

Master formula for one-loop renormalization of bosonic SMEFT operators

– In Search of New Physics Using SMEFT, Argonne National Laboratory –

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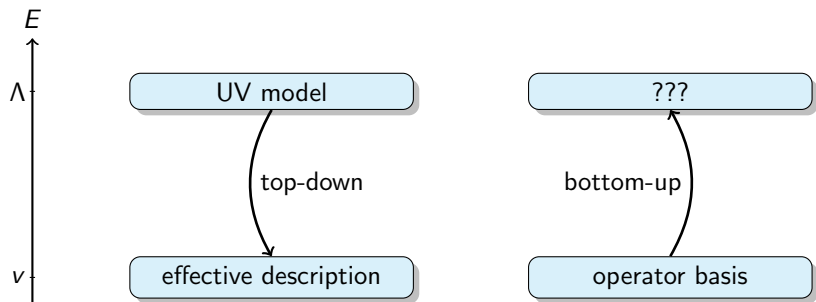


Alexander von Humboldt
Stiftung/Foundation

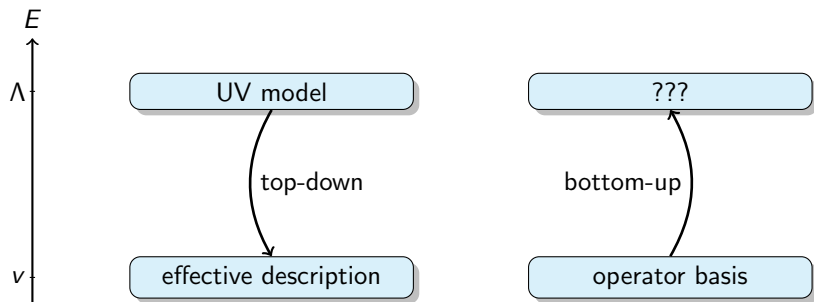


In collaboration with: Gerhard Buchalla, Alejandro Celis, and Jan-Niklas Toelstede
arXiv: 1904.07840

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- We match to UV-models at scale Λ , but measure at scale v .
 - The necessary RGEs also introduce an important operator mixing that was computed by Alonso/Jenkins/Manohar/Trott [1308.2627,1310.4838,1312.2014; JHEP].
 - The RGEs are also implemented in DSixTools: [1704.04504,EPJC] and Wilson: [1804.05033, EPJC]
- ⇒ An independent cross check with a different approach would be very beneficial!

Master formula for one-loop renormalization of bosonic SMEFT operators

Part I: The SMEFT in our notation

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{L}^{(6)} + \dots$$

$$\frac{1}{12} \Lambda^{\mu\nu} \Lambda_{\mu\nu} + \frac{1}{2} \Sigma^2$$

Part II: The Master Formula for 1-loop divergences
[1710.06412,1904.07840]

Part III: The result
[1904.07840]

$$\mathcal{Q}_\phi = (\phi^\dagger \phi)^3$$

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{L}^{(6)} + \dots$$
 I: We have a mass gap to NP.

We assume there is a gap between the SM at v and the new physics (NP) at Λ .

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We work with the real representation of the Higgs doublet

$$H \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \tilde{\phi} \\ \phi \end{pmatrix} = i\tau^j \varphi_j,$$

with $\tau^0 = -\frac{i}{2} \mathbf{1}$ and $\tau^i = \frac{\sigma^i}{2}$. The covariant derivative then becomes

$$(D_\mu \varphi)_i = \partial_\mu \varphi_i + ig W_\mu^a t_{Lij}^a \varphi_j + ig' B_\mu t_{Rij}^3 \varphi_j$$

t_L and t_R are generators of $SO(4)$ with the algebra

$$\begin{aligned} [t_L^a, t_L^b] &= i\epsilon^{abc} t_L^c \\ [t_R^a, t_R^b] &= i\epsilon^{abc} t_R^c \end{aligned}$$

$$\begin{aligned} \{t_L^a, t_L^b\} &= \frac{1}{2} \delta^{ab} \\ \{t_R^a, t_R^b\} &= \frac{1}{2} \delta^{ab} \end{aligned}$$

$$[t_L^a, t_R^b] = 0$$

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$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{L}^{(6)} + \dots$$
 | Our notation for the SM.

In this notation, we have

$$\begin{aligned} \mathcal{L}_{\text{SM}} = & -\frac{1}{4} G_{\mu\nu}^A G^{A\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ & + \frac{1}{2} (D_\mu \varphi)_i (D^\mu \varphi)_i + \frac{m^2}{2} \varphi_i \varphi_i - \frac{\lambda}{8} (\varphi_i \varphi_i)^2 \\ & + \bar{\psi} i \not{D} \psi - \left(\bar{\psi} \sqrt{2} H \mathcal{Y} P_R \psi + \text{h.c.} \right) \end{aligned}$$

with $\psi = (u, d, \nu, e)^T$ and $\mathcal{Y} = \text{diag}(\mathcal{Y}_u, \mathcal{Y}_d, 0, \mathcal{Y}_e)$.

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{L}^{(6)} + \dots \quad \text{I: Our notation for the SMEFT.}$$

We use the operators defined in the Warsaw basis and focus on the bosonic ones first. Grzadkowski/Iskrzynski/Misiak/Rosiek [1008.4884, JHEP]

\mathcal{Q}_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	\mathcal{Q}_φ	$\frac{1}{8}(\varphi\varphi)^3$	$\mathcal{Q}_{\psi\varphi}$	$\frac{1}{\sqrt{2}}(\varphi\varphi)\bar{\psi}_L H\psi_R$
$\mathcal{Q}_{\tilde{G}}$	$f^{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\mathcal{Q}_{\varphi\Box}$	$\frac{1}{4}(\varphi\varphi)\Box(\varphi\varphi)$	$\mathcal{Q}_{\psi G}$	$\sqrt{2}\bar{\psi}\sigma^{\mu\nu} G_{\mu\nu}^A T^A H\psi$
\mathcal{Q}_W	$\epsilon_{ijk} W_\mu^{i\nu} W_\nu^{j\rho} W_\rho^{k\mu}$	$\mathcal{Q}_{\varphi D}$	$\frac{1}{4}(\varphi D_\mu\varphi)(\varphi D^\mu\varphi)$	$\mathcal{Q}_{\psi W}$	$\sqrt{2}\bar{\psi}\sigma^{\mu\nu} W_{\mu\nu}^i \sigma^i H\psi$
$\mathcal{Q}_{\tilde{W}}$	$\epsilon_{ijk} \tilde{W}_\mu^{i\nu} W_\nu^{j\rho} W_\rho^{k\mu}$		$-(\varphi t_R^3 D_\mu\varphi)(\varphi t_R^3 D^\mu\varphi)$	$\mathcal{Q}_{\psi B}$	$\sqrt{2}\bar{\psi}\sigma^{\mu\nu} B_{\mu\nu} H\psi$
$\mathcal{Q}_{\varphi X}$	$\frac{1}{2}(\varphi\varphi)X_{\mu\nu}^a X^{a\mu\nu}$	\mathcal{Q}_{WB}	$2\varphi t_L^i t_R^3 \varphi W_{\mu\nu}^i B^{\mu\nu}$	$\mathcal{Q}_{\varphi\psi}^{(1)}$	$(\bar{\psi}_L \gamma^\mu \psi_L)(\varphi i \overleftrightarrow{D}_\mu \varphi)$
$\mathcal{Q}_{\varphi \tilde{X}}$	$\frac{1}{2}(\varphi\varphi)\tilde{X}_{\mu\nu}^a X^{a\mu\nu}$			$\mathcal{Q}_{\varphi\psi}^{(3)}$	$(\bar{\psi}_L \sigma^i \gamma^\mu \psi_L)(\varphi i \overleftrightarrow{D}_\mu^i \varphi)$
	$X = G, W, B$	$\mathcal{Q}_{\tilde{W}B}$	$2\varphi t_L^i t_R^3 \varphi \tilde{W}_{\mu\nu}^i B^{\mu\nu}$	$\mathcal{Q}_{\varphi\psi}$	$(\bar{\psi}_R \gamma^\mu \psi_R)(\varphi i \overleftrightarrow{D}_\mu \varphi)$
				$\mathcal{Q}_{\varphi ud}$	$-(\varphi(it_R^1 + t_R^2)D_\mu\varphi)(\bar{u}_R \gamma^\mu d_R)$

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Consider the 15 bosonic operators first

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{L}^{(6)} + \dots$$

I: The 1-loop SMEFT RGEs.

- n LO vertices \rightarrow LO operators

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1 NLO vertex
ops. } LO + NLO

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$$-32\pi^2 \epsilon \delta \mathcal{L}_{\text{div}} = \frac{C_j}{\Lambda^2} (K_{(i)} \delta_{ij} + K_{ij}) Q_i$$

$$\beta_i \equiv 16\pi^2 \frac{dC_i}{d \ln \mu} = K_{ij} C_j$$

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- running of dim-4 altered by $\beta(C_{\text{SM}}) \sim \frac{m^2}{\Lambda^2} C_{\text{dim-6}} C_{\text{SM}}^n$
- running of dim-6 given by $\beta(C_{\text{dim-6}}) \sim C_{\text{dim-6}} C_{\text{SM}}^n$

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starting from the generating functional:

$$Z[j, \rho, \bar{\rho}] = e^{iW[j, \rho, \bar{\rho}]} = \int [d\phi d\psi d\bar{\psi}] e^{i(S[\phi, \psi, \bar{\psi}] + j\phi + \bar{\psi}\rho + \bar{\rho}\psi)},$$

$$\phi = \hat{\phi} + \phi_{qu},$$

$$\psi = \hat{\psi} + \psi_{qu},$$

$$\left(\frac{\delta S}{\delta \phi} + j\right)_{\phi=\hat{\phi}} = 0, \quad \left(\frac{\delta S}{\delta \bar{\psi}} + \rho\right)_{\bar{\psi}=\hat{\psi}} = 0, \quad \left(\frac{\delta S}{\delta \psi} - \bar{\rho}\right)_{\psi=\hat{\psi}} = 0$$

$$\Rightarrow e^{iW_{L=1}} = \int [d\phi_{qu} d\psi_{qu} d\bar{\psi}_{qu}] e^{iS^{(2)}[\hat{\phi}, \hat{\psi}, \hat{\bar{\psi}}; \phi_{qu}, \psi_{qu}, \bar{\psi}_{qu}]}$$

Abbott '81

$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2$ We use the Background-Field Method...

Quantum gauge fixing:

$$\mathcal{L}_{\text{g-f}} = -\frac{1}{2} \left[\left(\hat{D}_G^\mu G_\mu^A \right)^2 + \left(\partial_\mu B^\mu - ig' \varphi_i t_{Rij}^3 \hat{\varphi}_j \right)^2 + \left(\hat{D}_W^\mu W_\mu^a - ig \varphi_i t_{Lij}^a \hat{\varphi}_j \right)^2 \right]$$

Dittmaier/Grosse-Knetter [hep-ph/9505266]; Helset/Paraskevas/Trott [1803.08001,PRL]

- We work in the Feynman gauge.
- The terms proportional to φ will make the next steps easier.

Using the background covariant derivative

$$\hat{D}_\mu^W X = \partial_\mu X + ig[\hat{W}_\mu, X]$$

maintains background gauge invariance.

$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2$ We use the Background-Field Method...

evaluating the one-loop functional

Neufeld/Gasser/Ecker hep-ph/9806436

$$e^{iW_{L=1}} = \int [d\phi d\psi d\bar{\psi}] e^{iS^{(2)}[\hat{\phi}, \hat{\psi}, \hat{\bar{\psi}}; \phi, \psi, \bar{\psi}]}$$

$$S^{(2)} = \frac{1}{2}\phi A\phi + \bar{\psi} B\psi + \phi\bar{\Gamma}\psi + \bar{\psi}\Gamma\phi$$

$$W_{L=1} = \frac{i}{2} \text{Tr} \ln A - i \text{Tr} \ln B - \frac{i}{2} \sum_{n=0}^{\infty} \frac{1}{n} \text{Tr} (A^{-1}\bar{\Gamma}B^{-1}\Gamma - A^{-1}\Gamma^T B^{-1,T}\bar{\Gamma}^T)^n$$

$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2$... and Super-Heat-Kernel Expansion...

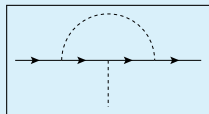
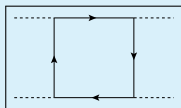
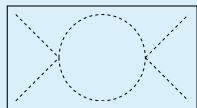
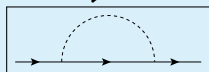
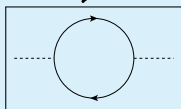
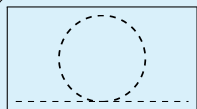
evaluating the one-loop functional

Neufeld/Gasser/Ecker hep-ph/9806436

$$e^{iW_{L=1}} = \int [d\phi d\psi d\bar{\psi}] e^{iS^{(2)}[\hat{\phi}, \hat{\psi}, \hat{\bar{\psi}}; \phi, \psi, \bar{\psi}]}$$

$$S^{(2)} = \frac{1}{2}\phi A\phi + \bar{\psi} B\psi + \phi\bar{\Gamma}\psi + \bar{\psi}\Gamma\phi$$

$$W_{L=1} = \frac{i}{2} \text{Tr} \ln A - i \text{Tr} \ln B - \frac{i}{2} \sum_{n=0}^{\infty} \frac{1}{n} \text{Tr} (A^{-1}\bar{\Gamma}B^{-1}\Gamma - A^{-1}\Gamma^T B^{-1,T}\bar{\Gamma}^T)^n$$



$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2$... and Super-Heat-Kernel Expansion...

Introducing supermatrix algebra:

Neufeld/Gasser/Ecker hep-ph/9806436

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$\text{sdet } M = \det(A - BD^{-1}C) \det D^{-1}$$

$$\text{str } M = \text{tr } A - \text{tr } D$$

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The one-loop functional of $S^{(2)} = \frac{1}{2}\phi A\phi + \bar{\psi} B\psi + \phi\bar{\Gamma}\psi + \bar{\psi}\Gamma\phi$ becomes:

$$W_{L=1} = \frac{i}{2} \text{Str } \ln \Delta, \quad \Delta = \begin{pmatrix} A & \bar{\Gamma} & -\Gamma^T \\ -\bar{\Gamma}^T & 0 & -B^T \\ \Gamma & B & 0 \end{pmatrix}$$

$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2$... and Super-Heat-Kernel Expansion...

Applying the Heat-Kernel Expansion:

Donoghue/Golowich/Holstein '92

Neufeld/Gasser/Ecker hep-ph/9806436

$$\begin{aligned}W_{L=1} &= \frac{i}{2} \text{Str} \ln \Delta \\ &= -\frac{i}{2} \int_0^\infty \frac{d\tau}{\tau} \int d^d x \text{str} \langle x | e^{-\tau\Delta} | x \rangle\end{aligned}$$

with the expansion in Seeley-DeWitt coefficients

$$\langle x | e^{-\tau\Delta} | x \rangle = \frac{i}{(4\pi)^{d/2}} \frac{e^{-\tau m^2}}{\tau^{d/2}} \sum_{n=0}^{\infty} a_n(x) \tau^n$$

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- The a_n can be computed, knowing the form of Δ .
- The UV-divergences of $W_{L=1}$ are the poles in $\frac{1}{\tau}$.
⇒ only a_2 contributes!

$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2$... and Super-Heat-Kernel Expansion...

The Heat-Kernel Expansion extracts the $\frac{1}{\epsilon}$ -poles of $W_{L=1}$.

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With

Donoghue/Golowich/Holstein '92; Neufeld/Gasser/Ecker [hep-ph/9806436,PLB]

$$\Delta = (\partial_\mu + \Lambda_\mu)(\partial^\mu + \Lambda^\mu) + \Sigma$$

we get

$$W_{L=1,div} = \frac{1}{32\pi^2\epsilon} \int d^4x \text{str} \left[\frac{1}{12}\Lambda_{\mu\nu}\Lambda^{\mu\nu} + \frac{1}{2}\Sigma\Sigma \right].$$

$$\Lambda_{\mu\nu} = \partial_\mu\Lambda_\nu - \partial_\nu\Lambda_\mu + [\Lambda_\mu, \Lambda_\nu]$$

- Specifying the Dirac structure of $S^{(2)}$, we can further evaluate the Dirac-traces.
- The resulting Master-Formula is purely algebraic (Matrix multiplication and traces).

'tHooft '73,NPB

$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2$... to find the Master Formula.

In the SM (and the EWChL), we have

$$\mathcal{L}_2^{\text{SM}} = -\frac{1}{2}\phi^i A_i^j \phi_j + \bar{\chi} (i\not{\partial} - G) \chi + \bar{\chi} \Gamma^i \phi_i + \phi^i \bar{\Gamma}_i \chi,$$

with $A = (\partial^\mu + N^\mu)(\partial_\mu + N_\mu) + Y$ and $G \equiv (r + \rho_\mu \gamma^\mu)P_R + (l + \lambda_\mu \gamma^\mu)P_L$.

This gives

$$\mathcal{L}_{\text{div}}^{\text{SM}} = \frac{1}{32\pi^2\varepsilon} \left(\text{tr} \left[\frac{1}{12} N^{\mu\nu} N_{\mu\nu} + \frac{1}{2} Y^2 - \frac{1}{3} (\lambda^{\mu\nu} \lambda_{\mu\nu} + \rho^{\mu\nu} \rho_{\mu\nu}) \right] \right. \\ \left. + \text{tr} [2D^\mu l D_\mu r - 2l r l r] + \bar{\Gamma} \left(i\not{\partial} + i\not{N} + \frac{1}{2} \gamma^\mu G \gamma_\mu \right) \Gamma \right)$$

with

$$N_{\mu\nu} \equiv \partial_\mu N_\nu - \partial_\nu N_\mu + [N_\mu, N_\nu], \\ \lambda_{\mu\nu} \equiv \partial_\mu \lambda_\nu - \partial_\nu \lambda_\mu + i[\lambda_\mu, \lambda_\nu], \quad \rho_{\mu\nu} \equiv \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + i[\rho_\mu, \rho_\nu], \\ D_\mu l \equiv \partial_\mu l + i\rho_\mu l - il\lambda_\mu, \quad D_\mu r \equiv \partial_\mu r + i\lambda_\mu r - ir\rho_\mu.$$

'tHooft '73,NPB; Buchalla/Catà/Celis/Knecht/CK [1710.06412,NPB]

$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2$... to find the Master Formula.

For the bosonic SMEFT operators in the Warsaw Basis, we have

$$\mathcal{L}_2^{\text{SMEFT}} = \mathcal{L}_2^{\text{SM}} + \phi^i (a_{\mu\nu}^{ij} D^\mu D^\nu + 2b_\mu^{ij} D^\mu + c^{ij}) \phi_j,$$

with $a_{\mu\nu}, b_\mu, c \sim \frac{1}{\Lambda^2}$.

This gives

$$\begin{aligned} \mathcal{L}_{\text{div}}^{\text{SMEFT}} = & \mathcal{L}_{\text{div}}^{\text{SM}} + \frac{1}{32\pi^2\varepsilon} \left(\text{tr} \left[cY + \frac{1}{3} N_{\mu\nu} [D^\mu, b^\nu] + i\bar{\Gamma} b\Gamma - \frac{1}{6} \bar{\Gamma} i\overleftrightarrow{D} a\Gamma \right] \right. \\ & + \text{tr} \left[\frac{1}{6} a^{\mu\nu} N_{\mu\lambda} N_\nu^\lambda - \frac{1}{24} a_\lambda^\lambda N_{\mu\nu} N^{\mu\nu} + \frac{1}{6} N_{\mu\lambda} [D_\nu, [D^\lambda, a^{\mu\nu}]] \right] \\ & \left. + \text{tr} \left[\frac{1}{3} Y [D_\mu, [D_\nu, a^{\mu\nu}]] - \frac{1}{4} a_\lambda^\lambda Y^2 - \frac{1}{12} Y [D_\mu, [D^\mu, a_\lambda^\lambda]] \right] \right). \end{aligned}$$

Buchalla/Celis/CK/Toelstede [1904.07840]

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Part I: The SMEFT in our notation

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{L}^{(6)} + \dots$$

$$\frac{1}{12} \Lambda^{\mu\nu} \Lambda_{\mu\nu} + \frac{1}{2} \Sigma^2$$

Part II: The Master Formula for 1-loop divergences
[1710.06412,1904.07840]

Part III: The result
[1904.07840]

$$\mathcal{Q}_\phi = (\phi^\dagger \phi)^3$$

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An explicit example

Starting from

$$\mathcal{Q}_\phi = (\phi^\dagger \phi)^3 = \frac{1}{8} (\varphi_i \varphi_i)^3,$$

we find

$$a_{ij}^{\mu\nu} = 0, \quad b_{ij}^\mu = 0, \quad c_{ij} = -\frac{3}{4} (\hat{\varphi}_a \hat{\varphi}_a)^2 \delta_{ij} - 3 (\hat{\varphi}_a \hat{\varphi}_a) \hat{\varphi}_i \hat{\varphi}_j,$$
$$Y_{ij} = \left(\left(\frac{\lambda}{2} + \frac{g^2}{4} \right) \hat{\varphi}_a \hat{\varphi}_a - m^2 \right) \delta_{ij} + \left(\lambda - \frac{g^2}{4} \right) \hat{\varphi}_i \hat{\varphi}_j - g'^2 (t_R^3 \hat{\varphi})_i (t_R^3 \hat{\varphi})_j.$$

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Therefore

$$\text{tr } cY = - \left(54\lambda + \frac{9}{2}g^2 + \frac{3}{2}g'^2 \right) (\phi^\dagger \phi)^3 + 24m^2 (\phi^\dagger \phi)^2.$$

gives with $K_{(\phi)} = 6(3g^2 + g'^2 - \gamma_\phi)$

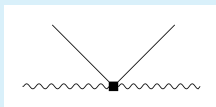
$$\beta_\phi \supseteq \left(54\lambda - \frac{27}{2}g^2 - \frac{9}{2}g'^2 + 6\gamma_\phi \right) C_\phi, \quad \text{and} \quad \beta_\lambda \supseteq 48 \frac{m^2}{\Lambda^2} C_\phi.$$

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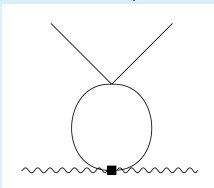
Operators mix under renormalization.

example of operator mixing: $\mathcal{O}_{\phi G} = \phi^\dagger \phi G_{\mu\nu}^A G^{A\mu\nu}$

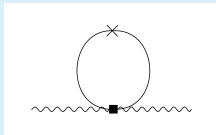
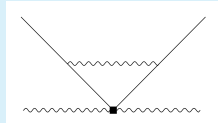
$$c_{\phi G} \phi^\dagger \phi G_{\mu\nu}^A G^{A\mu\nu}$$



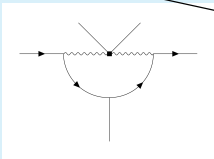
$$c_{\phi G} \lambda \phi^\dagger \phi G_{\mu\nu}^A G^{A\mu\nu}$$



$$c_{\phi G} g^2 \phi^\dagger \phi G_{\mu\nu}^A G^{A\mu\nu}$$

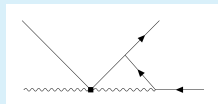


$$c_{\phi G} \frac{\mu_\phi^2}{\Lambda^2} G_{\mu\nu}^A G^{A\mu\nu}$$



$$c_{\phi G} g_s^2 \gamma (\bar{q}_L q_R \phi)(\phi^\dagger \phi)$$

$$c_{\phi G} g_s \gamma (\bar{q}_L \sigma_{\mu\nu} G^{\mu\nu} q_R \phi)$$



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Our results agree with the literature.

Buchalla/Celis/CK/Toelstede [1904.07840]

- We performed independent computations to cross check our results.
- We reproduce all RGE contributions of the 15 bosonic operators computed in

Alonso/Jenkins/Manohar/Trott [1308.2627,1310.4838,1312.2014,JHEP]

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To compute the RGEs of the remaining operators, we have to extend our master formula by:

- Four-Fermion operators
- Fermionic tensor currents
- Mixed bosonic/fermionic derivatives

⇒ Stay tuned!

Master formula for one-loop renormalization of bosonic SMEFT operators — Summary —

- We derived a master formula for the $1/\epsilon$ -poles based on the super-heat-kernel.

$$\frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{1}{2}\Sigma^2$$

- It can be applied to a broad class of theories, like the SM, the bosonic sector of the SMEFT, or even the EWChL.
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- We reproduce the RGE contributions of the 15 bosonic operators that have been previously computed by Alonso/Jenkins/Manohar/Trott [1308.2627,1310.4838,1312.2014,JHEP]
- We are currently extending the master formula to include the full set of SMEFT operators.