Impact of Resonance Modeling on Neutrino-Nucleus Scattering Measurements

Clarence Wret

NuSTEC workshop
Neutrino-Nucleus Pion Production in the Resonance Region
2 October 2019

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Impact of Neutrino-Nucleus Scattering Measurements on Resonance Modeling?

Pretty huge topic, decided to focus on some recent results

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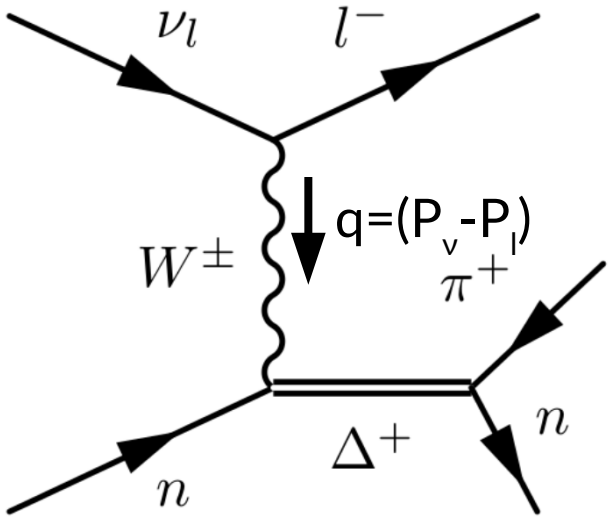
Neutrino interaction 101

This is a neutrino interacting with a neutron, producing a Δ^+ resonance from a neutron, which decays into a charged pion and a neutron



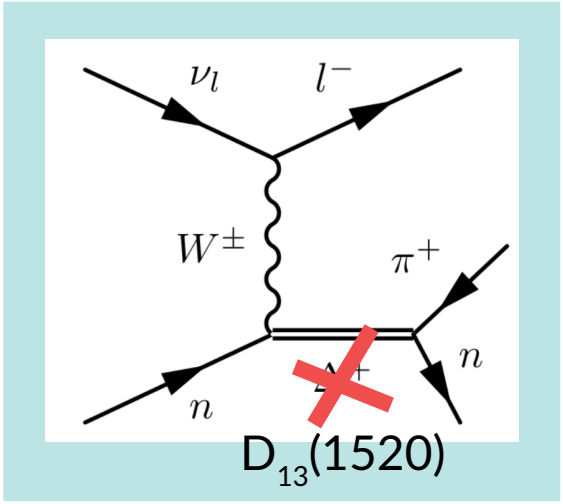
Define the usual kinematic variables

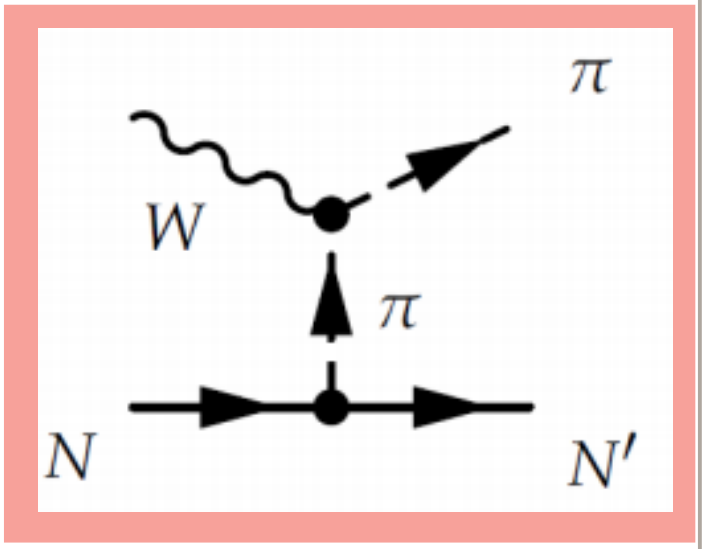
e.g.
$Q^2 = -q^2 = -(P_\nu - P_l)$ (four vectors)
$q_0 = E_\nu - E_l$
$\mathbf{q}_3 = \mathbf{p}_\nu - \mathbf{p}_l$
$W^2 = (P_n + q)^2$

This "clear picture" actually not at all clear, for example

- Additional resonances
- Their interferences
- Non-resonant diagrams
- And how to eject hadrons

$(A+B)^2 \neq A^2+B^2$





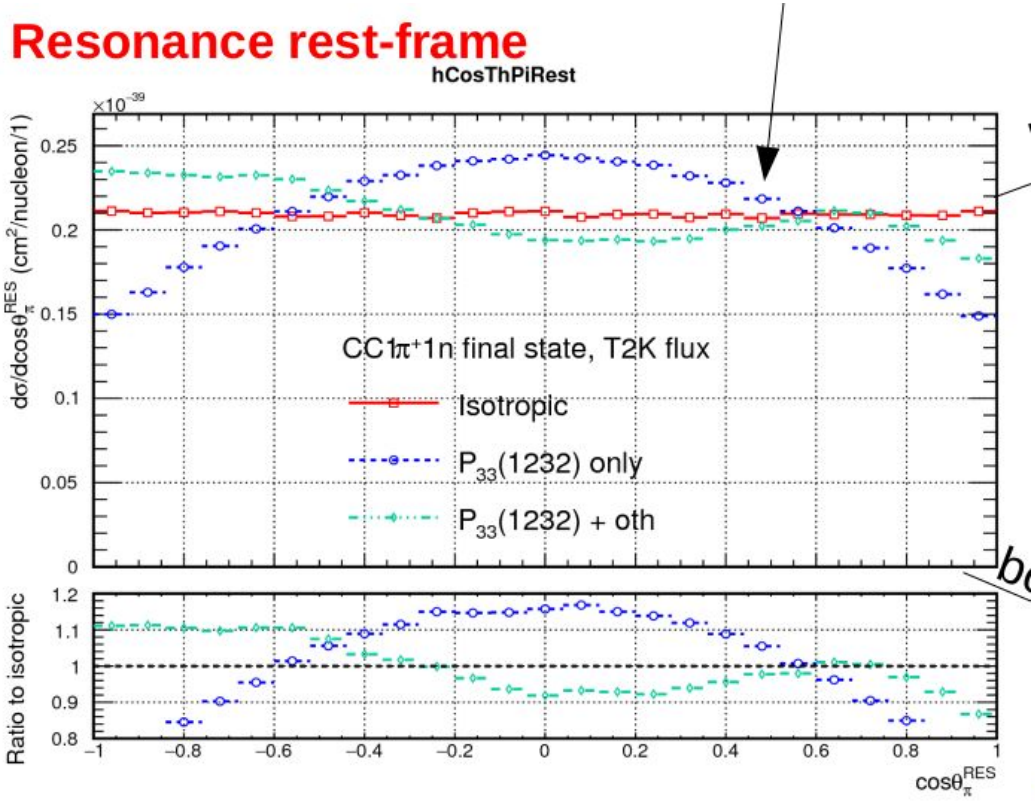
Neutrino interaction 101

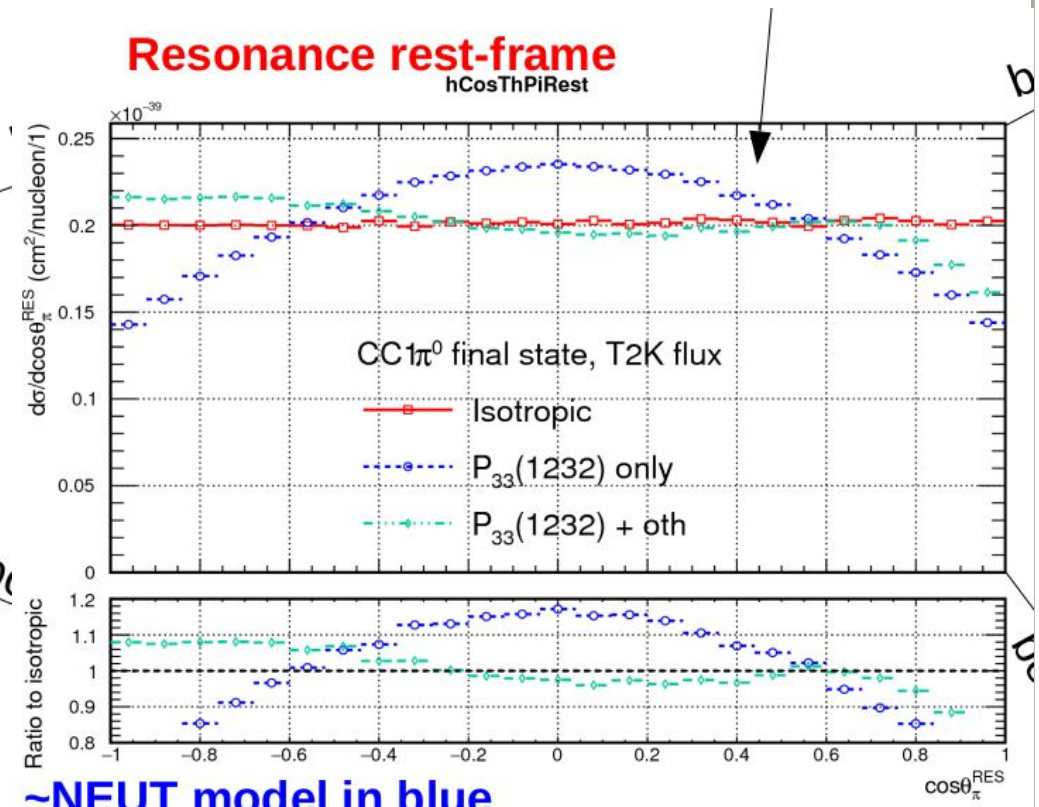
- Ejecting the hadrons is often done "(non-)isotropically"
  - Most of the time this means accounting for the Delta or not
- Rein-Sehgal include recipes for including the dominant resonances and their interference in all channels

In turning to the mixed isospin channel ($\nu n \to \mu n \pi^+$) or likewise ($\bar{\nu} p \to \bar{\mu} p \pi^-$), which is easier to measure, we have to envisage the possibility of even more than two nearby resonances interfering strongly with the leading Δ-resonance. Actually we have taken into account altogether four prominent resonances. The angular distribution formula then gets rather lengthy involving also higher rank spherical harmonics. It will be given in the Appendix.

Neutrino interaction 101

- Shameless advertising
  - I included these calculations in NEUT, found interesting differences to Delta only and isotropic, even at T2K E_ν





~NEUT model in blue
~GENIE model in red

~NEUT model in blue
~GENIE model in red

- **Multiple resonances with resonance-resonance interference:**
  - $P_{33}(1232) + P_{33}(1640)$ – two main resonances in $I_{3/2}$ channel
  - $P_{33}(1232) + D_{13}(1525) + P_{11}(1450) + S_{11}(1540)$ – main resonances in mixed channel interactions

Quick aside…

- I enjoy this section of the FKR paper, which models the resonances as quark oscillators
  - Rein-Seghal and Berger-Sehgal uses the FKR matrix elements

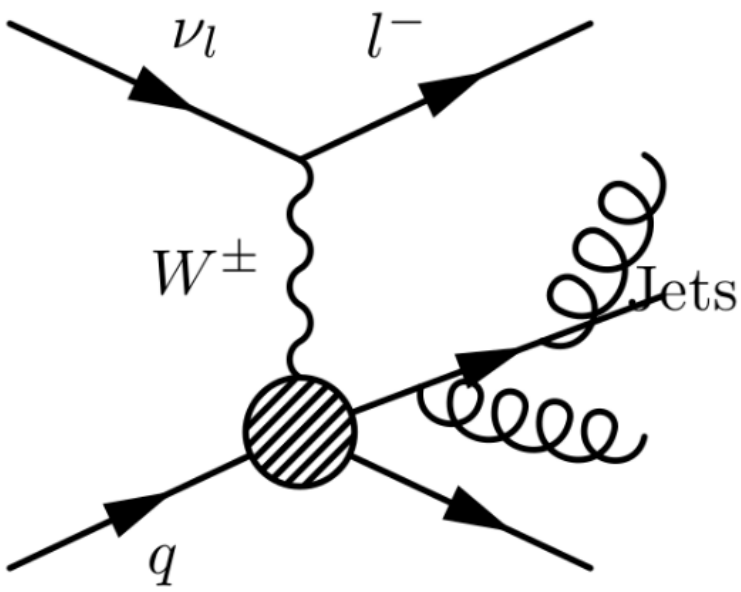
the slope of the Regge trajectories, and the masses of the particles, 75 matrix elements are calculated, of which more than $\frac{3}{4}$ agree with the experimental values within 40%. The problems of extending this calculational scheme to a viable physical theory are discussed.

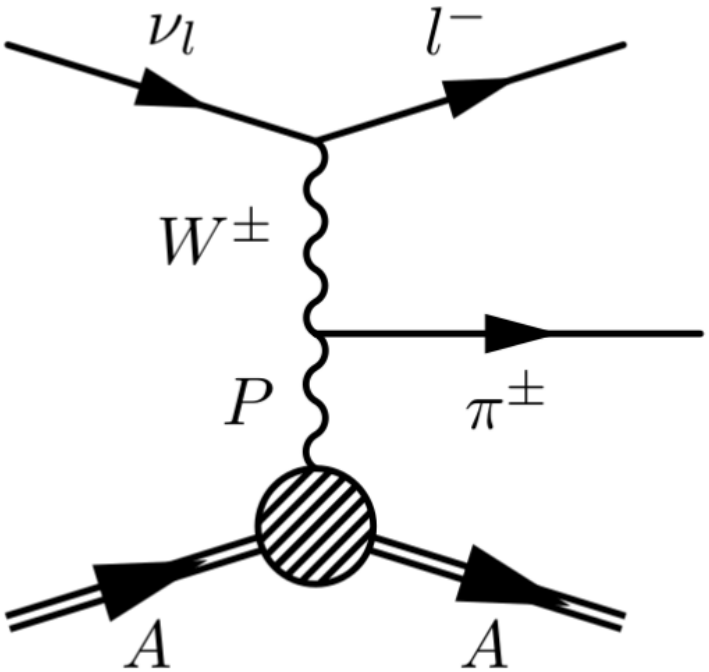
- I am in no way criticising FKR
- …I am more criticising that we expect our 1π model to match data knowing the above

ficing theoretical adequacy for simplicity. We shall choose a relativistic theory which is naive and obviously wrong in its simplicity, but which is definite and in which we can calculate as many things as possible – not expecting the results to agree exactly with experiment, but to see how closely our "shadow of the truth" equation gives a partial reflection of reality. In our attempt to maintain simplicity, we shall evidently have to violate known principles of a complete relativistic field theory (for example, unitarity). We shall attempt to modify our calculated results in a general way to allow, in a vague way, for these errors.

Neutrino interaction 101

- When we move to the nucleus, it gets more complicated
- Often define the CC1π^+ final state (any number of nucleons)
  - Opens up to contributions from FSI, CCQE, DIS, multi-π, and coherent

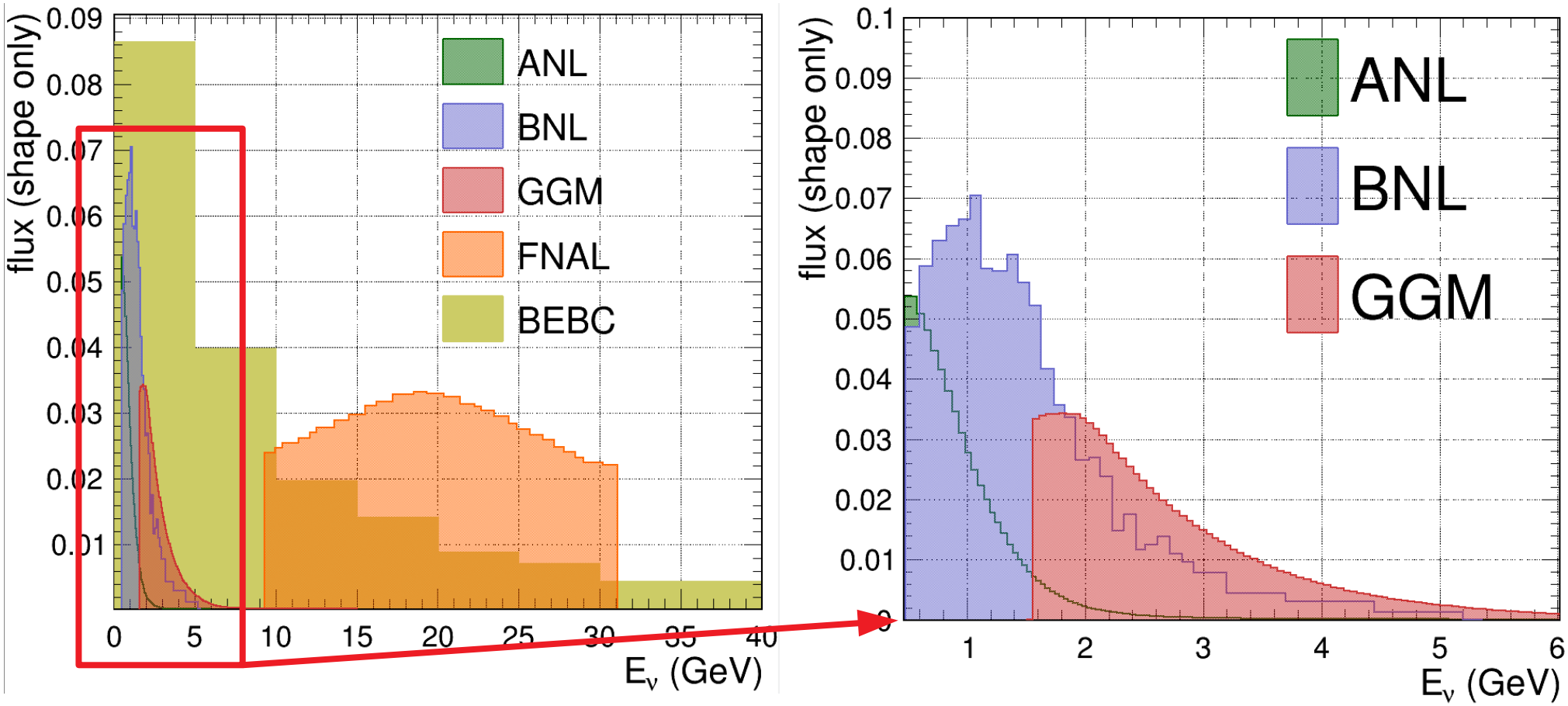




- Resonances need to be propagated through the nucleus
- Pauli blocking of outgoing nucleons
- No easy physics, none of which is <u>well modelled</u>!
- A good nucleon model does not guarantee a good nuclear model

Experiment landscape, bubble chambers

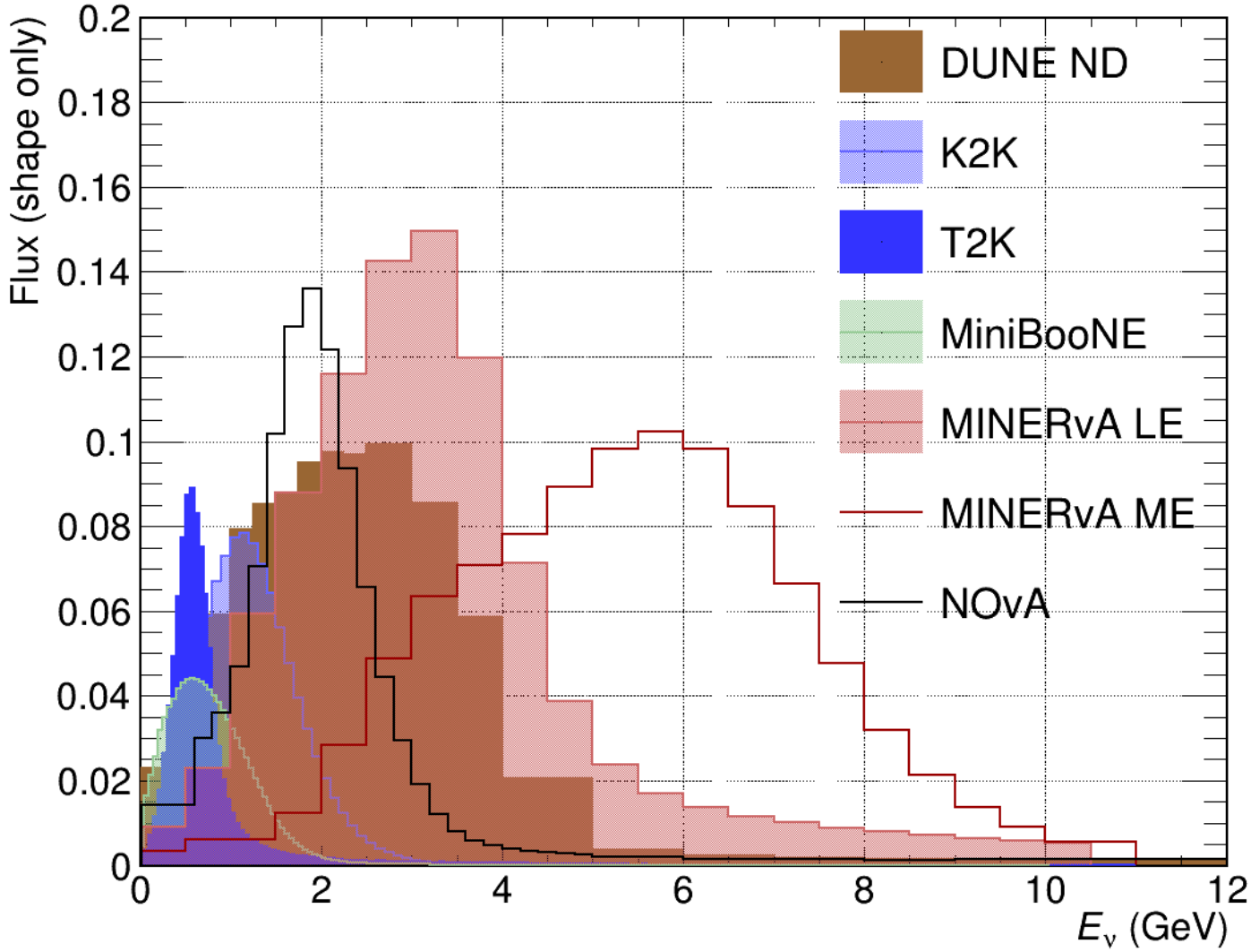
- Reducing bubble chambers to experiments with H_2/D_2 fills
  - Largely void of nuclear effects (although D_2 has Q^2 suppression!)



- ANL, BNL and Gargamelle are the relevant GeV region experiments, all of which have CCQE and 1π measurements

Experiment landscape, nuclear

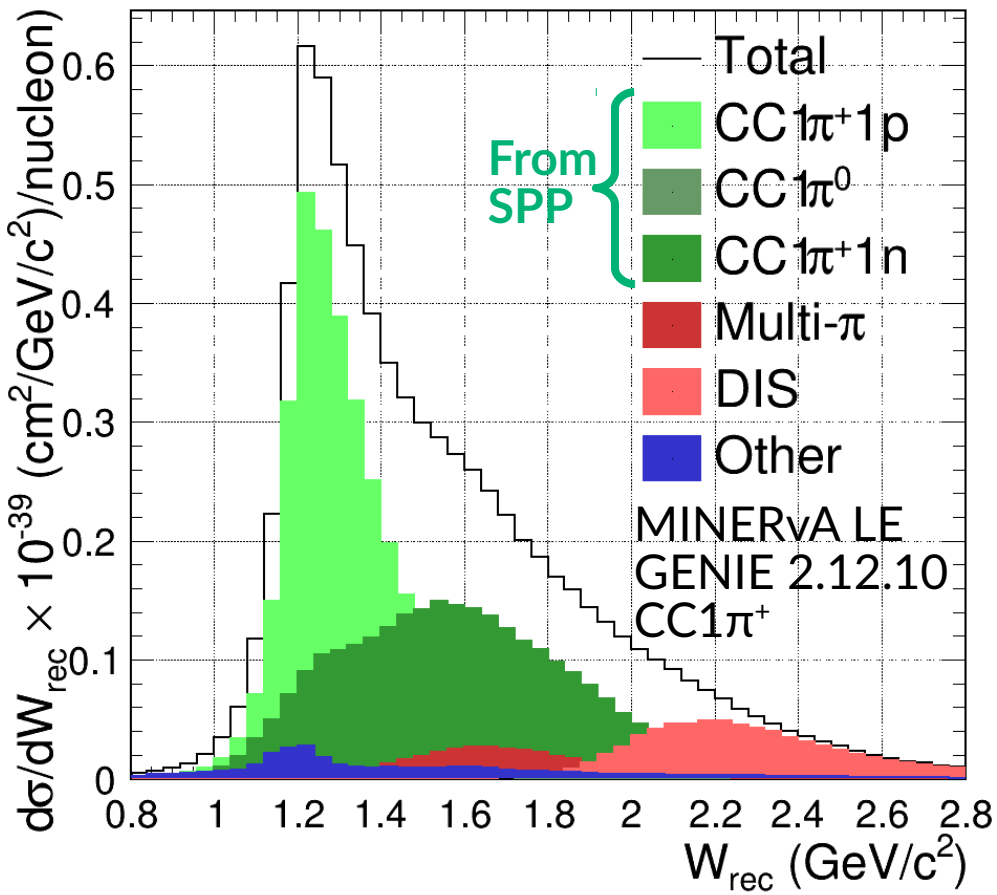
- A variety of targets: CH, CH_2, H_2O and Ar

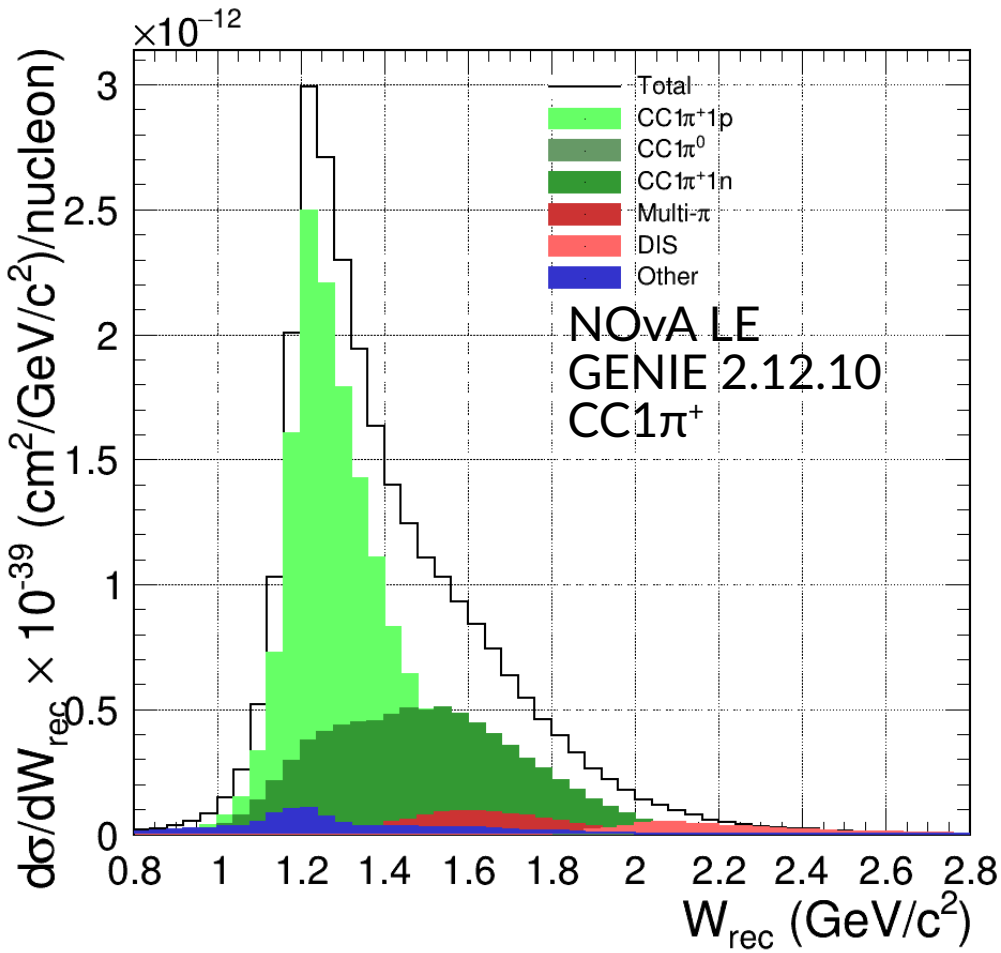


- Different experiments will see vastly different mechanisms for producing their 1π cross-sections

Not all pions are created equally

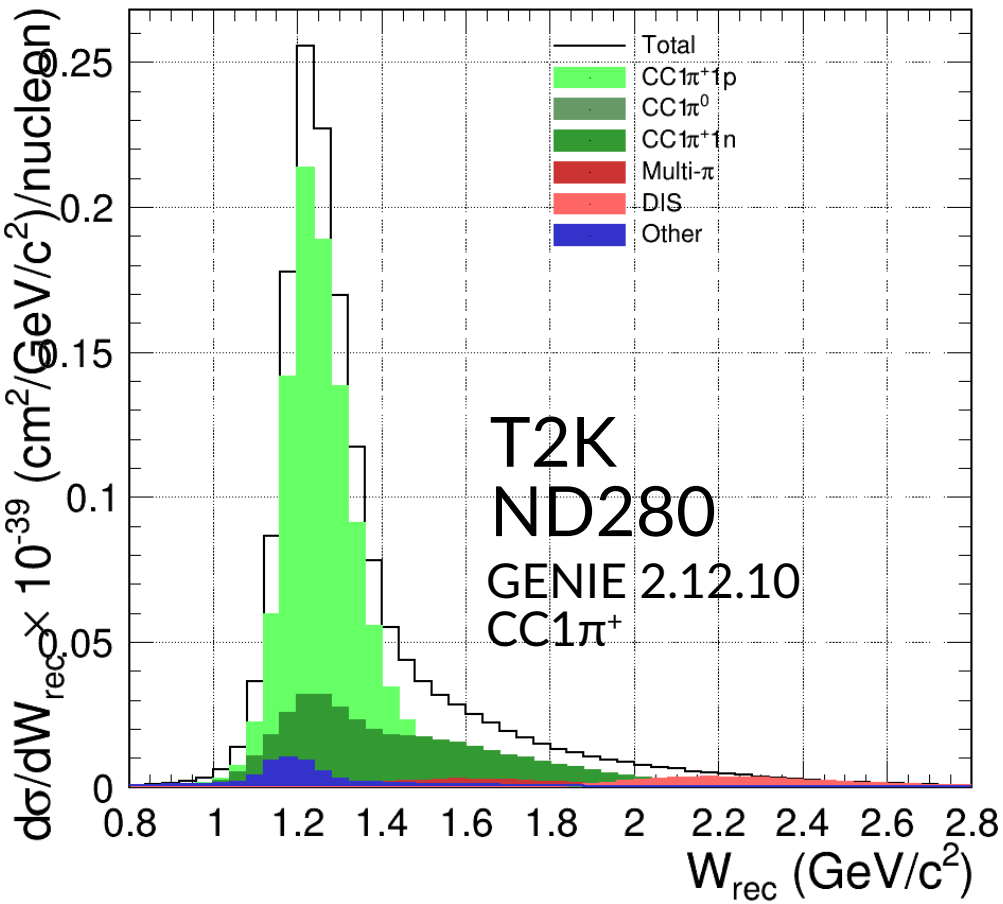
- Predictions in observables depend on true interaction mode (e.g. resonance, multi-π)
- Largely function of W, the hadronic rest mass
  - Prediction for CC1π⁺ final state shows contributions

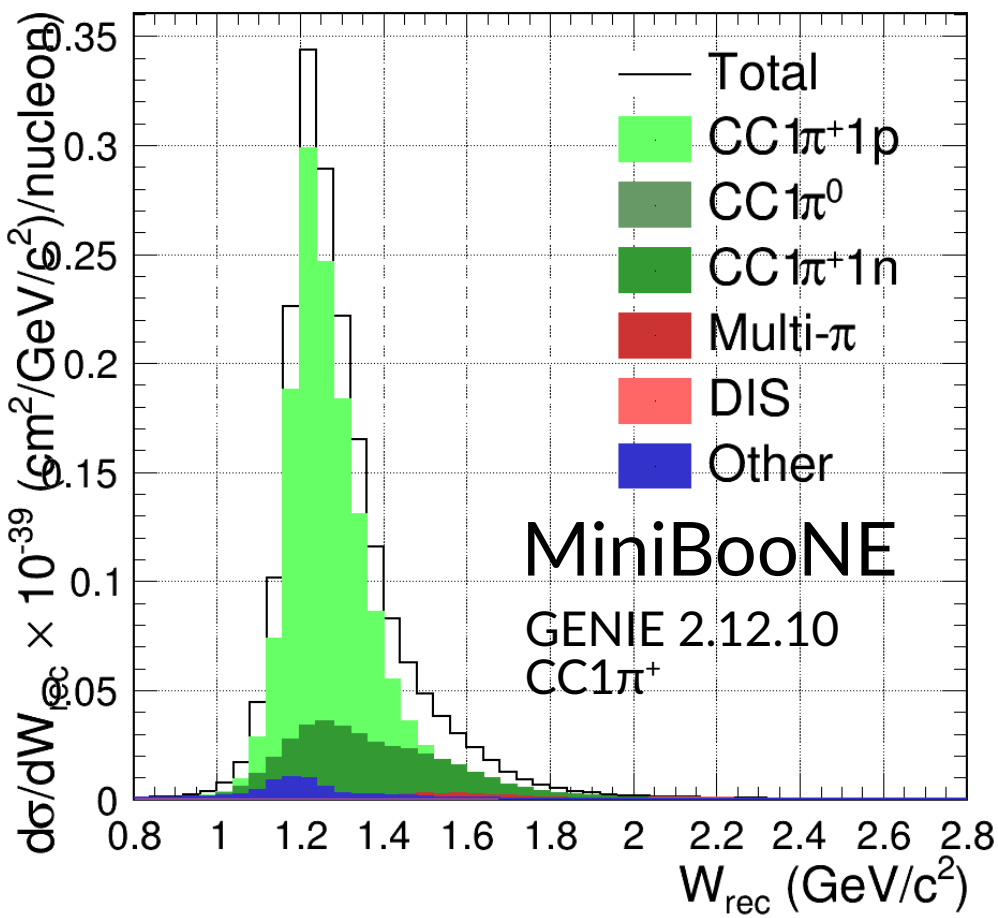




Not all pions are created equally

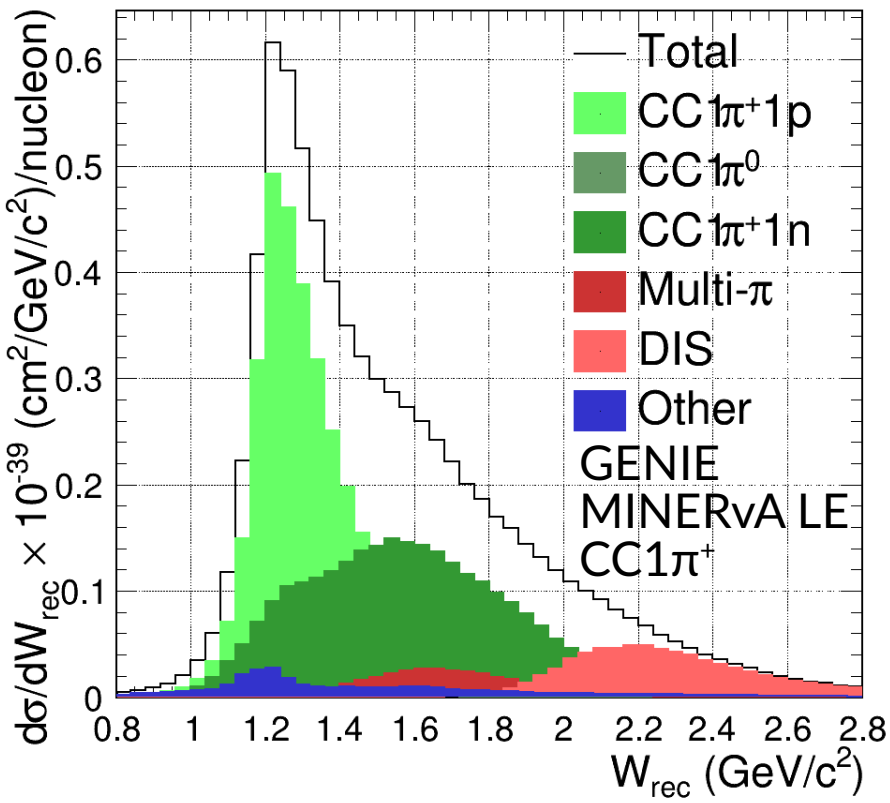
- T2K and MiniBooNE are Delta dominated for $CC1\pi^+$
  - Barely any higher resonances
  - Barely any multi-π or DIS contributions

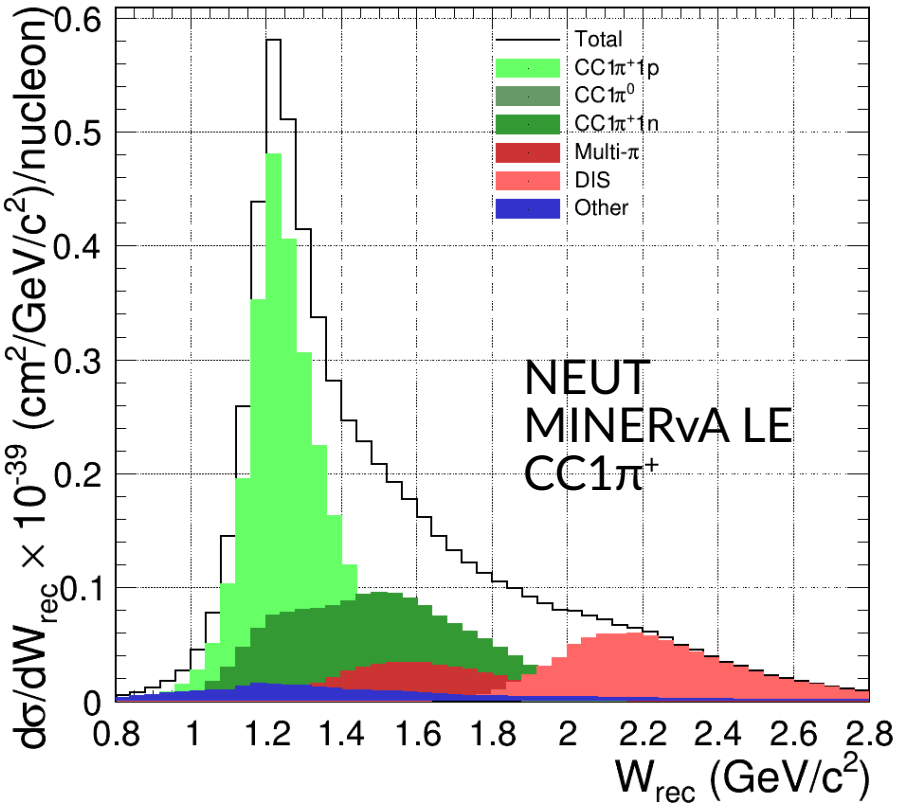




Not all pions are created equally

- The generators also assign different strength to different processes, but approximately similar total
  - Different resonance implementation
  - Different multi-π and DIS models

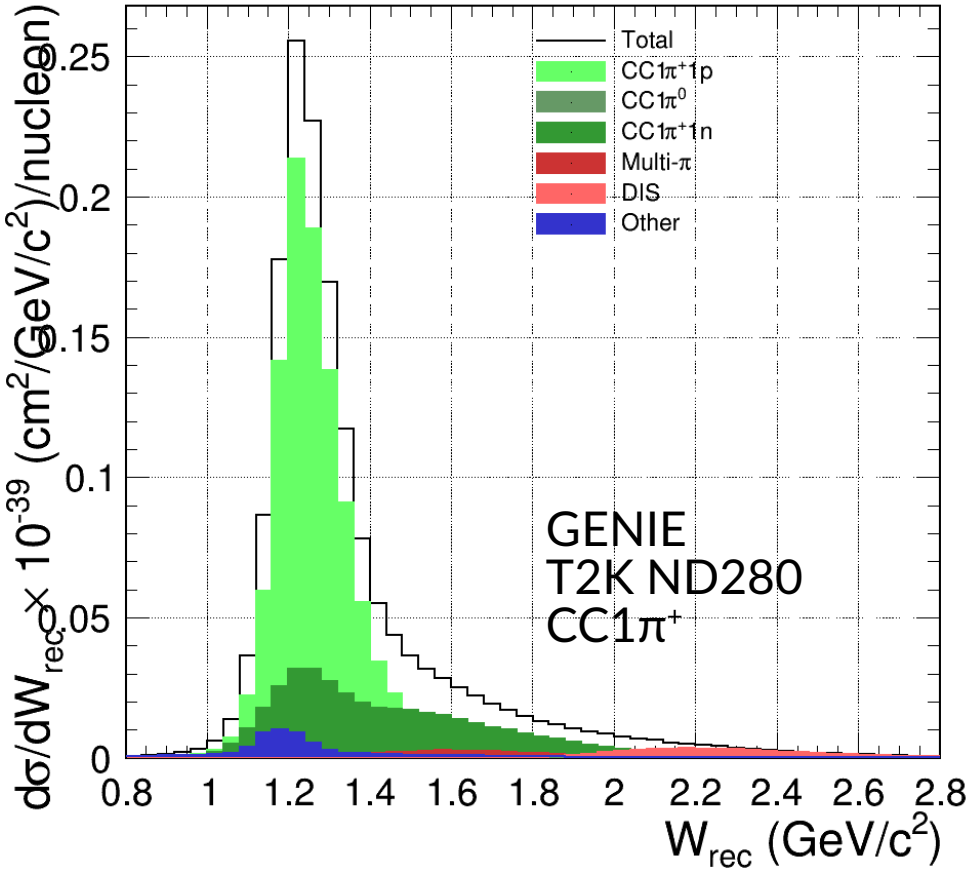


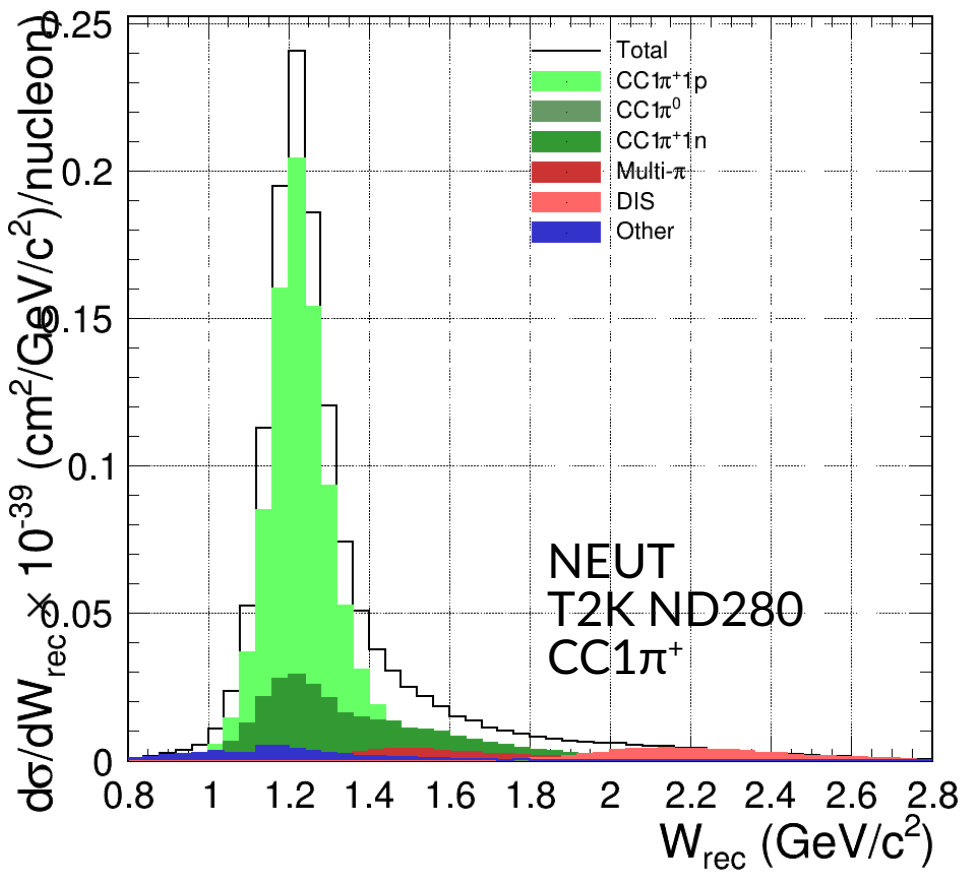


- Especially important for higher E_ν experiments

Not all pions are created equally

- T2K has lower E_ν → Delta only (+non-resonant?)
  - GENIE and NEUT have similar treatment

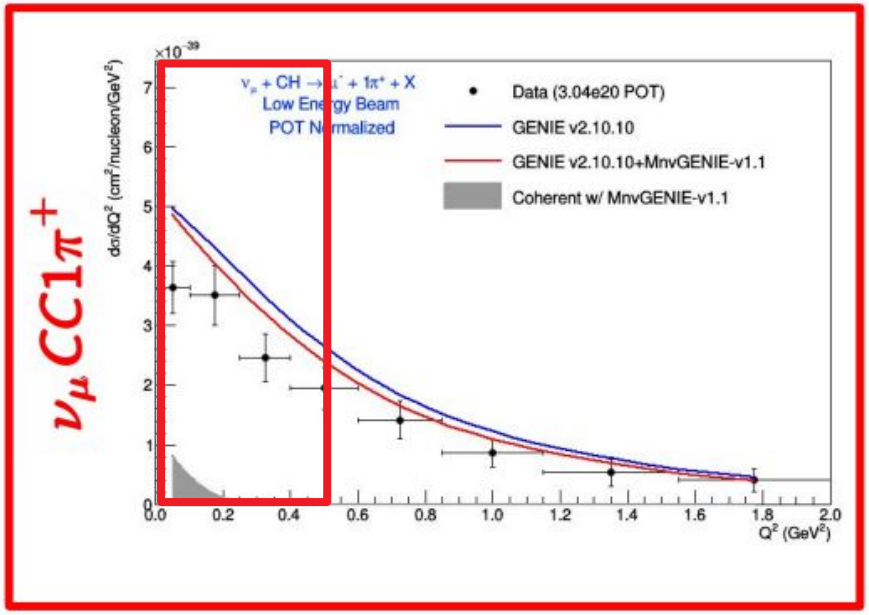


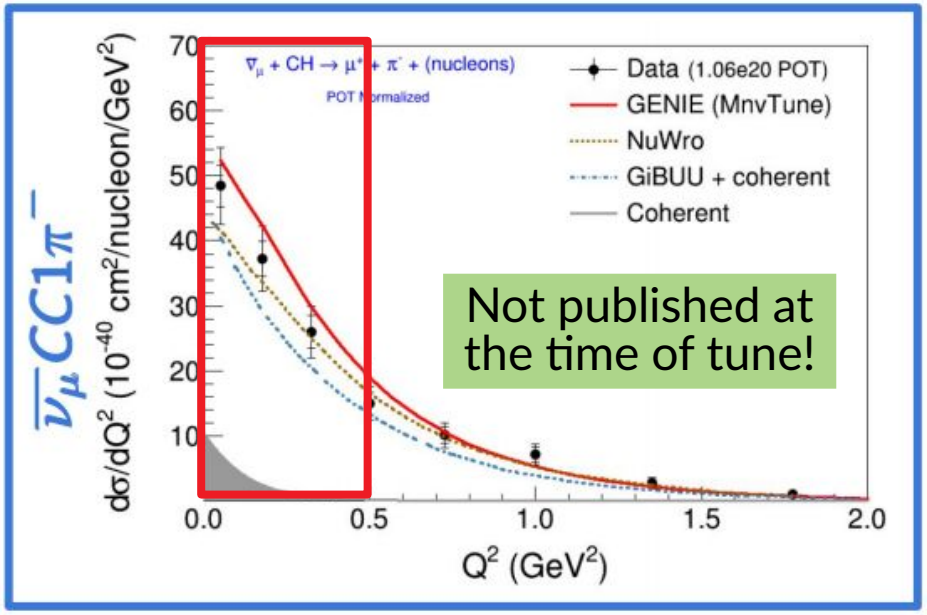


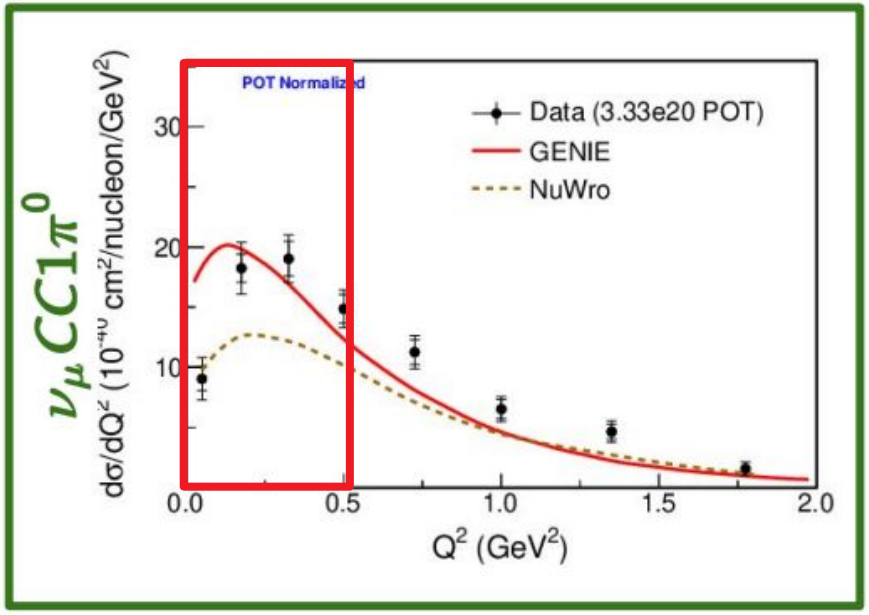
- Results from one experiment does not necessarily map to a different experiment for data <u>and</u> MC

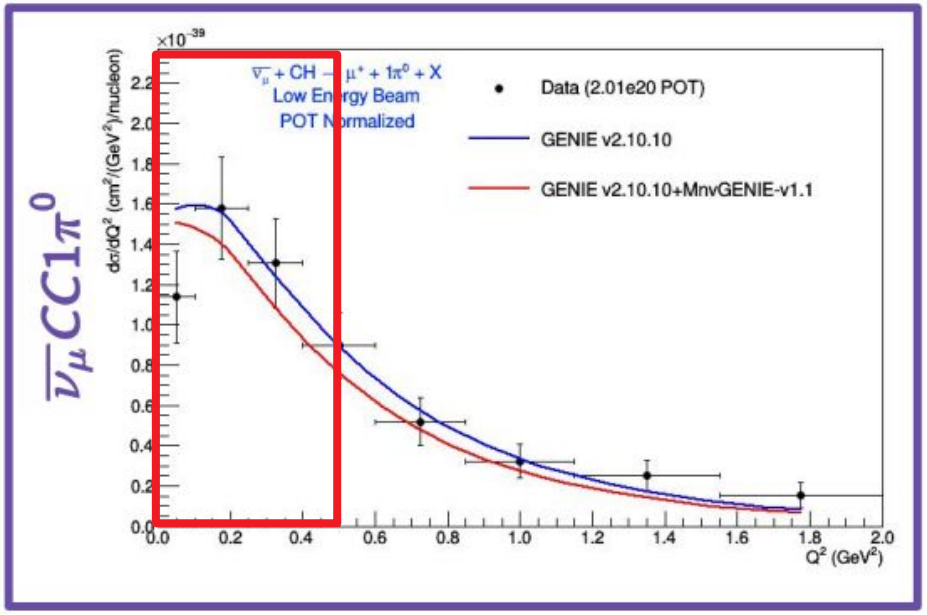
What does the data imply?

- MINERvA sees indications in most channels





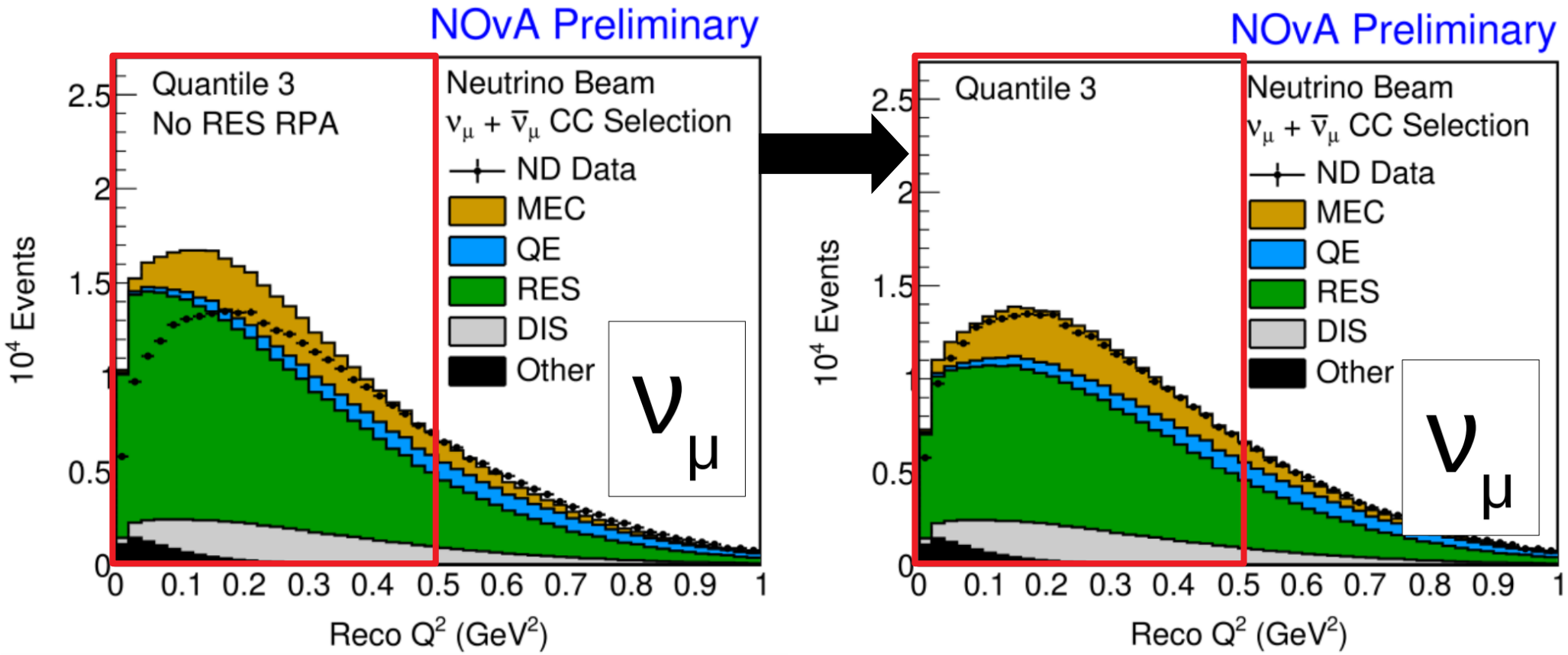




A. Bercellie

What does the data imply?

- We never quiet get single pion modelling right
- NOvA currently applies 1p1h Nieves RPA correction to resonant events

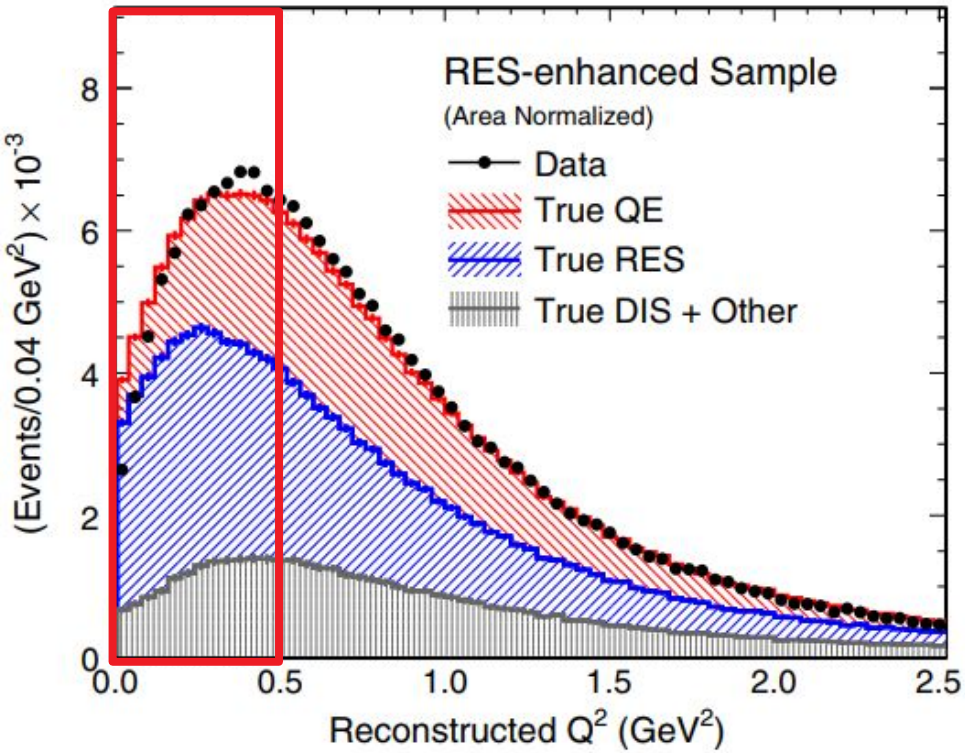


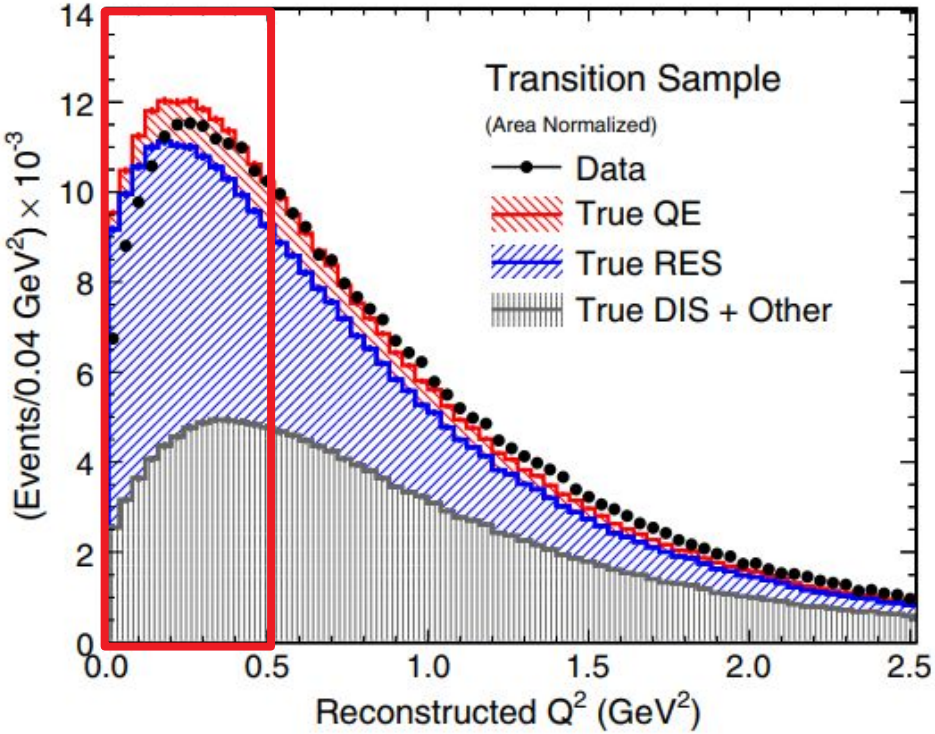
- Quartiles are in E_{had}/E_ν

Jeremy Wolcott

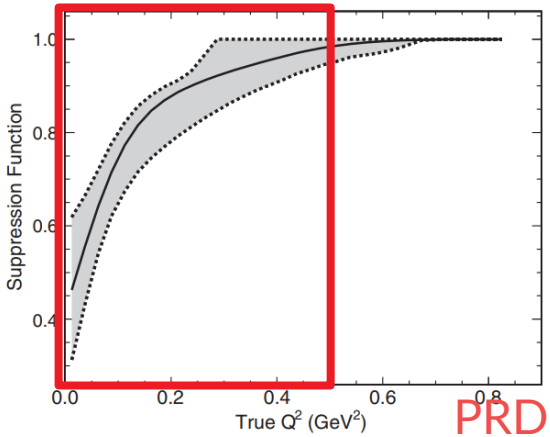
What does the data imply?

- MINOS CCQE analysis saw consistent low-Q^2 mismodelling in resonant-enhanced sidebands





- Developed Q^2 dependent suppression for that analysis



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What does the data imply?

- The source of this mismodelling is (probably) a complex combination of missing known effects
  - e.g. lepton mass effects, non-resonant background modelling, resonance in-medium propagation, poor nucleon model, multi-pion/DIS transition model, FSI
- And unknown effects!

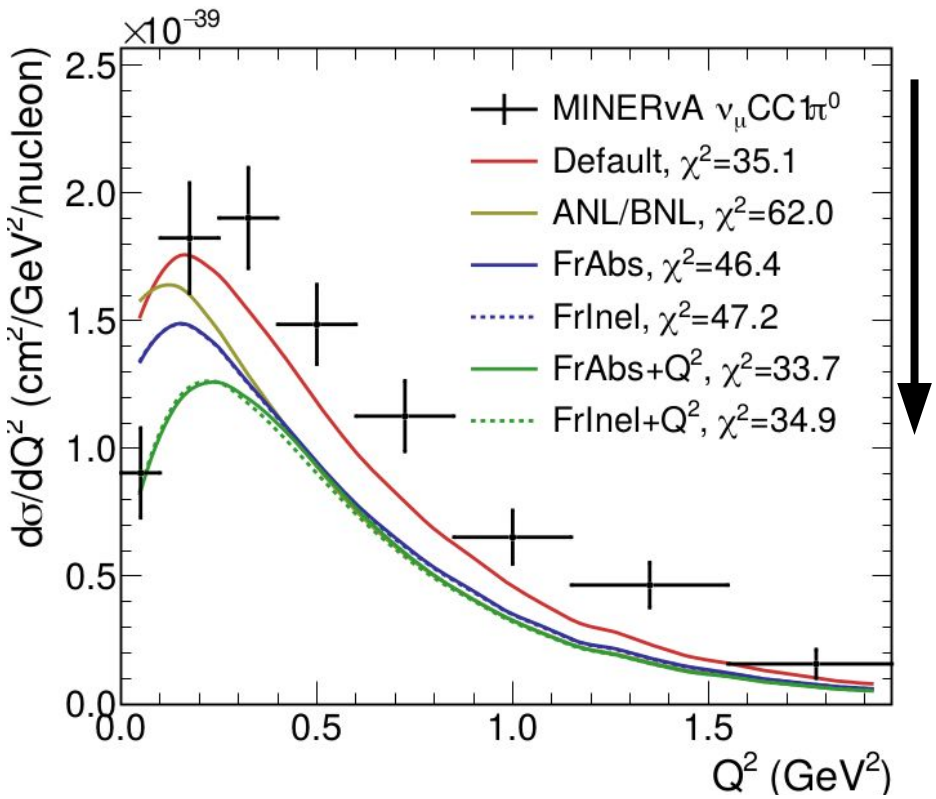
- We are <u>not trying to assess where the effect comes from</u>, we're just providing a tune to data
  - Provides experiments with data driven model and uncertainties
  - Much better than ignoring the problem
  - <u>But certainly not a complete solution!</u>

Procedure

- Default GENIE + MINERvA coherent tune
  - $E_\pi < 0.45$ GeV → 0.5 norm, $E_\pi > 0.45$ GeV → 1.0 norm
- Apply ANL/BNL tune from paper

K. Eur. Phys. J. C (2016)

- Identify and tune theory parameters
- Introduce empirical low Q^2 tune

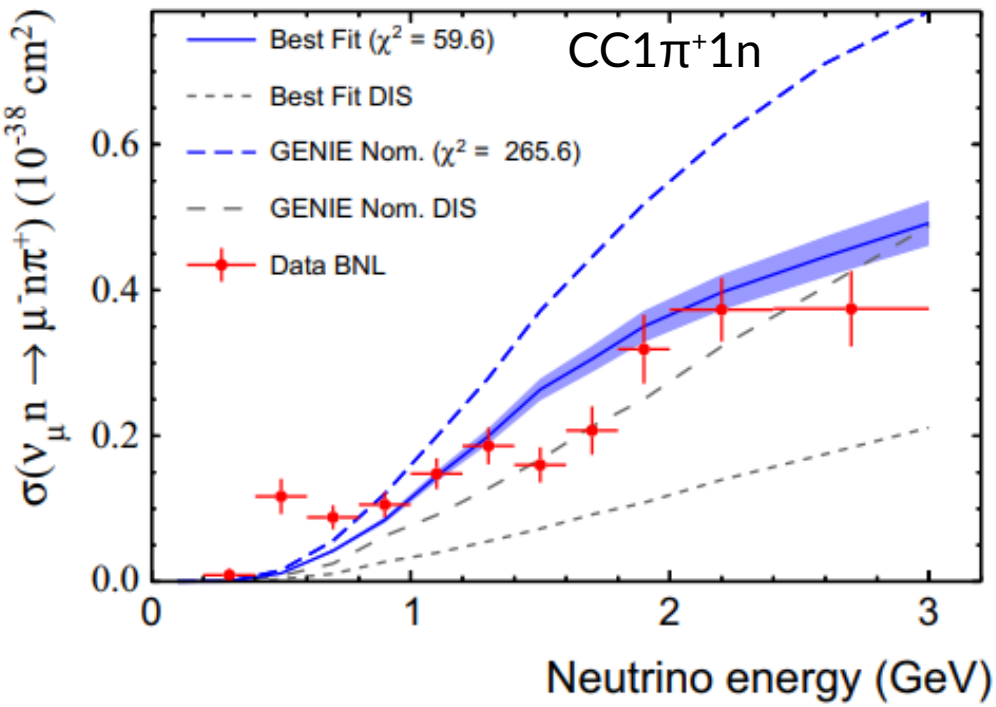


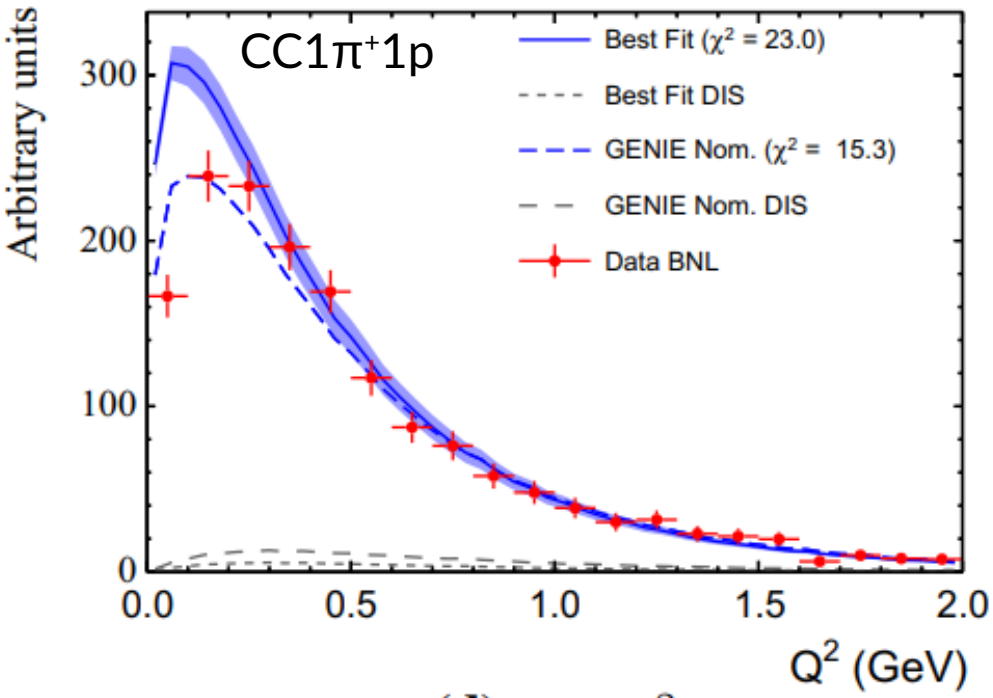
Different tunes

Paper

Single pion tune ANL+BNL 101

- GENIE's Rein-Sehgal model overestimated large amounts of reanalysed single pion data from bubble chamber
- Performed simultaneous fit of M_a^{res}, RES norm., DIS norm. to all CC1π channels from ANL and BNL in E_ν and Q^2





(d) BNL Q^2

- DIS norm (making up the non-res background in GENIE) pulled down to **43%** of nominal, resonant normalisation up by **15%** and M_a^{res} from **1.12→0.94 GeV**

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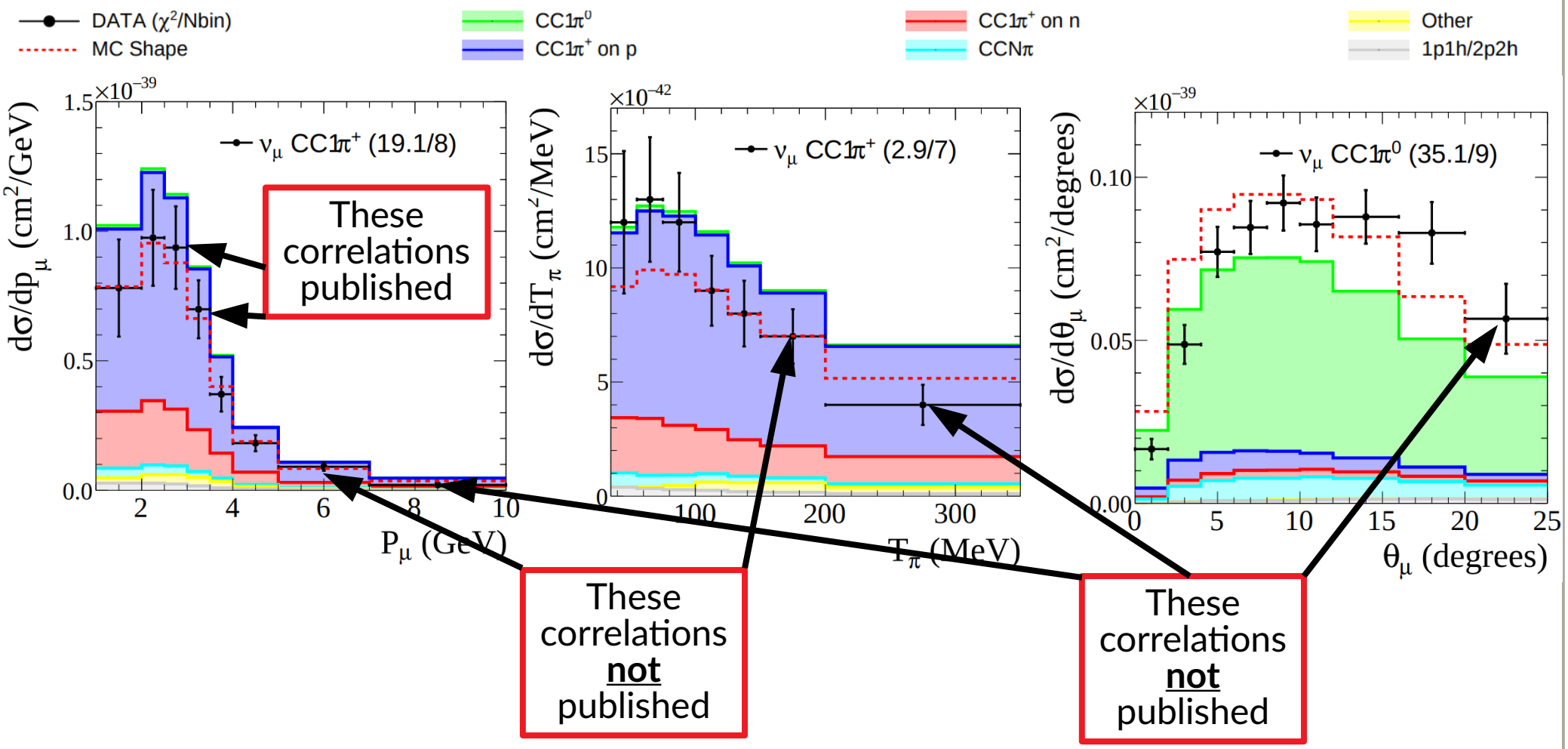
Method

- Didn't want to use measurements in "theory variables", e.g. Q^2_{True}
  - Possible interaction model dependence in data
- Use observed kinematic distributions
  - Straight-forward smearing
  - Less reliant on experiment's theory systematics

Channel	ν_μCC1$\pi^\pm$ [19]	ν_μCC$N\pi^\pm$ [20]	ν_μCC1π^0 [21]	$\bar{\nu}_\mu$CC1π^0 [20]
N_{bins} p_μ	8	9	8	9
N_{bins} θ_μ	9	9	9	9
N_{bins} T_π	7	7	7	7
N_{bins} θ_π	14	14	11	11
N_{bins} total	38	39	35	36
Signal definition	$1\pi^\pm$, $\geq 0\pi^0$	$> 0\pi^\pm$, $\geq 0\pi^0$	$1\pi^0$, $0\pi^\pm$	$1\pi^0$, $0\pi^\pm$
	$1\mu^-$	$1\mu^-$	$1\mu^-$	$1\mu^+$
	$W_{rec} < 1.4$ GeV	$W_{rec} < 1.8$ GeV	$W_{rec} < 1.8$ GeV	$W_{rec} < 1.8$ GeV
	—	—	$\theta_\mu < 25°$	—

Correlations in data

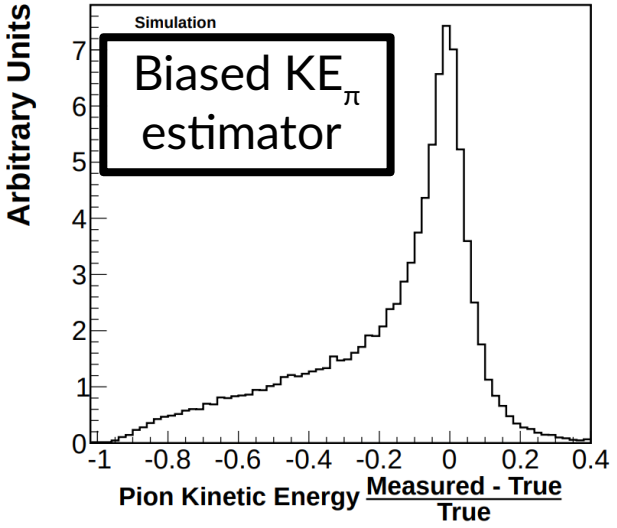
- All data (so far) are single dimension cross-sections



- Have correlations for each individual distribution
- No cross-correlations between distributions

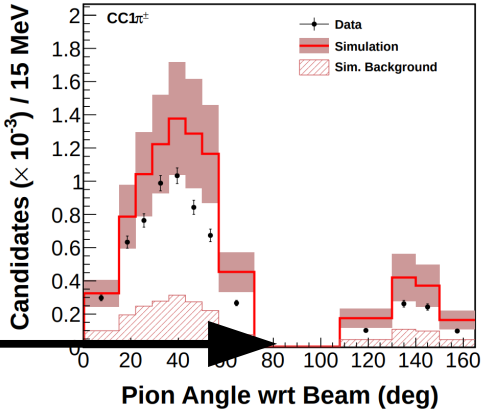
Correlations in data

- <u>Only correct way</u> is to re-run analyses simultaneously, keeping track of the correlated universes
  - No volunteers in MINERvA, so wasn't done
- Pick a distribution which controls the normalisation (rate), use the others as shape
  - We chose p_μ because
    - Clean in MINERvA
    - Pretty flat efficiency
    - Pretty good smearing
    - Largely insensitive to shape variations of fitting parameters



- Chose to use one p_μ distribution per topology
  - Could've done one p_μ in total
- Doesn't fully mitigate problem

Holes of efficiency in $\theta_{\nu\pi}$



Applying ANL/BNL tune

- Chose a decent set of GENIE systematics to weight in
  - M_A^{res}, CC_{RES}^{Norm}, Non-Res Norm, 2π norm, (non)isotropic RS
- Apply tuning from ANL/BNL paper

CC1π^0 gets uniformly worse

Distribution	Channel	N_{bins}	Default	ANL/BNL
p_μ (Rate)	$\nu_\mu CC1\pi^+$	8	19.1	13.8
	$\nu_\mu CCN\pi^+$	9	35.4	19.5
	$\nu_\mu CC1\pi^0$	8	11.1	19.6
	$\bar{\nu}_\mu CC1\pi^0$	9	7.4	6.4
θ_μ (Shape)	$\nu_\mu CC1\pi^+$	9	7.1	12.4
	$\nu_\mu CCN\pi^+$	9	4.5	10.4
	$\nu_\mu CC1\pi^0$	9	35.1	71.5
	$\bar{\nu}_\mu CC1\pi^0$	9	9.3	14.0
T_π (Shape)	$\nu_\mu CC1\pi^+$	7	2.9	2.6
	$\nu_\mu CCN\pi^+$	7	39.8	34.7
	$\nu_\mu CC1\pi^0$	7	28.3	31.4
	$\bar{\nu}_\mu CC1\pi^0$	7	19.3	17.9
θ_π (Shape)	$\nu_\mu CC1\pi^+$	14	25.4	26.5
	$\nu_\mu CCN\pi^+$	14	11.7	11.1
	$\nu_\mu CC1\pi^0$	11	13.5	15.0
	$\bar{\nu}_\mu CC1\pi^0$	11	5.7	5.9
Total χ^2		148	275.6	312.7

Rate χ^2 improves?

All θ_μ shape distributions are worse

Pretty much everything else gets worse

Total χ^2 is bad with and without

Tensions in applying nucleon fits to nuclear data

Fitting, part I

- Both FrAbs and FrInel fits converge to similar parameter values and test-statistics, with clear improvements in χ^2

Parameter	Default Value	ANL/BNL Value	FrAbs Fit Result	FrInel Result
M_A^{res} (GeV)	1.12 ± 0.22	0.94 ± 0.05	1.07 ± 0.04	1.08 ± 0.04
NormRes (%)	100 ± 30	115 ± 7	94 ± 6	92 ± 6
NonRes1π (%)	100 ± 50	43 ± 4	44 ± 4	44 ± 4
NonRes2π (%)	100 ± 50	-	166 ± 32	161 ± 33
π-iso	$0 =$ RS	-	$1 =$ Iso (limit)	$1 =$ Iso (limit)
FrAbs (%)	100 ± 30	-	109 ± 16	-
FrInel (%)	100 ± 40	-	-	109 ± 24
MINERνA χ^2	275.6	312.7	242.3	240.7
χ^2_{pen}	299.3	0.0	9.3	11.1
Total χ^2	574.8	312.7	251.6	251.8
N_{DoF}	148	148	145	145

- As expected, ANL/BNL parameters are contended in the fit
- The fit moves closer to the GENIE nominal, except for the non-resonant background

Fitting, part I

- Fit individual cross-section topologies to gauge which is pulling

Parameter	ν_μCC1π^+	ν_μCC$N\pi^+$	ν_μCC1π^0	$\bar{\nu}_\mu$CC1π^0
M_A^{res} (GeV)	0.97 ± 0.05	0.97 ± 0.05	1.02 ± 0.05	0.96 ± 0.05
NormRes (%)	110 ± 7	110 ± 7	104 ± 7	111 ± 7
NonRes1π (%)	43 ± 4	42 ± 4	44 ± 4	43 ± 4
NonRes2π (%)	300 (limit)	99±30	300 (limit)	300 (limit)
π-iso	1 = Iso (limit)	1 = Iso (limit)	1 = Iso (limit)	1 = Iso (limit)
FrAbs (%)	156 ± 53	128 ± 34	126 ± 17	82 ± 31
MINERνA χ^2	36.6	64.1	92.3	34.6
χ^2_{pen}	0.5	0.7	3.2	0.3
Total χ^2	37.1	64.8	95.5	34.9
N_{DoF}	35	36	32	33

- CC1π^0 channel does not agree well with prior
  - Anti-neutrino pulls to different FSI parameter value
- Parameters largely agree for the fits, no huge pulls
  - NonRes2π barely has an effect, which is why +300%

Have we learnt anything?

Arguably, yes.

ANL/BNL prior does not agree with MINERvA 1π data

Largest pull from CC1π^0

Be careful with your priors and uncertainties: same model is OK for CC1π^+ but not CC1π^0

Fitting, part II

- Including the Q^2-dependent suppression alleviates the tension with the ANL and BNL tuning

Parameter	FrAbs Tune	FrAbs + low-Q^2 Tune	FrInel Tune	FrInel + low-Q^2 Tune
M_A^{res} (GeV)	1.07 ± 0.04	0.92 ± 0.02	1.08 ± 0.04	0.93 ± 0.05
NormRes (%)	94 ± 6	116 ± 3	92 ± 6	116 ± 7
NonRes1π (%)	43 ± 4	46 ± 4	44 ± 4	46 ± 4
NonRes2π (%)	166 ± 32	99 ± 31	161 ± 33	120 ± 32
π-iso	1.0 (limit)	1.0 (limit)	1.0 (limit)	1.0 (limit)
FrAbs (%)	109 ± 16	48 ± 21	-	-
FrInel (%)	-	-	109 ± 24	132 ± 27
Lag. R_1	-	0.32 ± 0.06	-	0.37 ± 0.09
Lag. R_2	-	0.5 (limit)	-	0.60 ± 0.16
MINERνA χ^2	242.3	212.2	240.7	215.7
χ^2_{pen}	9.3	0.7	11.1	0.5
Totalχ^2	251.6	212.9	251.8	216.2
N_{DoF}	145	143	145	143

- And improves the χ^2 from the MINERvA data-sets
- Absorption and inelastic tune ~agree, although R_2 sits at the limit
  - Still not a great χ^2, and tension may be artificially relieved

Fitting, part II

- Looking at individual distributions' χ^2
  - Sometimes $1\pi^+$ improves with Q^2 tune, whereas $1\pi^0$ worsens

Distribution	Channel	N_{bins}	FrAbs Tune	FrAbs + low-Q^2 Tune	FrInel Tune	FrInel + low-Q^2 Tune
p_μ (Rate)	$\nu_\mu CC1\pi^\pm$	8	12.0	10.8	12.3	10.9
	$\nu_\mu CCN\pi^\pm$	9	26.1	16.2	26.8	17.9
	$\nu_\mu CC1\pi^0$	8	19.0	26.2	19.3	26.9
	$\bar{\nu}_\mu CC1\pi^0$	9	6.2	7.1	6.3	7.2
θ_μ (Shape)	$\nu_\mu CC1\pi^\pm$	9	7.5	7.4	7.4	7.1
	$\nu_\mu CCN\pi^\pm$	9	4.0	6.3	4.1	5.6
	$\nu_\mu CC1\pi^0$	9	44.5	20.0	45.6	20.5
	$\bar{\nu}_\mu CC1\pi^0$	9	10.2	7.0	10.3	6.9
T_π (Shape)	$\nu_\mu CC1\pi^\pm$	7	2.5	2.5	2.3	2.4
	$\nu_\mu CCN\pi^\pm$	7	31.2	28.9	29.4	27.7
	$\nu_\mu CC1\pi^0$	7	30.9	27.1	29.9	32.0
	$\bar{\nu}_\mu CC1\pi^0$	7	16.6	15.7	16.0	18.7
θ_π (Shape)	$\nu_\mu CC1\pi^\pm$	14	13.0	13.4	12.6	12.6
	$\nu_\mu CCN\pi^\pm$	14	6.9	7.0	6.2	6.3
	$\nu_\mu CC1\pi^0$	11	8.3	12.2	8.9	9.4
	$\bar{\nu}_\mu CC1\pi^0$	11	3.4	4.4	3.5	3.7
Total χ^2		148	242.3	212.2	240.7	215.7

Fitting, part II

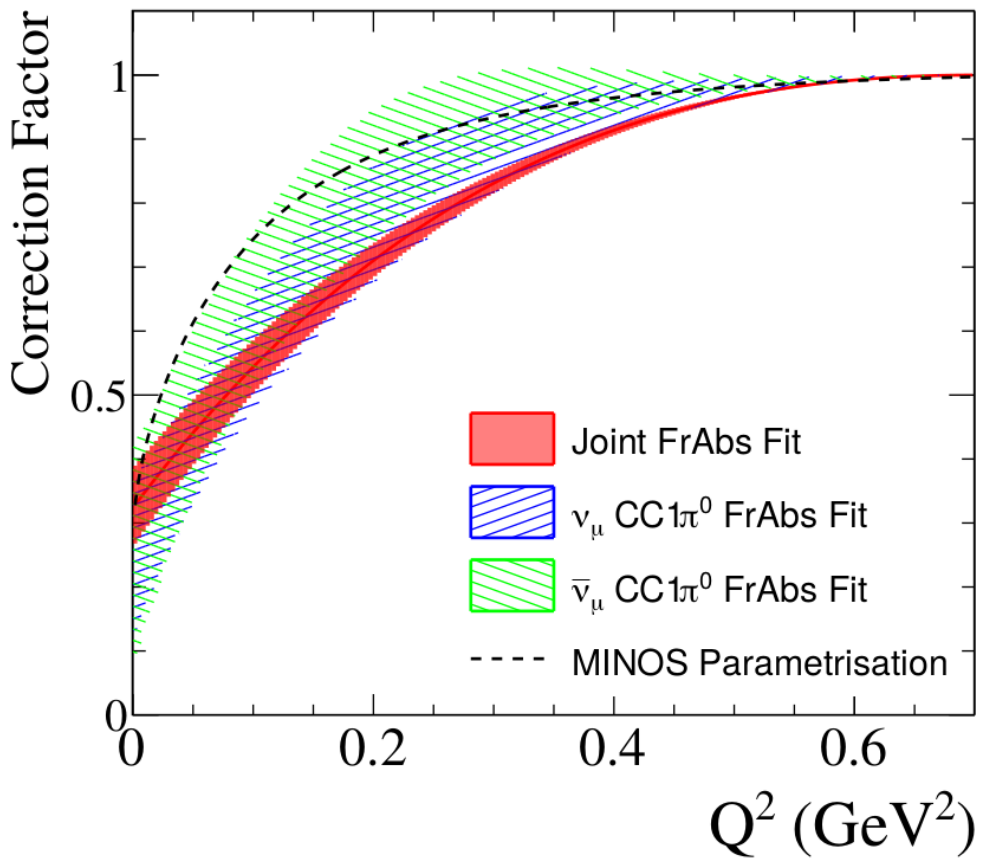
- ANL/BNL penalty term steers the nucleon parameters
  - Mismodelling absorbed in very different R_1 and R_2

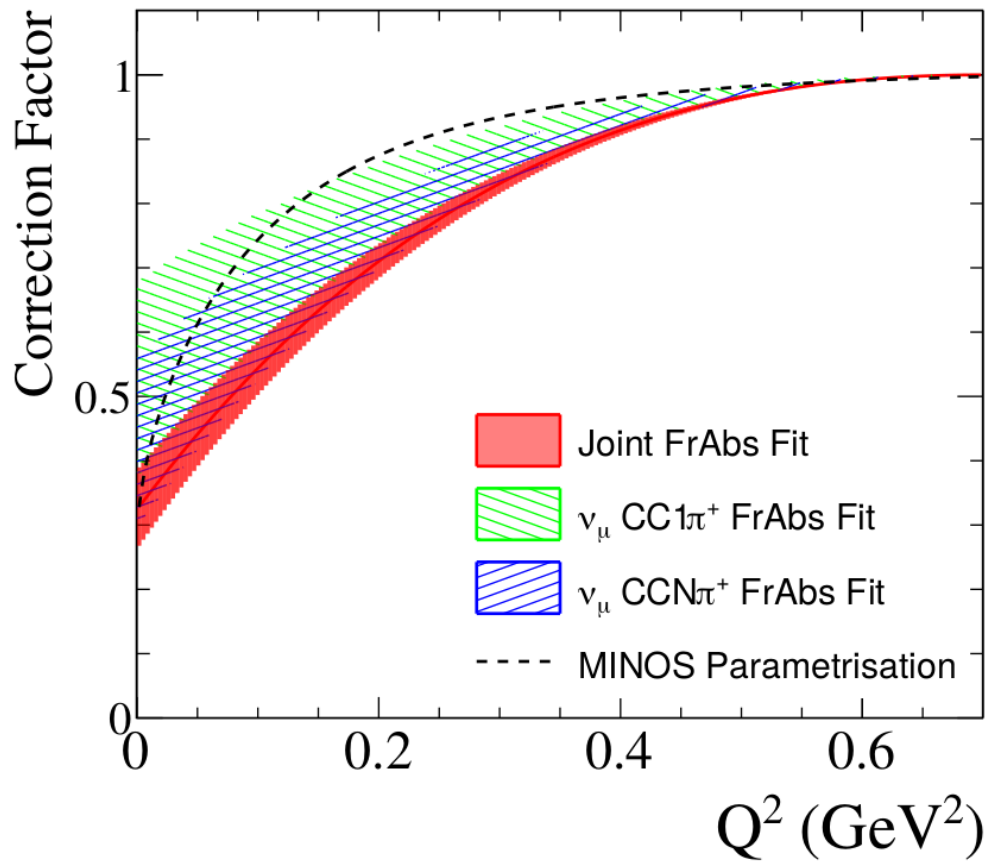
Parameter	ν_μCC1π^+	ν_μCC$N\pi^+$	ν_μCC1π^0	$\bar{\nu}_\mu$CC1π^0
M_A^{res} (GeV)	0.93 ± 0.02	0.92 ± 0.02	0.96 ± 0.05	0.94 ± 0.05
NormRes (%)	115 ± 3	117 ± 3	114 ± 7	115 ± 7
NonRes1π (%)	43 ± 4	43 ± 4	45 ± 4	43 ± 4
NonRes2π (%)	300 (limit)	70 ± 28	300 (limit)	300 (limit)
π-iso	1 = Iso (limit)	1 = Iso (limit)	1 = Iso (limit)	1 = Iso (limit)
FrAbs (%)	92 ± 65	79 ± 40	74 ± 22	34 ± 35
Lag. R_1	0.53 ± 0.16	0.43 ± 0.13	0.21 ± 0.14	0.14 ± 0.22
Lag. R_2	0.50 (limit)	0.50 (limit)	0.63 ± 0.31	1.00 (limit)
MINERνA χ^2	32.2	55.7	71.2	27.7
χ^2_{pen}	0.1	0.4	0.5	0.0
Total χ^2	32.3	56.1	71.7	27.7
N_{DoF}	33	34	30	31

- At times at the limit for R_2
- Not enough power in data? Insufficient model freedom?

Q^2 corrections from MINERvA data

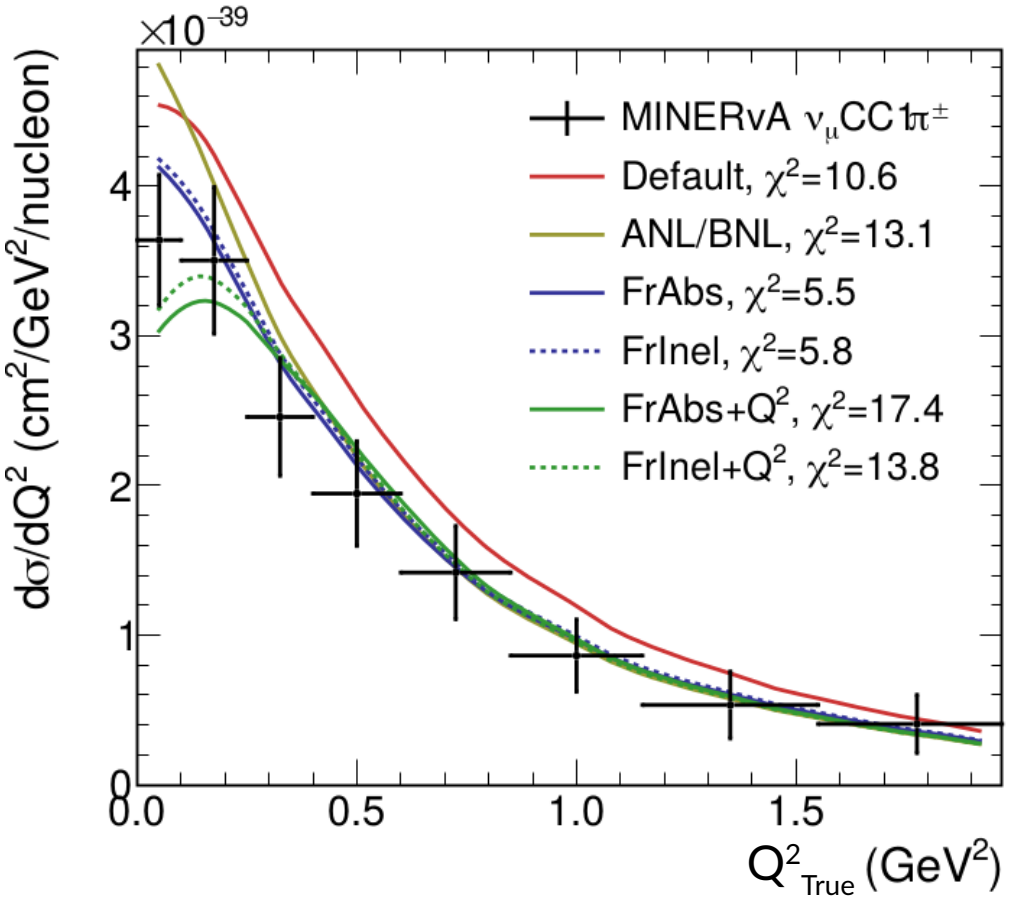
- Perform the tune individually to different data sets and combined
- Indicates different preferred tuning for $CC1\pi^0$ and $CC1\pi^+$ cross-sections

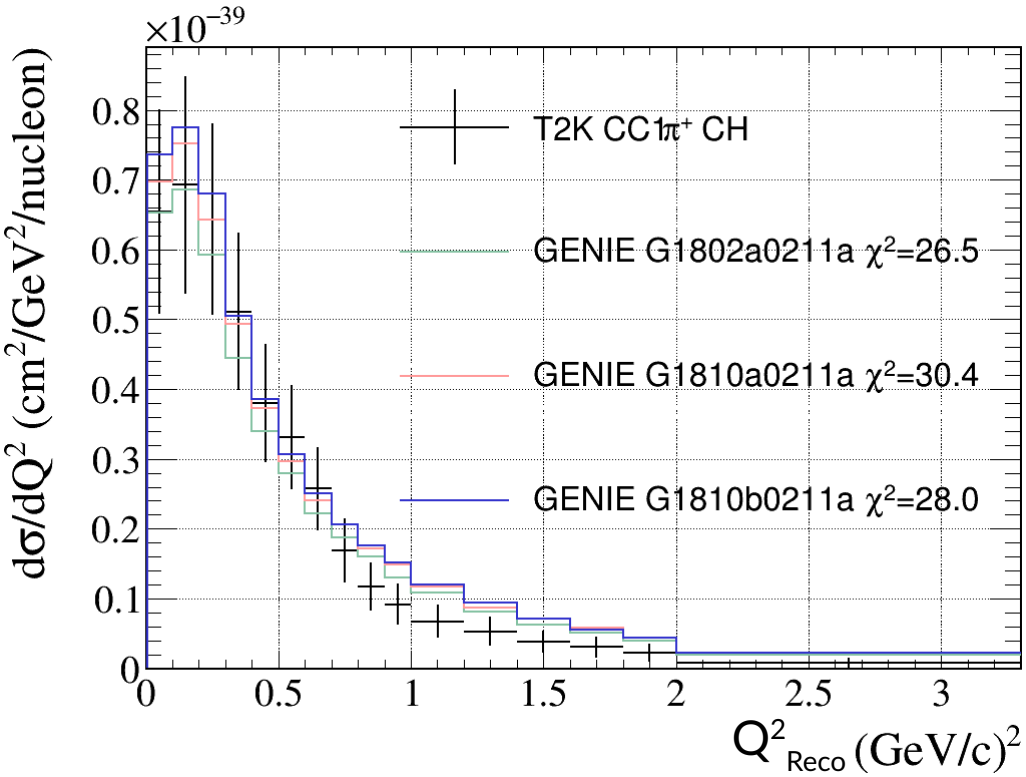




What does data seem to imply?

- Not entirely convinced that CC1π^+ needs a low Q^2 suppression, looking at Q^2_{True}

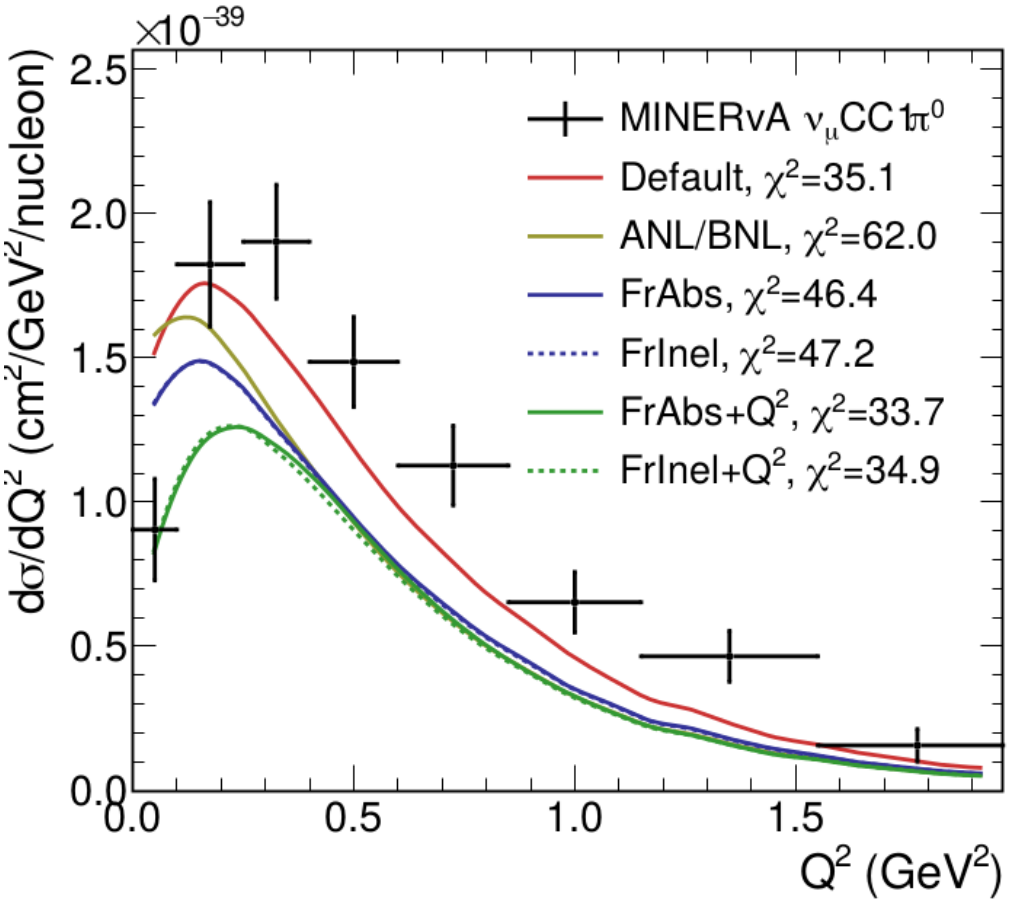


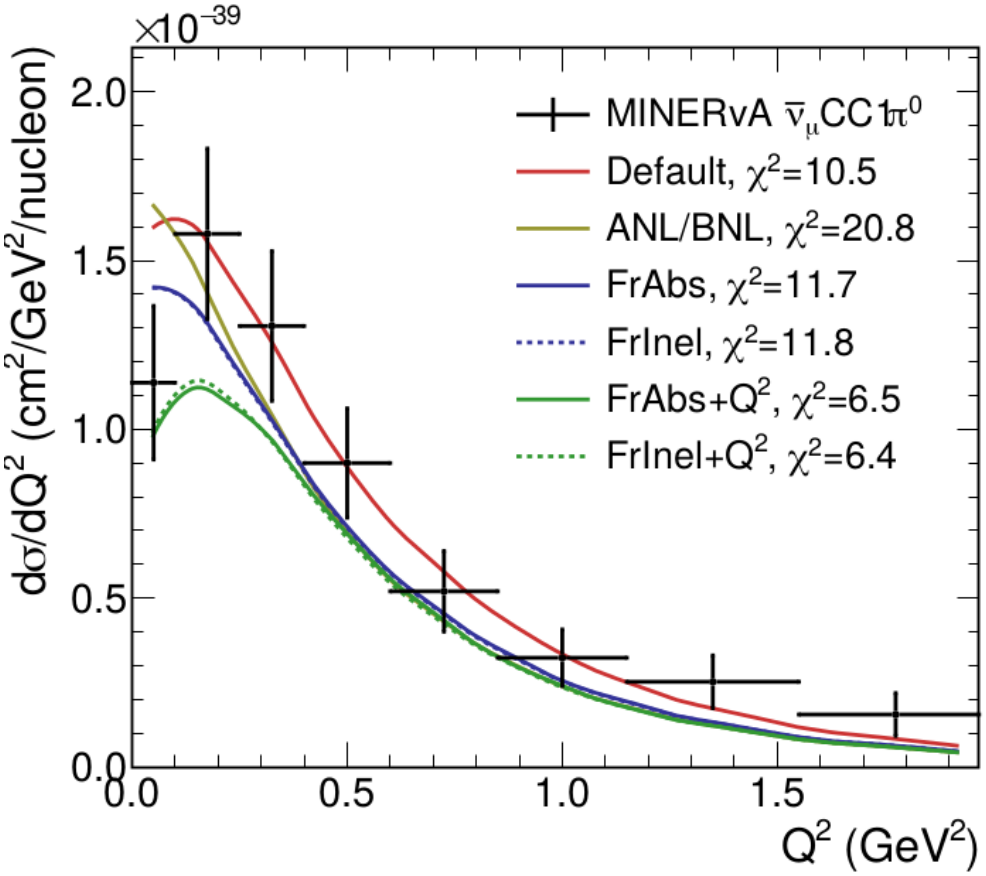


- Similarly looking at T2K (right) no suppression appears to be needed

What does data seem to imply?

- Focussing instead on the CC1π^0, a more consistent picture emerges

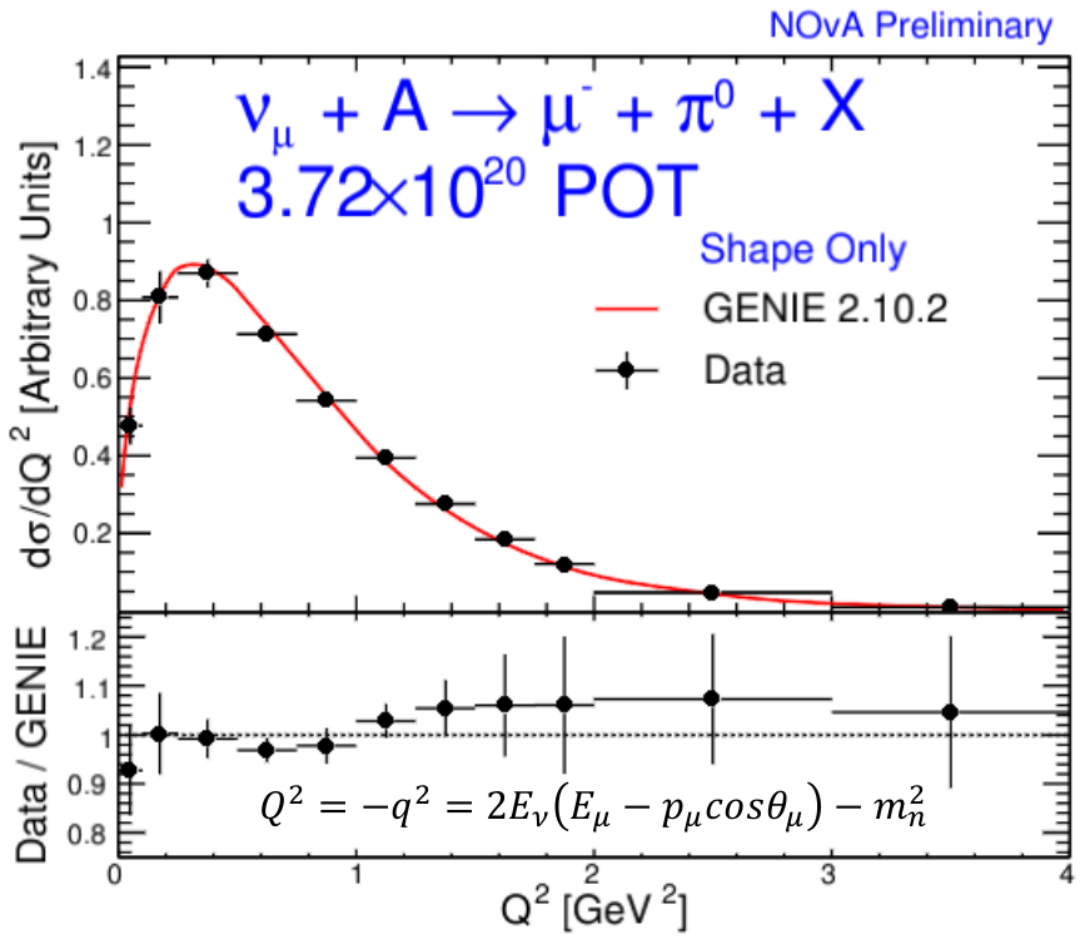




- The low Q^2 suppression is clearly favoured

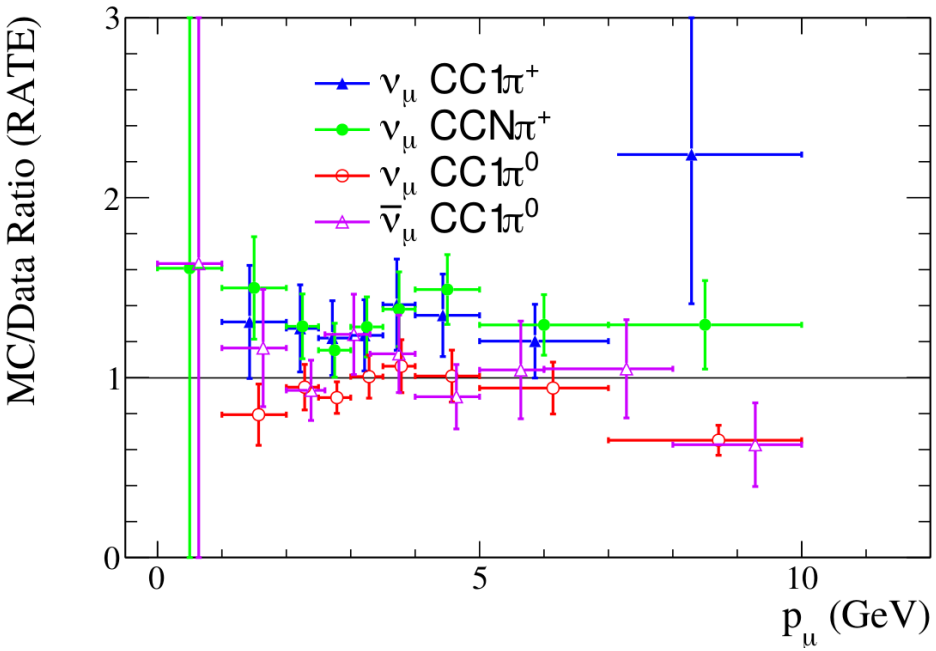
What does data seem to imply?

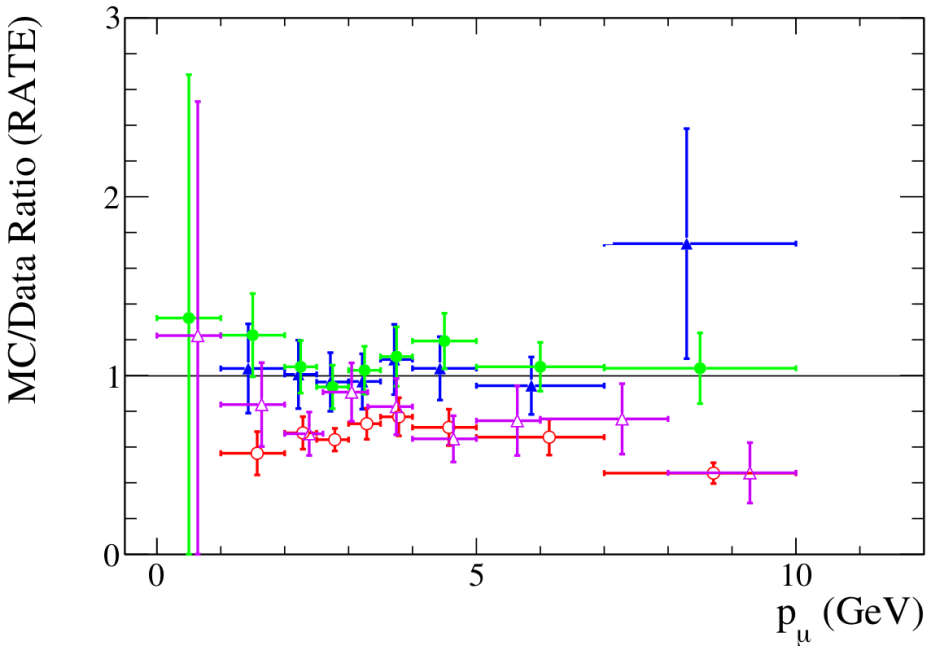
- This isn't necessarily consistent with MINOS and NOvA results
- Largest contribution to MINOS resonant sideband should be CC1π^+, which wanted suppression
- NOvA CC1π^0 appears to not need low Q^2 suppression



Fitting, part II

- The pion distributions are largely invariant to the tune
- We've changed nucleon physics and made a Q^2 tuning
  - Nothing explicitly working on the pions other than FSI and (non-)isotropic parameter





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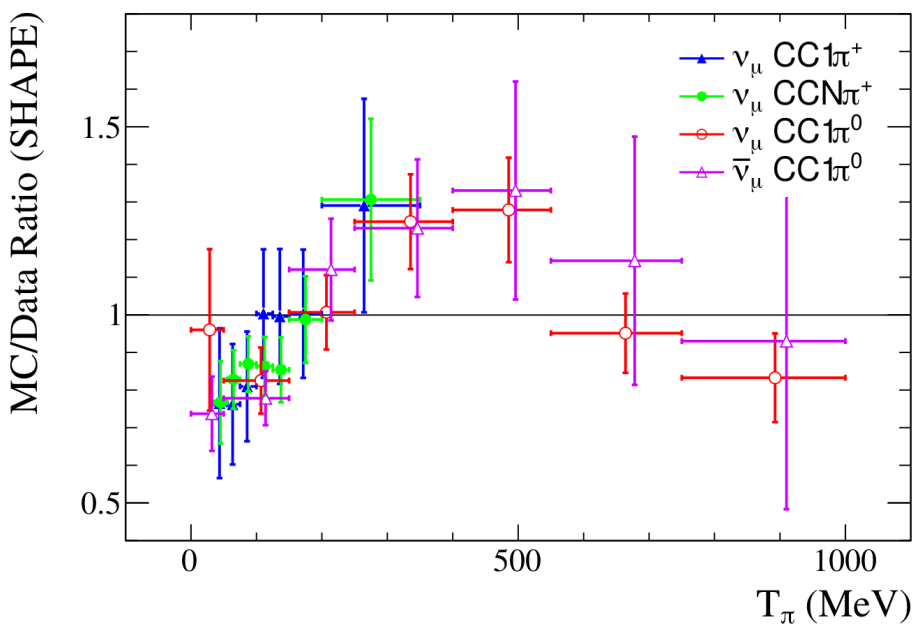

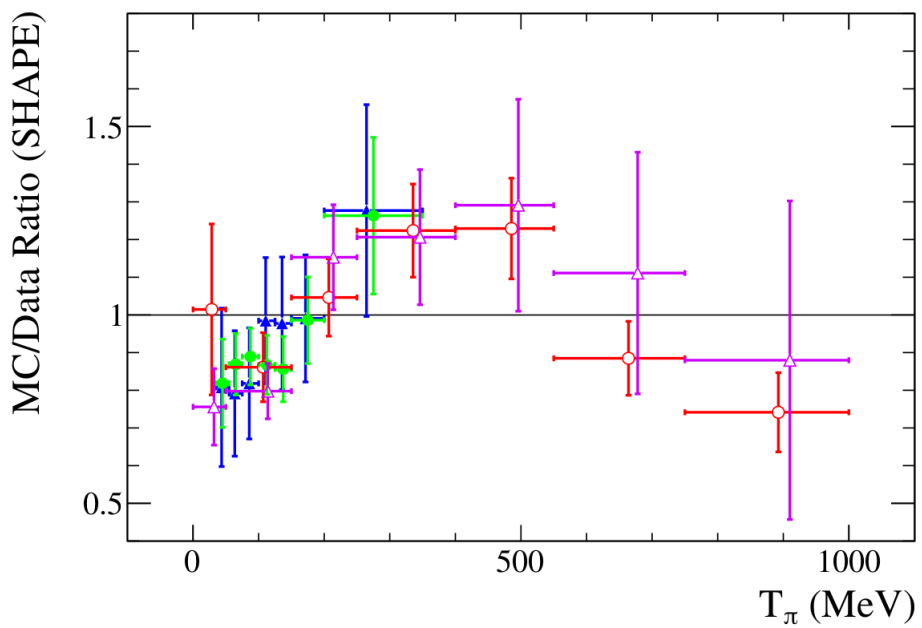

What's needed to disentangle

- 2D measurements and/or cross-correlations between distributions
- Measurements to separate single pion from multi-π+FSI is currently only achievable by comparing experiments
  - Or need more clever choice of variables?
- Correlation between muon and pion (e.g. $\cos\theta_{\mu\pi}$) is very interesting for nuclear dynamics
- Comparisons against lower energy, e.g. T2K, MiniBooNE?

- Your ideas here

Conclusions

- Single pion modelling on the nucleon can only be considered "well understood" on the Delta resonance
  - W<1.4 GeV data from ANL and BNL inform this
- Used MINERvA data to tune GENIE single pion production
- Tuning GENIE to nucleon level data worsens the prediction for the MINERvA single pion data
- Tuning the nucleon level parameters and pion FSI pulls the nucleon parameters closer to GENIE nominal: clear tension
- MINERvA CC1π^0 data in tension with other distributions
- Introduce Q^2 dependent correction, looking for a nuclear effect
- Alleviates tension with nucleon tune, but far from perfect

- Pion variables still aren't well described
- Unclear how this is E_ν or W dependent

Thanks for listening!

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Backups

Selected measurements

- ANL and BNL bubble chambers
- GGM, FNAL, BEBC

Why might 1pi be hard? Experiment

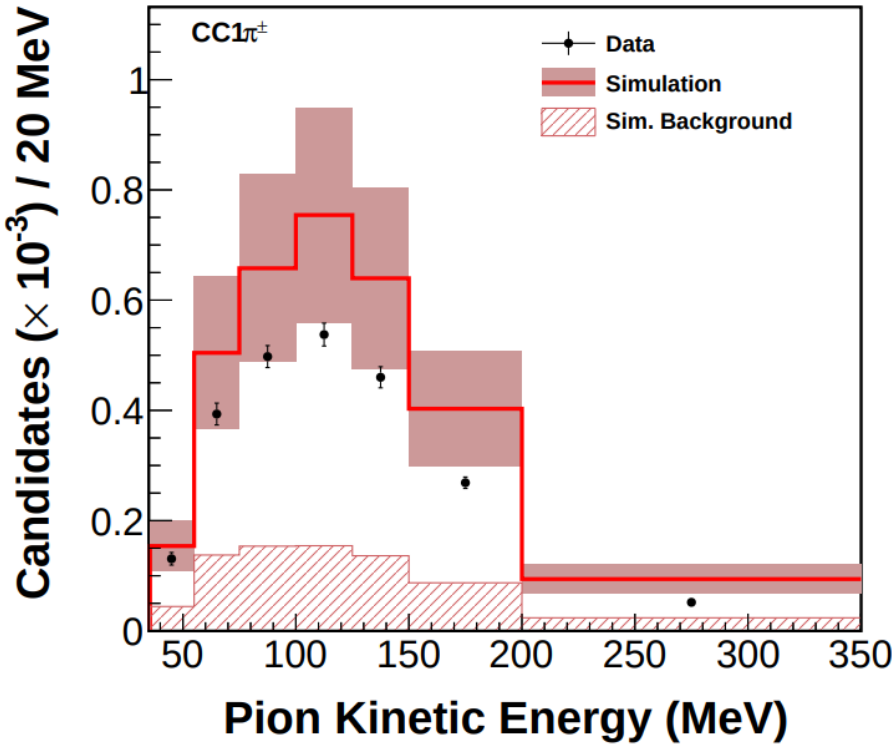
- Multi-particle topology
  - Efficiency correct in multi-dimension?
  - Plot in reduced phase space in one variable still means including those “cut” events in other variable
- Selecting muon vs pions
- Different experiments use different “tagging” methods

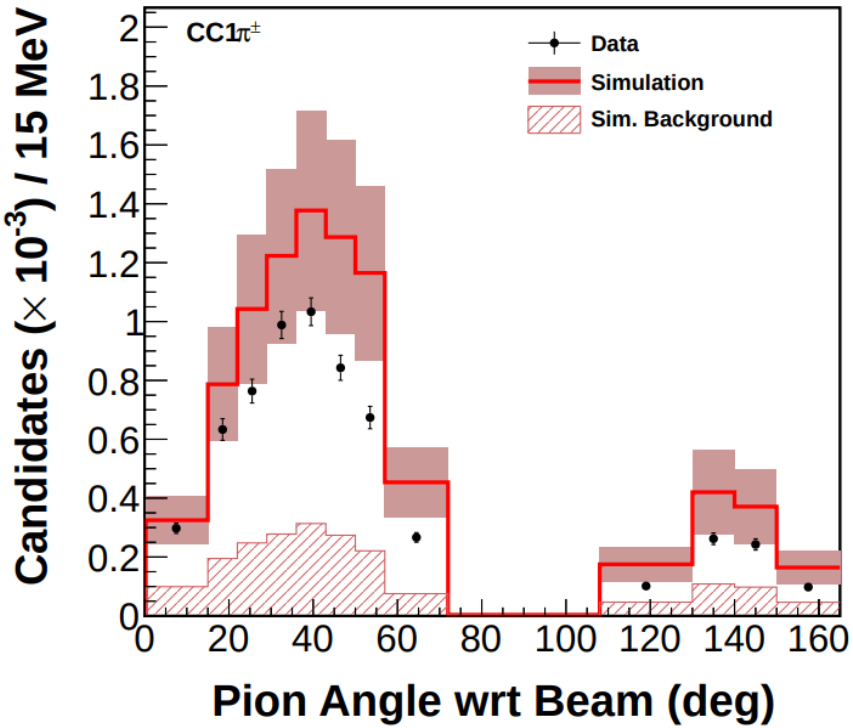
Selected measurements

- MiniBooNE CC1pi+
  - Large statistics, and 2D
  - Missing covariance matrix, W < 1.35 GeV selection cut
  - No reduced phase space signal definition
- MINERvA CC1pi+
  - Updated data release (but no paper describing it)
  - My gripe: no reduced phase space signal definition
    - Selects 50-350 MeV kinetic energy pions explictly, but corrects for this with GENIE
    - Can't see pions at high-angle, corrects for this with GENIE
    - Unclear where these corrected events appear in e.g. pmu or theta mu distributions
    - Will improve in next analysis

Selected measurements

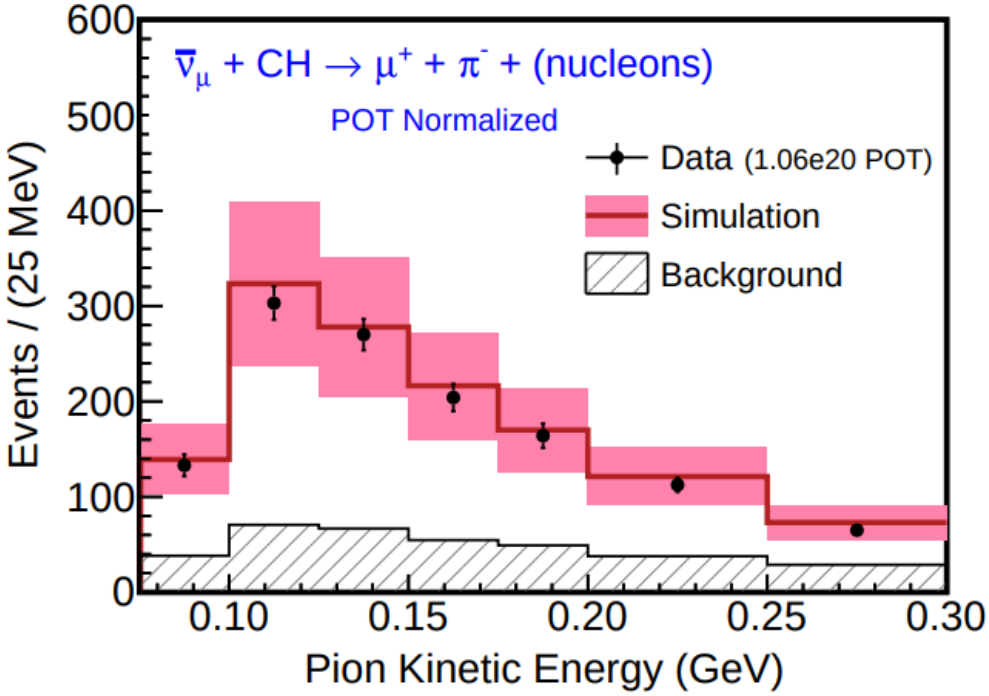
- MINERvA CC1pi+

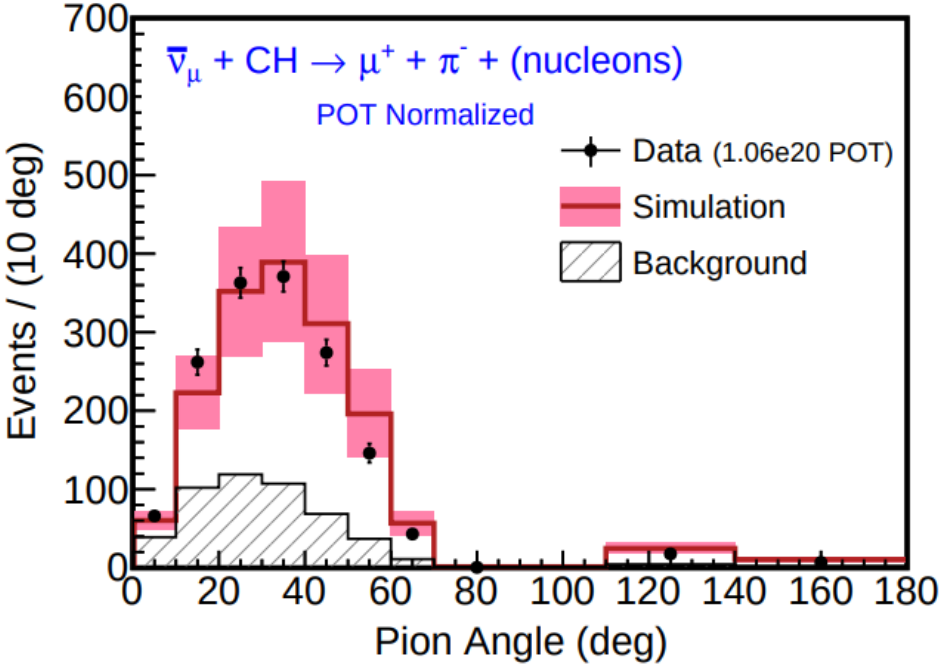




Selected measurements

- MiniBooNE CC1pi0
  - Covariance matrix not always trustworthy (some don't decompose)
  - Large statistics
- MINERvA CC1pi-
  - Repeats CC1pi+ corrections





Fitting, part I

- Maybe it's all in FSI parameters?
- Apply a penalty on nucleon parameters from ANL/BNL tuning, no penalty on remaining parameters

Parameter	Default Value	GENIE-RW Name
CC Resonant Axial Mass (M_A^{res})	1.12 ± 0.22 GeV	MaCCRES
CC Resonant Normalization (NormRes)	100 ± 20 %	NormCCRES
CC1π Nonresonant Normalization (NonRes1π)	100 ± 50 %	NonRESBGvnCC1pi NonRESBGvpCC1pi NonRESBGvbarnCC1pi NonRESBGvbarpCC1pi
CC2π Nonresonant Normalization (NonRes2π)	100 ± 50 %	NonRESBGvnCC2pi NonRESBGvpCC1pi NonRESBGvbarnCC1pi NonRESBGvbarpCC1pi
Pion Angular Emission (π-iso)	0 (RS)	Theta_Delta2Npi
Pion Absorption FSI Fraction (FrAbs)	100 ± 30 %	FrAbs_pi
Pion Inelastic FSI Fraction (FrInel)	100 ± 40 %	FrInel_pi

Nucleon parameters from ANL/BNL

Freely fitted parameters

Fitting, part I

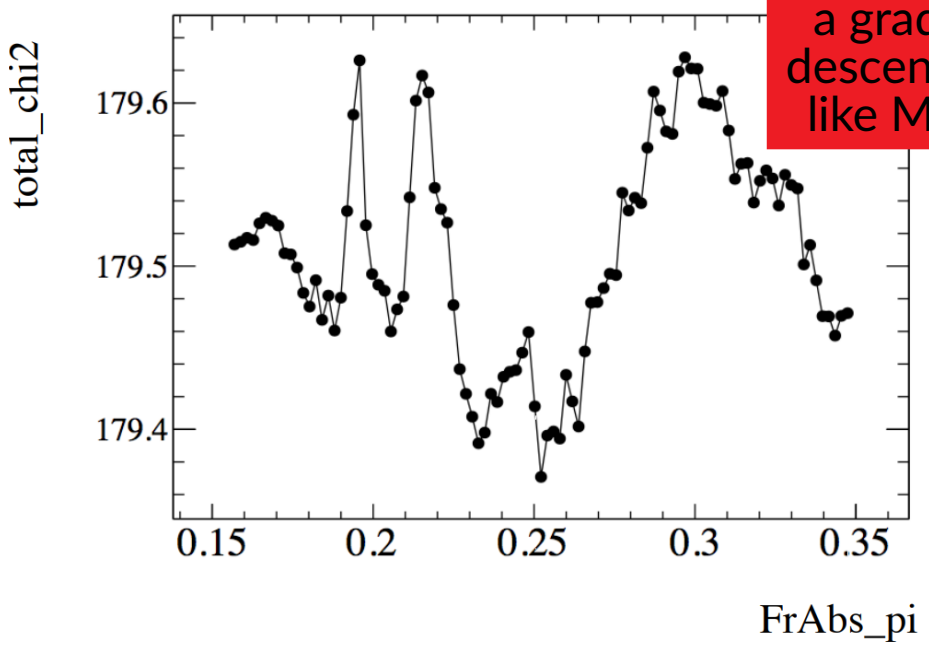
- Hold on, two FSI parameters?! Well spotted!

Pion Absorption FSI Fraction (FrAbs)	100 ± 30 %	`FrAbs_pi`
Pion Inelastic FSI Fraction (FrInel)	100 ± 40 %	`FrInel_pi`

- Initially tried fitting all FSI parameters simultaneously
- Tiny errors from strange behaviour in the test-statistic
  - Not present when varying one FSI parameter at a time
  - Or any other parameter simultaneously

jointPION-kinematic-fit
M_A^{RES} : 0.93 ± 0.02 (GeV)
NormRES : 114.0 ± 3.0 %
NormDIS : 46.2 ± 3.7 %
Theta_Delta2Npi : 1.0 ± 0.0 σ
MINERvARW_MINOSRPA_Apply : 1.0 ± 0.0 σ
FrCEx_pi : 1.332 ± 0.0061 σ
FrInel_pi : 0.9481 ± 0.0047 σ
FrAbs_pi : 0.2522 ± 0.0192 σ
FrPiProd_pi : 0.1226 ± 0.005 σ
NormDIS2PI : 93.8 ± 31.9 %
CHI2 : 182.686
NDOF : 115.0
CHI2/NDOF : 1.589

Tiny errors



Very very difficult for a gradient descent algo like Minuit

Fitting, part II

- MINOS and MiniBooNE have both seen this before
  - MINOS imposed an empirical Q^2 dependent tuning
- NOvA currently see this
  - Apply the RPA correction from CCQE
- Empirical Q^2 dependent tuning could absorb missing nuclear effect, but difficult to diagnose where it is from
  - There's so much missing in single pion production models
- Develop our own form for the Q^2 dependent suppression

$$w(Q^2) = 1 - (1 - R_1)(1 - R(Q^2))^2$$

Lagrange interpolating function in Q^2

$$R(Q^2 < x_3) = \frac{R_2(Q^2 - x_1)(Q^2 - x_3)}{(x_2 - x_1)(x_2 - x_3)} + \frac{(Q^2 - x_1)(Q^2 - x_2)}{(x_3 - x_1)(x_3 - x_2)}.$$

Cut-offs at x_1, x_2, x_3; **tune R_1 and R_2**