MK single pion production model

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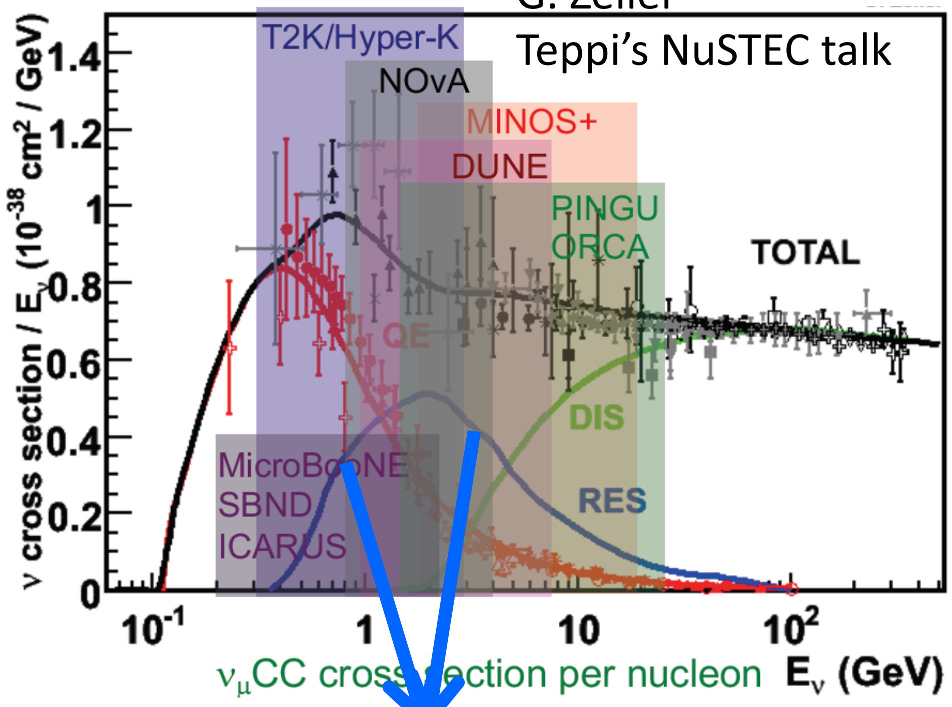
NUSTEC Workshop

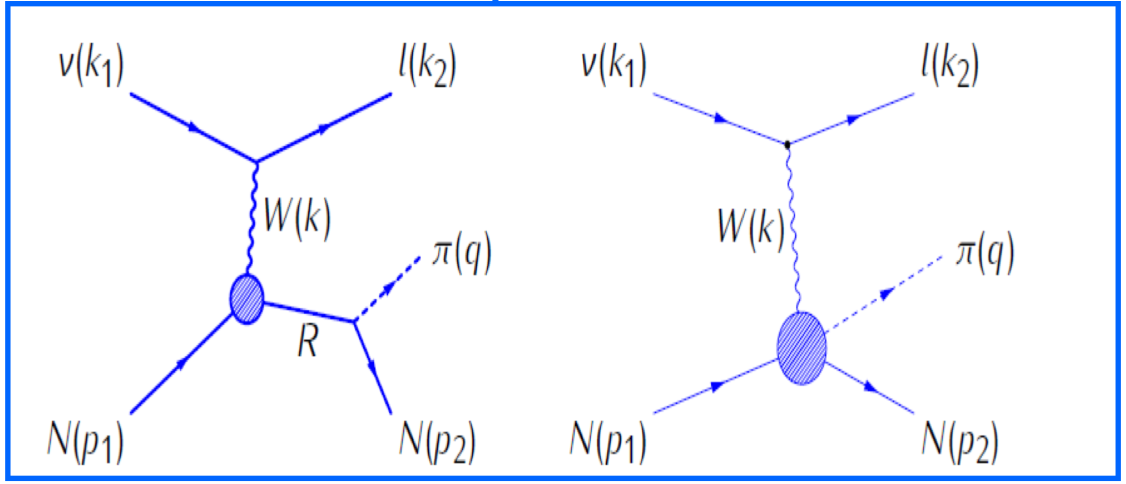
Oct. 4, 2019

Why Single pion production?

- For electron appearance experiments neutrino must be at difficult intermediate energy where single pion production has a significant contribution.
- Signal process for NOvA/DUNE.
- Single pion can be produced via decay of resonance excitations or non-resonant interactions.
- Current NEUT(GENIE) has no reliable model for non-resonant interaction.

G. Zeller
Teppi's NuSTEC talk



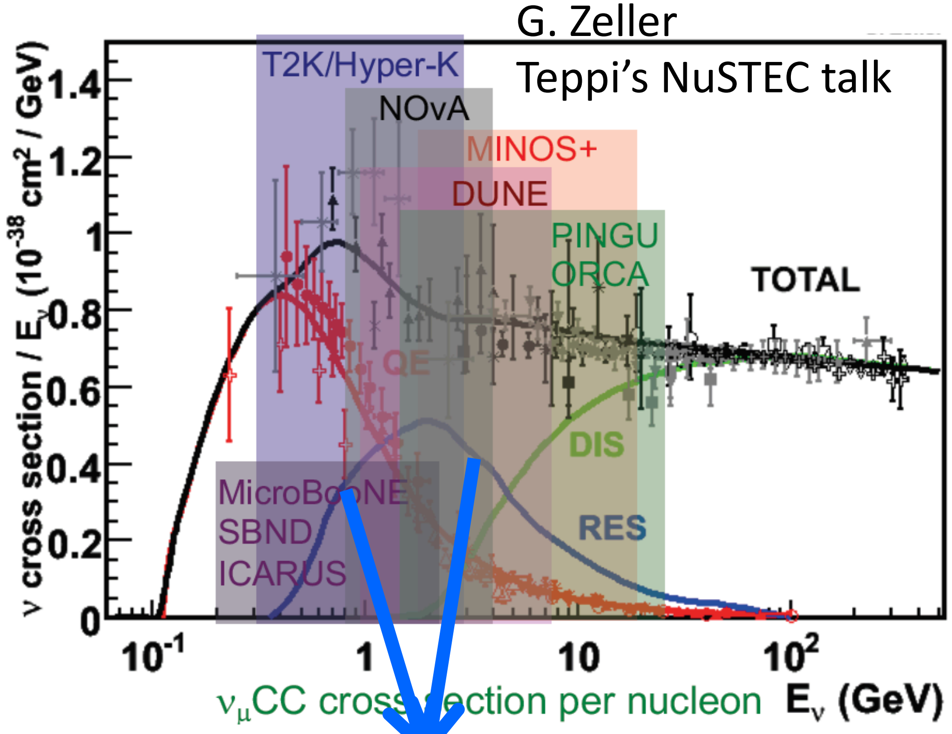


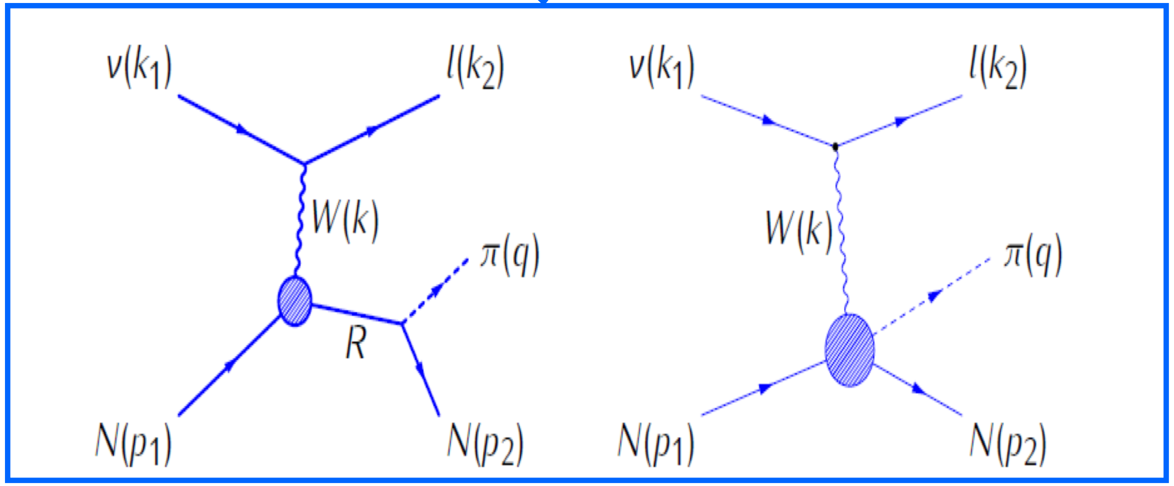
Some recent work on pion production
http://inspirehep.net/record/1746270?ln=en

Why Single pion production?

	ν	$\bar{\nu}$
CC	$\nu p \rightarrow \mu^- p \pi^+$	$\bar{\nu} n \rightarrow \mu^+ n \pi^-$
	$\nu n \rightarrow \mu^- p \pi^0$	$\bar{\nu} p \rightarrow \mu^+ n \pi^0$
	$\nu n \rightarrow \mu^- n \pi^+$	$\bar{\nu} p \rightarrow \mu^+ p \pi^-$
NC	$\nu p \rightarrow \nu p \pi^0$	$\bar{\nu} p \rightarrow \bar{\nu} p \pi^0$
	$\nu p \rightarrow \nu n \pi^+$	$\bar{\nu} p \rightarrow \bar{\nu} n \pi^+$
	$\nu n \rightarrow \nu n \pi^0$	$\bar{\nu} n \rightarrow \bar{\nu} n \pi^0$
	$\nu n \rightarrow \nu p \pi^-$	$\bar{\nu} n \rightarrow \bar{\nu} p \pi^-$

From
C. Wilkinson



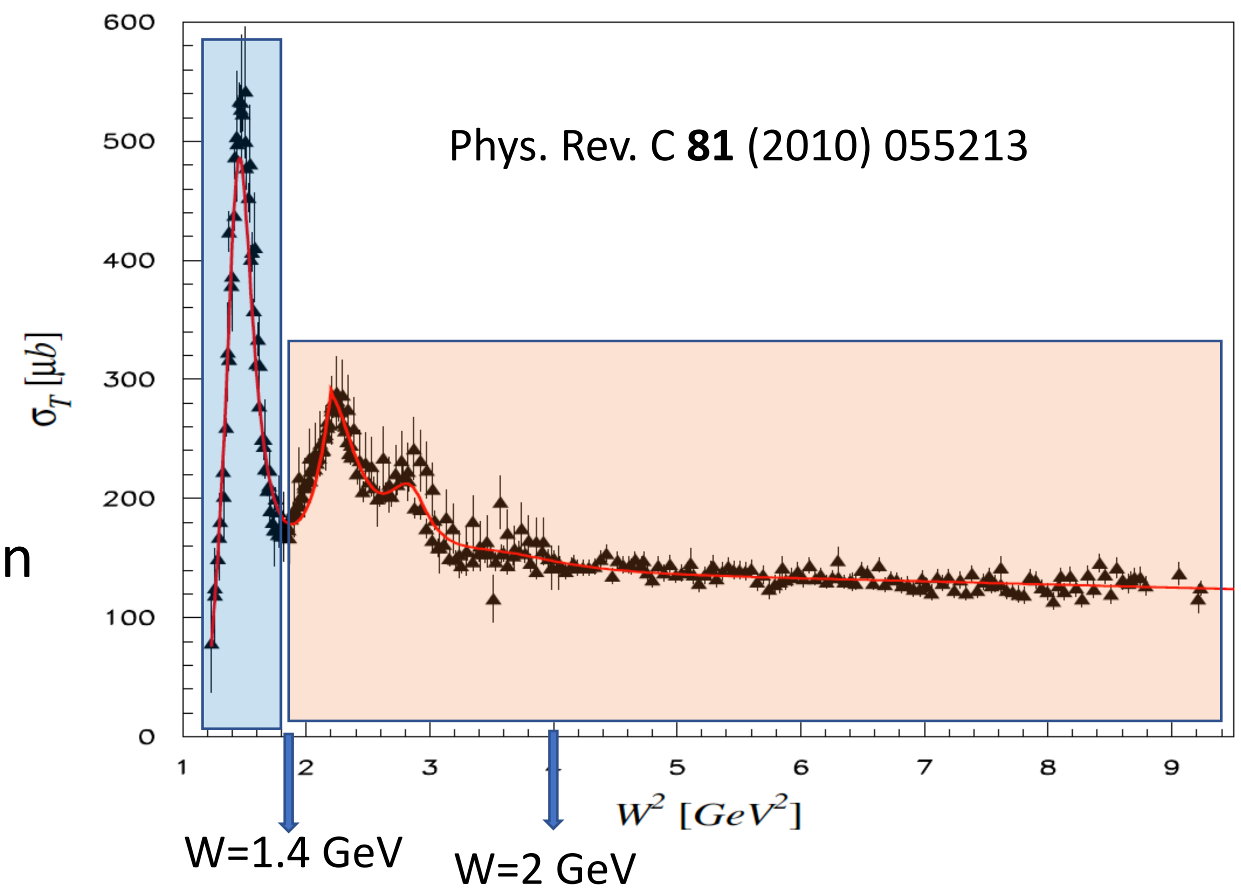


Inclusive electron scattering data

- For E_ν <1 GeV only Δ resonance contributes but for higher energy (DUNE) all resonances contribute to single pion production.

Δ(1232) region
(1.08GeV<W<1.4 GeV)

- Δ resonance dominates
- Only single pion can be produced



Beyond Δ region
W>1.4 GeV

- No single resonance dominate
- Several comparable resonances overlap
- Multi-pion and other mesons can be produced

Rein-Sehgal model (1981)

D. Rein and L. M. Sehgal, Annals Phys. 133 (1981) 79.

Rein-Sehgal[1] is based on helicity amplitudes deriven in relativistic quark model. It is a default model in the **NEUT** and **GENIE.**

- Easy to be implemented in generators.
- It covers all resonances up to W= 2 GeV.
- It does not cover non-resonant interaction
- Not a full kinematic model.
  The helicity amplitudes are **not** a function of pion angles $d\sigma/dW\,dQ^2$
- Pion angles are described by density matrix.
  NEUT and GENIE **only** implemented the Δ resonance.

Resonance	M_R	Γ_0	χ_E
$P_{33}(1232)$	1232	117	1
$P_{11}(1440)$	1430	350	0.65
$D_{13}(1520)$	1515	115	0.60
$S_{11}(1535)$	1535	150	0.45
$P_{33}(1600)$	1600	320	0.18
$S_{31}(1620)$	1630	140	0.25
$S_{11}(1650)$	1655	140	0.70
$D_{15}(1675)$	1675	150	0.40
$F_{15}(1680)$	1685	130	0.67
$D_{13}(1700)$	1700	150	0.12
$D_{33}(1700)$	1700	300	0.15
$P_{11}(1710)$	1710	100	0.12
$P_{13}(1720)$	1720	250	0.11
$F_{35}(1905)$	1880	330	0.12
$P_{31}(1910)$	1890	280	0.22
$P_{33}(1920)$	1920	260	0.12
$F_{37}(1950)$	1930	285	0.40

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The RS model is improved by including the pion angles and non-resonant interactions

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RS model in NEUT

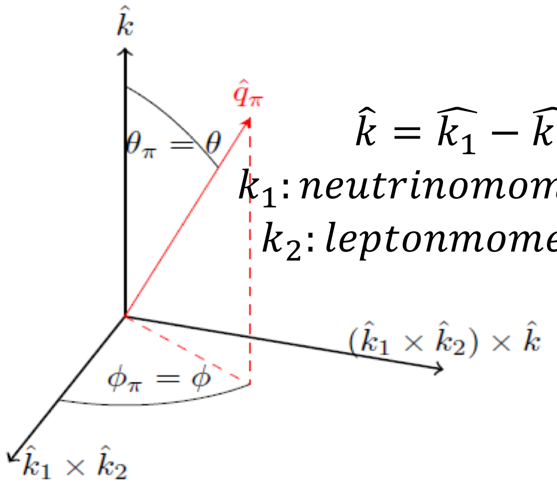
- RS model for 18 resonances and their interferences as it is in the original paper.
- Lepton mass is included based on BS paper in both model and phase space.
- Two options for resonance interaction: RS and GS (default)
- Pion angles are described by density matrix proposed in RS model.
- Except Delta, other resonances are ignored.
- For non-resonant contribution, an ad hoc term for I=1/2 with adjustable coefficient based of RS paper is implemented

RS model in GENIE (from the manual 2015)

- RS model for 16 resonances. Interference between resonances has been ignored.
- Lepton mass **is** included in the model but it is included in the phase space.
- It seems GENIE has only one option for resonance form-factor i.e. only RS FF
- Pion angles are described by density matrix proposed in RS model.
- Except Delta, other resonances are ignored.
- No neutrino-nucleon non-resonance background.

Rein Model (1987)

D. Rein,
Z.Phys. C – Particles and Fields 35,43-64 (1987)



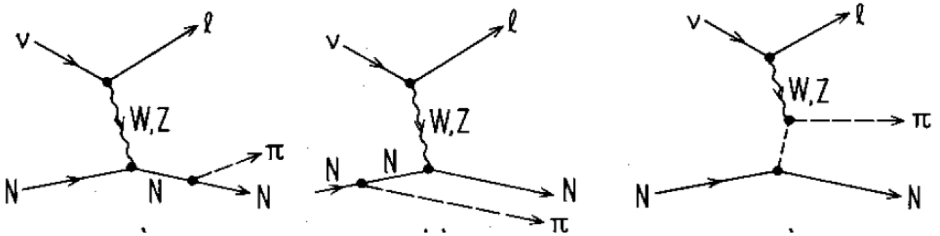
$\hat{k} = \widehat{k_1} - \widehat{k_2}$
k_1: *neutrinomomentum*
k_2: *leptonmomentun*

- Define a suitable framework; Adler frame
- Calculate both resonant and non-resonant interactions
- Add them coherently to include the interference effects

→Resonant Interaction

- Uses **Rein-Sehgal** model which is based on **helicity amplitudes.**

→Nonresonant Interaction



- **Born graphs** based on linear sigma model.

→It is NOT a full kinematic model

- It is not suitable for event generator. Very CPU concuming.

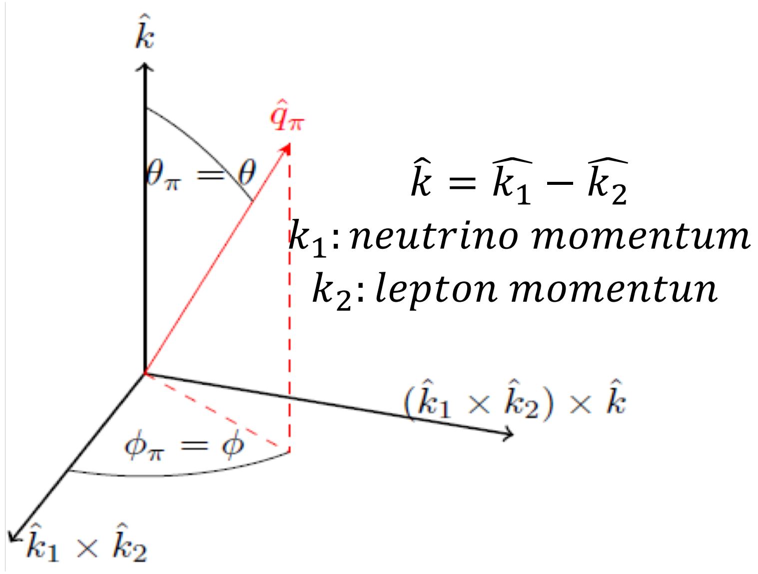
→The lepton is assumed to be massless

General framework: how to calculate the helicity amplitudes?

$$\mathcal{M}_{CC}(\nu N \to l_\lambda N'\pi) = \frac{G_F}{\sqrt{2}} \cos\theta_C \, \langle N'\pi | \, \varepsilon^\rho_\lambda J_\rho \, | N \rangle$$

$$= \frac{G_F}{\sqrt{2}} \cos\theta_C \, \langle N'\pi | \, C_{L_\lambda} e^\rho_L J_\rho + C_{R_\lambda} e^\rho_R J_\rho + C_\lambda e^\rho_\lambda J_\rho | N \rangle$$

Hadron current $J_\alpha = J^V_\alpha - J^A_\alpha$



Lepton current $\epsilon^\alpha = \bar{u}_l(k_2)\gamma^\alpha(1-\gamma_5)u_\nu(k_1)$

can be interpreted as the intermediate gauge boson's polarization vector.

$$\epsilon^\rho_\lambda = C_{L_\lambda} e^\rho_L + C_{R_\lambda} e^\rho_R + C_\lambda e^\rho_\lambda$$

Four different polarizations $e^\rho_{\lambda_k}$

$$e^\rho_L = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & -i & 0 \end{pmatrix}$$

$$e^\rho_R = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -1 & -i & 0 \end{pmatrix}$$

$$e^\rho_\lambda = \frac{1}{\sqrt{|(\varepsilon^0_\lambda)^2 - (\varepsilon^3_\lambda)^2|}}\begin{pmatrix} \varepsilon^0_\lambda & 0 & 0 & \varepsilon^3_\lambda \end{pmatrix}$$

Hadronic Current

- Dirac equation allows us to have 16 independent Lorentz covariance
- Conservation of vector current reduce the number of O(V_i) to six.

$$J_V^\rho e_\rho^{\lambda_k} = \sum_{i=1}^{6} V_i \, \bar{u}_N(p_2) O^{\lambda_k}(V_i) u_N(p_1)$$

Dirac Spinors

$$J_A^\rho e_\rho^{\lambda_k} = \sum_{i=1}^{8} A_i \, \bar{u}_N(p_2) O^{\lambda_k}(A_i) u_N(p_1)$$

4×4 matrices

2×2 matrices

$$J_V^\rho e_\rho^{\lambda_k} = \sum_{i=1}^{6} \mathscr{F}_i \, \chi_2^* \, \Sigma_i^{\lambda_k} \, \chi_1$$

Pauli spinors

$$J_A^\rho e_\rho^{\lambda_k} = \sum_{i=1}^{8} \mathscr{G}_i \, \chi_2^* \, \Lambda_i^{\lambda_k} \, \chi_1$$

O(V) and O(A) are 4*4 matrices in terms of Dirac matrices and particle's 4-momenta.

Helicity Amplitudes

$$\tilde{F}_{\lambda_2,\lambda_1}^{\lambda_k} = \langle \, N\pi | \, e_{\lambda_k}^\rho \, J_\rho^V \, |N \, \rangle$$

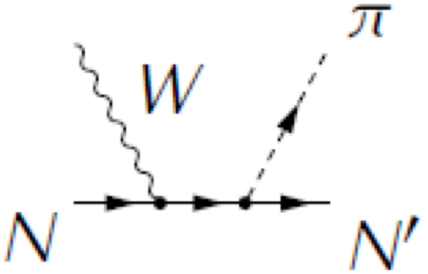
$$\tilde{G}_{\lambda_2,\lambda_1}^{\lambda_k} = \langle \, N\pi | \, e_{\lambda_k}^\rho \, J_\rho^A \, |N \, \rangle$$

Can be defined by knowing the helicity of incident and outgoing nucleons and gauge boson's polarization. 16 helicity amplitudes for each vector and axial.

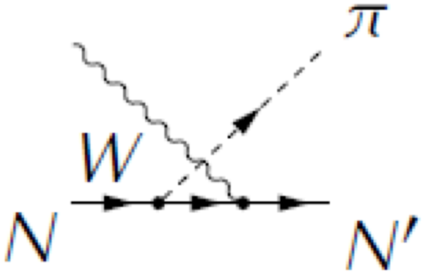
non-resonant background

E. Hernandez, J. Nieves and M. Valverde, Phys. Rev. D **76** (2007) 033005

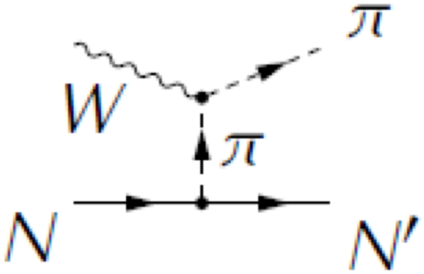
Defined by a set of diagrams determined by HNV model based on non-linear sigma model.



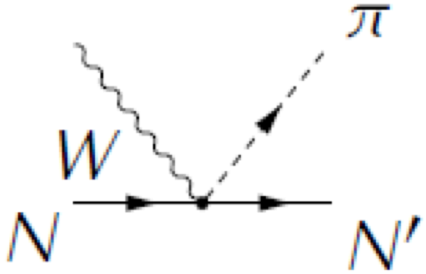
NP



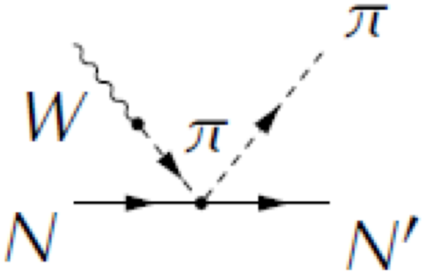
CNP



PF



CT



PP

$$\mathcal{M}_{CC}^{NP} = C^{NP}\cos\theta_C \frac{g_A}{\sqrt{2}f_\pi}\frac{1}{s-M^2}\,\bar{u}(p_2)\;\not{q}\,\gamma_5(\not{p}_1+\not{k}+M)\varepsilon^\mu[F_\mu^V - F_\mu^A]\,u(p_1),$$

$$\mathcal{M}_{CC}^{CNP} = C^{CNP}\cos\theta_C \frac{g_A}{\sqrt{2}f_\pi}\frac{1}{u-M^2}\,\bar{u}(p_2)\,\varepsilon^\mu[F_\mu^V - F_\mu^A]\,(\not{p}_2-\not{k}+M)\;\not{q}\gamma_5\,u(p_1)$$

Helicity amplitudes of above diagrams are calculated in the Adler frame.

Cross-Section

One needs to calculate the helicity amplitudes for resonant and nonresonant interactions.

$$\frac{d\sigma(\nu N \to lN\pi)}{dk^2 dW d\Omega_\pi} = \frac{G_F^2}{2}\frac{1}{(2\pi)^4}\frac{|\mathbf{q}|}{4}\frac{-k^2}{(k^L)^2}\sum_{\lambda_2,\lambda_1}\Big\{ \left|C_{L_-}(\tilde{F}^{eL}_{\lambda_2\lambda_1} - \tilde{G}^{eL}_{\lambda_2\lambda_1}) + C_{R_-}(\tilde{F}^{eR}_{\lambda_2\lambda_1} - \tilde{G}^{eR}_{\lambda_2\lambda_1}) + C_-(\tilde{F}^{e-}_{\lambda_2\lambda_1} - \tilde{G}^{e-}_{\lambda_2\lambda_1})\right|^2 + \left|C_{L_+}(\tilde{F}^{eL}_{\lambda_2\lambda_1} - \tilde{G}^{eL}_{\lambda_2\lambda_1}) + C_{R_+}(\tilde{F}^{eR}_{\lambda_2\lambda_1} - \tilde{G}^{eR}_{\lambda_2\lambda_1}) + C_+(\tilde{F}^{e+}_{\lambda_2\lambda_1} - \tilde{G}^{e+}_{\lambda_2\lambda_1})\right|^2 \Big\}$$

- Helicity amplitudes are a functions of W, Q^2 and pion angles in the Adler frame (θ, ϕ)
- The cross-section is used in several papers, but the lepton mass was ignored.

Angular calculation

- Resonant interactions has an intermediate resonance with definite quantum numbers right before pion production, while it is not the case for nonresonant-background.

$$\langle N\pi,\lambda_2|\epsilon^\alpha J_\alpha|N,\lambda_1\rangle = \langle N\pi,\lambda_2|R\lambda_R\rangle\langle R\lambda_R|\epsilon^\alpha J_\alpha|N\lambda_1\rangle$$

- FKR (RS) model provides us with the helicity amplitude for individual resonance with definite angular momentum not in terms of pion angles, but there is a relation between them.

$$F_{\mu\lambda}(\theta,\phi)=\sum_j F^j_{\mu\lambda}(2j+1)\,d^j_{\lambda\mu}(\theta)\,e^{i(\lambda-\mu)\phi}$$

$$G_{\mu\lambda}(\theta,\phi)=\sum_j G^j_{\mu\lambda}(2j+1)\,d^j_{\lambda\mu}(\theta)\,e^{i(\lambda-\mu)\phi}$$

$$\mu=\lambda_q-\lambda_2=-\lambda_2 \qquad \lambda=\lambda_k-\lambda_1$$

$$j=l+\tfrac{1}{2}$$

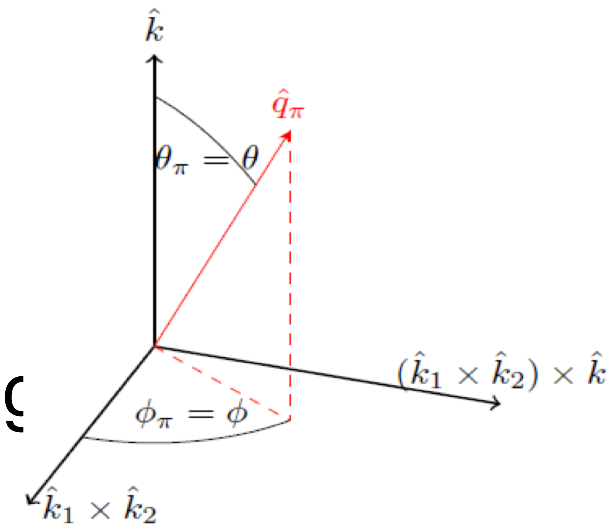
$$d^j_{\frac{1}{2}\frac{1}{2}}=(l+1)^{-1}\cos\frac{\theta}{2}(P'_{l+1}-P'_l)$$

$$d^j_{-\frac{1}{2}\frac{1}{2}}=(l+1)^{-1}\sin\frac{\theta}{2}(P'_{l+1}+P'_l)$$

From Jacob& Wick paper (1959)
Annals Phys. **7** (1959) 404

MK-model (2017)

M. Kabirnezhad,
Phys. Rev. D **97**, 013002



- MK model is a model for single pion production i.e. resonant and non-resonant interactions including **the interference effects**.
- Uses Rein-Sehgal model with GS form-factors to describe resonant interaction (17 resonances) up to W=2 GeV.
- Lepton mass is included.
- non-resonant background is defined by a set of diagrams determined by HNV model.

E. Hernandez, J. Nieves and M. Valverde, Phys. Rev. D **76** (2007) 033005

Output of the MK-model

$$d\,\sigma/dW\,dQ^2 d\Omega_\pi$$

The MK-model in NEUT

(Nucleon target)

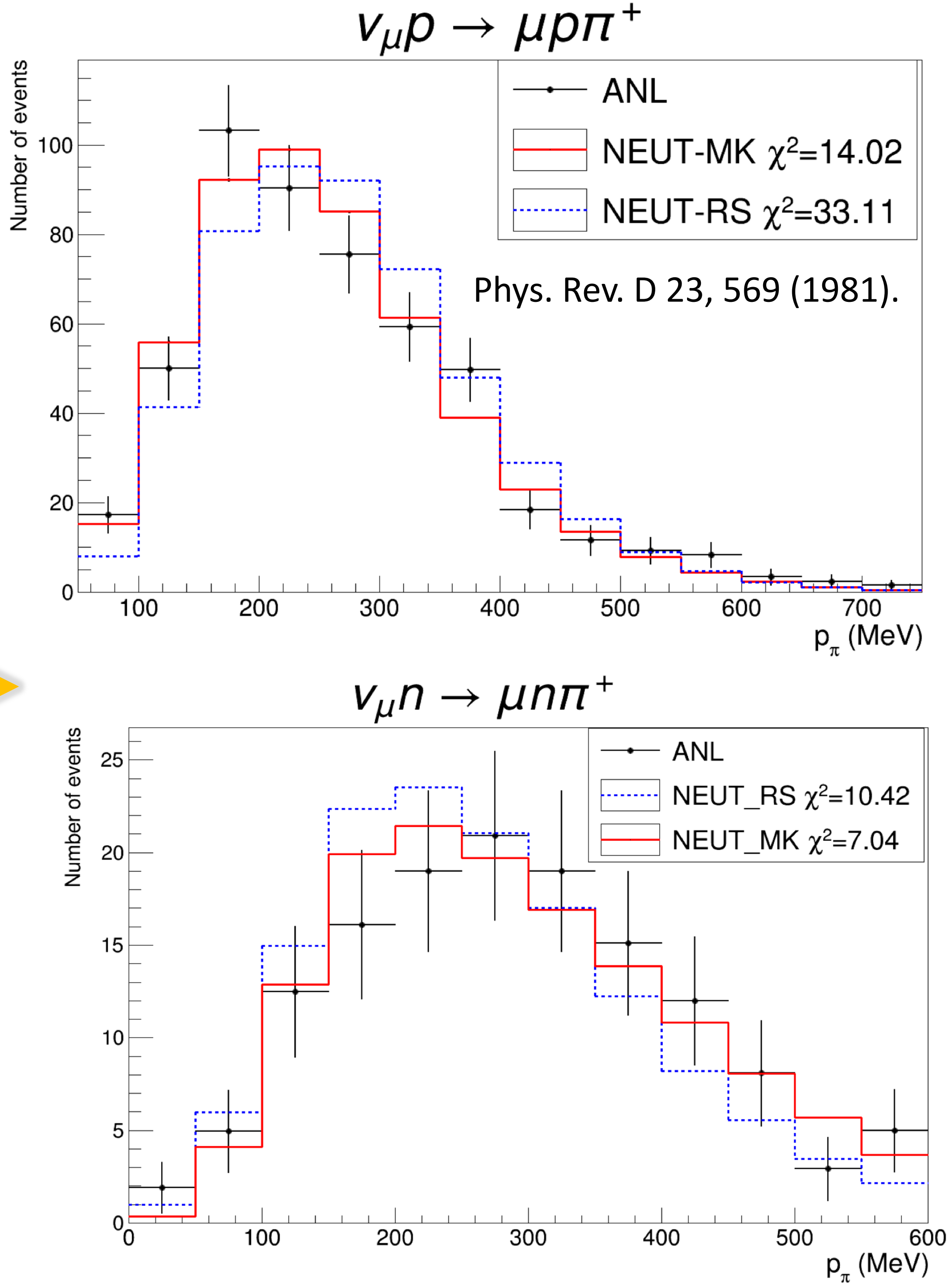
Thanks C. Wret

Data comparison can be done

- In any frame
- In any physical region
- With any target
- With any selection
- Wide energy range

pion momentum

lab frame



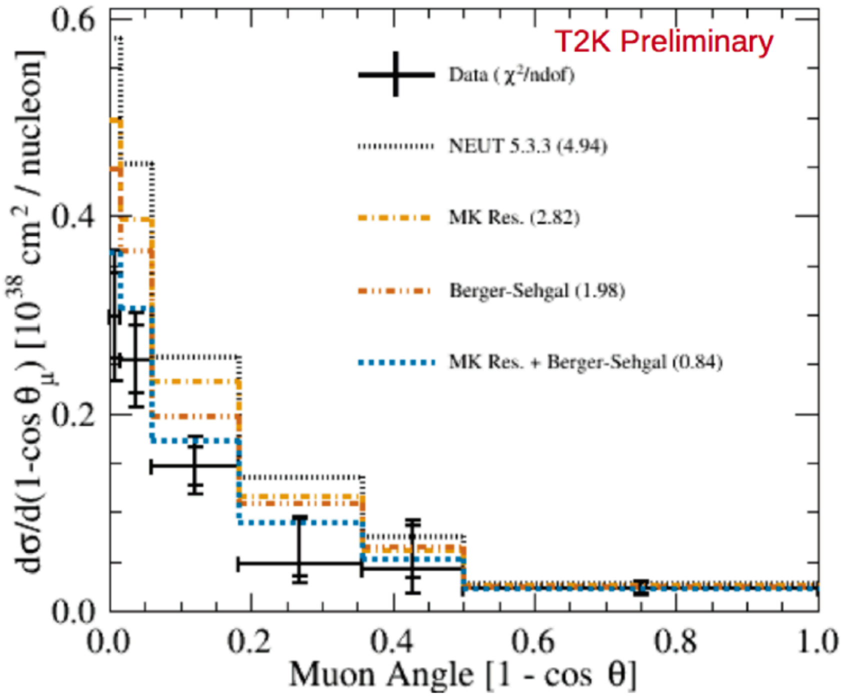
Data comparison with

NUISANCE

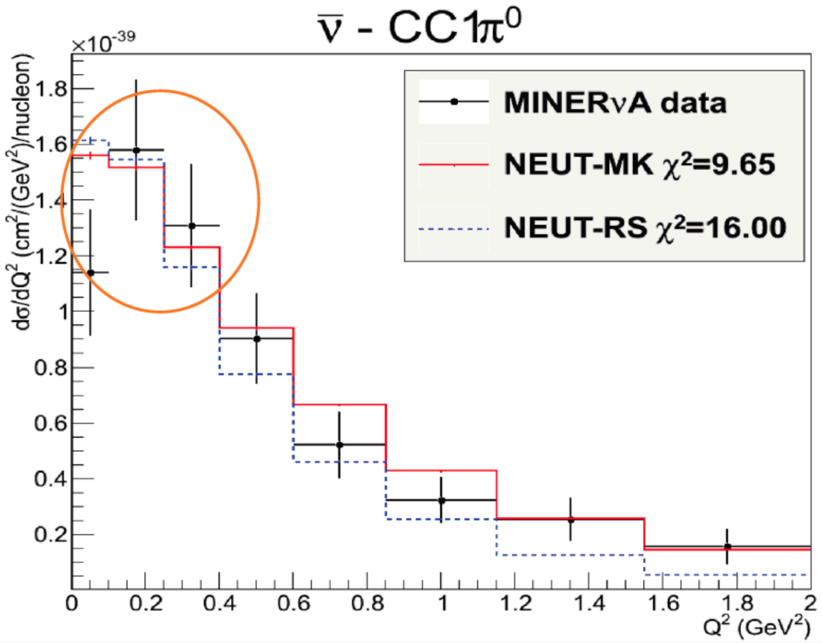
P. Stowell et al., JINST 12 (2017) no.01, P01016

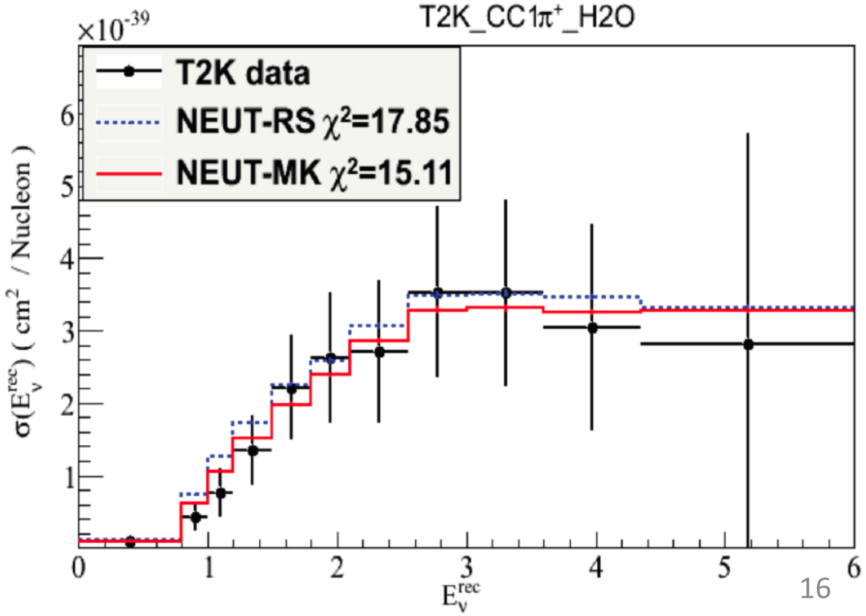
The MK-model in NEUT
(Nuclear target)

- NEUT comparisons with nuclear data shows improvement with MK-model but it is <u>not perfect</u> sometimes!
- NEUT prediction with nuclear target is pion production + nuclear model + FSI.



This is probably due to nuclear effects





Verifying the model is difficult with limited neutrino data sets!

- In principle, there are many adjustable parameters in MK model that use other model's fits. Like HNV model and GiBUU that use Lalakulich fit.
- Existing neutrino data on "free" nucleon is scarce and it is doubtful that it will be improved. ☹

A practical solution is to split the model

1. Vector part (electron scattering)
2. Axial part (pion scattering)

Phys. Rev. D 92, 074024 (2015)

Motivation

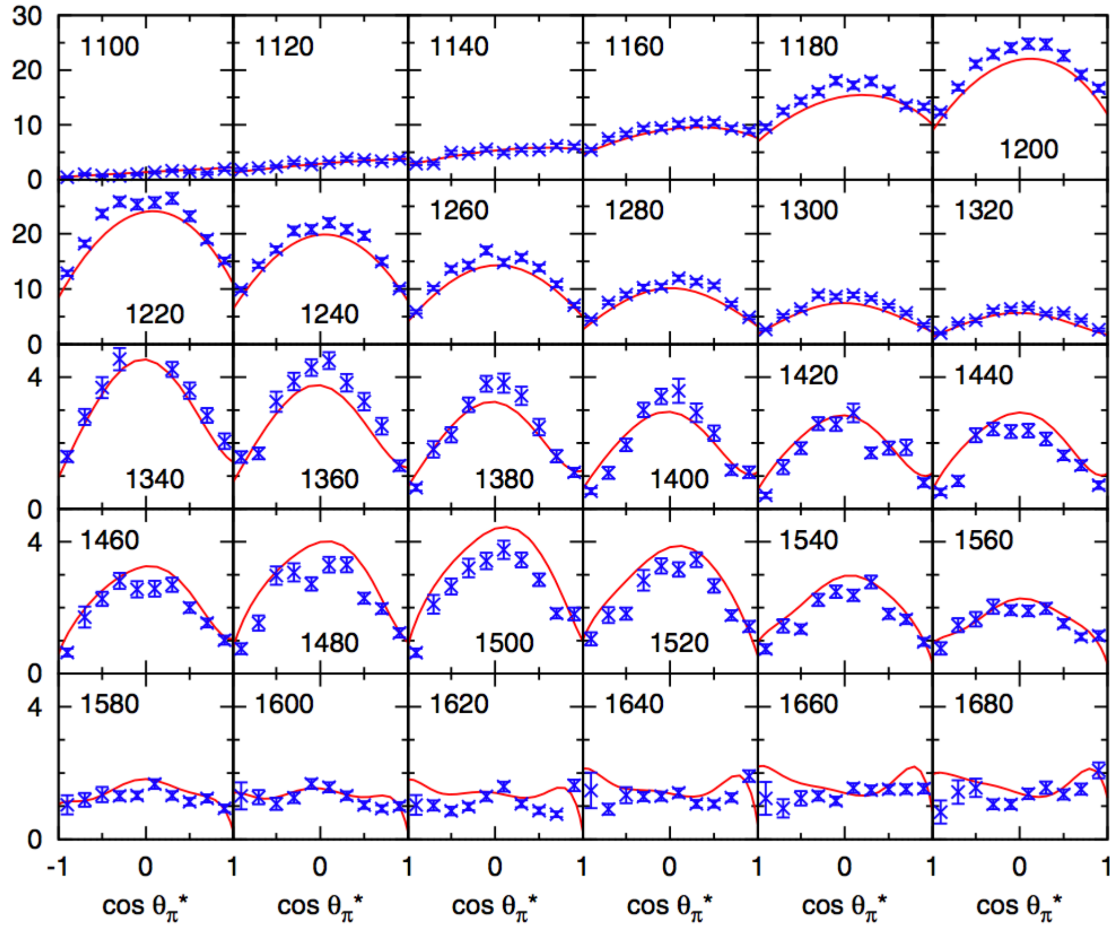
Dynamical coupled-channels (DCC) model

- Fully combined analysis of $\gamma N, \pi N$ ➔ $\pi N, \eta N, K\Lambda, K\Sigma$ data

- More than 440 parameters are determined to fit the obtained vector form factors.

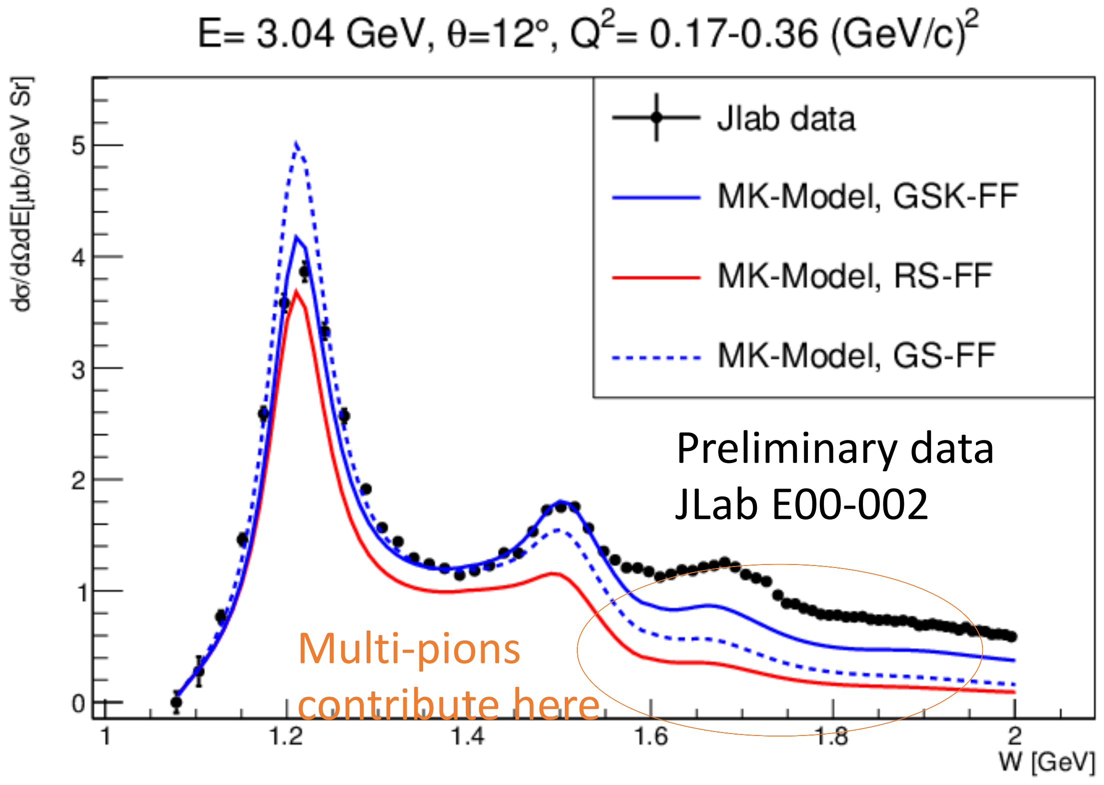
- All the other (406) parameters such as resonance parameters (masses & decay widths) and relative phases between resonant and nonresonant amplitudes have been fitted to data.

- Systematic error is not estimated. There is no freedom in parameters!

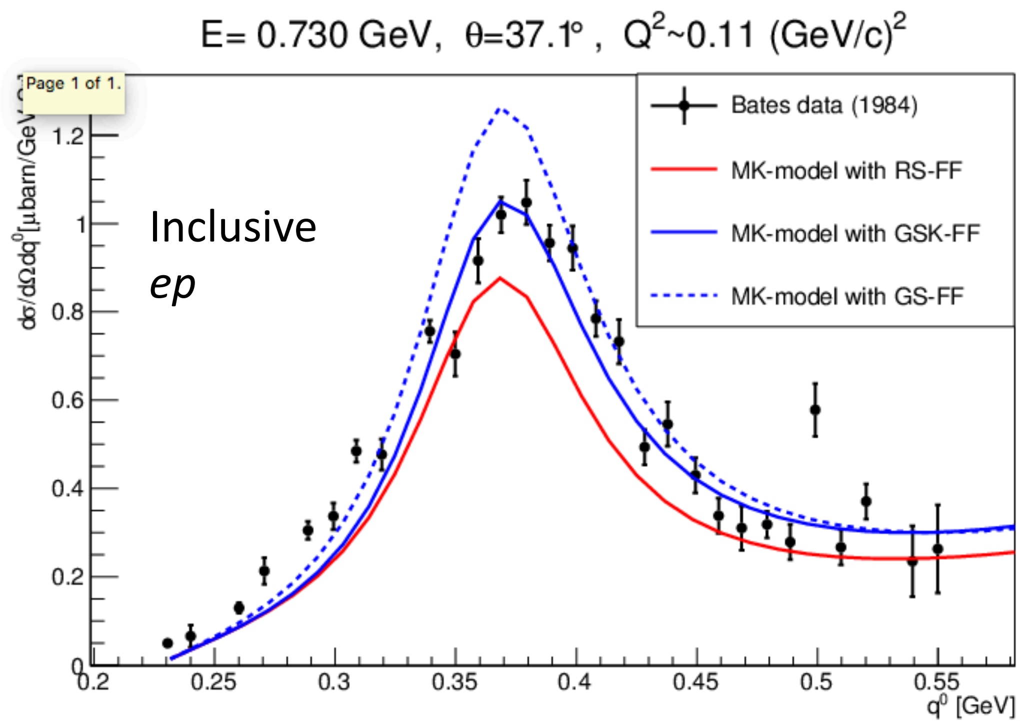


MK-model (Vector part)

Inclusive electron data



More data comparison in backup



- Among 3 vector form-factors (RS, GS, GSK), GSK form-factor (the actual solutions in GS paper) has the best agreement with electron scattering data.

Graczyk-Sobczyk form-factor

They equivalent the helicity amplitudes of RS model with Rarita-Schwinger formalism and extract a new form-factor for the RS model.

General definition of the helicity amplitudes

$$f_{+3}^{\Delta,V} \equiv (2\pi)^3 \sqrt{\frac{E_{p,res}}{M}} \left\langle \Delta, p'_{res}, s' = \frac{3}{2} \middle| \mathcal{J}_{+}^{V} \middle| N, p_{res}, s = \frac{1}{2} \right\rangle ,$$

$$f_{+1}^{\Delta,V} \equiv (2\pi)^3 \sqrt{\frac{E_{p,res}}{M}} \left\langle \Delta, p'_{res}, s' = \frac{1}{2} \middle| \mathcal{J}_{+}^{V} \middle| N, p_{res}, s = -\frac{1}{2} \right\rangle ,$$

$$f_{+0}^{\Delta,V} \equiv (2\pi)^3 \sqrt{\frac{E_{p,res}}{M}} \left\langle \Delta, p'_{res}, s' = \frac{1}{2} \middle| \mathcal{J}_{\underline{0}}^{V} \middle| N, p_{res}, s = \frac{1}{2} \right\rangle .$$

Graczyk-Sobczyk form-factor

They equivalent the helicity amplitudes of RS model with Rarita-Schwinger formalism and extract a new form-factor for the RS model.

Helicity amplitudes for Δ resonance in Rarita-Schwinger **and** **Rein-Sehgal mode**

$$f_{+3}^{\Delta,V} = -N_{q_{res}} \frac{q_{res}}{M+E_{q_{res}}} \left\{ \frac{C_4^V}{M^2} p'_\mu q^\mu + \frac{C_5^V}{M^2} p_\mu q^\mu + \frac{C_3^V}{M}(W+M) \right\},$$

$$f_{+1}^{\Delta,V} = \sqrt{\frac{1}{3}} N_{q_{res}} \frac{q_{res}}{M+E_{q_{res}}} \left\{ \frac{C_4^V}{M^2} p'_\mu q^\mu + \frac{C_5^V}{M^2} p_\mu q^\mu + \frac{C_3^V}{M}(W+M-2(M+E_{q_{res}})) \right\},$$

$$f_{+0}^{\Delta,V} = -\sqrt{\frac{2}{3}} N_{q_{res}} \frac{q_{res}}{M+E_{q_{res}}} \sqrt{Q^2} \left\{ \frac{C_4^V}{M^2} W + \frac{C_5^V}{M^2} \frac{M(M+W)}{W} + \frac{C_3^V}{M} \right\},$$

$$f_{+3}^{\Delta,V,RS} = -\sqrt{6}\sqrt{\frac{W}{M}} R,$$

$$f_{+1}^{\Delta,V,RS} = -\sqrt{2}\sqrt{\frac{W}{M}} R,$$

$$f_{+0}^{\Delta,V,RS} = 0,$$

$$R \equiv \sqrt{2} \frac{M}{W} \frac{q(M+W)}{Q^2+(W+M)^2} G_V^{RS}$$

Graczyk-Sobczyk form-factor

They equivalent the helicity amplitudes of RS model with Rarita-Schwinger formalism and extract a new form-factor for the RS model.

$$G_V^{RS}(Q^2,W) = \frac{1}{2\sqrt{3}}\left(1+\frac{Q^2}{(M+W)^2}\right)^{\frac{1}{2}}\left[C_4^V\frac{W^2-Q^2-M^2}{2M^2}+C_5^V\frac{W^2+Q^2-M^2}{2M^2}+\frac{C_3^V}{M}(W+M)\right], \rightarrow G_V^{f_3}(\mathrm{W},\mathrm{Q}^2)$$

$$G_V^{RS}(Q^2,W) = -\frac{1}{2\sqrt{3}}\left(1+\frac{Q^2}{(M+W)^2}\right)^{\frac{1}{2}}\left[C_4^V\frac{W^2-Q^2-M^2}{2M^2}+C_5^V\frac{W^2+Q^2-M^2}{2M^2}-C_3^V\frac{(M+W)M+Q^2}{MW}\right] \rightarrow G_V^{f_1}(\mathrm{W},\mathrm{Q}^2)$$

$$0 = C_4^V\frac{W}{M^2}+\frac{C_5^V}{M}\frac{(M+W)}{W}+\frac{C_3^V}{M}.$$

The partial solution of the above equations did not agree with data. The proposed GS form-factor is:

$$G_V^{RS,new}(W,Q^2) = \frac{1}{2}\sqrt{3\left(G_V^{f_3}(W,Q^2)\right)^2+\left(G_V^{f_1}(W,Q^2)\right)^2}$$

GS use the Lalakulich fit to the Maid data like Nieves 1p model

J-Lab data on hydrogen target

- Not all data set have reliable estimates for systematic uncertainty!

Index	Data Set	Beam Energy	Npoints	Q^2 Range	W Range	SID	PID	Status
1	e1e-smith	1.046	3903	0.16-0.32	1.10-1.34	1,2,3	$p\pi^0$,$n\pi^+$	Unpublished
2	e1e-markov	2.036	5040	0.45-0.95	1.1125-1.7875	1,2,3	$p\pi^0$	Review Complete
3	e1b-joo	1.645,2.445	10140	0.4-1.8	1.10-1.68	1,2,3	$p\pi^0$	PRL
4	e1b-joo	1.515	240	0.4,0.65	1.11-1.66	4	$p\pi^0$,$n\pi^+$	PRC
5	e1c-egiyan	1.515	2361	0.3-0.6	1.11-1.55	1,2,3	$n\pi^+$	PRC
6	e16-park	5.754	4781	1.72-4.16	1.15-1.67	1,2,3	$n\pi^+$	PRC
7	e1f-park	5.499	1350	1.8-4.0	1.62-2.01	1,2,3	$n\pi^+$	PRC (submitted)
8	e16-ungaro-1	5.754	2250	3.0-6.0	1.11-1.39	1,2,3	$p\pi^0$	PRL
9	e16-ungaro-2	5.754	4500	2.4-5.0	1.11-1.69	1,2,3	$p\pi^0$	Unpublished
10	e1c-carman-3str	2.567,4.056	1527	0.65-2.55	1.65-2.25	1,2,3	$K^+\Lambda^0$, $K^+\Sigma^0$	PRC
11	e1c-carman-4str	2.567,4.056	168	1.0	1.65-1.95	2,3,5,6	$K^+\Lambda^0$, $K^+\Sigma^0$	PRC

$$\frac{d\sigma_{em}}{d\Omega' dE' d\Omega^*_\pi} = \Gamma_{em}\Big\{\sigma_T + \varepsilon\sigma_L + \sqrt{2\varepsilon(1+\varepsilon)}\sigma_{LT}\cos\phi^*_\pi$$

$$+ h\sqrt{2\varepsilon(1-\varepsilon)}\sigma_{LT'}\sin\phi^*_\pi + \varepsilon\sigma_{TT}\cos 2\phi^*_\pi\Big\}$$

MK-model (Vector part)

- There are 13 resonances with vector current contribution for proton target.
- Used Galster functional form-factor for nonresonant bkg, but I do not use theirs fit. I have 4 adjustable parameters for nonresonant vector form-factor.
- I tried to fit MK-model with GS form-factor to Jlab data.

$$G_V^{RS,new}(W,Q^2) = \frac{1}{2}\left(1+\frac{Q^2}{(M+W)^2}\right)^{\frac{1}{2}}\left(1+\frac{Q^2}{4M^2}\right)^{-N}\left[3(G_3(W,Q^2))^2+(G_1(W,Q^2))^2\right]^{\frac{1}{2}}$$

Where N is 1 or 2 for resonances higher than \Delta

But minimizer did not converge!

- Tried alternative form-factors in the market but I failed.
- I decided to try new approaches (form-factors).

Updated Parameters

PDG 2019

Resonance	M_R	Γ_0	χ_E
$P_{33}(1232)$	1232	117	0.994
$P_{11}(1440)$	1440	350	0.65
$D_{13}(1520)$	1515	115	0.60
$S_{11}(1535)$	1530	150	0.45
$P_{33}(1600)$	1570	250	0.16
$S_{31}(1620)$	1610	130	0.3
$F_{15}(1680)$	1685	120	0.65
$D_{33}(1700)$	1710	300	0.15
$P_{11}(1710)$	1710	140	0.11
$P_{13}(1720)$	1720	250	0.11
$F_{35}(1905)$	1880	330	0.12
$P_{31}(1910)$	1900	300	0.22
$P_{33}(1920)$	1920	300	0.12
$F_{37}(1950)$	1930	285	0.40

Options for Res vector Form-factor

1. Use GS form-factor with the same functional form as in Lalakulich et al. however, this can be only valid for \Delta resonance.

$$C_3^V(Q^2) = \frac{C_3^V(0)}{\left(1+D\cdot\frac{Q^2}{M_v^2}\right)^2}\frac{1}{1+A\frac{Q^2}{M_v^2}}$$

$$C_4^V(Q^2) = \frac{C_4^V(0)}{\left(1+D\cdot\frac{Q^2}{M_v^2}\right)^2}\frac{1}{1+A\frac{Q^2}{M_v^2}}$$

$$C_5^V(Q^2) = \frac{C_5^V(0)}{\left(1+D\cdot\frac{Q^2}{M_v^2}\right)^2}\frac{1}{1+B\frac{Q^2}{M_v^2}}.$$

2. There is other parametrization proposed by J. Zmuda & K. Graczyk

$$C_3^V(Q^2) = \frac{C_3^V(0)}{1+AQ^2+BQ^4+CQ^6}\cdot(1+K_1Q^2)$$

$$C_4^V(Q^2) = -\frac{M_p}{W}C_3^V(Q^2)\cdot\frac{1+K_2Q^2}{1+K_1Q^2}$$

$$C_5^V(Q^2) = \frac{C_5^V(0)}{\left(1+D\frac{Q^2}{M_V^2}\right)^2}.$$

3. Use vector form-factor proposed in DCC model

$$F^V_{NN^*}(Q^2) \sim \sum_{n=0}^{\mathcal{N}} c_n^N (Q^2)^n$$

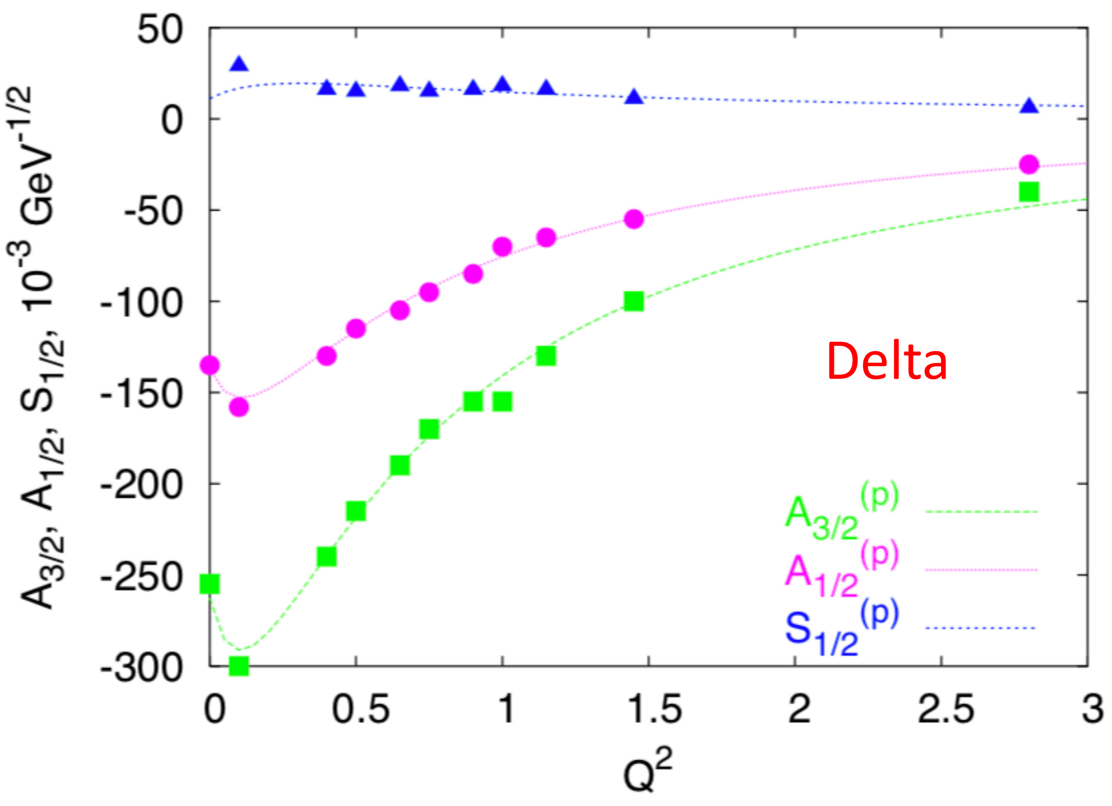
4. Z – expansion?

Olga Lalakulich et al (2006)
Resonance P_{33}(1232)

$$A^{P_{33}}_{3/2} = -\sqrt{N}\frac{q^z}{p'^0+M_R}\left[\frac{C_3^{(em)}}{m_N}(m_N+M_R) + \frac{C_4^{(em)}}{m_N^2}q\cdot p' + \frac{C_5^{(em)}}{m_N^2}q\cdot p\right] \qquad \text{(IV.17)}$$

$$A^{P_{33}}_{1/2} = \sqrt{\frac{N}{3}}\left[\frac{C_3^{(em)}}{m_N}\left(m_N+M_R-2\frac{m_N}{M_R}(p'^0+M_R)\right) + \frac{C_4^{(em)}}{m_N^2}q\cdot p' + \frac{C_5^{(em)}}{m_N^2}q\cdot p\right]\frac{q^z}{p'^0+M_R} \qquad \text{(IV.18)}$$

$$S^{P_{33}}_{1/2} = \sqrt{\frac{2N}{3}}\frac{q_z^2}{M_R(p'^0+M_R)}\left[\frac{C_3^{(em)}}{m_N}M_R + \frac{C_4^{(em)}}{m_N^2}W^2 + \frac{C_5^{(em)}}{m_N^2}m_N(m_N+q^0)\right] \qquad \text{(IV.19)}$$



The vector form factors are extracted from Maid data (helicity amplitudes).

$$C_3^{(p)} = \frac{2.13/D_V}{1+Q^2/4M_V^2},$$

$$C_4^{(p)} = \frac{-1.51/D_V}{1+Q^2/4M_V^2},$$

$$C_5^{(p)} = \frac{0.48/D_V}{1+Q^2/0.776M_V^2}.$$

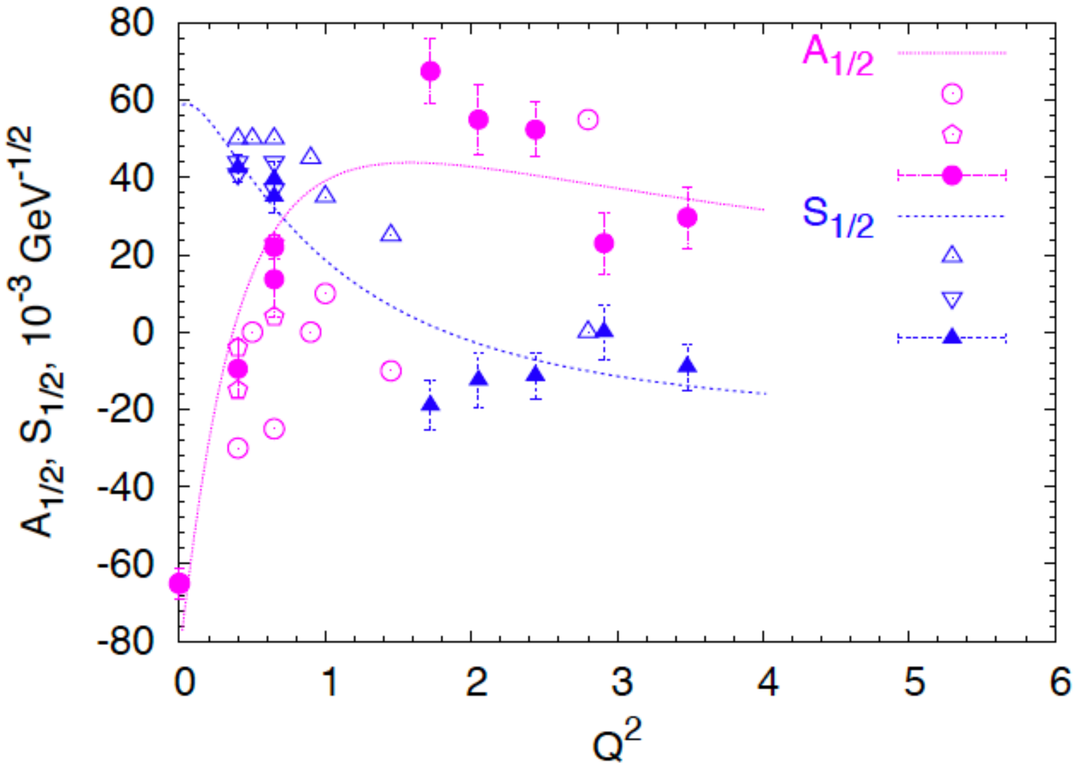
$$D_V = (1+Q^2/M_V^2)^2$$

Lalakulich *et al*
Resonance $P_{11}(1440)$

$$A_{1/2}^{P_{11}} = \sqrt{N}\frac{\sqrt{2}q^z}{p'^0 + M_R}\left[\frac{g_1^{(em)}}{\mu^2}Q^2 + \frac{g_2^{(em)}}{\mu}(M_R + m_N)\right]$$

Helicity amplitudes

$$S_{1/2}^{P_{11}} = \sqrt{N}\frac{q_z^2}{p'^0 + M_R}\left[\frac{g_1^{(em)}}{\mu^2}(M_R + m_N) - \frac{g_2^{(em)}}{\mu}\right]$$



$$g_1^{(p)} = \frac{2.3/D_V}{1 + Q^2/4.3M_V^2},$$
$$g_2^{(p)} = \frac{-0.76}{D_V}\left[1 - 2.8\ln\left(1 + \frac{Q^2}{1\ \mathrm{GeV}^2}\right)\right]$$

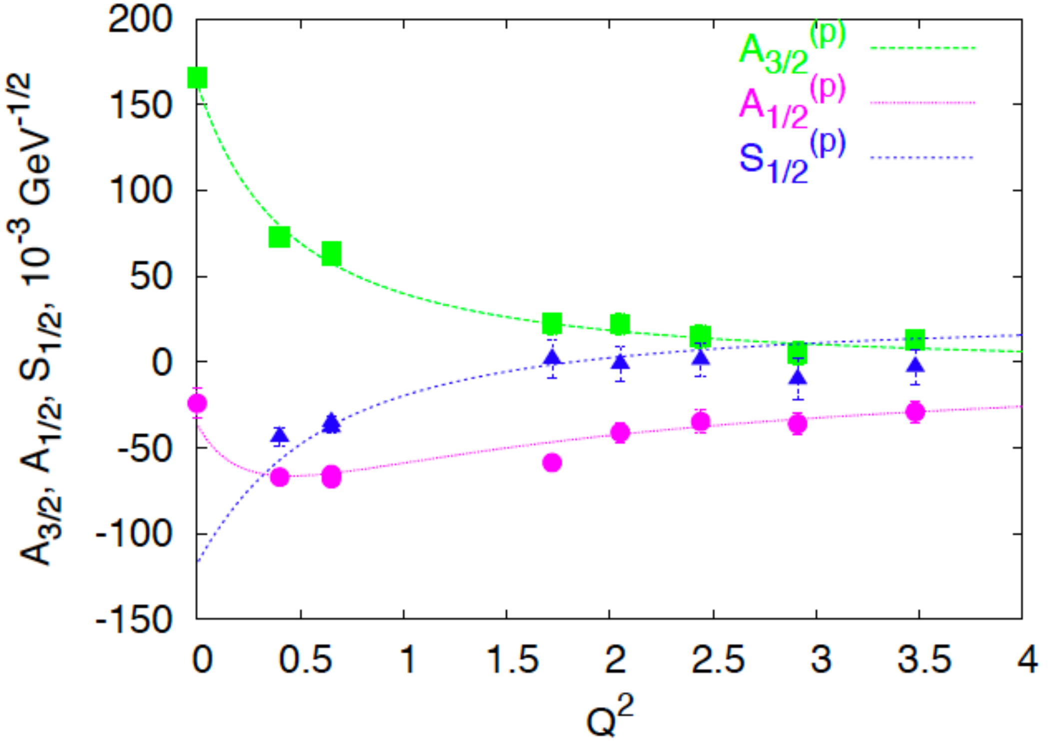
These numbers can be different for varies models

Lalakulich *et al*
Resonance $D_{13}(1520)$

$$A_{3/2}^{D_{13}} = \sqrt{N}\left[\frac{C_3^{(em)}}{m_N}(M_R - m_N) + \frac{C_4^{(em)}}{m_N^2} q\cdot p' + \frac{C_5^{(em)}}{m_N^2} q\cdot p\right]$$

$$A_{1/2}^{D_{13}} = \sqrt{\frac{N}{3}}\left[\frac{C_3^{(em)}}{m_N}\left(M_R - m_N - \frac{2m_N}{M_R}\frac{q_z^2}{p'^0 + M_R}\right) + \frac{C_4^{(em)}}{m_N^2} q\cdot p' + \frac{C_5^{(em)}}{m_N^2} q\cdot p\right] \tag{IV.6}$$

$$S_{1/2}^{D_{13}} = \sqrt{\frac{2N}{3}}\frac{q^z}{M_R}\left[\frac{C_3^{(em)}}{m_N}(-M_R) + \frac{C_4^{(em)}}{m_N^2}(Q^2 - 2m_N q^0 - m_N^2) - \frac{C_5^{(em)}}{m_N}(q^0 + m_N)\right] \tag{IV.7}$$

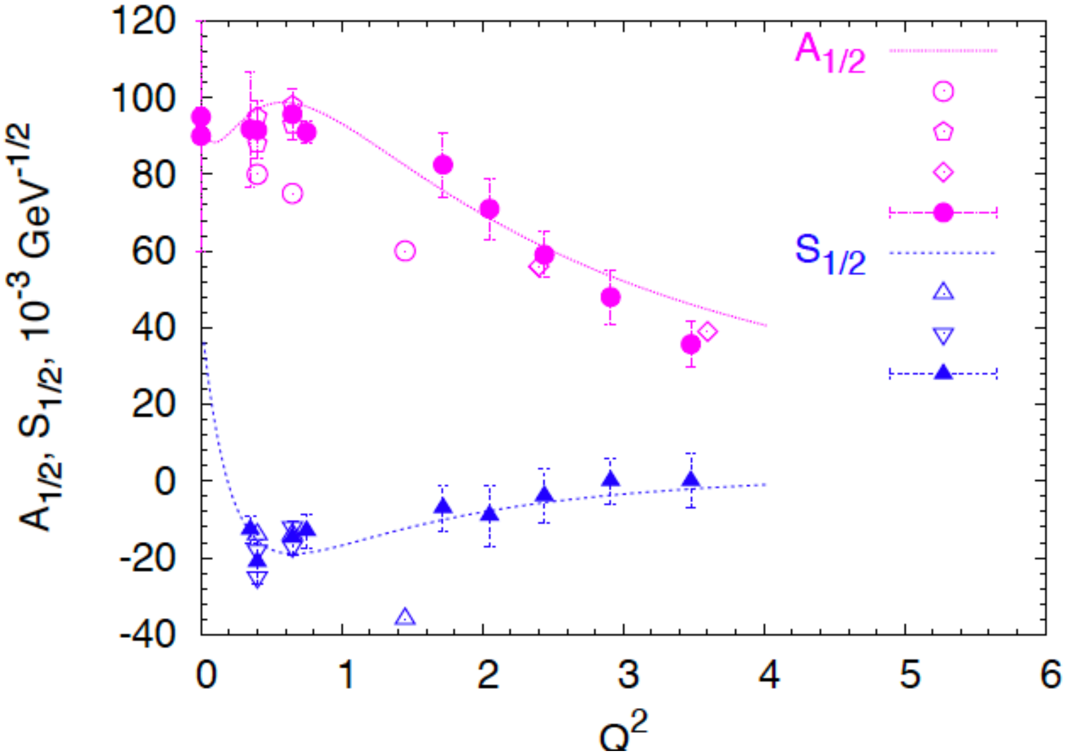


$$C_3^{(p)} = \frac{2.95/D_V}{1 + Q^2/8.9M_V^2},$$
$$C_4^{(p)} = \frac{-1.05/D_V}{1 + Q^2/8.9M_V^2},$$
$$C_5^{(p)} = \frac{-0.48}{D_V}.$$
$$D_V = (1 + Q^2/M_V^2)^2$$

Lalakulich *et al*
Resonance $S_{11}(1535)$

$$A_{1/2}^{S_{11}} = \sqrt{2N}\left[\frac{g_1^{(em)}}{\mu^2}Q^2 + \frac{g_2^{(em)}}{\mu}(M_R - m_N)\right]$$

$$S_{1/2}^{S_{11}} = \sqrt{N}q_z\left[-\frac{g_1^{(em)}}{\mu^2}(M_R - m_N) + \frac{g_2^{(em)}}{\mu}\right]$$



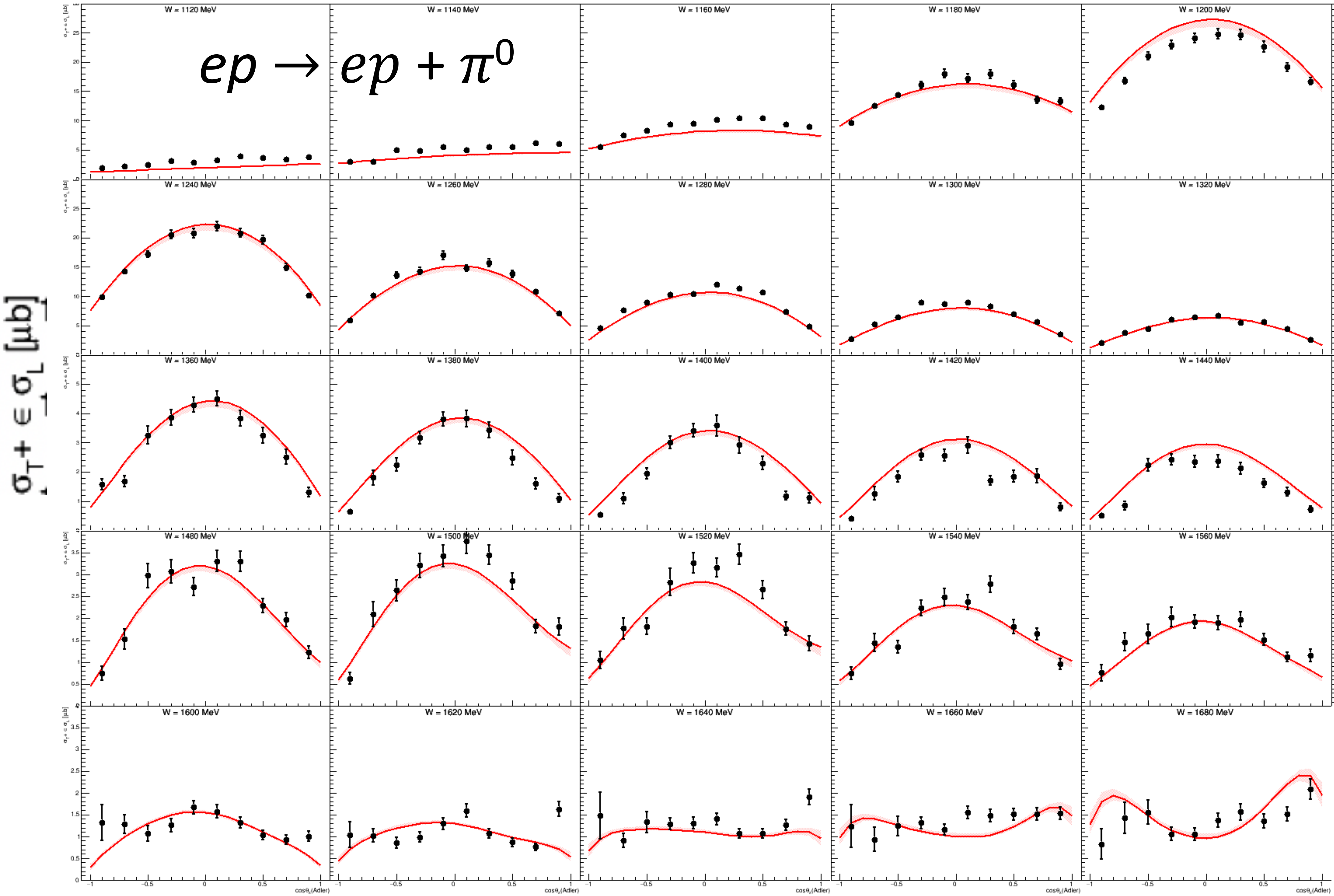
$$S_{11}(1535):$$

$$g_1^{(p)} = \frac{2.0/D_V}{1 + Q^2/1.2M_V^2}\left[1 + 7.2\ln\left(1 + \frac{Q^2}{1\ \mathrm{GeV}^2}\right)\right]$$

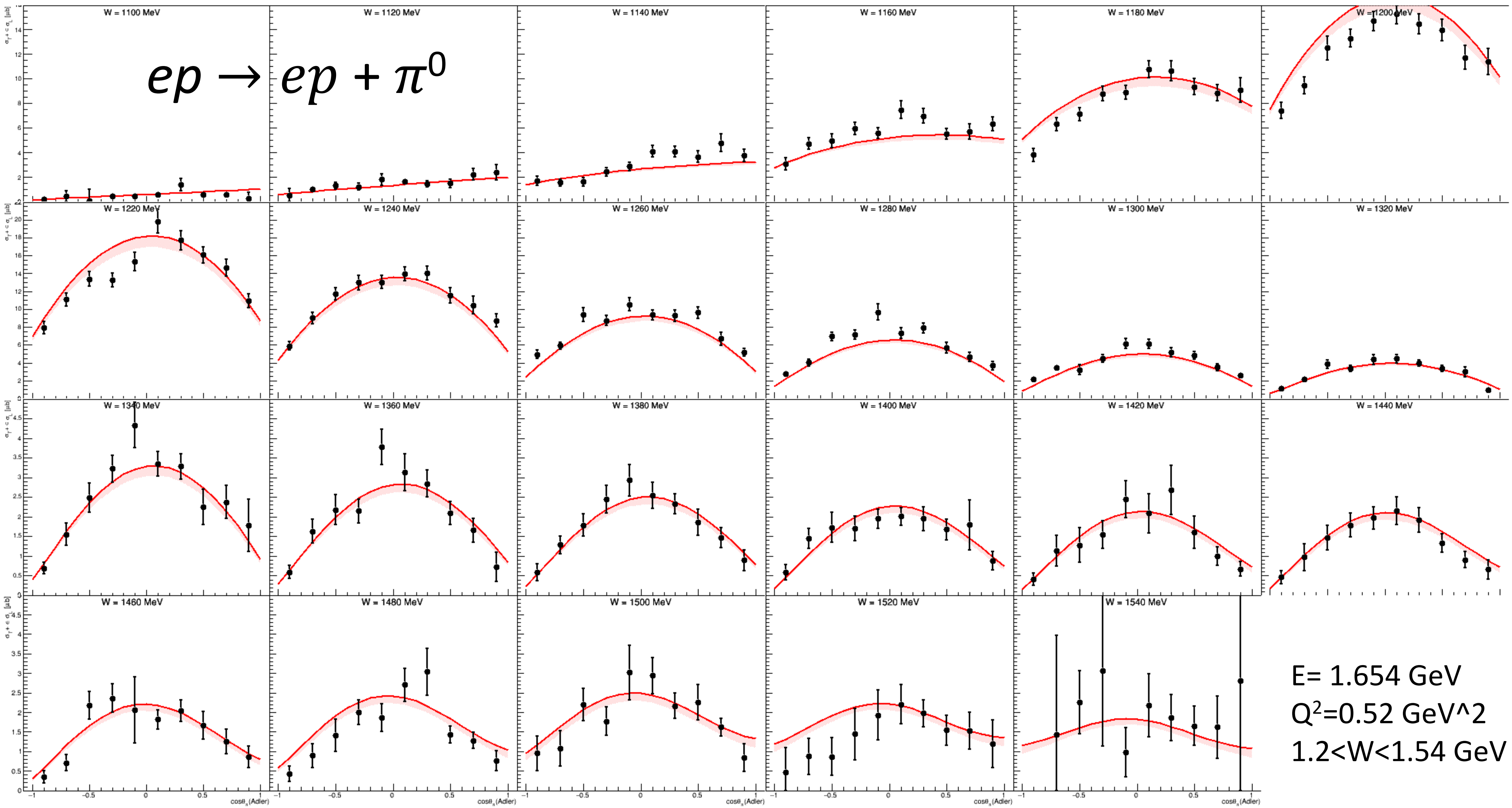
$$g_2^{(p)} = \frac{0.84}{D_V}\left[1 + 0.11\ln\left(1 + \frac{Q^2}{1\ \mathrm{GeV}^2}\right)\right].$$

MK-model (Update)

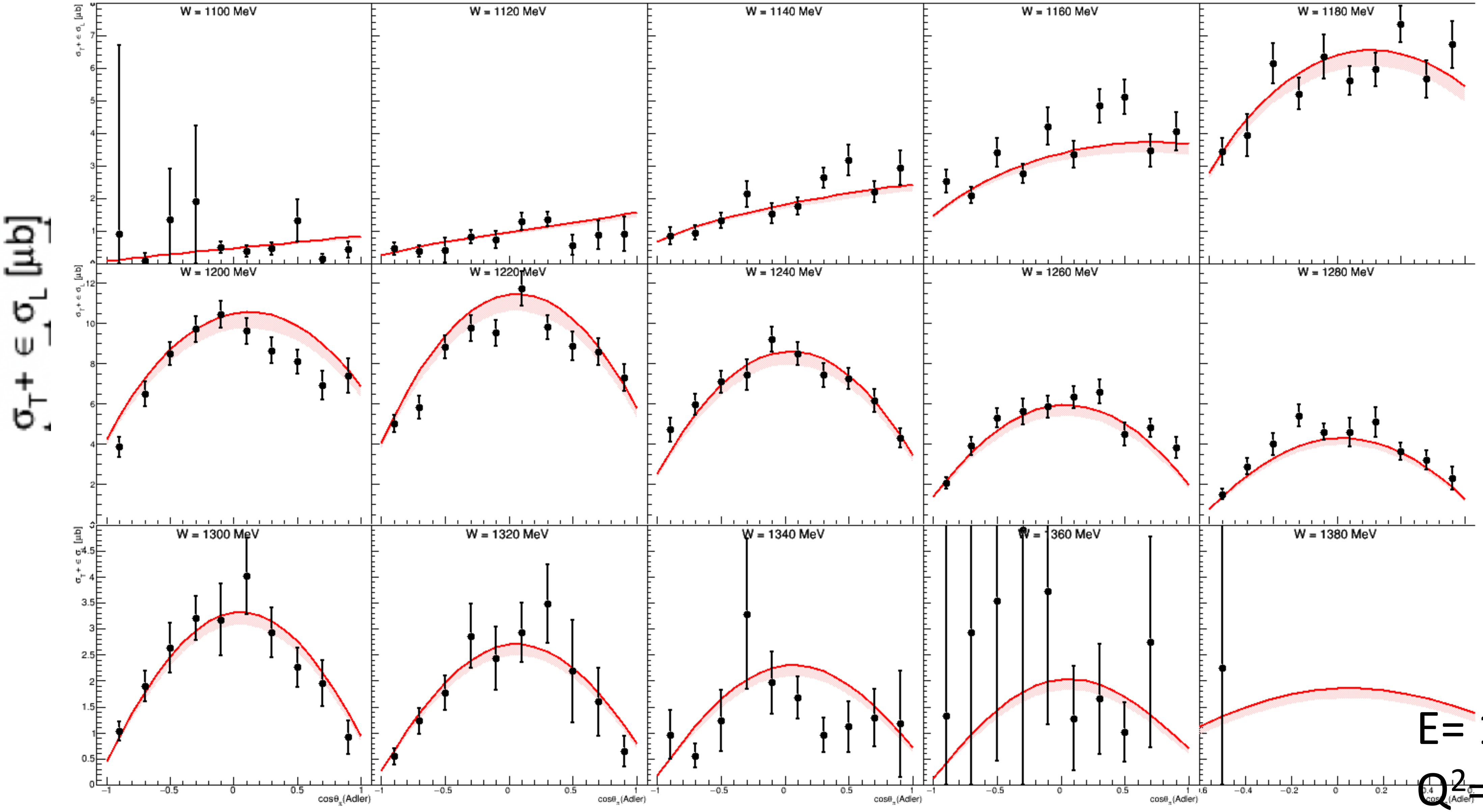
- The helicity amplitudes for Δ, $P_{11}(1440)$, $D_{13}(1520)$ & $S_{11}(1535)$ resonances are substituted by the helicity amplitudes in Lalakulich et al. with adjustable parameters. These resonances cover the first and the second resonance regions.
- The nonresonant helicity amplitudes with Galster form-factor have five adjustable parameters.
- There is an adjustable phase between resonances and nonresonant helicity amplitudes.
- The helicity amplitudes for the rest of resonances (third resonance region) is the RS helicity amplitudes and dipole form-factor with an adjustable coefficient for each resonance’s form-factor.
- The J-lab data for $ep \rightarrow ep + \pi^0$ and $ep \rightarrow en + \pi^+$ channels with 1.1<W<1.68 GeV, and different Q^2 are used to fit all the free parameters in the vector part.
- Estimate $1\boldsymbol{\sigma}$ error for my fit.



$ep \rightarrow ep + \pi^0$
$\sigma_T + \epsilon\, \sigma_L$ [μb]
W = 1120 MeV
W = 1140 MeV
W = 1160 MeV
W = 1180 MeV
W = 1200 MeV
W = 1220 MeV
W = 1240 MeV
W = 1260 MeV
W = 1280 MeV
W = 1300 MeV
W = 1320 MeV
W = 1340 MeV
W = 1360 MeV
W = 1380 MeV
W = 1400 MeV
W = 1420 MeV
W = 1440 MeV
W = 1460 MeV
W = 1480 MeV
W = 1500 MeV
W = 1520 MeV
W = 1540 MeV
W = 1560 MeV
W = 1580 MeV
W = 1600 MeV
W = 1620 MeV
W = 1640 MeV
W = 1660 MeV
W = 1680 MeV
E= 1.654 GeV
Q^2=0.4 GeV^2
1.2<W<1.68 GeV


ep → ep + π0
W = 1100 MeV
W = 1120 MeV
W = 1140 MeV
W = 1160 MeV
W = 1180 MeV
W = 1200 MeV
W = 1220 MeV
W = 1240 MeV
W = 1260 MeV
W = 1280 MeV
W = 1300 MeV
W = 1320 MeV
W = 1340 MeV
W = 1360 MeV
W = 1380 MeV
W = 1400 MeV
W = 1420 MeV
W = 1440 MeV
W = 1460 MeV
W = 1480 MeV
W = 1500 MeV
W = 1520 MeV
W = 1540 MeV
cosθπ (Adler)
E= 1.654 GeV
Q2=0.52 GeV^2
1.2<W<1.54 GeV

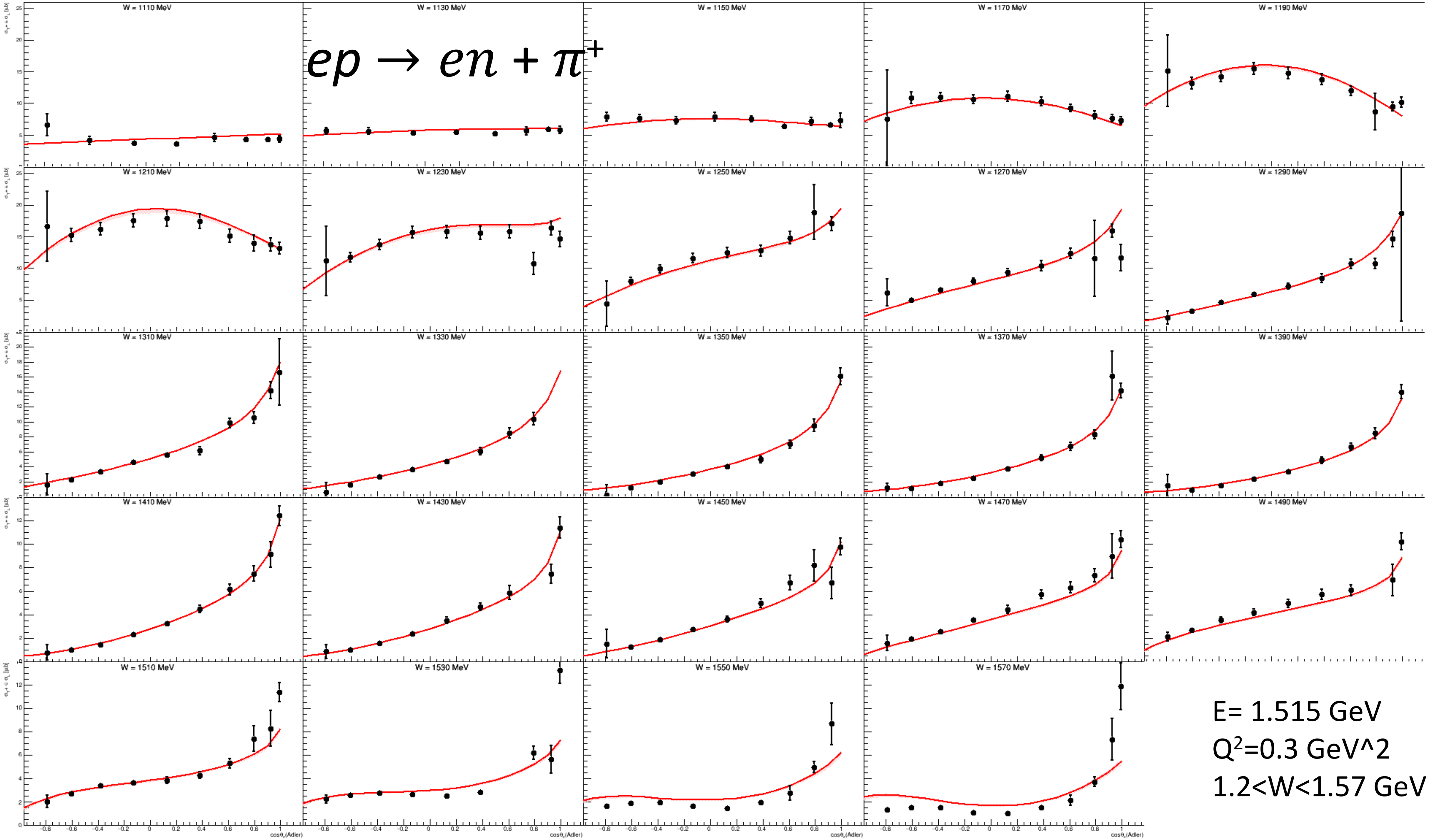
$ep \rightarrow ep + \pi^0$



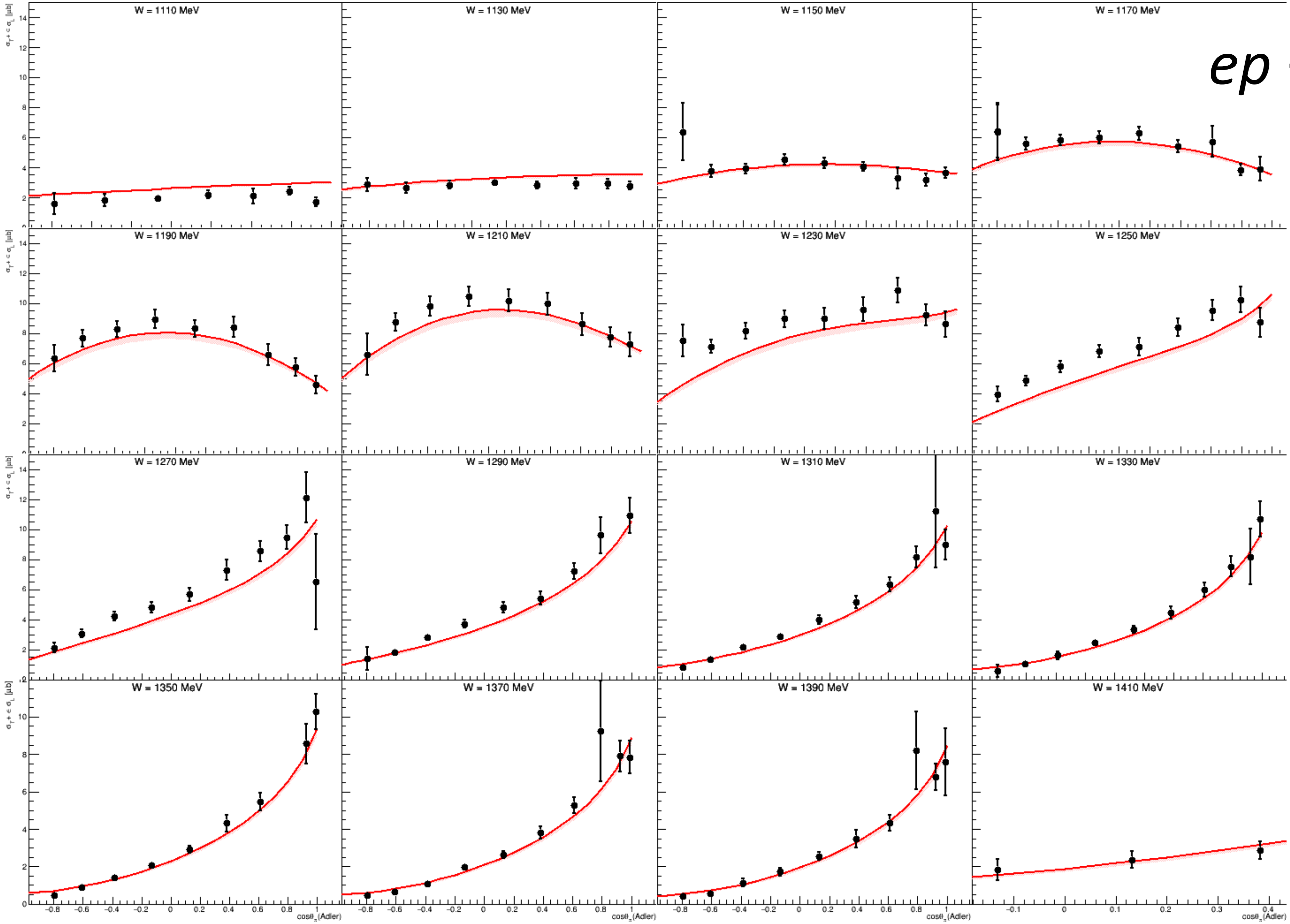
E= 1.654 GeV

Q^2=0.9 GeV^2

1.2<W<1.46 GeV

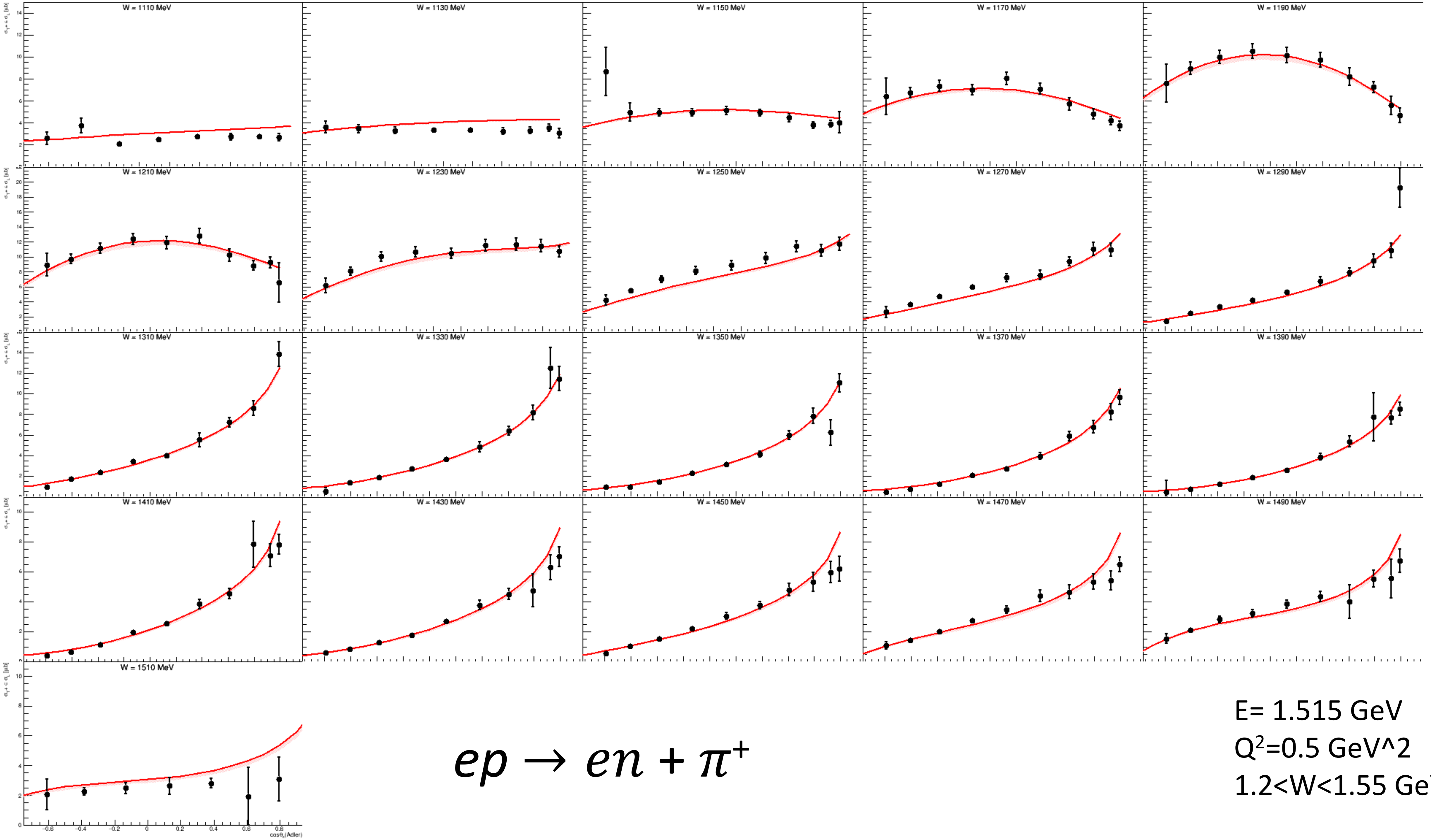


$ep \rightarrow en + \pi^+$
W = 1110 MeV
W = 1130 MeV
W = 1150 MeV
W = 1170 MeV
W = 1190 MeV
W = 1210 MeV
W = 1230 MeV
W = 1250 MeV
W = 1270 MeV
W = 1290 MeV
W = 1310 MeV
W = 1330 MeV
W = 1350 MeV
W = 1370 MeV
W = 1390 MeV
W = 1410 MeV
W = 1430 MeV
W = 1450 MeV
W = 1470 MeV
W = 1490 MeV
W = 1510 MeV
W = 1530 MeV
W = 1550 MeV
W = 1570 MeV
E= 1.515 GeV
Q^2=0.3 GeV^2
1.2<W<1.57 GeV



$ep \rightarrow en + \pi^{+}$

E= 1.515 GeV
Q^2=0.6 GeV^2
1.2<W<1.41 GeV



$$ep \rightarrow en + \pi^{+}$$

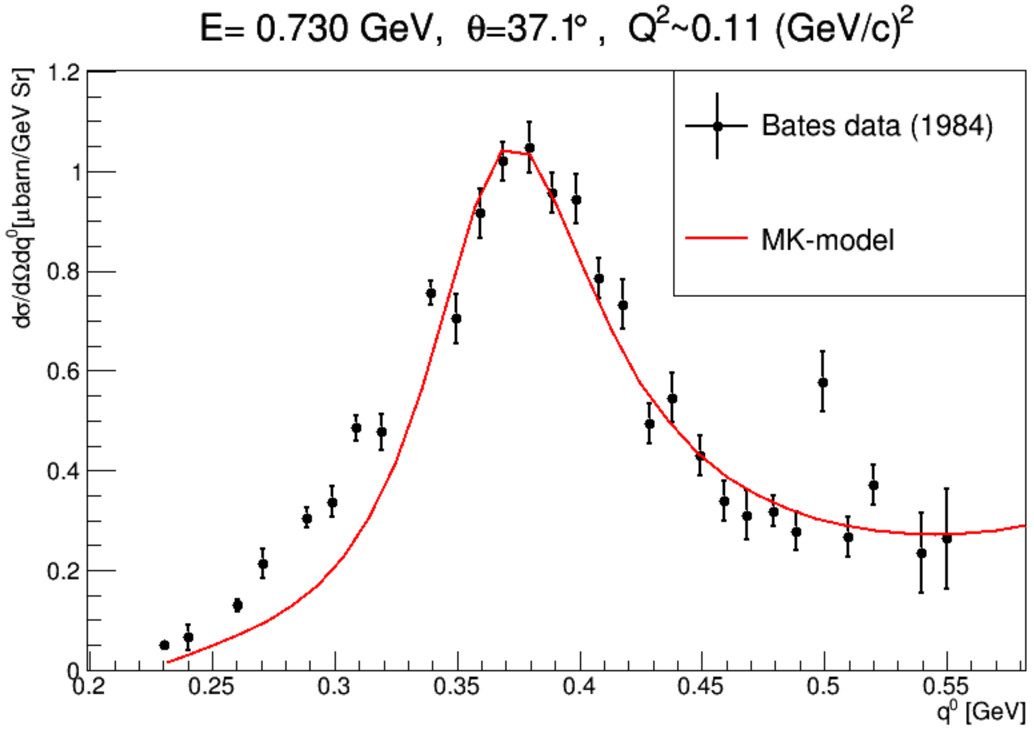
E= 1.515 GeV

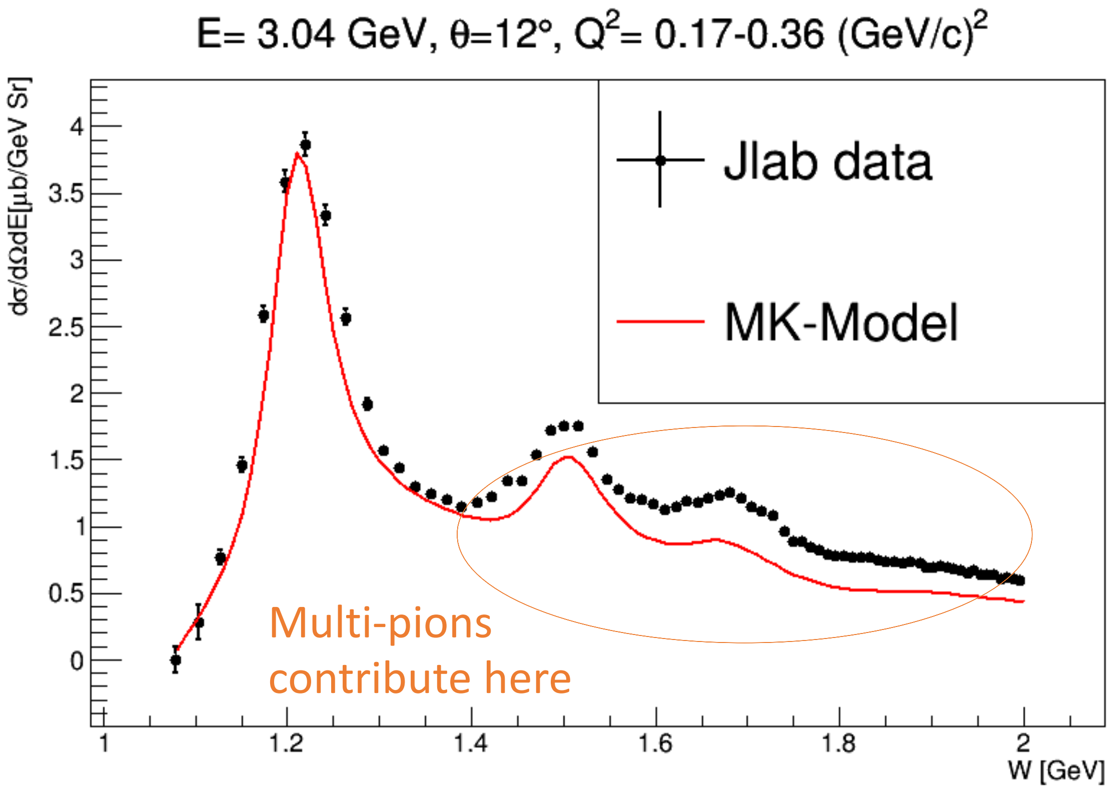
Q^2=0.5 GeV^2

1.2<W<1.55 GeV

Inclusive electron data

Validating this tune through comparisons to inclusive measurements.





Please find more inclusive and exclusive results in the backup!

Improving the axial part

- At low Q^2(<0.2 GeV), the axial current has the main contribution (due to the conservation of vector current).
- Data/MC disagree at this region.
- At this particular kinematics, the cross section is given by the divergence of the axial-current amplitude that is related to the πN amplitude through the PCAC relation.

Cross section at Q^2=0 and m_μ=0

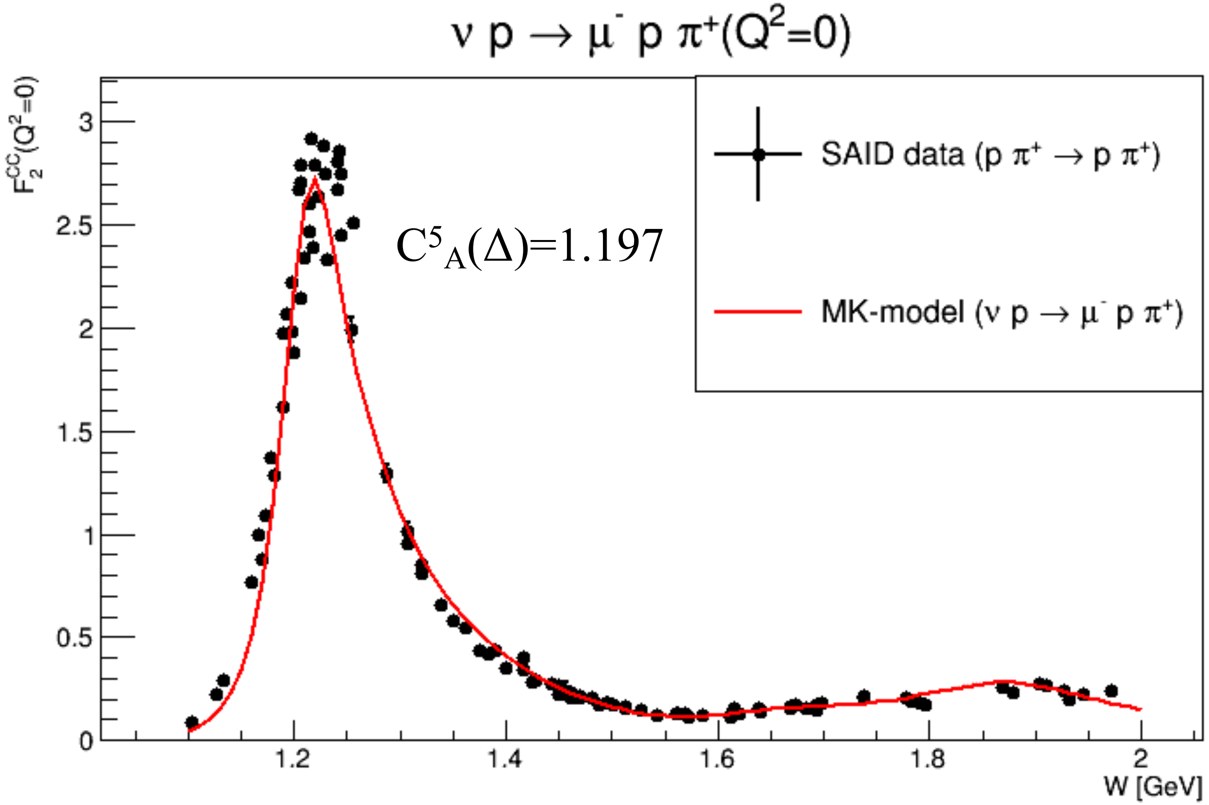
$$\frac{d\sigma^{CC}}{dE_l d\Omega_l} = \frac{G_F^2 V_{ud}^2}{2\pi^2} \frac{E'^2}{E - E'} F_2,$$

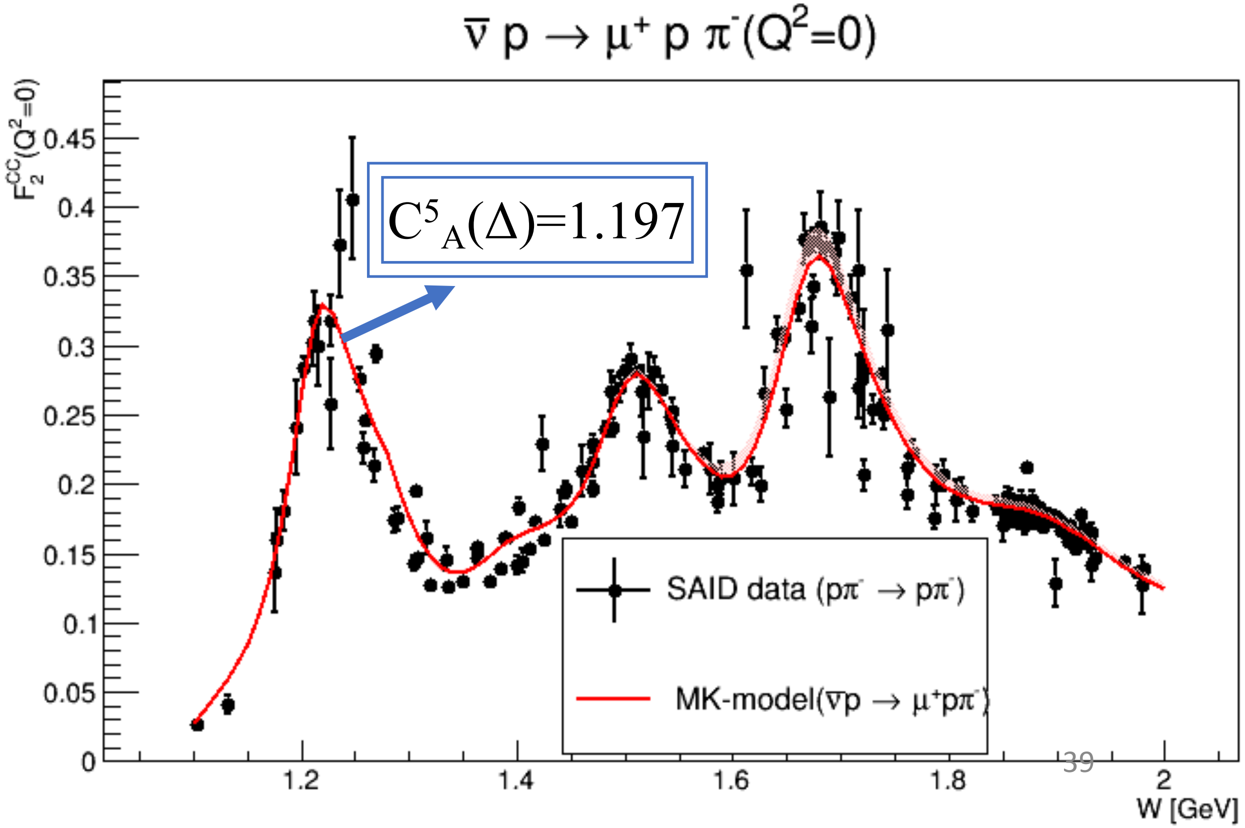
PCAC

$$F_2 = \frac{2f_\pi^2}{\pi} \sigma_{tot}(\pi + N)$$

Improving the axial part

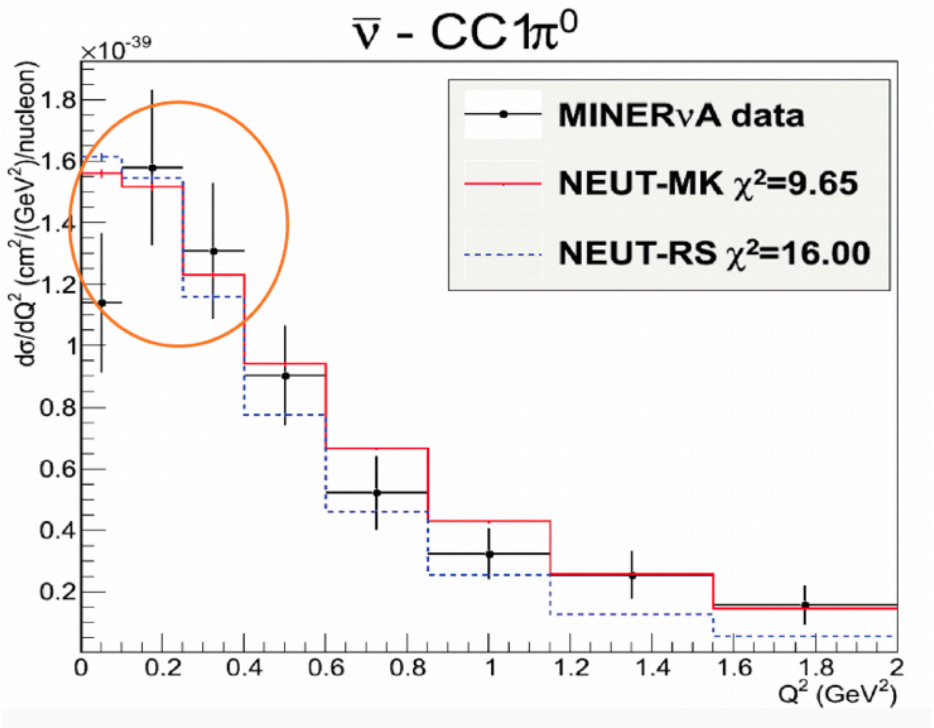
- fit (partially) MK axial form-factor (only $C^5_A(0)$) and the phase between resonant and nonresonant, using pion scattering (SAID) data on hydrogen at Q^2=0.
- This fit has been done for two channels (neutrino and anti-neutrino). It can be done for NC channel.
- Not able to fit my M_A.





Remarks!

Fitting the axial current at $Q^2=0$ reduced the axial contribution and will reduce the neutrino cross-section at low Q^2.



New fit will be checked in NEUT!

Summary

- MK model has its own vector form-factors for all resonances and nonresonan bkg. They are fitted to J-lab data points. Thanks to Phil Rodrigues!
- The vector phases between resonance and nonresonant bkg are fitted.
- Resonance parameters are updated (PDG 2019).
- The axial form-factors (normalization) and phases are fitted to the pion scattering data.
- Next step is to combine the vector and axial parts of the model and fit M_A to neutrino bubble chamber data (ANL) and update the NEUT code.
- Igor Kakorin is currently implementing the MK-model in GENIE.

Future plan

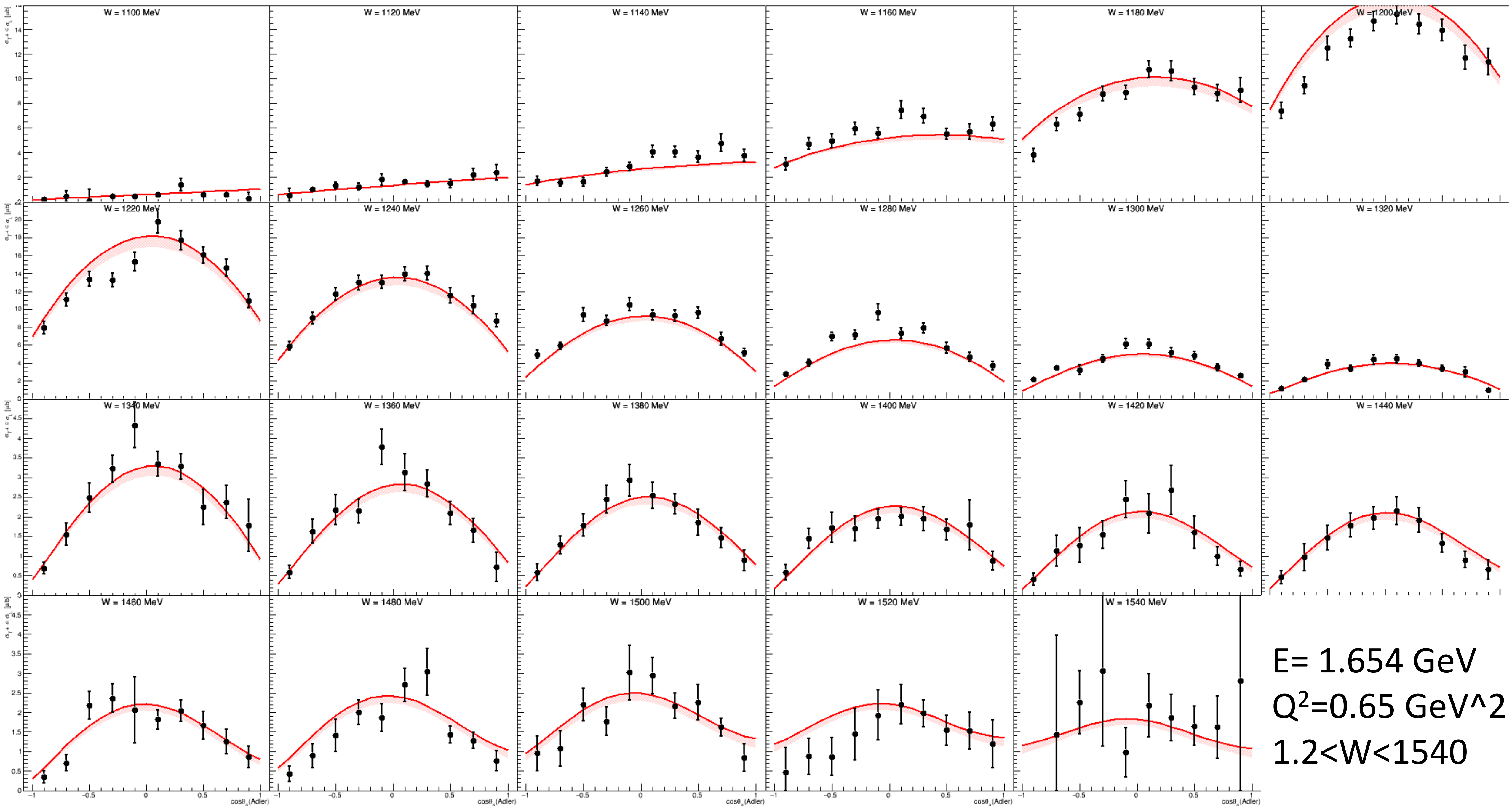
- Relativistic Mean Field theory (RMF) is calculated in a fully relativistic and quantum mechanical framework.
- I have had a two days meeting in Madrid to implement the MK model in the RMF model. I am going to collaborate with Raul Gonzalez-Jimenez who is the main developer of RMF in pion production.

$$J^{\mu} = \int d\mathbf{p}'_N \int \frac{d\mathbf{p}}{(2\pi)^{3/2}} \overline{\psi}_{S_f}(\mathbf{p}'_N, \mathbf{p}_N) \phi^*(\mathbf{k}'_\pi, \mathbf{k}_\pi) \mathcal{O}^{\mu}_{1\pi}(Q, P, K'_\pi, P'_N) \psi^{m_j}_{\kappa}(\mathbf{p}),$$

- It will be implemented in MEUT if everything goes well!

Many thanks to Fermilab for your continued support!

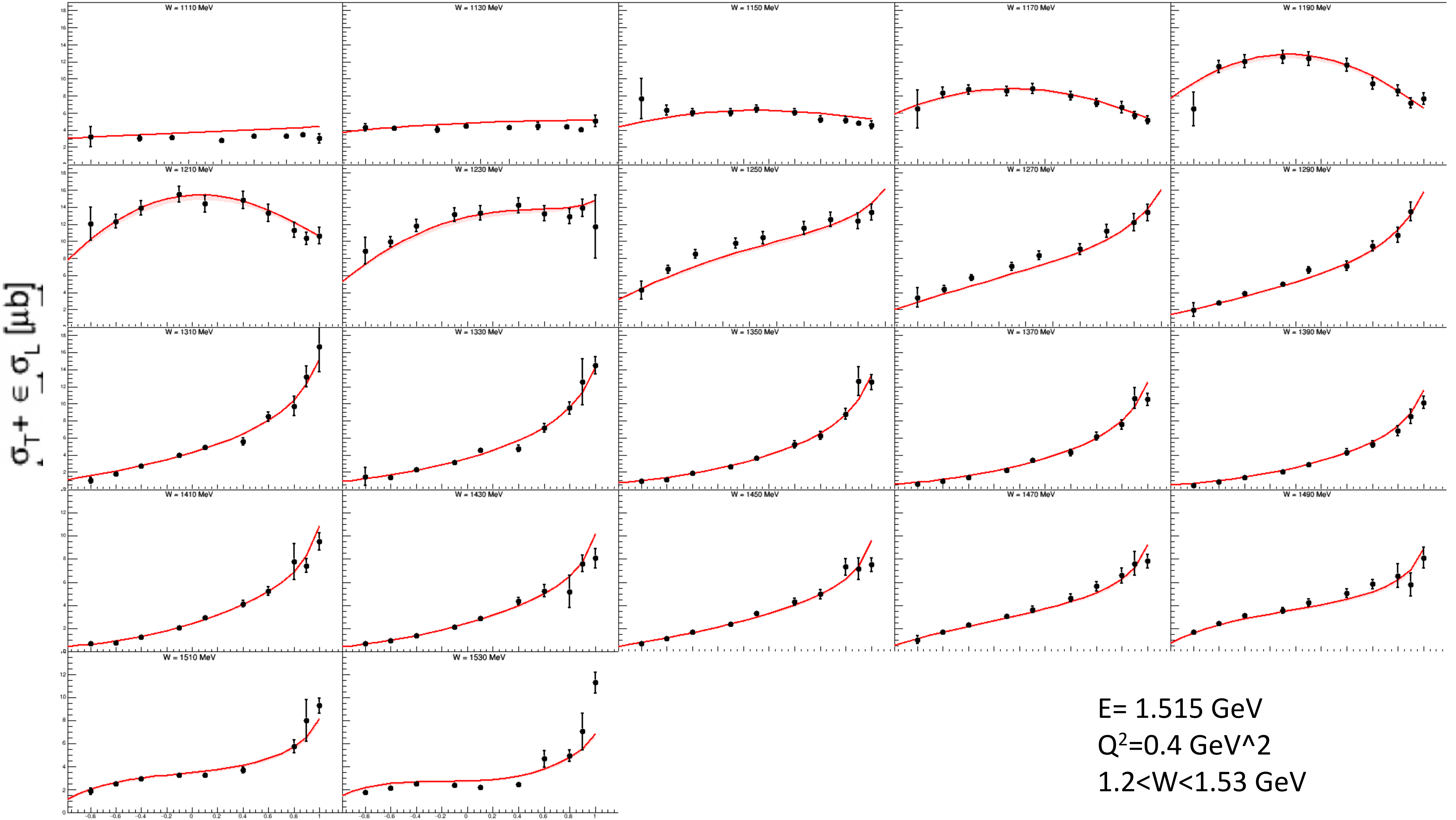
Backup



E= 1.654 GeV

Q^2=0.65 GeV^2

1.2<W<1540



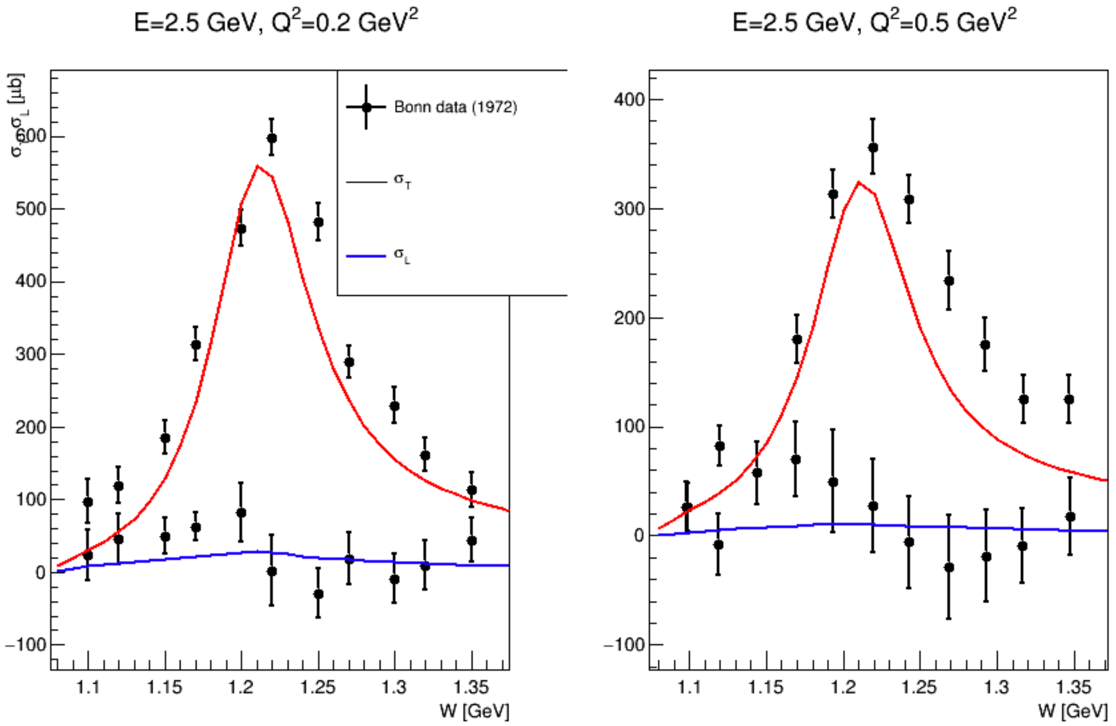
E= 1.515 GeV

Q^2=0.4 GeV^2

1.2<W<1.53 GeV

Inclusive electron data

Validating this tune through comparisons to inclusive measurements.



Graczyk-Sobczyk form-factor

Phys. Rev. D 77 (2008) 053001.

- They equivalent the RS model with Lalakiluch et al model (Rarita-Schwinger formalism)

$$G_V^{RS}(Q^2,W) = \frac{1}{2\sqrt{3}}\left(1+\frac{Q^2}{(M+W)^2}\right)^{\frac{1}{2}}\left[C_4^V\frac{W^2-Q^2-M^2}{2M^2}+C_5^V\frac{W^2+Q^2-M^2}{2M^2}+\frac{C_3^V}{M}(W+M)\right],$$

$$G_V^{RS}(Q^2,W) =-\frac{1}{2\sqrt{3}}\left(1+\frac{Q^2}{(M+W)^2}\right)^{\frac{1}{2}}\left[C_4^V\frac{W^2-Q^2-M^2}{2M^2}+C_5^V\frac{W^2+Q^2-M^2}{2M^2}-C_3^V\frac{(M+W)M+Q^2}{MW}\right]$$

$$0 = C_4^V\frac{W}{M^2}+\frac{C_5^V}{M}\frac{(M+W)}{W}+\frac{C_3^V}{M}.$$

A "partial" solution used by other models is:

$$C_5^V=0, \quad C_3^V=-\frac{W}{M}C_4^V$$

$$C_4^V(Q^2)=-4\sqrt{3}\left(\frac{M}{M+W}\right)^2\left(1+\frac{Q^2}{(M+W)^2}\right)^{-3/2}G_V^{RS}(Q^2)$$

it does not agree well with the existing electromagnetic data.

GS use the Lalakulich fit to e.m. data

$$C_3^V = 2.13\left(1+\frac{Q^2}{4M_V^2}\right)^{-1}\left(1+\frac{Q^2}{M_V^2}\right)^{-2},$$

$$C_4^V = -1.51\left(1+\frac{Q^2}{4M_V^2}\right)^{-1}\left(1+\frac{Q^2}{M_V^2}\right)^{-2},$$

$$C_5^V = 0.48\left(1+\frac{Q^2}{4M_V^2}\right)^{-1}\left(1+\frac{Q^2}{0.776M_V^2}\right)^{-2}$$

Is there a typo in C_5?

Graczyk-Sobczyk form-factor

- They equivalent the RS model with Lalakiluch et al model (Rarita-Schwinger formalism)

$$G_V^{RS}(Q^2,W) = \frac{1}{2\sqrt{3}}\left(1+\frac{Q^2}{(M+W)^2}\right)^{\frac{1}{2}}\left[C_4^V\frac{W^2-Q^2-M^2}{2M^2}+C_5^V\frac{W^2+Q^2-M^2}{2M^2}+\frac{C_3^V}{M}(W+M)\right],$$

$$G_V^{RS}(Q^2,W) = -\frac{1}{2\sqrt{3}}\left(1+\frac{Q^2}{(M+W)^2}\right)^{\frac{1}{2}}\left[C_4^V\frac{W^2-Q^2-M^2}{2M^2}+C_5^V\frac{W^2+Q^2-M^2}{2M^2}-C_3^V\frac{(M+W)M+Q^2}{MW}\right]$$

$$0 = C_4^V\frac{W}{M^2}+\frac{C_5^V}{M}\frac{(M+W)}{W}+\frac{C_3^V}{M}.$$

A "partial" solution used by other models is:

$$C_5^V = 0, \quad C_3^V = -\frac{W}{M}C_4^V$$

$$C_4^V(Q^2) = -4\sqrt{3}\left(\frac{M}{M+W}\right)^2\left(1+\frac{Q^2}{(M+W)^2}\right)^{-3/2}G_V^{RS}(Q^2)$$

it does not agree well with the existing electromagnetic data.

We should check the actual solution within MK-model

GSK form-factor

Cross-section definition in electron scattering

$$\frac{d\sigma_{em}}{d\Omega' dE' d\Omega_\pi^*} = \Gamma_{em}\left\{\sigma_T + \varepsilon\sigma_L + \sqrt{2\varepsilon(1+\varepsilon)}\sigma_{LT}\cos\phi_\pi^* + h\sqrt{2\varepsilon(1-\varepsilon)}\sigma_{LT'}\sin\phi_\pi^* + \varepsilon\sigma_{TT}\cos 2\phi_\pi^*\right\}$$

$$\Gamma \equiv \frac{\alpha}{2\pi^2}\frac{E'}{E}\frac{(W^2 - m_p^2)}{2m_p Q^2}\frac{1}{1-\epsilon}$$

$$\epsilon \equiv \left(1 + 2\frac{|\mathbf{q}|^2}{Q^2}\tan^2\frac{\theta_e}{2}\right)^{-1},$$

Γ is virtual photon flux factor

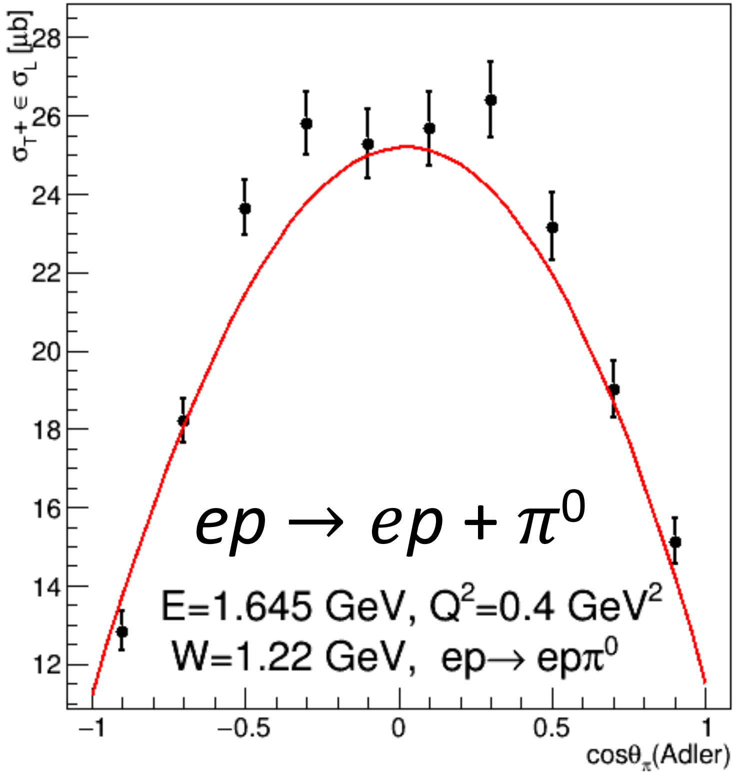
New Parameters:

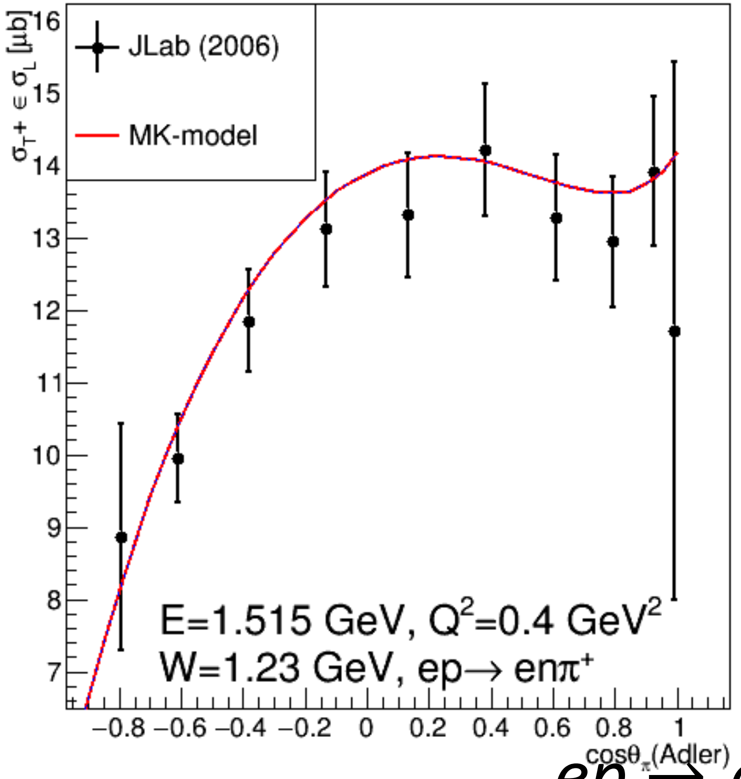
1. A coefficient to form-factor of individual resonances.

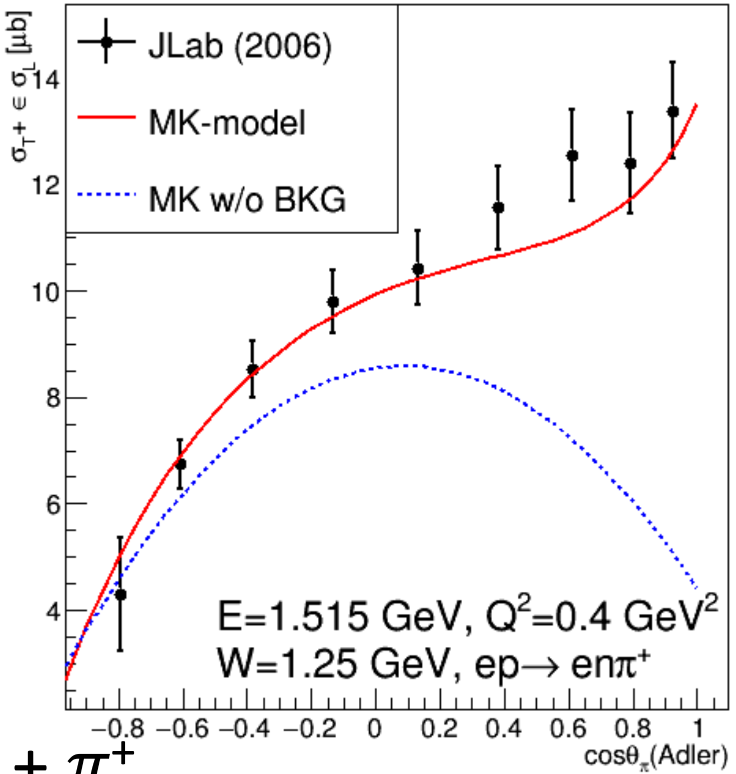
2. A phase between resonance and bkg amplitudes.

Mk model comparison with *ep* exclusive data
(pion polar angle)

Nonresonant Bkg has large contribution at forward bins

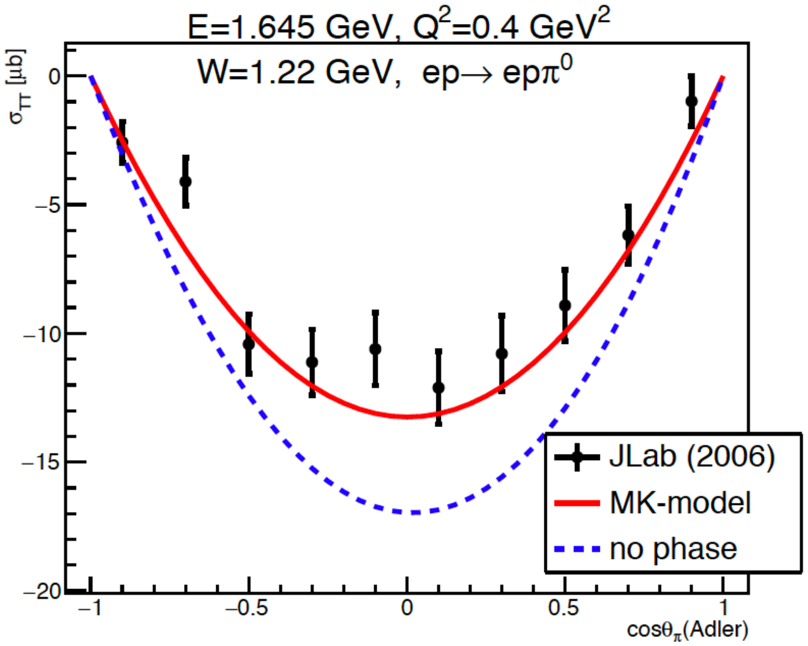


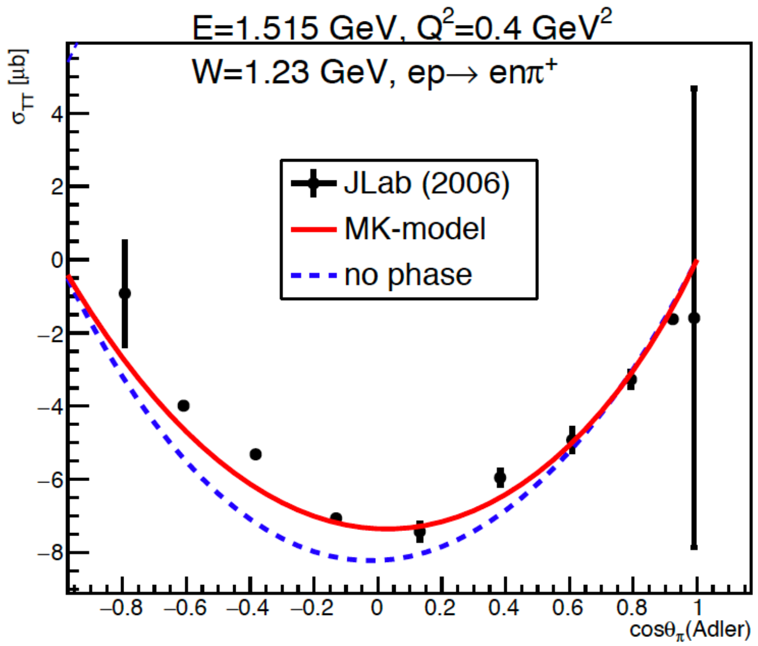




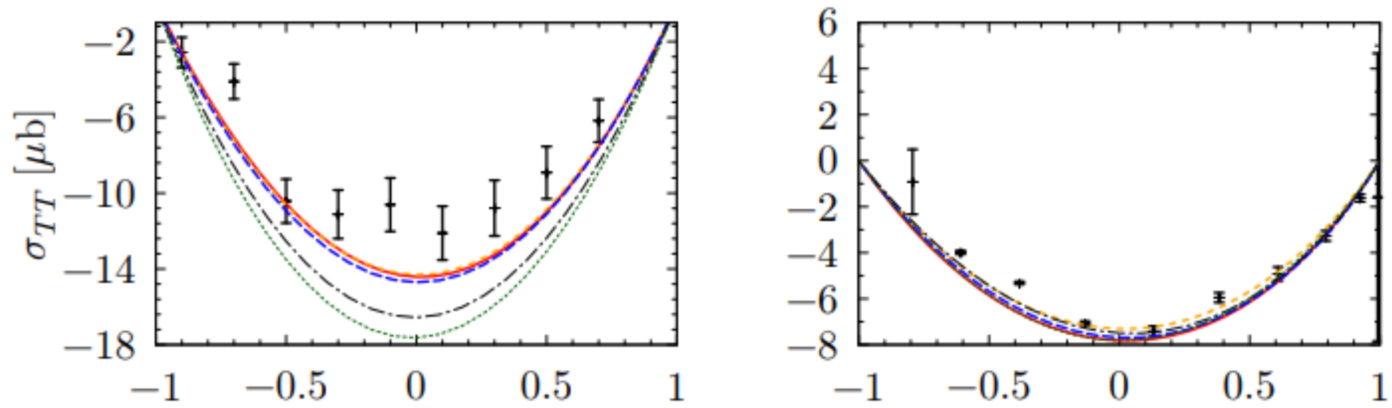
$ep \rightarrow en + \pi^+$

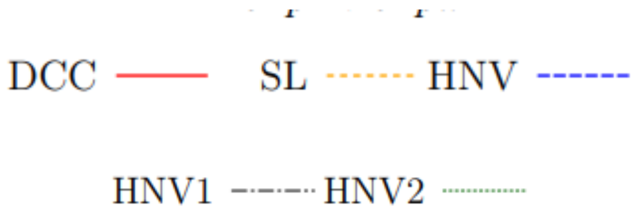
MK model comparison with J-lab data





σ_{TT}





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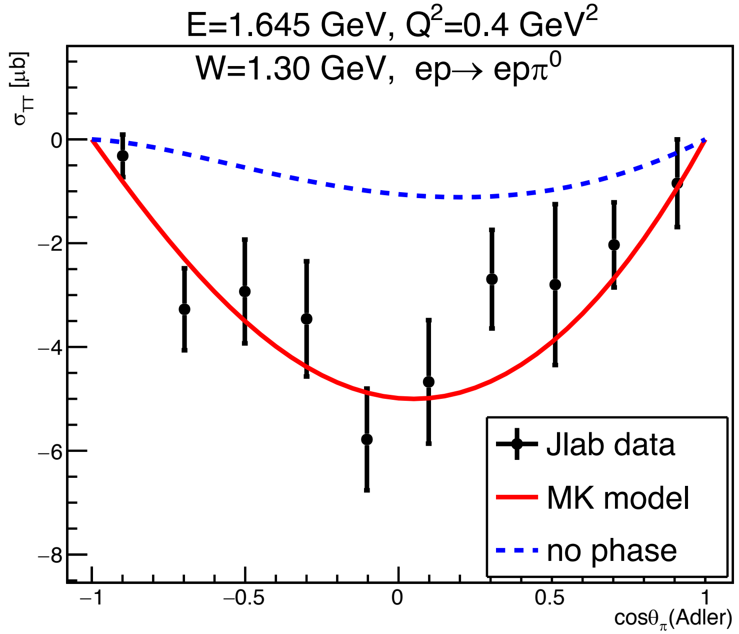
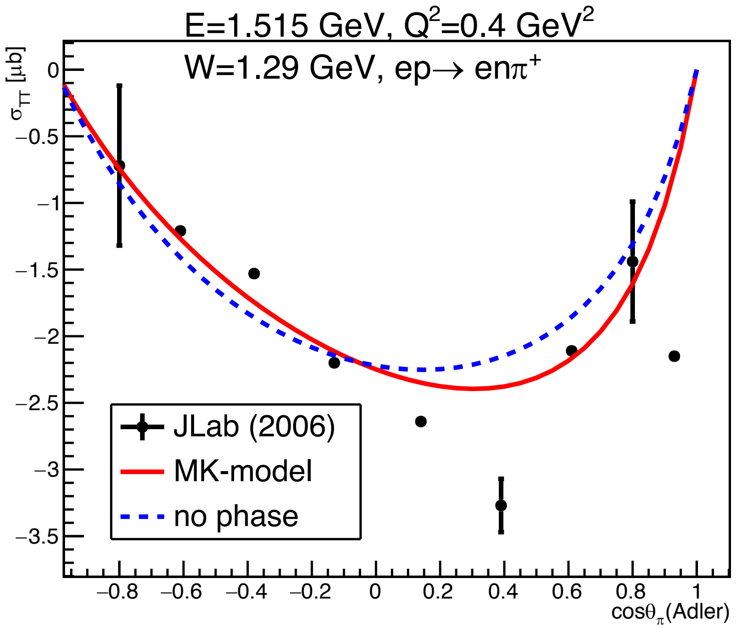
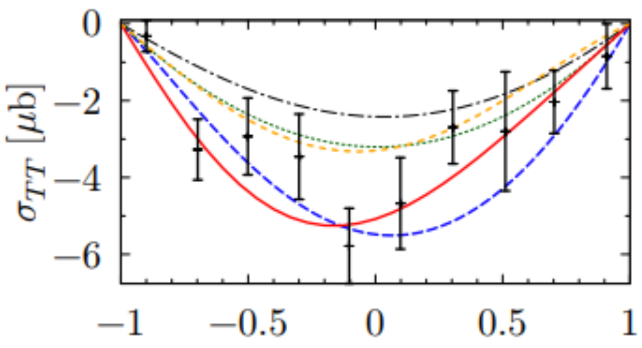
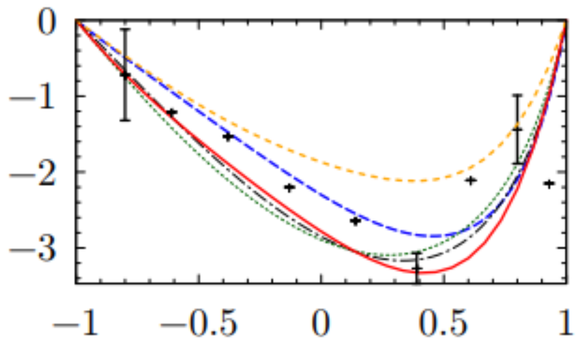
MK model comparison with J-lab data

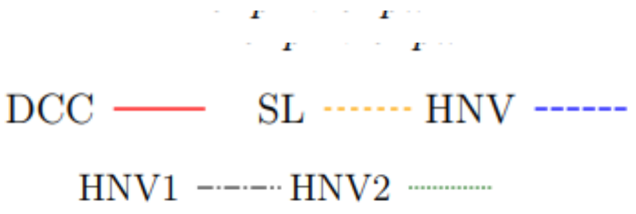




σ_{TT}







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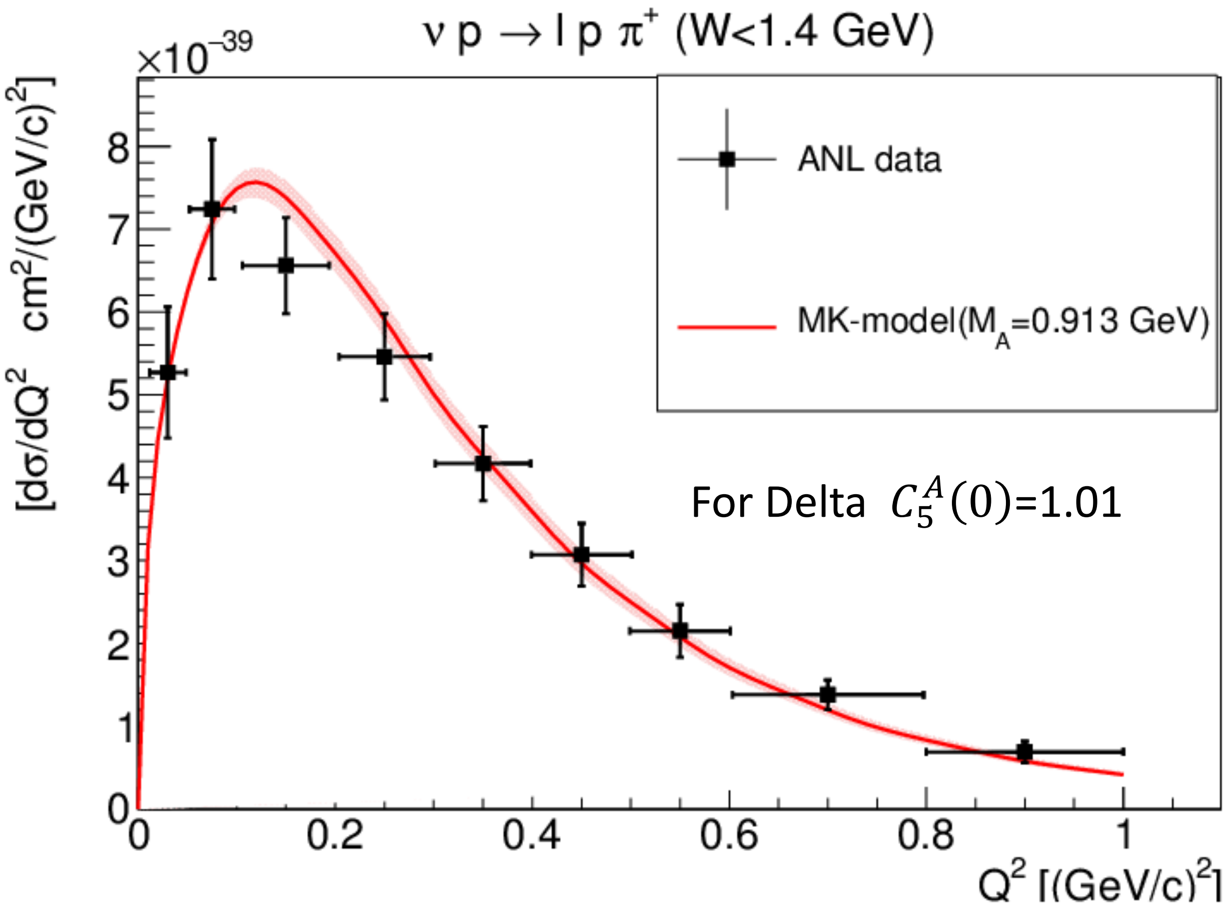
Fitting M_A

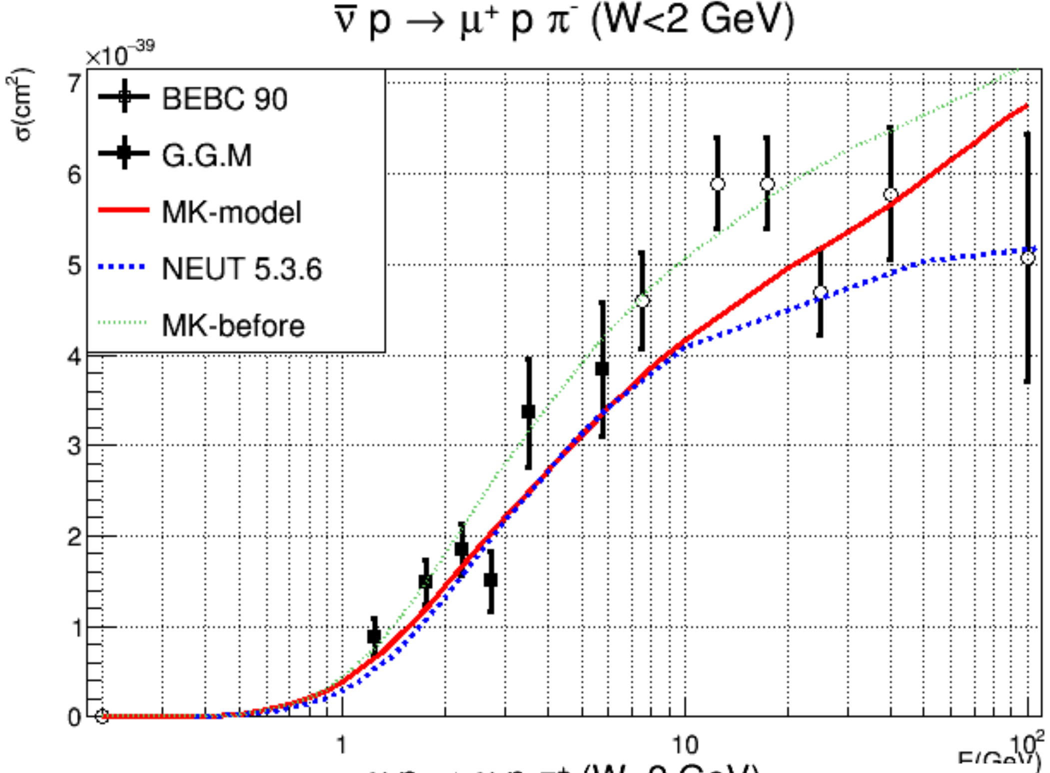
with ANL data

- For $Q^2 \neq 0$ we should only rely on neutrino data and fit M_A.
- $C_5^A(0)$ is already fitted to the pion scattering data.

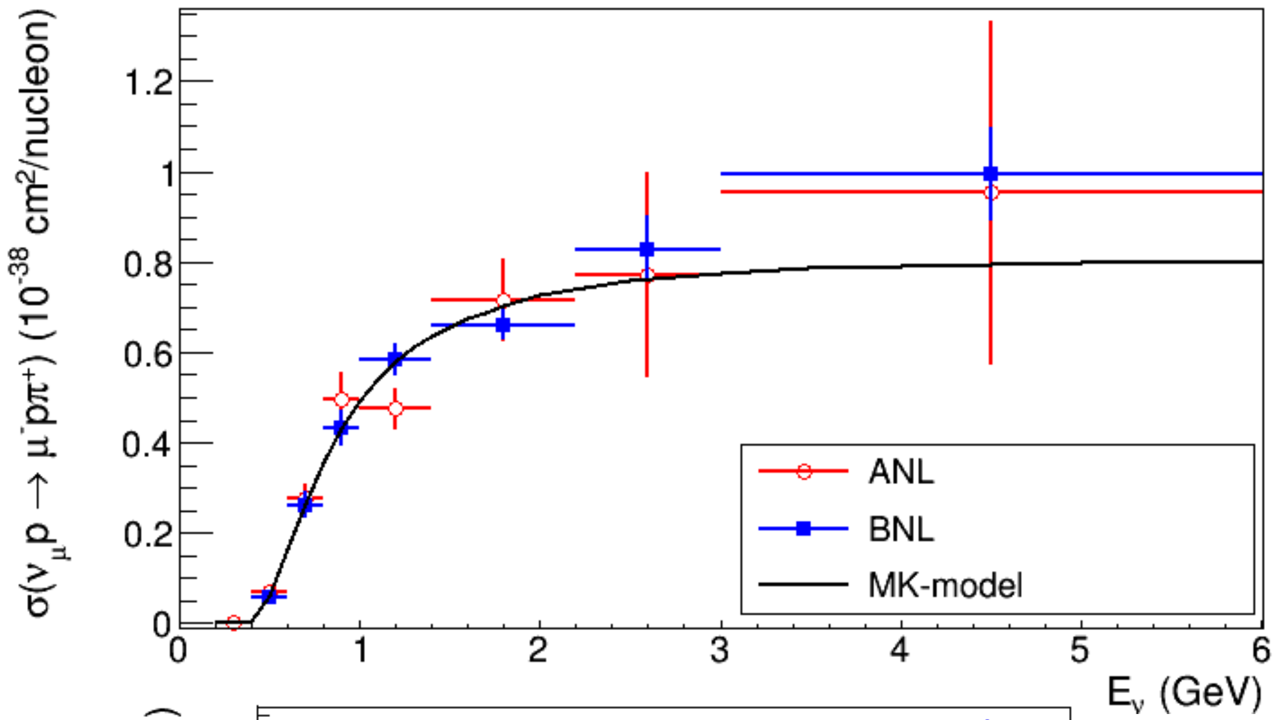
$$C_5^A(Q^2) = \frac{C_5^A(0)}{\left(1 + \frac{Q^2}{M_a^2}\right)^2}$$



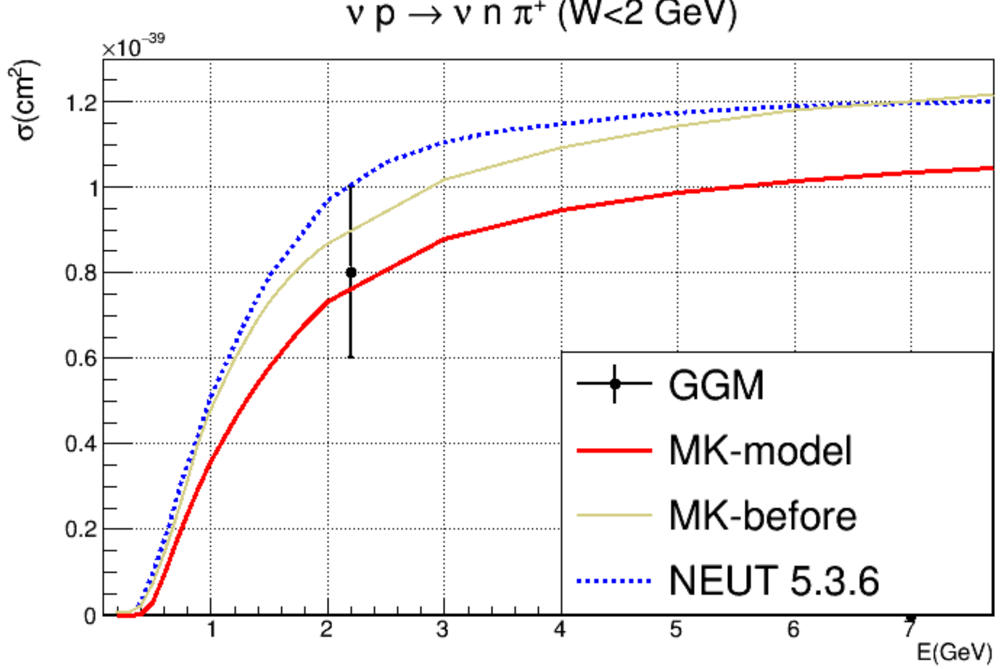
For Delta $C_5^A(0)$=1.01



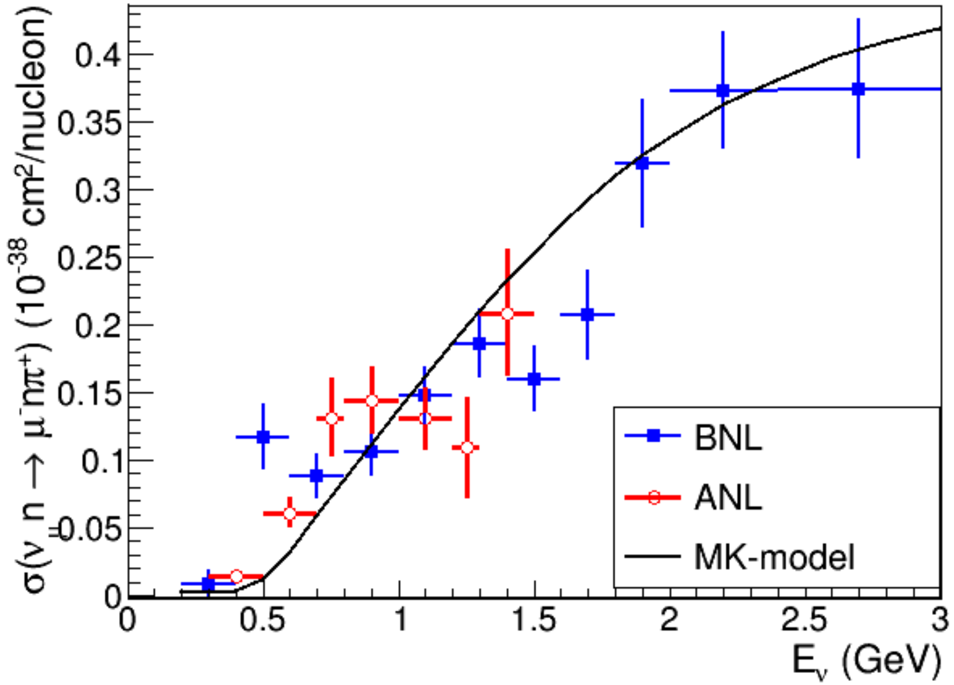
$\bar{\nu}$ p → μ^+ p π^- (W<2 GeV)
σ(cm²)
×10⁻³⁹
BEBC 90
G.G.M
MK-model
NEUT 5.3.6
MK-before
E(GeV)



σ(ν_μ p → μ^-pπ^+) (10^{-38} cm²/nucleon)
ANL
BNL
MK-model
E_ν (GeV)



ν p → ν n π^+ (W<2 GeV)
σ(cm²)
×10⁻³⁹
GGM
MK-model
MK-before
NEUT 5.3.6
E(GeV)

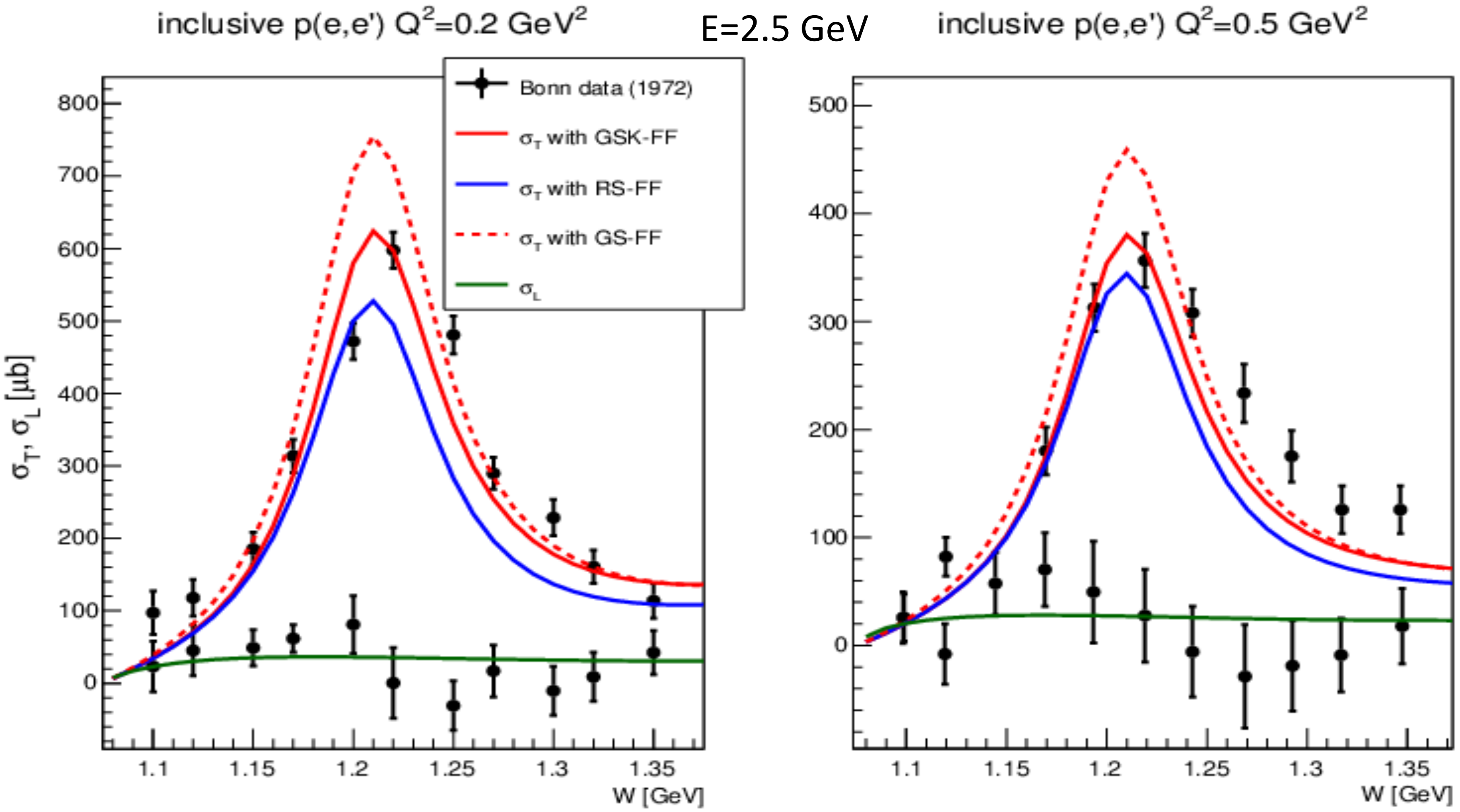


σ(ν_μ n → μ^-nπ^+) (10^{-38} cm²/nucleon)
BNL
ANL
MK-model
E_ν (GeV)

Bonn Inclusive *ep* Xsec

$$\frac{d^3\sigma_{ep\to e'X}}{dE_{e'}d\Omega_{e'}} = \Gamma_\gamma[\sigma_T(W,Q^2) + \epsilon\sigma_L(W,Q^2)]$$

virtual photon flux factor



$$\Gamma \equiv \frac{\alpha}{2\pi^2}\frac{E'}{E}\frac{(W^2-m_p^2)}{2m_pQ^2}\frac{1}{1-\epsilon}$$

$$\epsilon \equiv \left(1 + 2\frac{|\mathbf{q}|^2}{Q^2}\tan^2\frac{\theta_e}{2}\right)^{-1},$$

No adjustable parameter in vector form-factors

Vector and axial-vector currents

q.V (Q^2=0)=0 Conservation of Vector current (CVC)

q.A(Q^2=0) $\propto m_\pi^2 \neq 0$ axial current is not conserved. But it is partially conserved (PCAC) when $m_\pi \to 0$

→ Guiding principle to derive the axial current : PCAC relation with πN reaction amplitude

$$< X \mid q \cdot A(Q^2 \sim 0) \mid N > \sim i f_\pi < X \mid T \mid \pi N >$$

Phys. Rev. D 92, 074024 (2015)

Dynamical coupled-channels (DCC) model

DCC analysis of meson production data

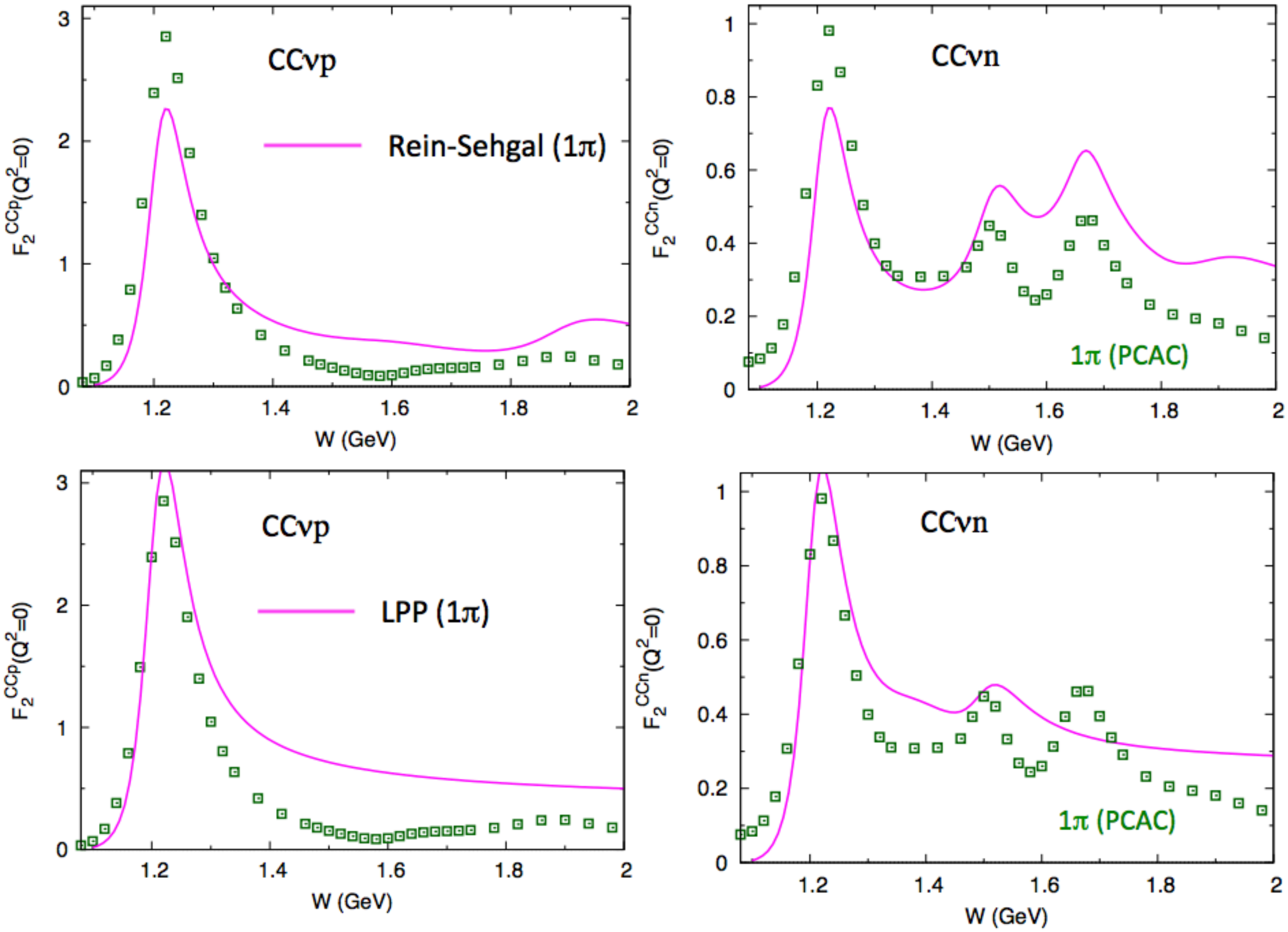
- Fully combined analysis of $\gamma N, \pi N \rightarrow \pi N, \eta N, K\Lambda, K\Sigma$ data

~ 27,000 data points are fitted

- In first analysis of the pion- and photon-induced meson production reactions, we have already constructed a DCC model for the strong interaction and the electromagnetic current of the proton at $Q^2 = 0$.
- More than 440 parameters are determined to fit the obtained vector form factors.

$$F^V_{NN^*}(Q^2) \sim \sum_{n=0}^{\mathcal{N}} c^N_n (Q^2)^n$$

- All the other (406) parameters such as resonance parameters (masses & decay widths) and relative phases between resonant and nonresonant amplitudes have been extracted from the DCC model.



Nakamura-san
NuSTEC 18

LPP: Lalakulich et al., PRD 74 (2006)