur. Phys. J. C **76** (2016) 474.

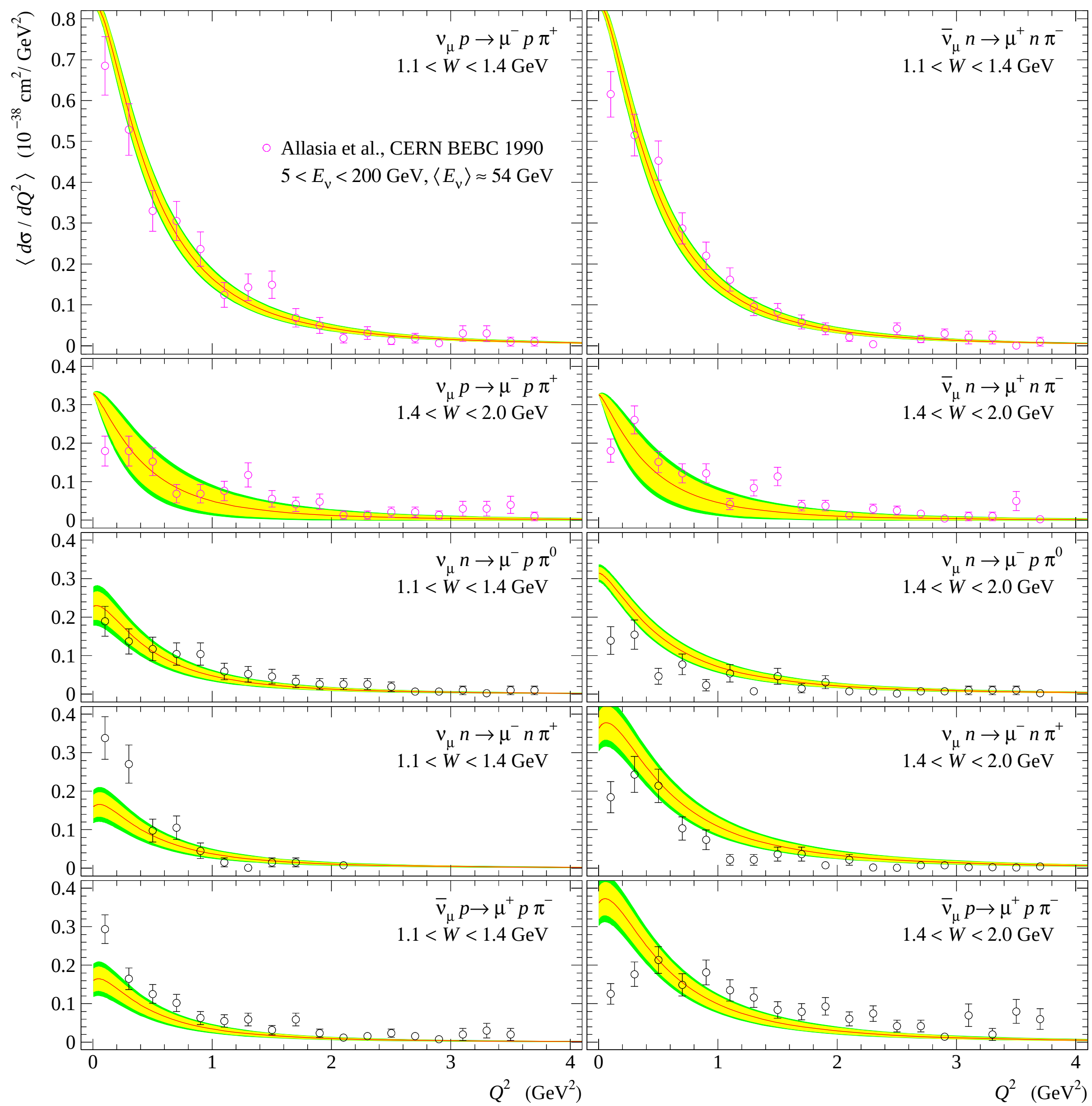


σ (10^-38 cm^2)
ν_μ n → μ^- p π^0
W < 1.4 GeV
H_2, D
(a)
ν_μ n → μ^- n π^+
W < 1.4 GeV
H_2, D
(e)
ν_μ n → μ^- p π^0
W < 1.6 GeV
H_2, D
(b)
ν_μ n → μ^- n π^+
W < 1.6 GeV
H_2, D
(f)
ν_μ n → μ^- p π^0
W < 2.0 GeV
D
(c)
ν_μ n → μ^- n π^+
W < 2.0 GeV
D
(g)
ν_μ n → μ^- p π^0
No cut
H_2, D
(d)
ν_μ n → μ^- n π^+
No cut
H_2, D
(h)
E_ν (GeV)
Derrick et al., ANL 1974
Barish et al., ANL 1976
Barnes et al., ANL 1978
Barish et al., ANL 1979
Radecky et al., ANL 1982
Kitagaki et al., BNL 1986
Barlag et al., CERN BEBC 1984
Allasia et al., CERN BEBC 1990
Krenz et al., CERN GGM 1978 (C_3H_8-CF_3Br),
W < 2.55 GeV, corrected to free nucleon

NuSTEC 2019



σ (ν_μ n → μ⁻ p π⁰) / σ^QES
No cut (reanalisys)
(a)
D
σ (ν_μ n → μ⁻ n π⁺) / σ^QES
No cut (reanalisys)
(e)
D
ν_μ n → μ⁻ p π⁰ + ν_μ n → μ⁻ n π⁺
W < 2.0 GeV
(b)
D
ν̄_μ p → μ⁺ p π⁻
W < 2.0 GeV
(f)
H_2, D
ν̄_μ p → μ⁺ p π⁻
W > 2.0 GeV
(c)
H_2
ν̄_μ p → μ⁺ p π⁻
W < 1.4 GeV
(g)
D
ν_μ p → μ⁻ p π⁺ + ν̄_μ p → μ⁺ p π⁻
W > 2.0 GeV
(d)
D
Ratio
σ (10⁻³⁸ cm²)
E_ν (GeV)
Furuno et al., ANL 2003
Furuno et al., BNL 2003
Barish et al., FNAL 1980, W < 1.32 GeV
Allasia et al., CERN BEBC 1983
Allen et al., CERN BEBC 1986
Jones et al., CERN BEBC 1989
Allasia et al., CERN BEBC 1990


$\langle d\sigma / dQ^2 \rangle$ (10^{-38} cm^2/ GeV^2)
$\nu_\mu p \to \mu^- p \pi^+$
1.1 < W < 1.4 GeV
Allasia et al., CERN BEBC 1990
5 < E_ν < 200 GeV, $\langle E_\nu \rangle \approx$ 54 GeV
$\bar{\nu}_\mu n \to \mu^+ n \pi^-$
1.1 < W < 1.4 GeV
$\nu_\mu p \to \mu^- p \pi^+$
1.4 < W < 2.0 GeV
$\bar{\nu}_\mu n \to \mu^+ n \pi^-$
1.4 < W < 2.0 GeV
$\nu_\mu n \to \mu^- p \pi^0$
1.1 < W < 1.4 GeV
$\nu_\mu n \to \mu^- p \pi^0$
1.4 < W < 2.0 GeV
$\nu_\mu n \to \mu^- n \pi^+$
1.1 < W < 1.4 GeV
$\nu_\mu n \to \mu^- n \pi^+$
1.4 < W < 2.0 GeV
$\bar{\nu}_\mu p \to \mu^+ p \pi^-$
1.1 < W < 1.4 GeV
$\bar{\nu}_\mu p \to \mu^+ p \pi^-$
1.4 < W < 2.0 GeV
Q^2 (GeV^2)

CONCLUSIONS

- We recommend to clarify current version of the KLN-BS model (using both non-interfering and interfering NRB) by getting rid of the normalizations of Breit-Wigner distributions of resonances and using the up-to-date values of parameters of resonances.
- The interference of resonance amplitudes should be taken into account in Monte Carlo neutrino event generators. This work is in progress.
- The values of resonance axial-vector mass and adjustable constant for fine tune non-interfering nonresonance background extracted from the global fit for ν_μ and $\overline{\nu}_\mu$ data on the total and differential charged-current cross sections for ν_μ and $\overline{\nu}_\mu$ reactions of 1π production through baryon resonances, measured by experiments with hydrogen and deuterium targets are

$$M_A = 1.176^{+0.071\,(0.087)}_{-0.067\,(0.083)}\ \text{GeV}, \quad f_{\text{NRB}} = 1.162^{+0.078\,(0.101)}_{-0.083\,(0.120)}.$$

The values extracted from the fit for the only ν_μ data yields the same. The low number of $\overline{\nu}_\mu$ data is not enough to obtain reliable results of the fits. It is need new modern experiments operating with hydrogen and deuterium targets for neutrino and antineutrino beams for more precise determination of the phenomenological parameters of the resonance reactions involved into the models based on the Rein-Sehgal approach.

BACKUP

The explicit form of the charged hadronic currents in the RS approach has been written in the resonance rest frame (RRF), below we will mark this frame with asterisk (*). So $\mathbf{q}^\star = -\mathbf{p}^\star$.

Let us denote

$$\mathcal{Q} = |\mathbf{q}| = \sqrt{E_\nu^2 - 2E_\nu P_\ell \cos\theta + P_\ell^2} = \sqrt{(E_\nu - P_\ell \cos\theta)^2 + P_\ell^2 \sin^2\theta}.$$

Then in RRF, the energy of the incoming neutrino, outgoing lepton, target nucleon and the 3-momentum transfer are, respectively,

$$E_\nu^\star = \frac{1}{2W}\left(2ME_\nu - Q^2 - m_\ell^2\right) = \frac{E_\nu}{W}\left[M - (E_\ell - P_\ell\cos\theta)\right],$$

$$E_\ell^\star = \frac{1}{2W}\left(2ME_\ell + Q^2 - m_\ell^2\right) = \frac{1}{W}\left[ME_\ell - m_\ell^2 + E_\nu\left(E_\ell - P_\ell\cos\theta\right)\right],$$

$$E_N^\star = W - (E_\nu^\star - E_\ell^\star) = \frac{M}{W}\left(M + E_\nu - E_\ell\right) \quad \text{and}$$

$$\mathcal{Q}^\star = |\mathbf{q}^\star| = \frac{M}{W}\mathcal{Q} = \frac{M}{W}\sqrt{E_\nu^2 - 2E_\nu P_\ell\cos\theta + P_\ell^2},$$

It is convenient to point the spatial axes of the RRF in such a way that $\mathbf{p}^\star = -\mathbf{q}^\star = (0, 0, -\mathcal{Q}^\star)$.

The elements of the polarization density matrix may be written as the superpositions of the partial cross sections (in the same definitions and similar notation as in paper by Rein and Sehgal, 1981) $\sigma_L^{\lambda\lambda'}$, $\sigma_R^{\lambda\lambda'}$ and $\sigma_S^{\lambda\lambda'}$:

$$\rho_{\lambda\lambda'} = \frac{\Sigma_{\lambda\lambda'}}{\Sigma_{++} + \Sigma_{--}}, \quad \Sigma_{\lambda\lambda'} = \sum_{i=L,R,S} c_i^{\lambda} c_i^{\lambda'} \sigma_i^{\lambda\lambda'},$$

where λ is the leptonic helicity and the differential cross section is given by

$$\frac{d^2\sigma}{dQ^2 dW^2} = \frac{G_F^2 \cos^2\theta_C Q^2}{2\pi^2 M \left|\mathbf{q}\right|^2} \left(\Sigma_{++} + \Sigma_{--}\right).$$

The partial cross sections are found to be the bilinear superpositions of the reduced amplitudes for producing a $N\pi$ final state with allowed isospin by a charged isovector current.

For ν induced reactions

$$\sigma_{L,R}^{\lambda\lambda'} = \frac{\pi W}{2M} \left(A_{\pm3}^{\lambda} A_{\pm3}^{\lambda'} + A_{\pm1}^{\lambda} A_{\pm1}^{\lambda'}\right), \quad \sigma_S^{\lambda\lambda'} = \frac{\pi M \left|\mathbf{q}\right|^2}{2WQ^2} \left(A_{0+}^{\lambda} A_{0+}^{\lambda'} + A_{0-}^{\lambda} A_{0-}^{\lambda'}\right).$$

Due to charge symmetry similar equations hold for charge conjugated $\overline{\nu}$ induced reactions with the interchange $L \leftrightarrow R$.

The amplitudes $A_{\varkappa}^{\lambda}$ (with $\varkappa = \pm 3,\ \pm 1$ or $0\pm$) for neutrino induced reactions are defined by

$$A_{\varkappa}^{\lambda}\left(p\pi^{+}\right) = \sqrt{3} \sum_{(I=3/2)} a_{\varkappa}^{\lambda}\left(N_3^{*+}\right),$$

$$A_{\varkappa}^{\lambda}\left(p\pi^{0}\right) = \sqrt{\frac{2}{3}} \sum_{(I=3/2)} a_{\varkappa}^{\lambda}\left(N_3^{*+}\right) - \sqrt{\frac{1}{3}} \sum_{(I=1/2)} a_{\varkappa}^{\lambda}\left(N_1^{*+}\right),$$

$$A_{\varkappa}^{\lambda}\left(n\pi^{+}\right) = \sqrt{\frac{1}{3}} \sum_{(I=3/2)} a_{\varkappa}^{\lambda}\left(N_3^{*+}\right) + \sqrt{\frac{2}{3}} \sum_{(I=1/2)} a_{\varkappa}^{\lambda}\left(N_1^{*+}\right).$$

Any amplitude $a_{\varkappa}^{\lambda}\left(N_\imath^{*+}\right)$ referring to a single resonance consists of two factors which describe the production and subsequent decay of the resonance $N_\imath^{*+}$:

$$a_{\varkappa}^{\lambda}\left(N_\imath^{*}\right) = f_{\varkappa}^{\lambda}\left(\nu N \to N_\imath^{*}\right) \eta(N_\imath^{*} \to N\pi) \equiv f_{\varkappa}^{\lambda(\imath)}\,\eta^{(\imath)}.$$

The decay amplitudes $\eta^{(\imath)}$ can be split into three factors:

$$\eta^{(\imath)} = \operatorname{sign}(N_\imath^{*})\ \sqrt{\chi_\imath}\ \eta_{BW}^{(\imath)}(W),$$

irrespective of isospin, charge or helicity. Here $\operatorname{sign}(N_\imath^{*})$ is the decay sign for resonance $N_\imath^{*}$, $\chi_\imath$ is the elasticity of the resonance taking care of the branching ratio into the πN final state, $\eta_{\mathrm{BW}}^{(\imath)}(W)$ is the properly normalized Breit-Wigner term with the running width specified by the πN partial wave from which the resonance arises.

The resonance production amplitudes, $f_{\varkappa}^{\lambda(\imath)}$, can be calculated within the FKR quark model in exactl with the only important difference: the coefficient functions involved into the definitions of the amplitudes have to be modified.

Indeed, since in our approach the structure of the polarization 4-vector $e_{(\lambda)}$ has been changed with respect to that of the original RS model, we have to recalculate its inner products with the vector and axial hadronic currents. To do this, we used the explicit form for the FKR currents given by Ravndal [a]. As a result, the coefficients S, B, and C (and thus the resonance production amplitudes) become parametrically dependent of the lepton mass and helicity:

$$S \to S_{\mathsf{KLN}} = \left(\nu^{\star}_{(\lambda)}\nu^{\star} - \mathcal{Q}^{\star}_{(\lambda)}\mathcal{Q}^{\star}\right)\left(1 - \frac{q^2}{M^2} - \frac{3W}{M}\right)\frac{G^V\left(q^2\right)}{6\mathcal{Q}^2},$$

$$B \to B_{\mathsf{KLN}} = \sqrt{\frac{\Omega}{2}}\left(\mathcal{Q}^{\star}_{(\lambda)} + \nu^{\star}_{(\lambda)}\frac{\mathcal{Q}^{\star}}{aM}\right)\frac{ZG^A\left(q^2\right)}{3W\mathcal{Q}^{\star}},$$

$$C \to C_{\mathsf{KLN}} = \left[\left(\mathcal{Q}^{\star}_{(\lambda)}\mathcal{Q}^{\star} - \nu^{\star}_{(\lambda)}\nu^{\star}\right)\left(\frac{1}{3} + \frac{\nu^{\star}}{aM}\right) + \nu^{\star}_{(\lambda)}\left(\frac{2}{3}W + \frac{q^2}{aM} + \frac{N\Omega}{3aM}\right)\right]\frac{ZG^A\left(q^2\right)}{2W\mathcal{Q}^{\star}}.$$

Here

$$\nu^{\star} = E^{\star}_{\nu} - E^{\star}_{\ell} = \frac{M\nu - Q^2}{W}, \quad a = 1 + \frac{W^2 + Q^2 + M^2}{2MW},$$

and the remaining notation is explained Rein and Sehgal. Other 5 coefficients are left unchanged.

[a] F. Ravndal, Nuovo Cim. **18A** (1973) 385.

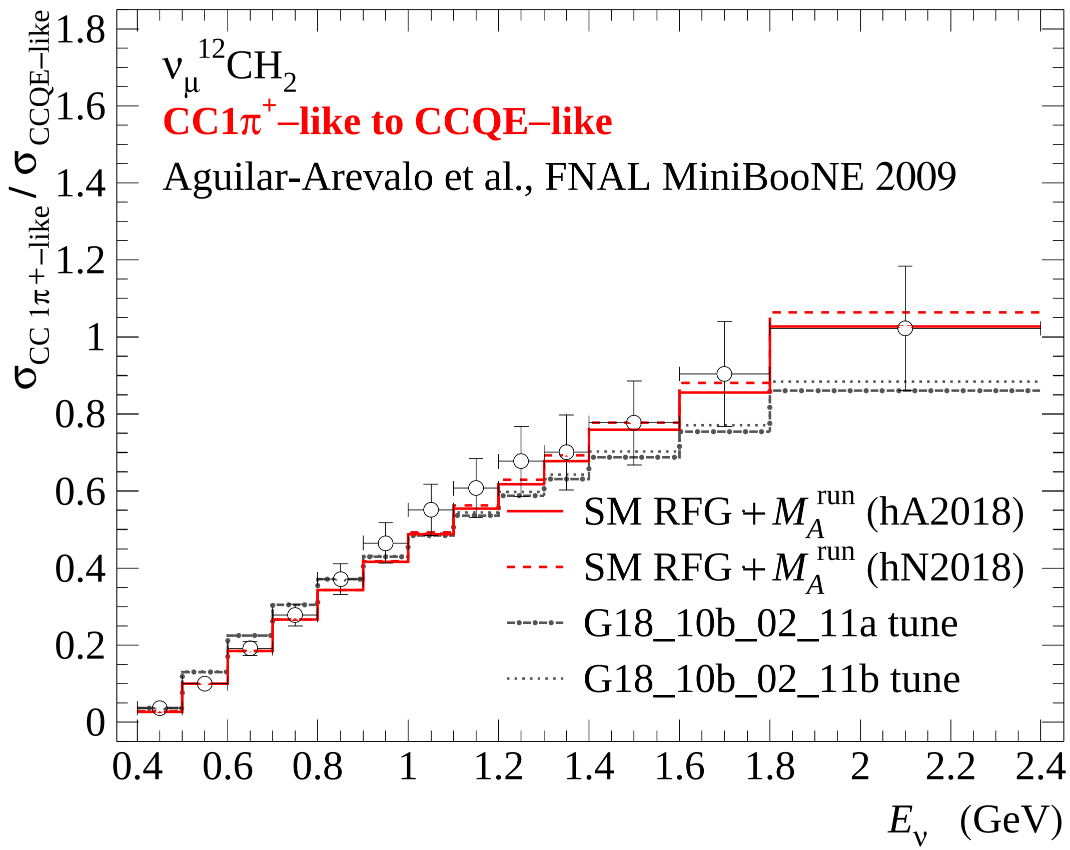
The axial hadronic current modifies the axial current as follows:

$$A_\mu \to \overline{A_\mu} = A_\mu + q_\mu \frac{q^\mu A_\mu}{m_\pi^2 + Q^2}$$

In the case of $m_\ell = 0$ the additional term in A_μ multiplied by the lepton current gives zero.
In the case of nonzero lepton mass the pion-pole term does contribute.
The effects of the pion-pole term can be incorporated by recalculating $e_\mu A^\mu$ and $q_\mu A^\mu$ terms.
The amplitudes B and C are modified

$$B_{\mathsf{BS}} = B_{\mathsf{KLN}} + \frac{ZG_A\left(q^2\right)}{2W\mathcal{Q}^\star}\left(\mathcal{Q}^\star_{(\lambda)}\nu^\star - \nu^\star_{(\lambda)}\mathcal{Q}^\star\right)\frac{\frac{2}{3}\sqrt{\frac{\Omega}{2}}\left(\nu^\star + \frac{\mathcal{Q}^{\star 2}}{aM_N}\right)}{m_\pi^2 + Q^2},$$

$$C_{\mathsf{BS}} = C_{\mathsf{KLN}} + \frac{ZG_A\left(q^2\right)}{2W\mathcal{Q}^\star}\left(\mathcal{Q}^\star_{(\lambda)}\nu^\star - \nu^\star_{(\lambda)}\mathcal{Q}^\star\right)\frac{\mathcal{Q}^\star\left(\frac{2}{3}W - \frac{Q^2}{aM_N} + \frac{n\Omega}{3aM_N}\right)}{m_\pi^2 + Q^2}.$$



Ratio of the total cross sections for CC $1\pi^+$ production and CCQE-like scattering on mineral oil measured with MiniBooNE [a] as a function of true neutrino energy. The vertical error bars represent the total errors including the normalization uncertainties. Solid and dashed histograms show the predictions of the SM RFG model with our tune of quasielastic axial mass model and the presented updates for KLN-BS model, with hA2018 and hN2018 models of FSI, respectively. Dashed-dotted and dotted histograms show GENIE3 tune G18_10a_02_11a and G18_10b_02_11a, respectively.

[a] A. A. Aguilar-Arevalo *et al.* (MiniBooNE Collaboration), Phys. Rev. Lett. **103** (2009) 081801.