Lattice QCD and Neutrino-Nucleus Scattering

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LQCD extension III Review
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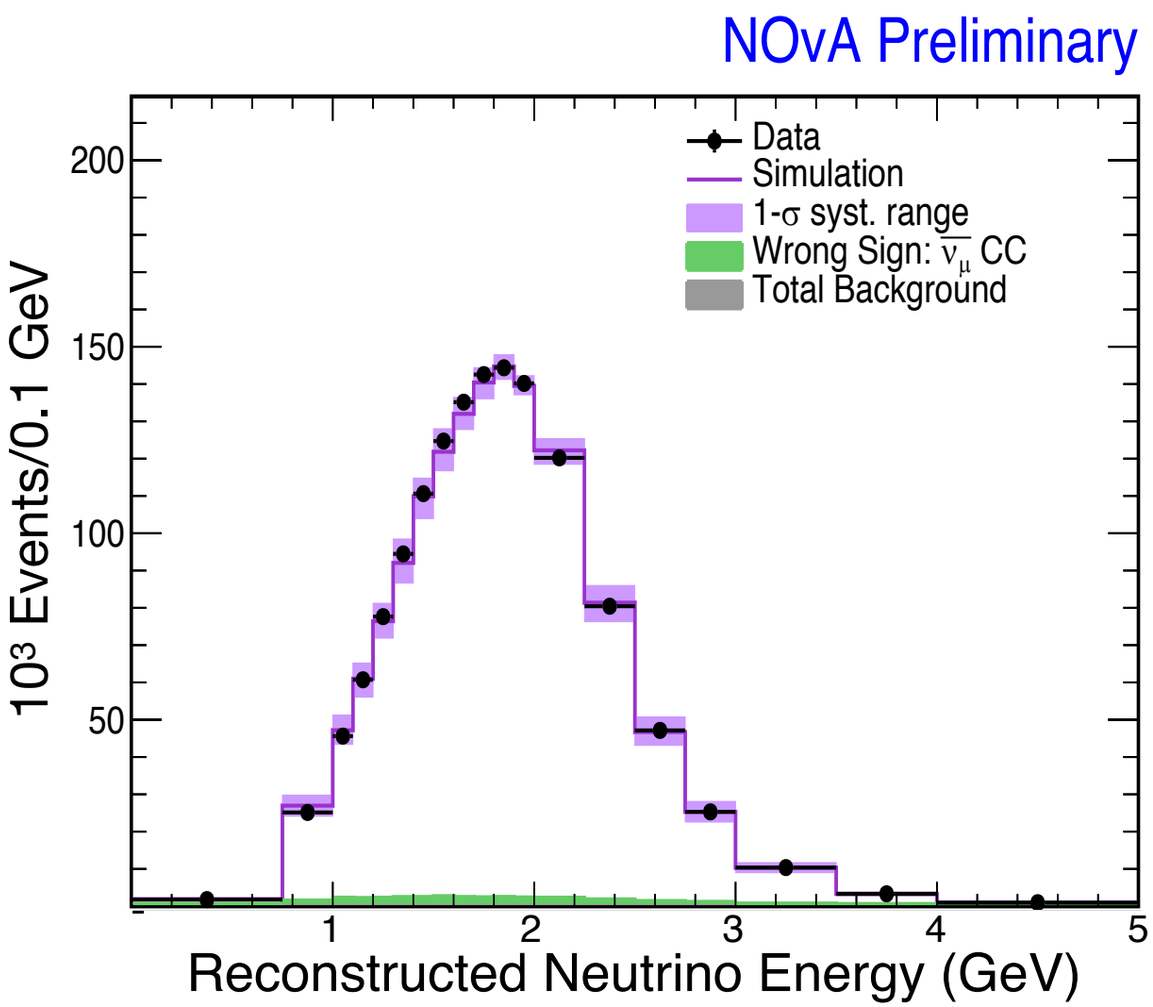
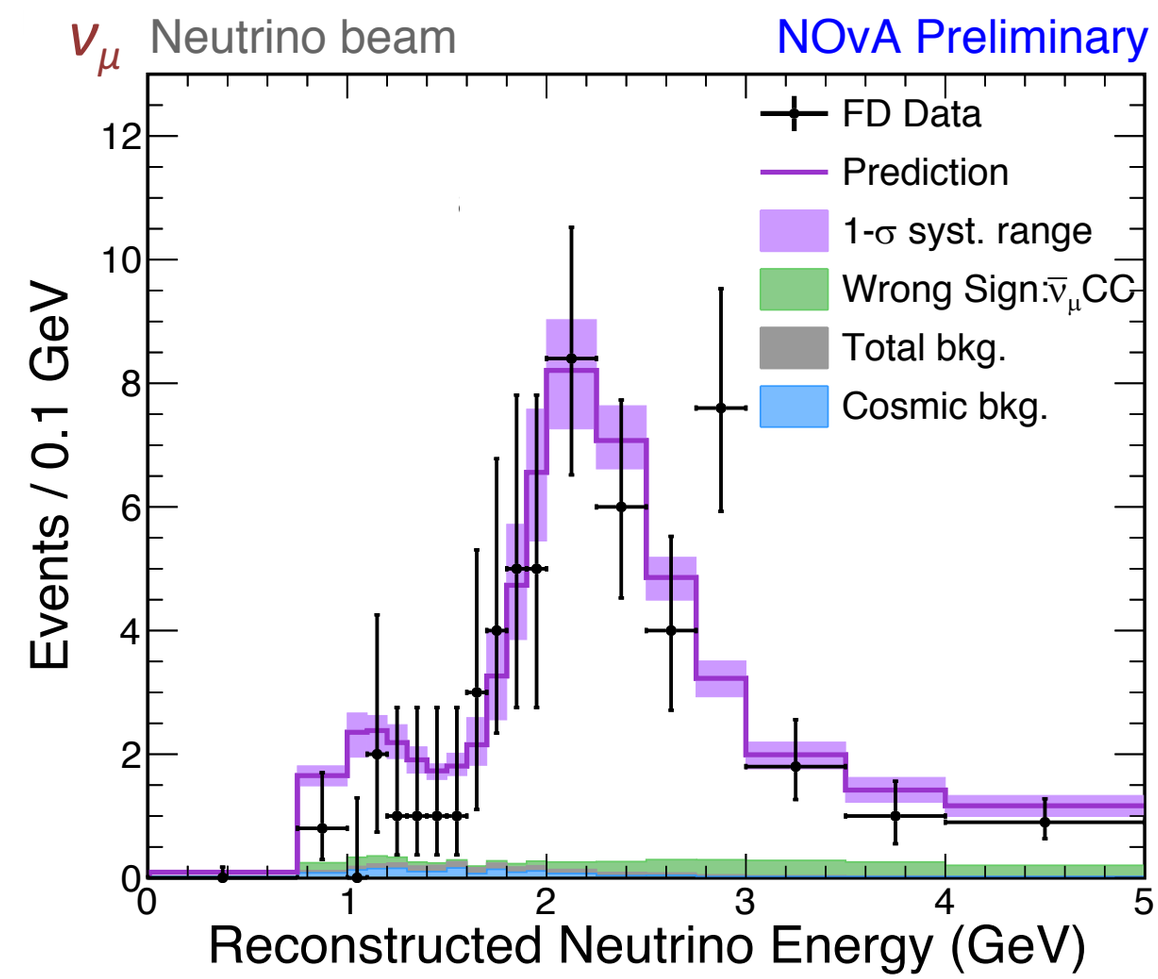
Neutrino Physics ⊂ Particle Physics

- Neutrinos have mass, change flavor ("oscillate"), and may violate *CP*.
- Ambitious experimental program to search for *CP* violation and to test the three-family paradigm:
  - $\$10^9$ for LBNF/DUNE;
  - numerous other experiments;
  - Japan; US + CERN.
- Oscillation experiments make a(n) (anti)neutrino beam and impinge it on nuclei such as ^{12}C, ^{16}O, ^{40}Ar.

Neutrino Oscillations

- Studied with a neutrino beam and two enormous vats of neutrons, one nearby and the other far away:





Alex Himmel, Wine & Cheese Seminar, 2018 June 15 @ Fermilab

Neutrino Oscillations

- The “disappearance” can be explained as an interference effect, if the muon couples to more than one (*i.e.*, three) mass eigenstates.

- Probability $P_{\alpha\beta}$ of a charged lepton of flavor α to be emitted at the source with a neutrino of mass m_j and energy E and to convert at the detector to a charged lepton of flavor β depends on mixing matrix U and distance L:

$$P_{\alpha\beta} = \sum_{j,k} U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k} \, e^{\pm i \Delta m^2_{jk} L/2E}$$

  with upper (lower) sign for ν ($\bar{\nu}$); ultrarelativistic neutrino $p \approx E - m^2/2E$.

- The key to the measurement is the unknown neutrino energy.

- It must be reconstructed from energy-momentum four-vectors of final-state particles: but four-momentum of nuclear remnant is not measured. 😒

Pontecorvo–Maki–Nakagawa–Sakata Matrix

- The PMNS matrix is to neutrino & lepton-flavor physics as the Cabibbo-Kobayashi-Maskawa (CKM) matrix is to quark-flavor physics:

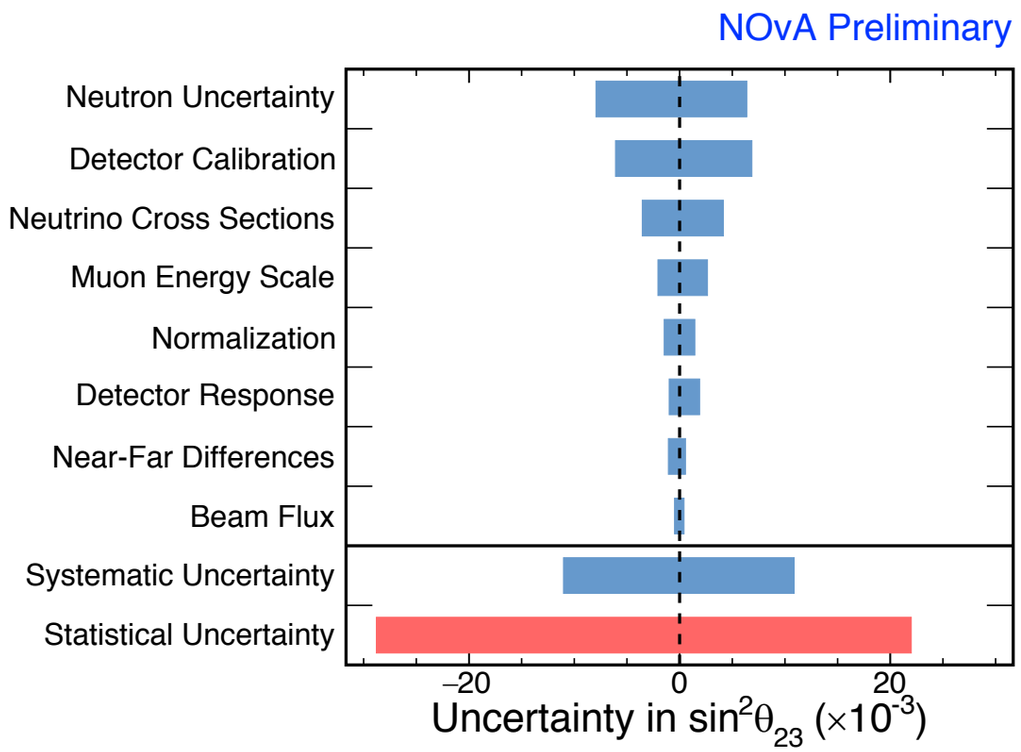
$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = [U]_{\ell j}$$

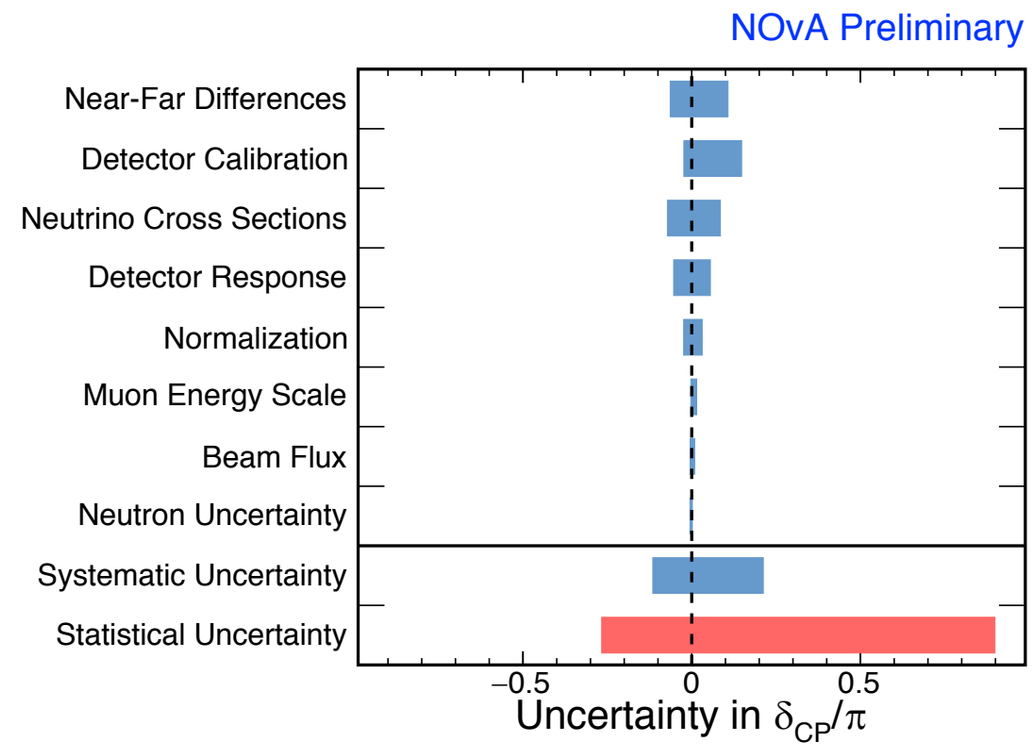
  where $\ell \in \{e, \mu, \tau\}$ labels flavor and $j \in \{1, 2, 3\}$ labels mass eigenstate.

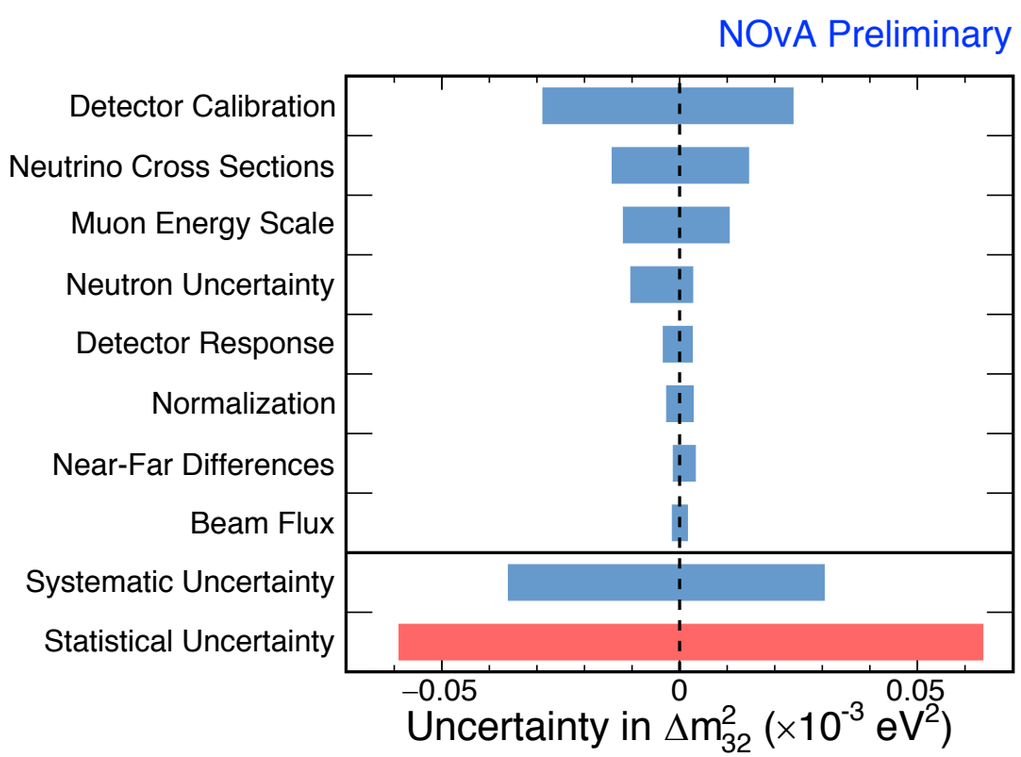
- Like CKM, PMNS has three mixing angles, θ_{lj}, & a *CP*-violating phase, δ.

- With Majorana mass terms, two fewer field phases absorbed into fields, so two more *CP*-violating phases, α_i, $i \in \{1, 2\}$, are physical ← $0\nu\beta\beta$.

Systematic Uncertainties



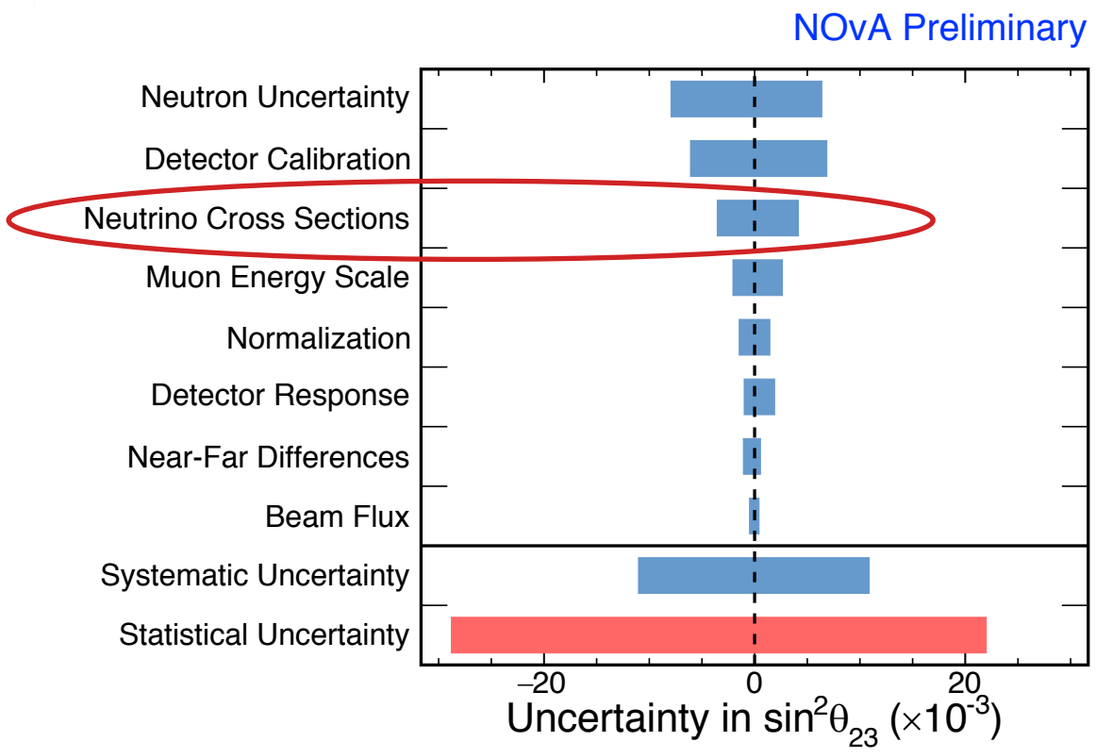


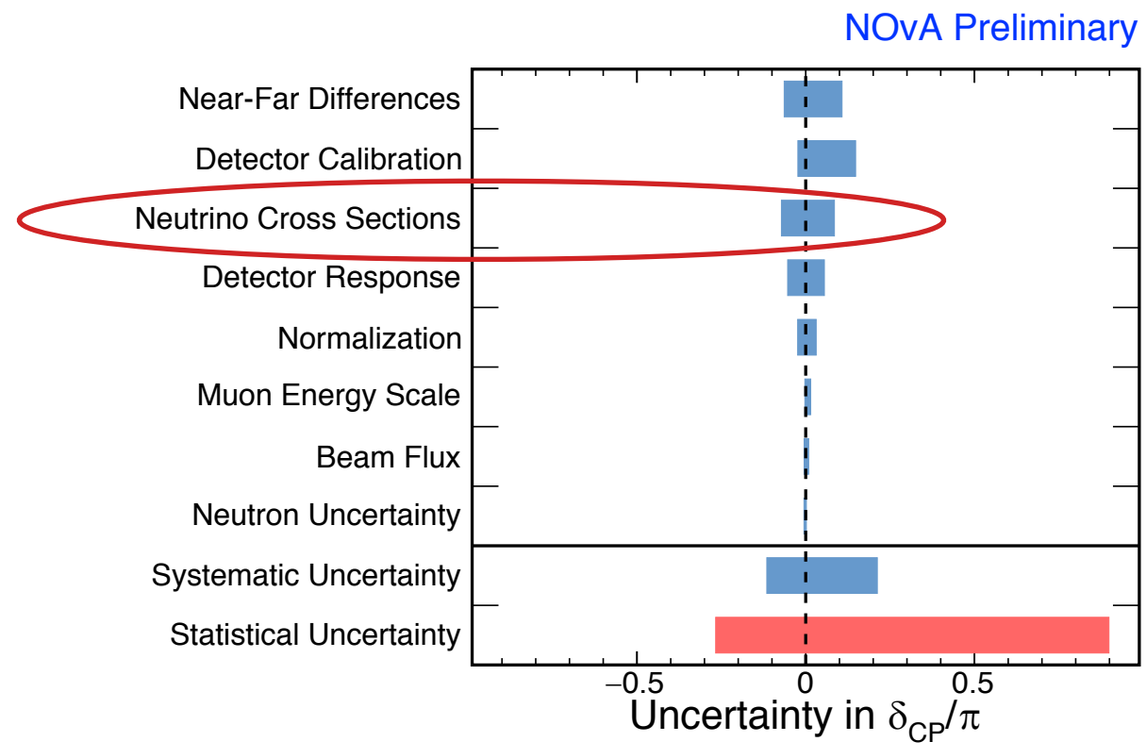


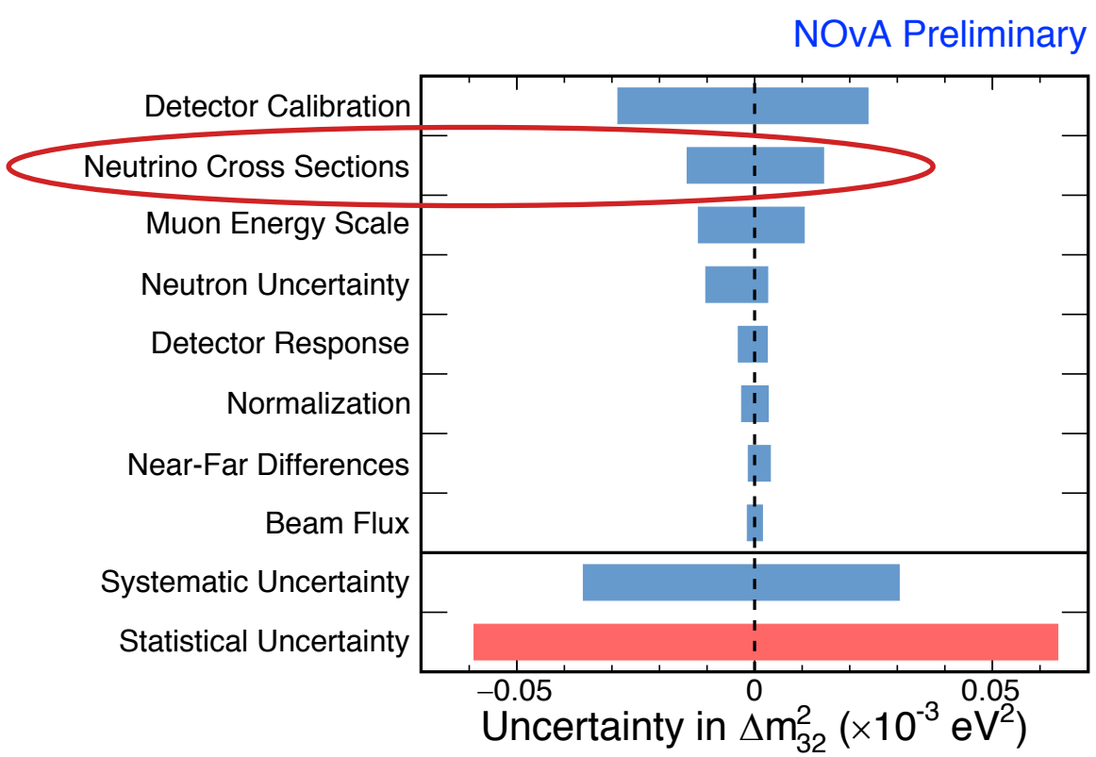
Most important systematics:

- Detector Calibration
  - Will be improved by the 2019 test beam program
- Neutrino cross sections
  - Particularly nuclear effects (RPA, MEC)
- Muon energy scale
- Neutron uncertainty – **new** with $\bar{\nu}$'s

Systematic Uncertainties







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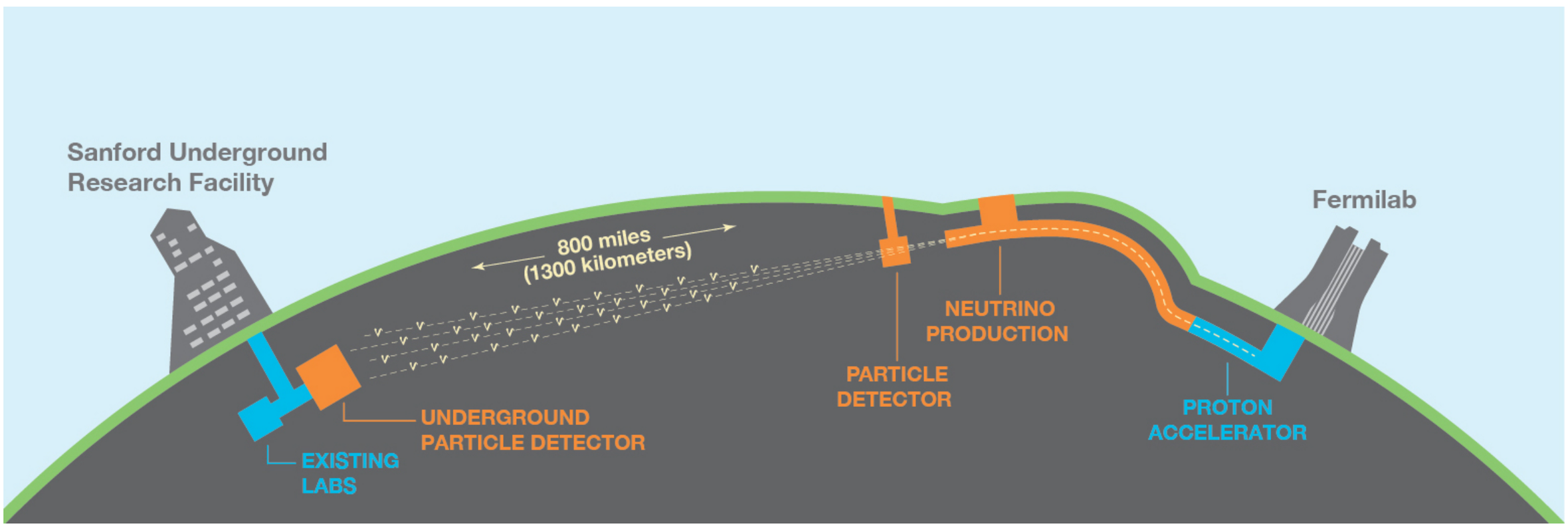
Looking Forward

- Statistical errors will be reduced via much larger exposure.
- Dominant systematic uncertainty will soon be that from νA cross section:
  - other large ones connected to statistics, test beam, and/or cross section itself.



Nuclear Structure

- A huge challenge (*cf*. Zohreh's talk):
  - nuclear theorists use a unified formalism describes many things and can be systematically improved—
    - many-body theory, with interactions that are tuned to data or—
    - is cast as a chiral effective theory (nucleons in a pion cloud);
    - not QCD, although sometimes called “*ab initio*”.
- Full error budgets are yet to come and a subject of intense research.
- Measure “data over here” to constrain nuclear model; then trust validated models “over there” where oscillation parameters are determined.

Hadronic Physics—νN Scattering

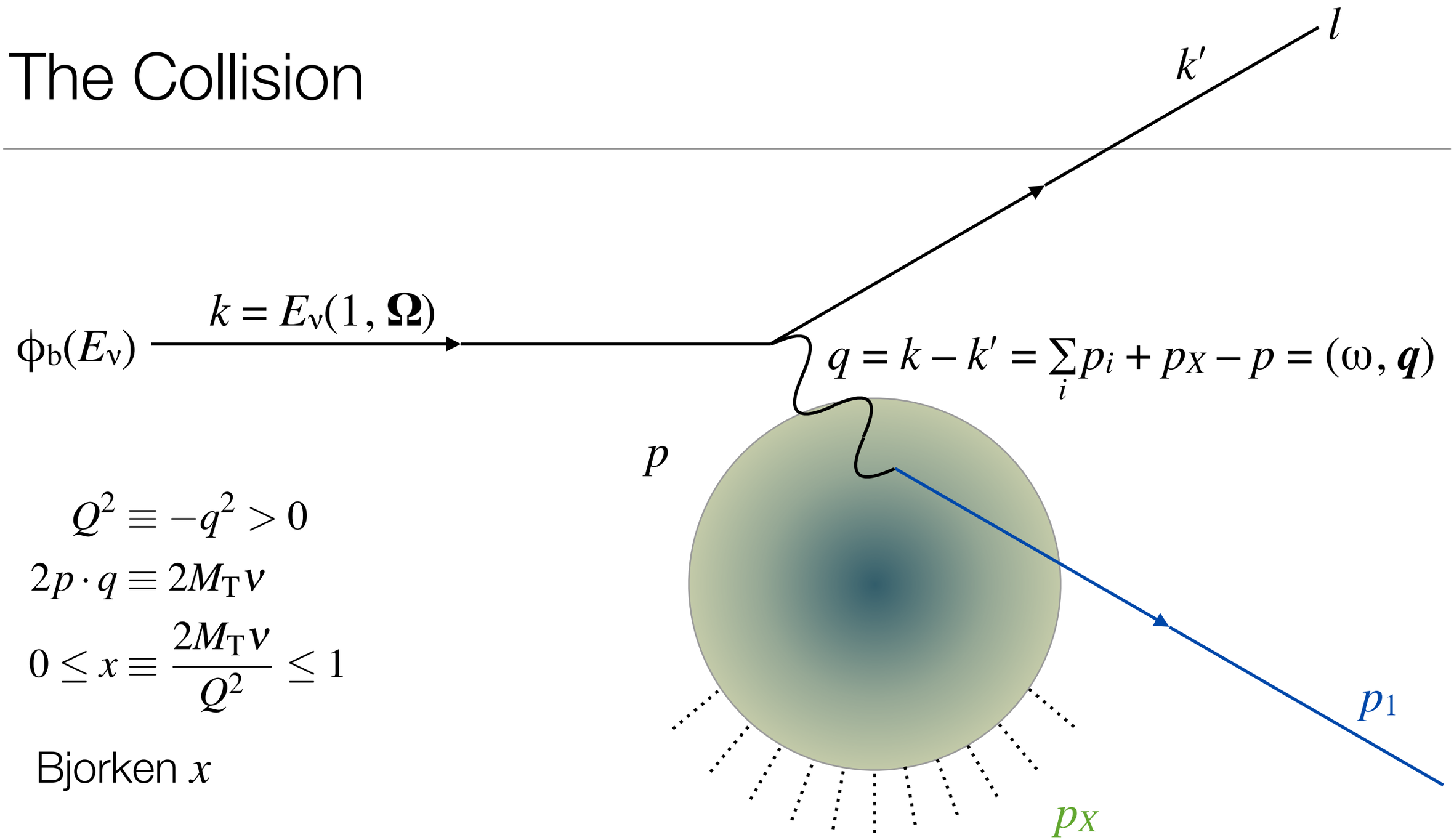
- Hard, but easy compared to nuclear theory.
- Avoid potential validation trap: nuclear model with nucleonic ingredients inconsistent with QCD:
  - even if tuned up “here” this procedure might be risky “over there”.
- Therefore, aim to get scattering amplitudes from first principles.
- Two approaches:
  - $\bar{\nu} p$ or νd scattering experiments in the Tokai, NuMI, or LBNF beam;
  - lattice QCD with error budgets as comprehensive as those for CKM.

Outline

- Introduction
- What calculations are needed by experiments?
- What has lattice QCD accomplished so far?
  - What will lattice QCD achieve in the future? $\Longleftarrow$ talk at 2:35 PM
- Summary

νA Scattering

The Collision



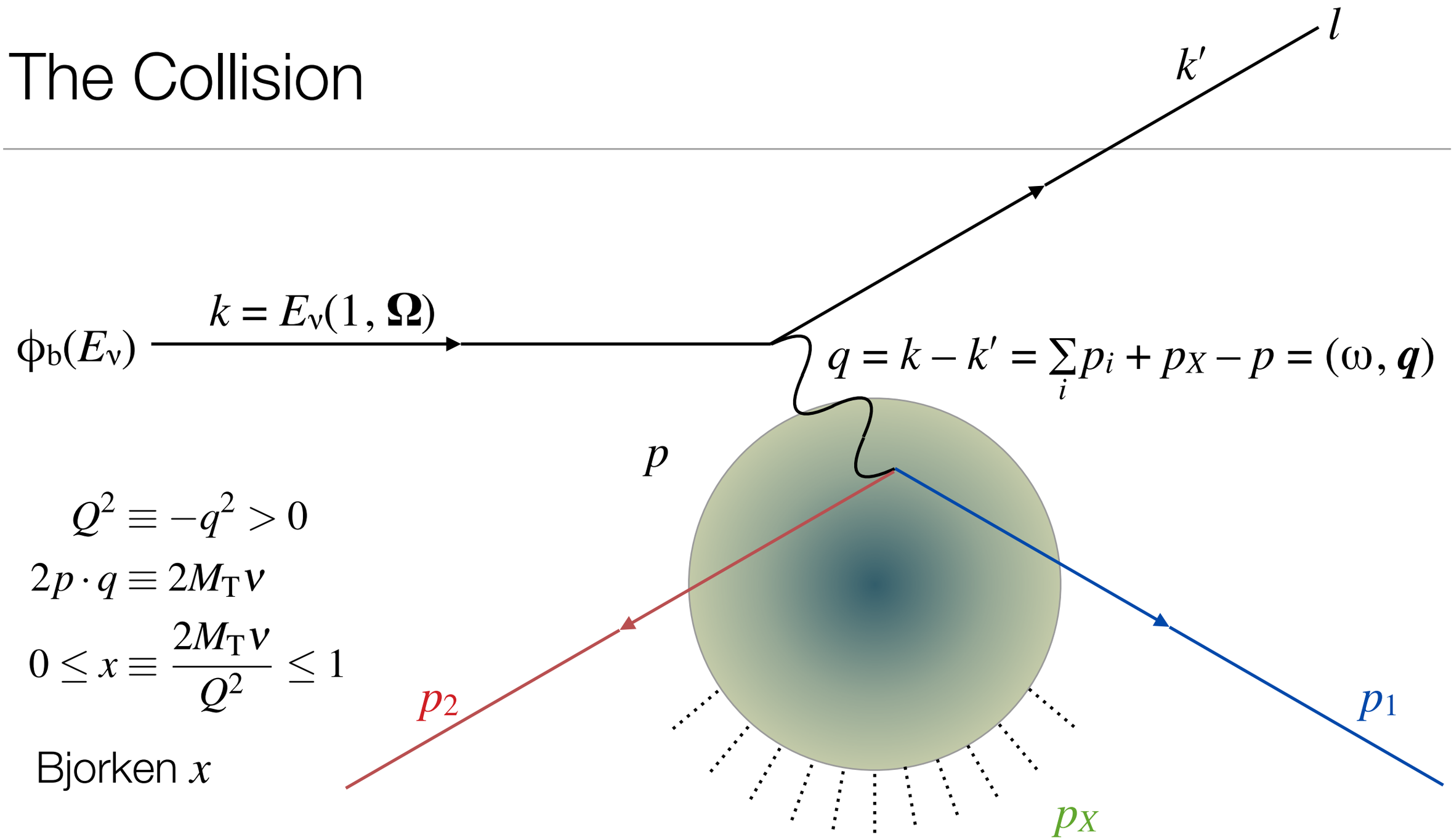
$$Q^2 \equiv -q^2 > 0$$

$$2p \cdot q \equiv 2M_T \nu$$

$$0 \leq x \equiv \frac{2M_T \nu}{Q^2} \leq 1$$

Bjorken x

The Collision



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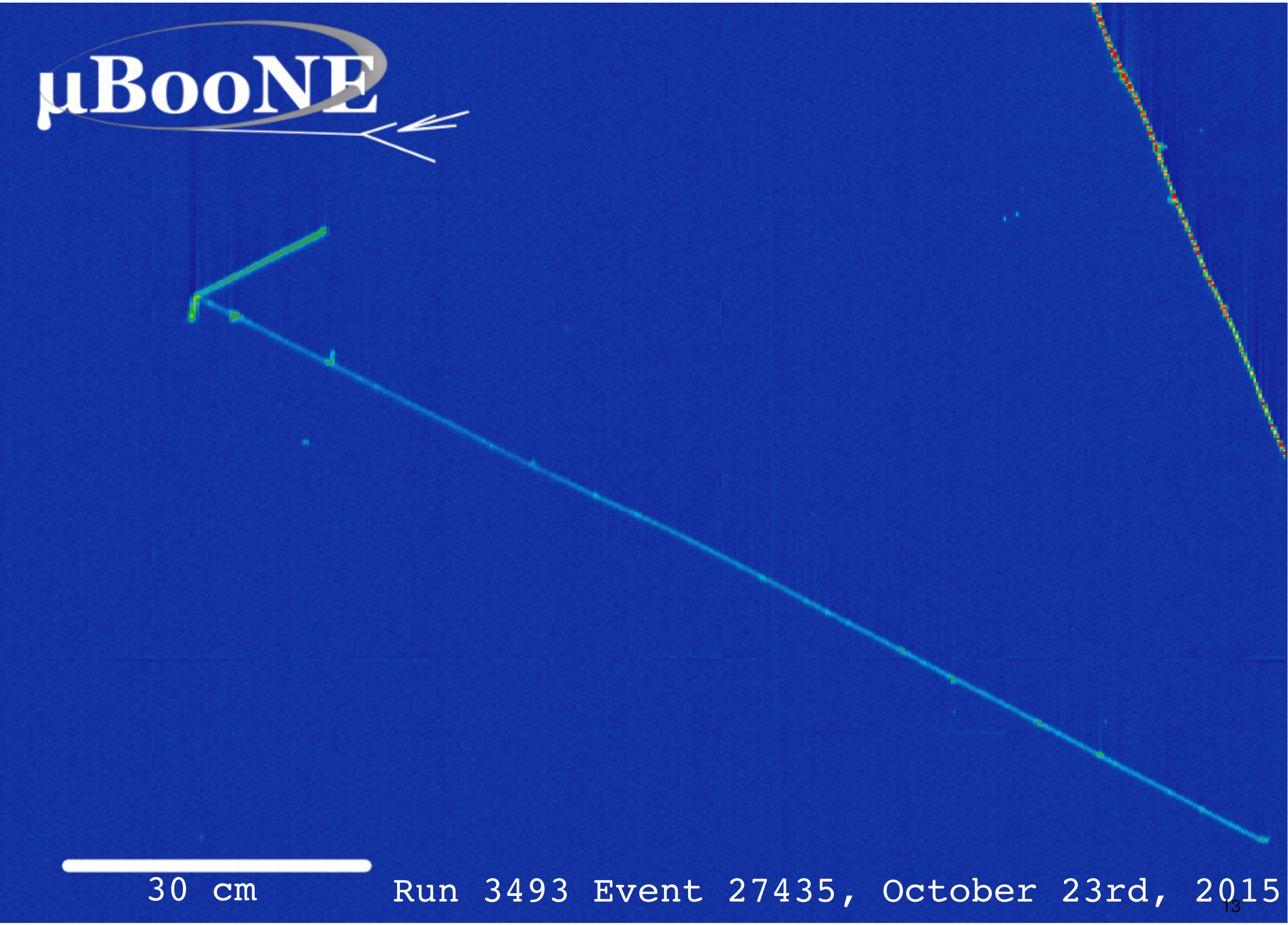
Bjorken x

knock out 2nd nucleon:
sign of “2p2h”


μBooNE
30 cm
Run 3493 Event 27435, October 23rd, 2015

Statement of the Problem I

- Measure 4-momentum of muon, probably of proton, maybe of a few more charged particles; just consider muon:

$$\frac{d\sigma[P_{\mathrm{b}}]}{dE_\mu d\cos\theta} = \int \frac{d\sigma(E_\nu)}{dE_\mu d\cos\theta} P_{\mathrm{b}}(E_\nu)\, dE_\nu$$

muon energy & direction

- Cross section proportional to lepton & hadron tensors:

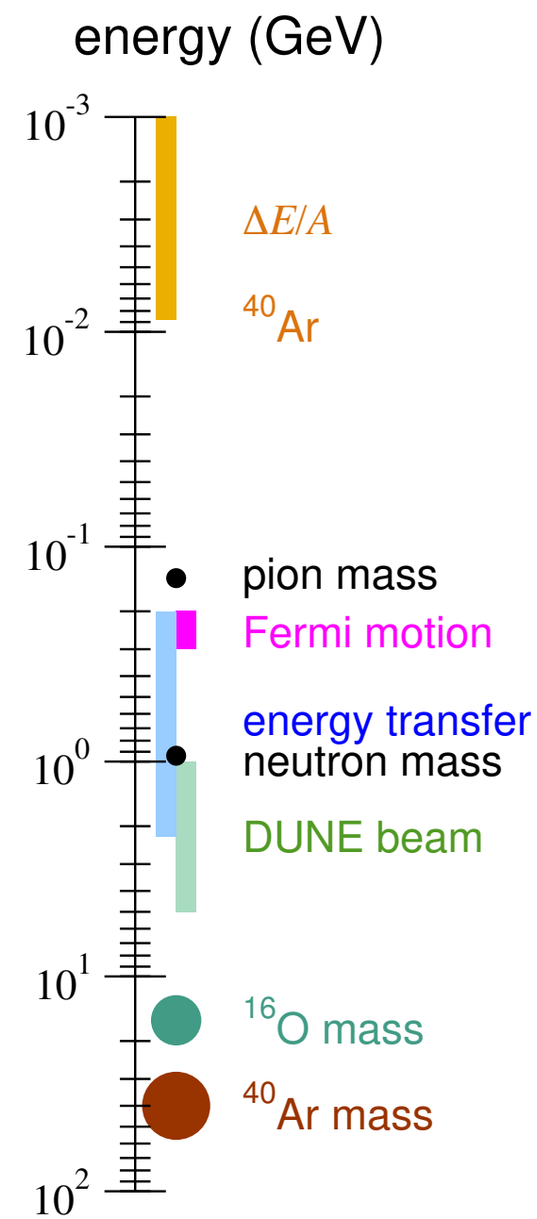
$$d\sigma(E_\nu) \propto L^{\mu\nu} W_{\mu\nu}$$

$$W_{\mu\nu} \propto \sum_{X_p} \langle A | \hat{J}_\nu^\dagger | X_p \rangle \langle X_p | \hat{J}_\mu | A \rangle$$

- The lepton tensor can be computed in leading-order electroweak perturbation theory.

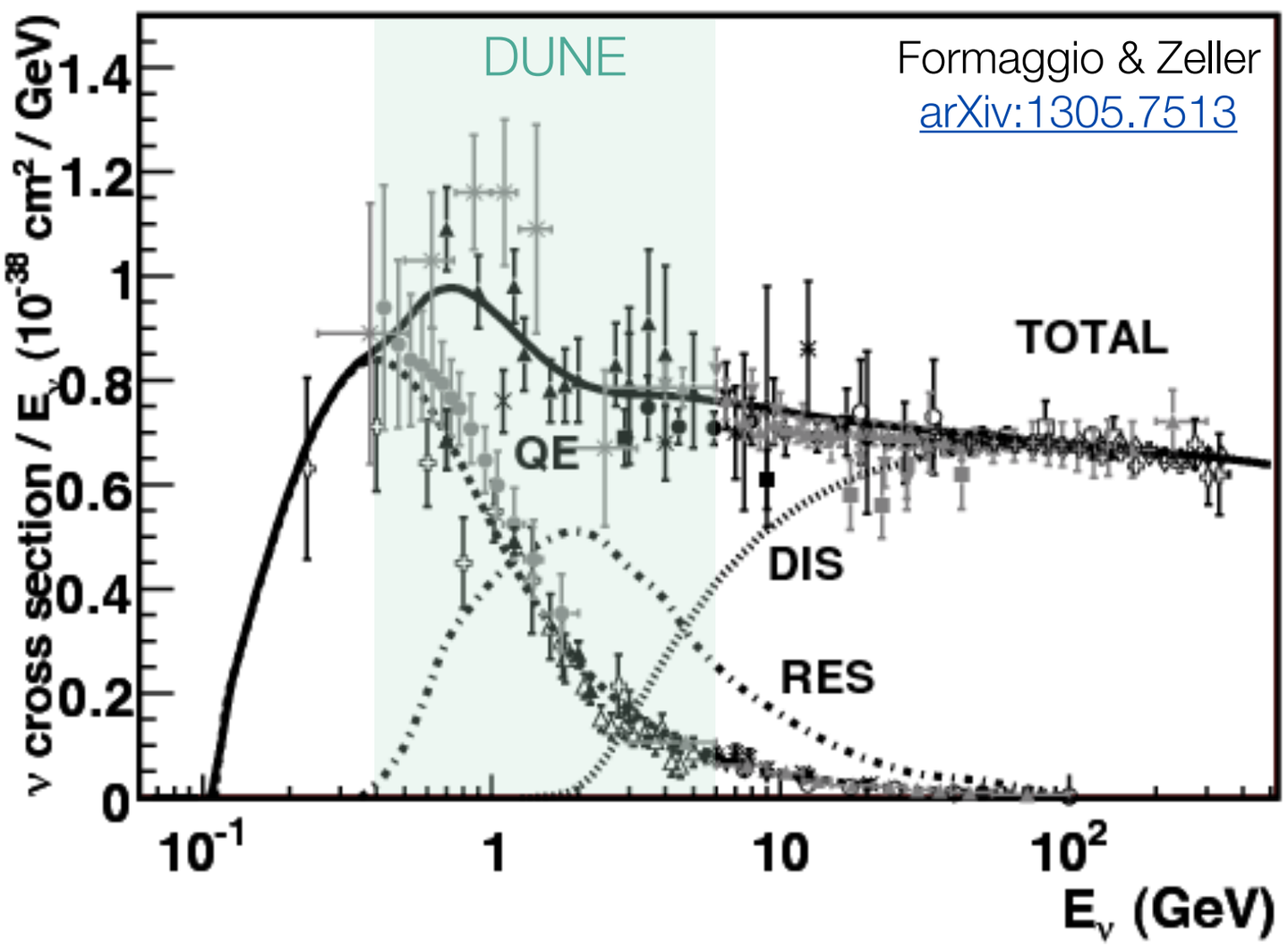
Energy Scales

- Thermal, molecular motion: $k_B T = 7.5$ meV, $16/A$ meV;
- Binding energy (per nucleon): 8.6 MeV (^{40}Ar);
- Pion mass: 140 MeV;
- Fermi motion of nucleons: 250 MeV;
- Energy transfer $\nu = E_\nu - E_l$: $200 \text{ MeV} < \omega < E_\nu - m_l$;
- Beam energy E_ν: $0.5 \text{ GeV} < E_\nu < 6 \text{ GeV}$ (DUNE);
- Nuclear target mass: $0.94A$ GeV, $A \in \{1, 2, 12, 16, 40\}$.



Different Calculations for Different Energy Transfers

- Have to know structure functions $W_{\mu\nu}$ reliably—a curve plus an (empirical?) error band—over an energy range where nuclear and nucleonic effects are



Different Calculations for Different Energy Transfers

- Have to know structure functions $W_{\mu\nu}$ reliably—a curve plus an (empirical?) error band—over an energy range where nuclear and nucleonic effects are both important:
  - quasielastic—scatter off nucleon;
  - resonance—enough energy for Δ, N^*, $N\pi$;
  - shallow inelastic scattering—many pions, but not enough for OPE;
  - deep inelastic scattering—but Q^2 not huge in oscillation experiments.

Different Calculations for Different Energy Transfers

- Have to know structure functions $W_{\mu\nu}$ reliably—a curve plus an (empirical?) error band—over an energy range where nuclear and nucleonic effects are both important:

• quasielastic—scatter off nucleon;	➡ elastic form factors;
• resonance—enough energy for Δ, N^*, $N\pi$;	➡ transition form factors with interference;
• shallow inelastic scattering—many pions, but not enough for OPE;	➡ hadron tensor of nucleon (small nuclei);
• deep inelastic scattering—but Q^2 not huge in oscillation experiments.	➡ parton distribution functions (also needed for LHC).

Lattice-QCD Calculations

Form Factors

- Hadronic matrix elements of the weak currents are decomposed into Lorentz covariant forms, multiplied by form factors:

$$\langle p(\boldsymbol{p})|\mathcal{V}^{\mu}_{\bar{u}d}|n(\boldsymbol{k})\rangle = \bar{u}_p(\boldsymbol{p})\gamma^{\mu}u_n(\boldsymbol{k})\,F_1(q^2) + \frac{q_\nu}{2M_N}\bar{u}_p(\boldsymbol{p})i\sigma^{\mu\nu}u_n(\boldsymbol{k})\,F_2(q^2),$$

$$\langle p(\boldsymbol{p})|\mathcal{A}^{\mu}_{\bar{u}d}|n(\boldsymbol{k})\rangle = \bar{u}_p(\boldsymbol{p})\gamma^{\mu}_{\perp}\gamma^5 u_n(\boldsymbol{k})\,F_A(q^2) + \frac{2M_N q^{\mu}}{q^2}\bar{u}_p(\boldsymbol{p})\gamma^5 u_n(\boldsymbol{k})\,F_P(q^2),$$

$$q^{\mu} = k^{\mu} - p^{\mu}, \quad Q^2 \equiv -q^2 > 0$$

aka G_A and G_P

- Some normalizations & definitions: $G_E = F_1 - Q^2\,F_2/M^2, \quad G_M = F_1 + F_2$

$$F_1(0) = 1, \quad F_1(0) + F_2(0) = \mu_p - \mu_n, \quad F_A(0) = g_A, \quad r_A^2 = \frac{6}{F_A(0)}\frac{dF_A}{dq^2}$$

Isovector Axial Charge Puzzle

- For neutrino experiments, known precisely from neutron β beta decay.

- However (*cf.*, Czarnecki, Marciano, Sirlin, arXiv:1802.01804):

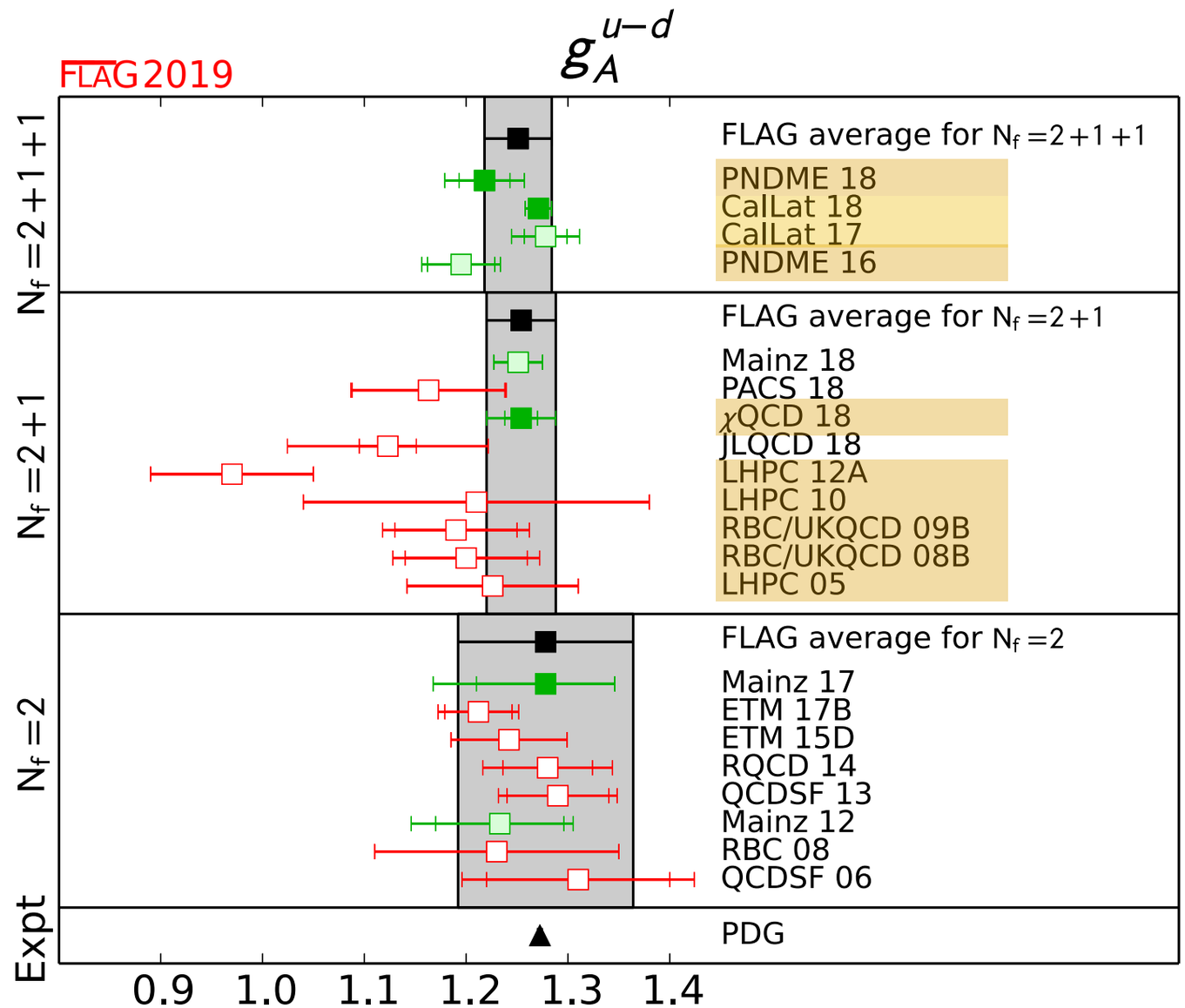
how	g_A
τ_n, trapped neutrons	1.2756 (5)
τ_n, neutron beams	1.2681(17)
asymmetries, $t < 2002$	1.2637(21)
asymmetries, $t > 2002$	1.2755(11)

so a set of lattice-QCD calculations with 0.5% error could be illuminating.

- An important benchmark in any case.

Isovector Axial Charge Status

- Compilation of Flavor Lattice Averaging Group $g_A^{u-d} = 1.251(0.033)$:
- USQCD results:
  - PNDME;
  - χQCD;
  - LHPC;
  - RBC/UKQCD.
- CalLat uses ensembles from MILC ⊂ USQCD.



Vector Form Factor from Lattice QCD

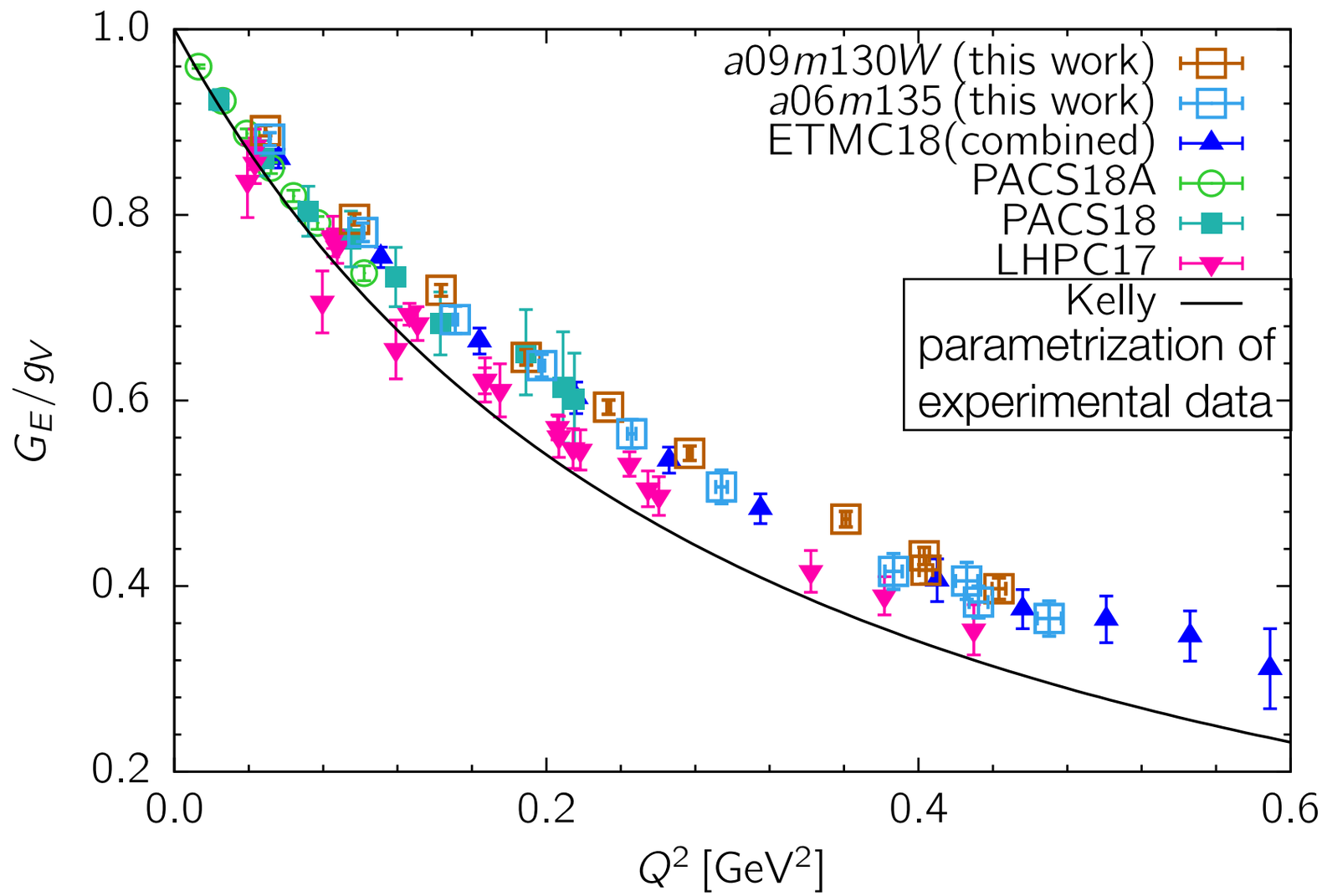
- PNDME ⊂ USQCD, arXiv:1901.00060 and arXiv:1906.07217:
- lattice QCD too high?
- In continuum limit:

  $r_E = 0.804(107)$ fm,

  vs CODATA 2018:

  $r_E = 0.8414(19)$ fm.*
- $G_E = F_1 - Q^2 F_2/M^2$.

*agrees with muonic hydrogen



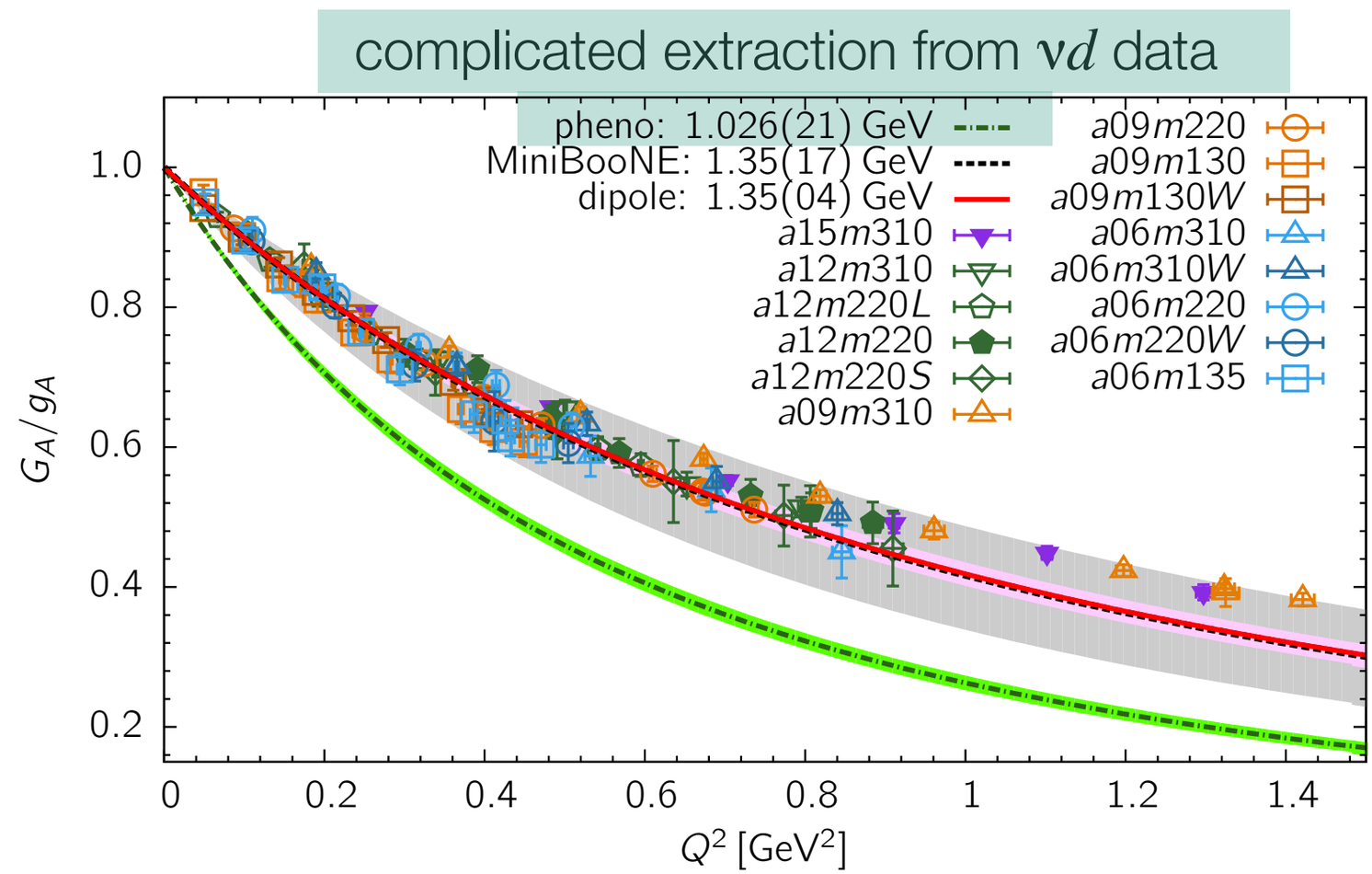
Axial Form Factor from Lattice QCD

- PNDME ⊂ USQCD, arXiv:1901.00060 and Lattice '19 talk:
- Lattice slopes smaller than that extracted from νd.
- Even continuum limit:

  $r_A = 0.481(85)$ fm,

  vs Meyer *et al.*:

  $r_A = 0.68(16)$ fm.
- *Caveat emptor* extraction from νd data.



Transition Form Factors

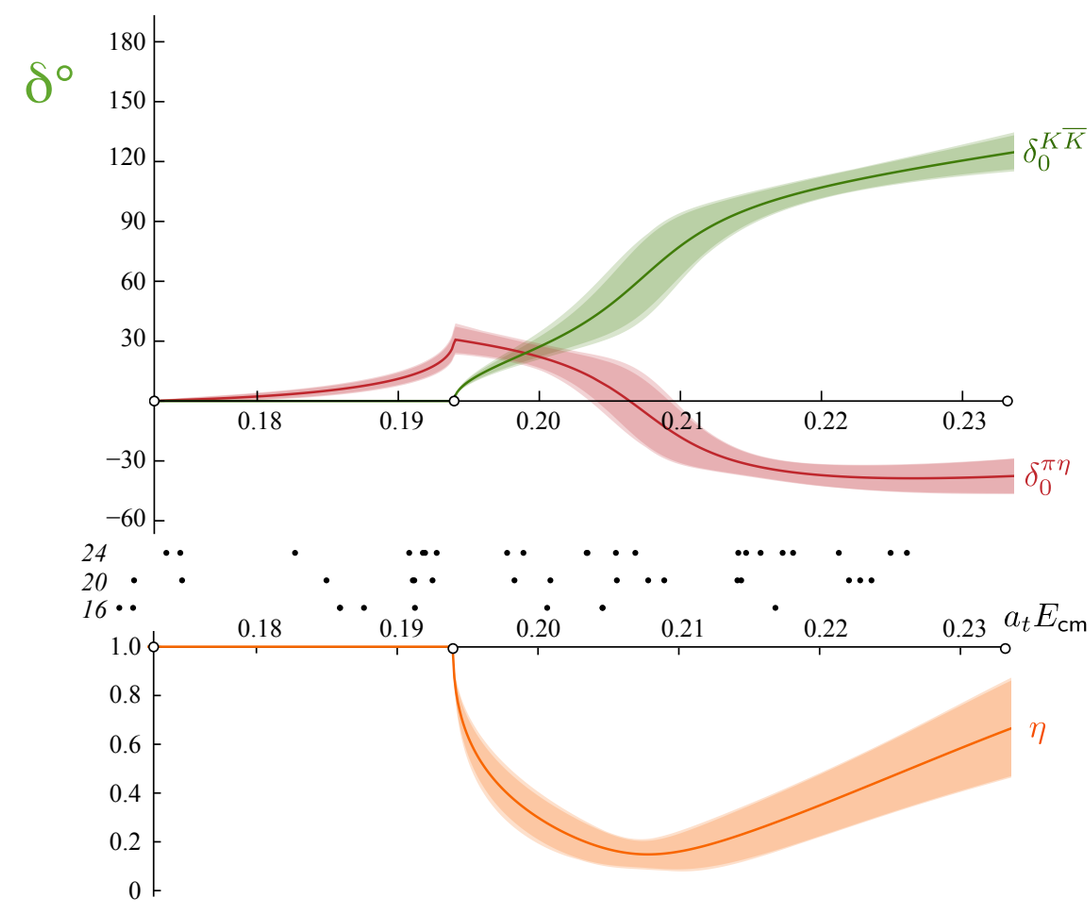
- At higher energy transfer, the struck nucleon can turn into an $N\pi$ state, possibly a resonance such as the Δ or the N^*.

- Finite-volume effects encode scattering & resonance information, even in Euclidean time (Lüscher, 1986).

- Example $a_0 \leftrightarrow \pi\eta \leftrightarrow K\bar{K} \leftrightarrow a_0$
  Hadron Spectrum ⊂ USQCD ☛

- Formalism for $JN \to N\pi$ available [arXiv:1502.04314] & being explored [arXiv:1807.08357 ($J\pi \to \pi\pi$)].

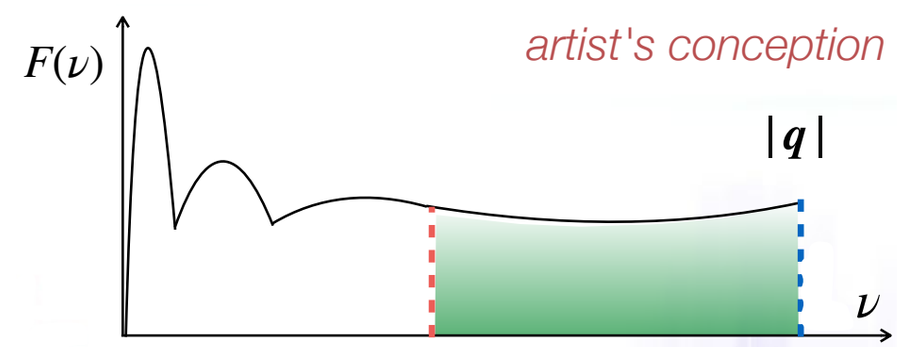
- Full error budget will be challenging.



Hadron Tensor

- The 4-point Euclidean correlation function $\langle N(w) J_\mu(x) J_\nu^\dagger(y) \bar{N}(z) \rangle$ contains information related to the hadron tensor.

- Can test in elastic region (form factor squared?) and proceed beyond resonance region. Can extend into DIS region $\Longrightarrow$ PDFs.



- Frame independent.

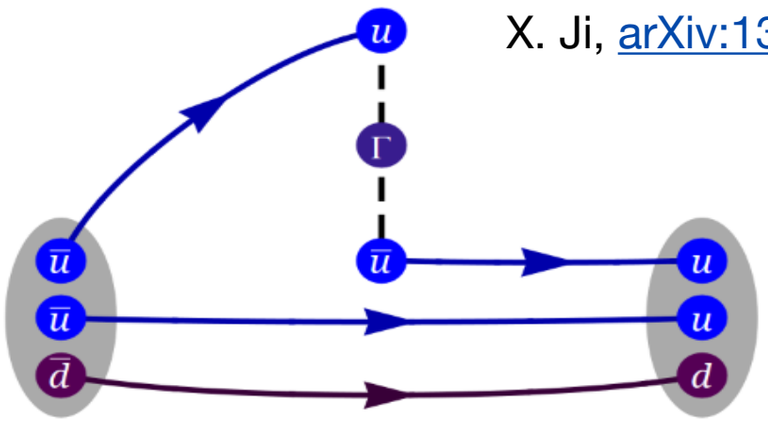
- Needs no (new) renormalization.

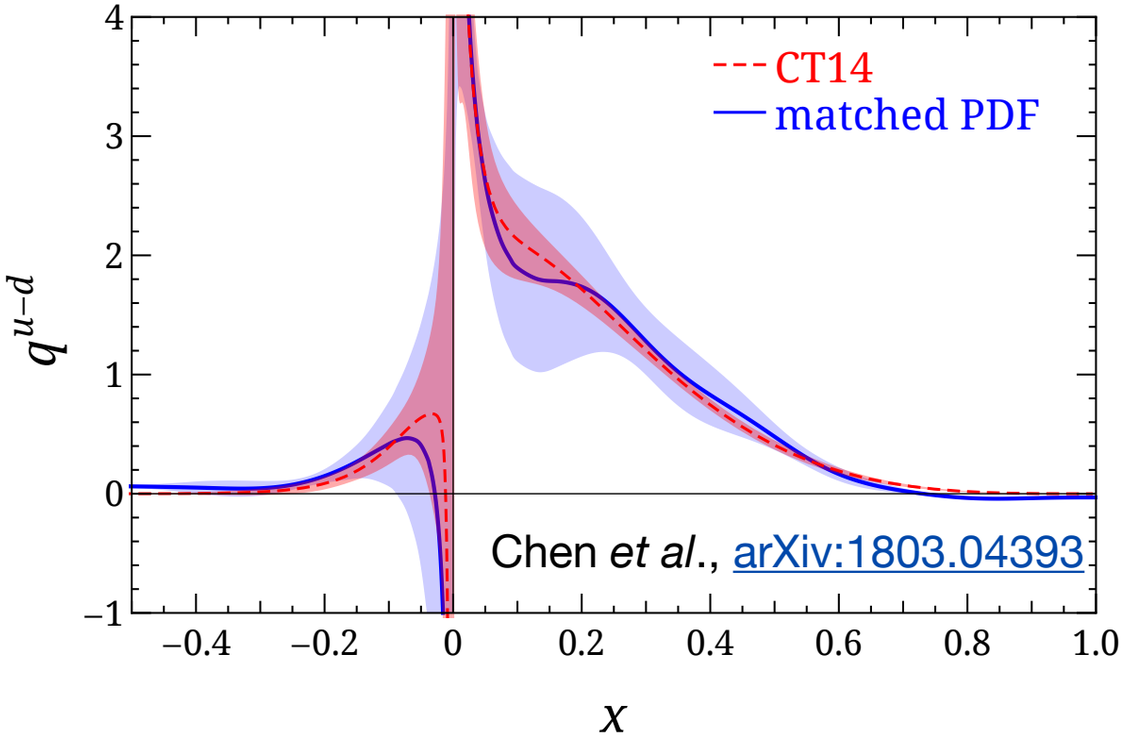
- Brand new USQCD project: Jian Liang (PI), Terrence Draper, Keh-Fei Liu, Alexander Rothkopf, Yi-Bo Yang—talk at Lattice 2019.

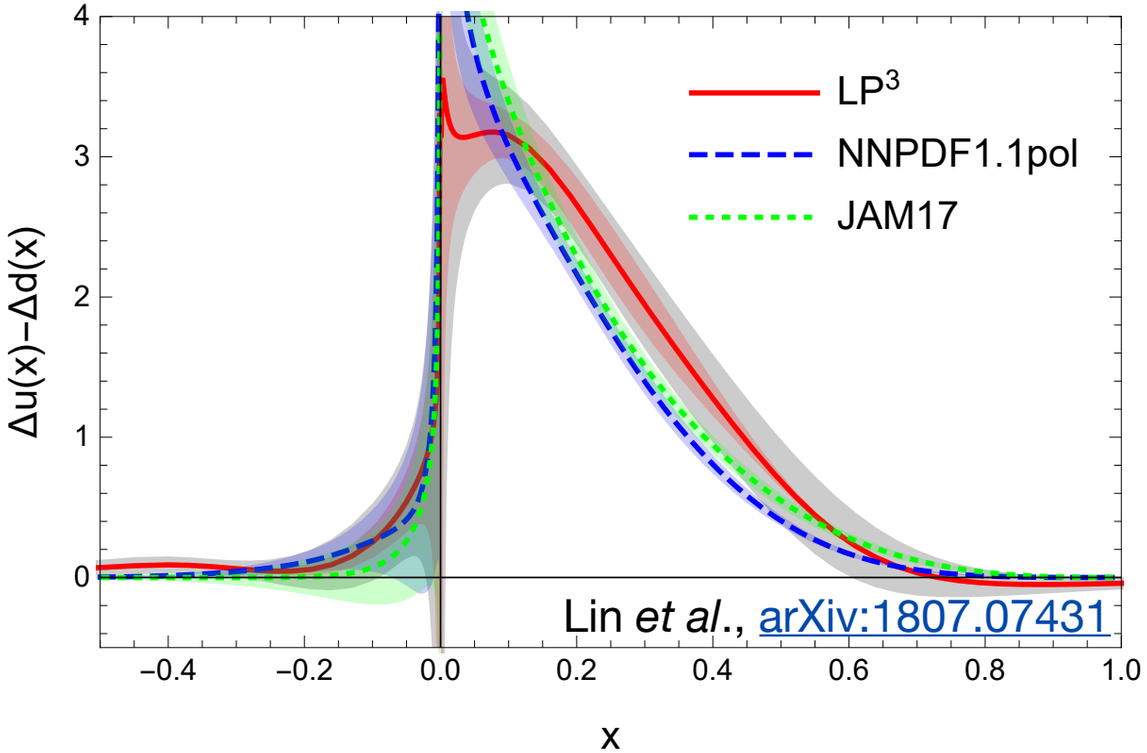
- Idea hatched in whitepaper discussions [arXiv:1904.09931].

Parton Distribution Functions

- Any improvement benefits LHC searches.
- Key measure of nucleon structure.
- Explosion of numerical & theoretical work ⟸ first USQCD studies ↓ ↘







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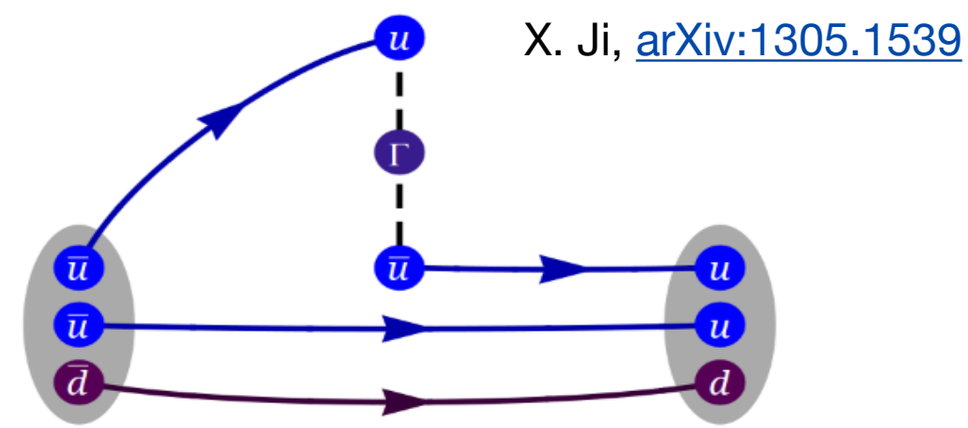
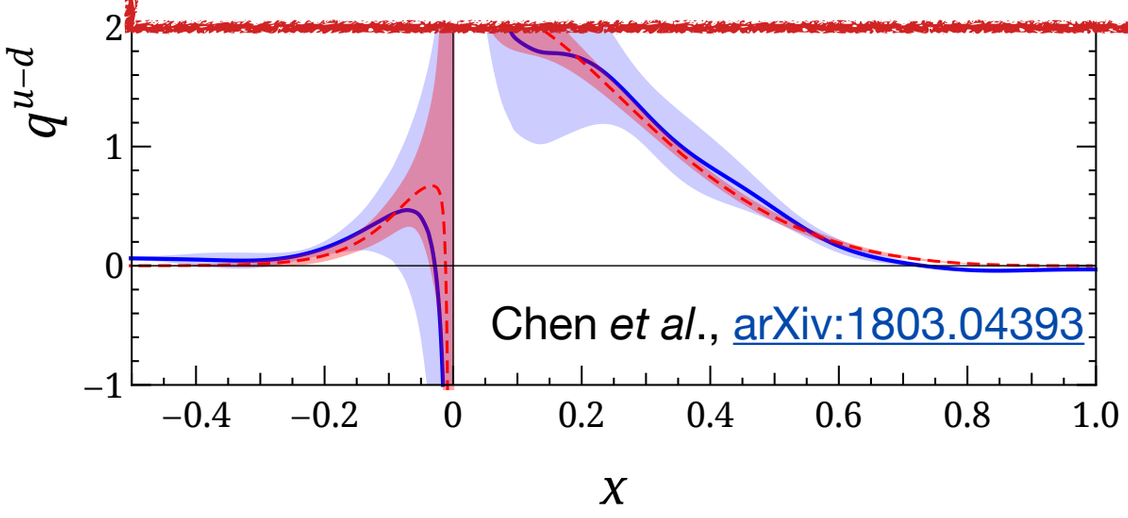
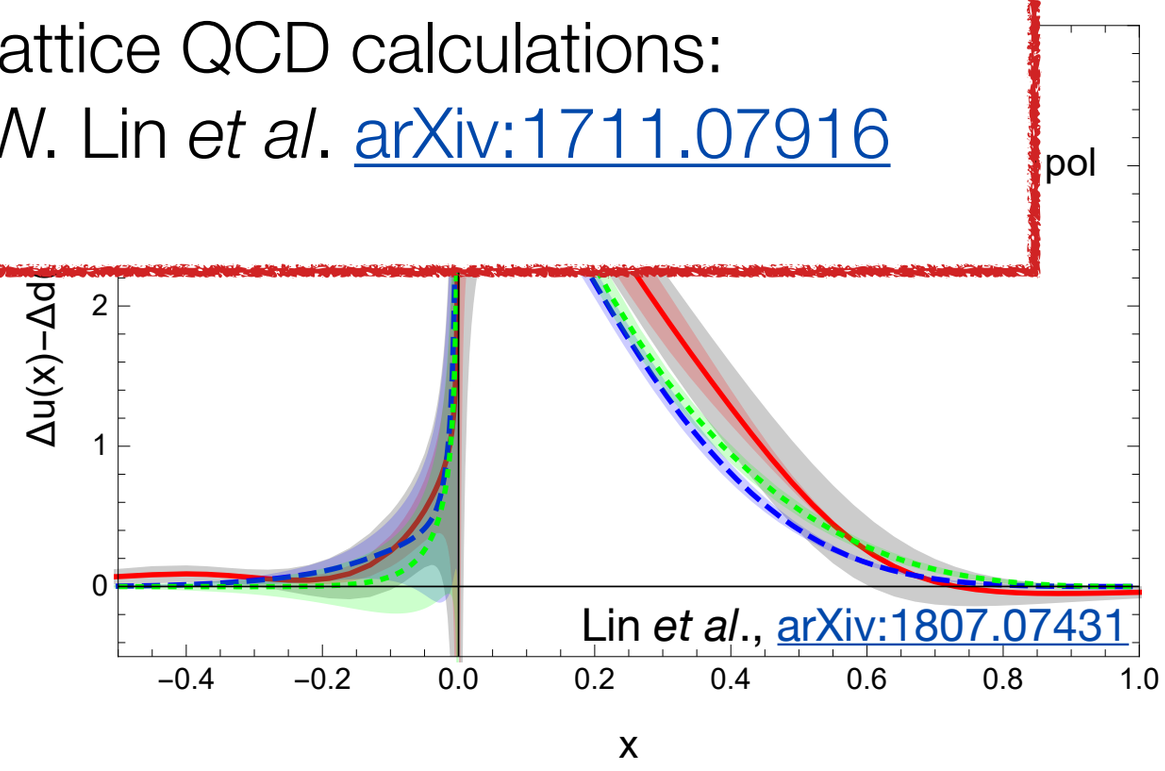


Parton distributions and lattice QCD calculations:
A community white paper, H.-W. Lin *et al*. arXiv:1711.07916





Summary

Neutrino-Nucleus Roadmap

- At the energies used by experiments measuring oscillation parameters, the nucleus is treated as a collection of weakly interacting nucleons:
  - use nuclear theory for nuclear structure;
  - use *ab initio* nucleon-level calculations for neutrino-nucleon transition matrix elements including, eventually—
    - $\nu N \to eX$, $X = N'$ or $N\pi$ or Δ or …, i.e., matrix element $\langle N|V-A|N\rangle$ &
    - two-body effects: $\langle NN|V-A|NN\rangle$, *cf*. arXiv:1812.10504;
  - use nuclear transport (e.g., GiBUU) theory for post-collision absorption/ secondary production of pions.

Lattice-QCD Roadmap

- Use lattice QCD to compute nucleon form factors with full error budgets.
- Explore plethora of lattice-QCD methods for parton distribution functions:
  - including hadron tensor in resonance/shallow inelastic region.
- Use lattice QCD to compute hadron tensor & PDFs with full error budgets.
- Compute matrix elements of small nuclei to test/pin down EFT (*cf*. Zohreh).
- Engage with experimentalists, generator developers, and nuclear theorists to solidify understanding of neutrino-nucleus cross sections.

Questions and Discussion

Backup

Parametrizations

- When data are poor (large uncertainties, limited q^2 range), any functional form will do, as long as the shape is not aggressively stupid.

- Nucleon form factors often have employed "dipole Ansatz":

$$f(t) = \frac{g_A}{\left(1 - t/M_A^2\right)^2}$$

[Llewellyn-Smith, 1972].

- Instead use properties of field theory: analyticity, unitarity, …, *e.g.*, the "z expansion", Padé approximants (dispersion relation is Stieltjes), ….

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not QCD

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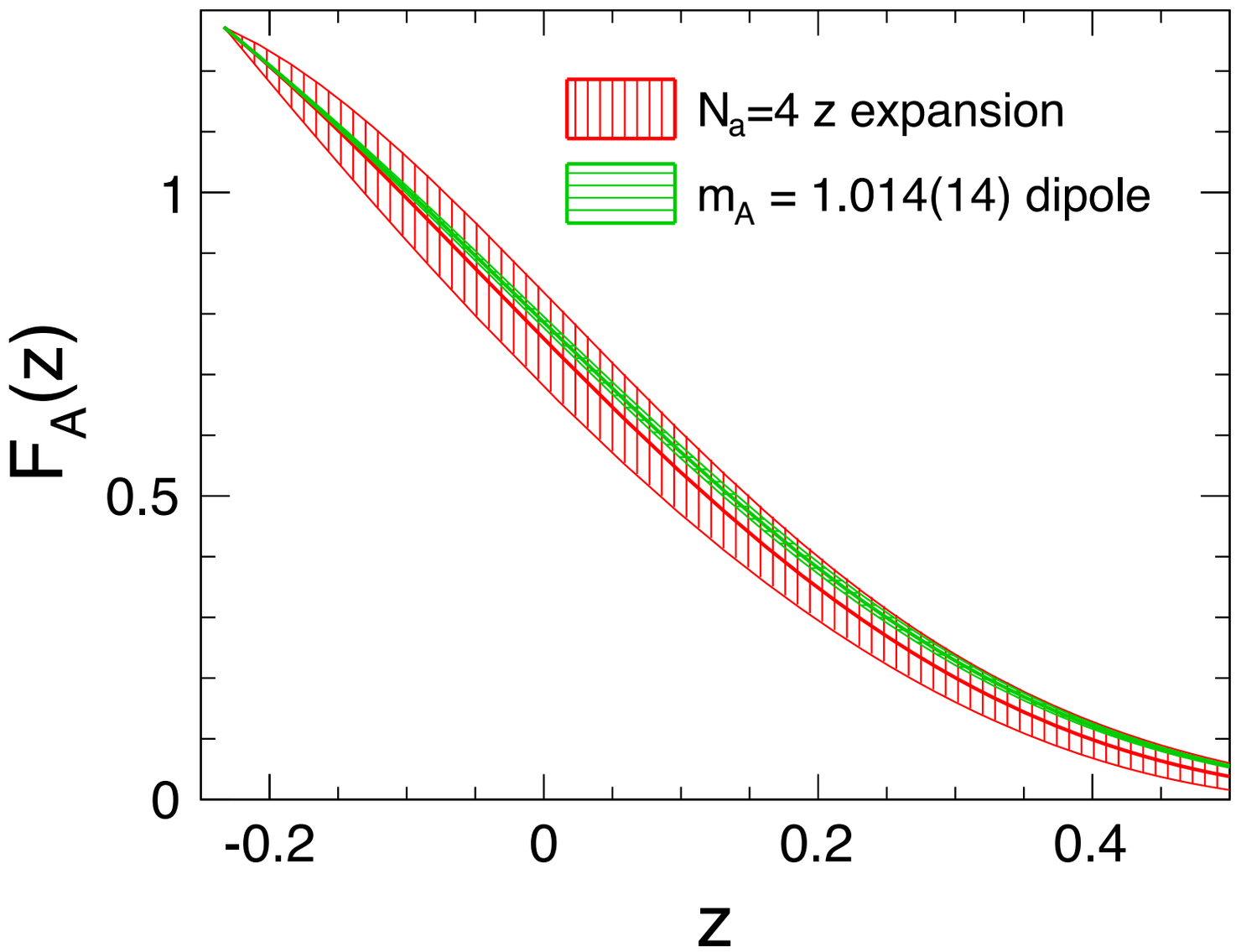
Axial Form Factor from Deuterium

- Meyer, Betancourt, Gran, Hill [arXiv:1603.03048]:



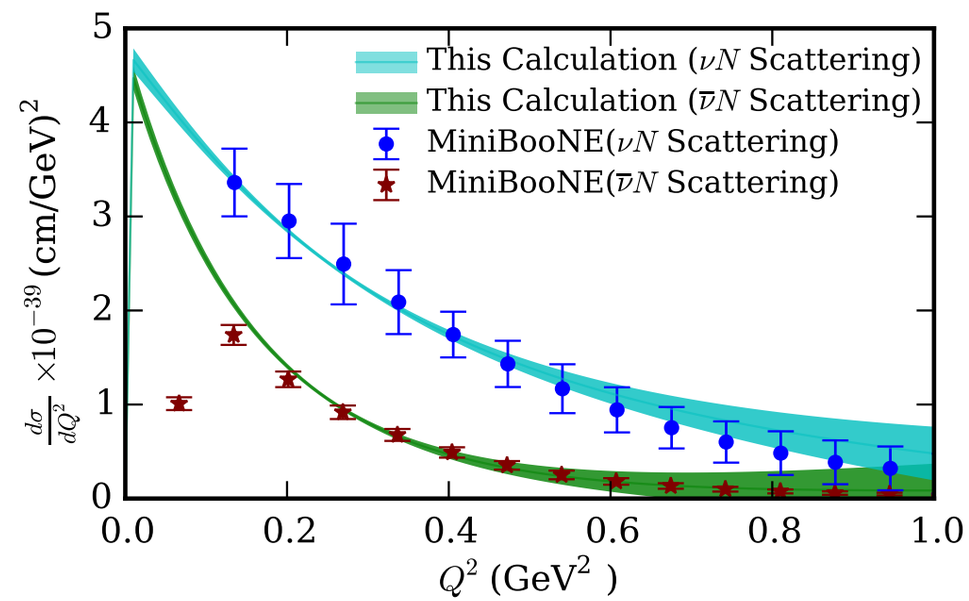
$$z = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

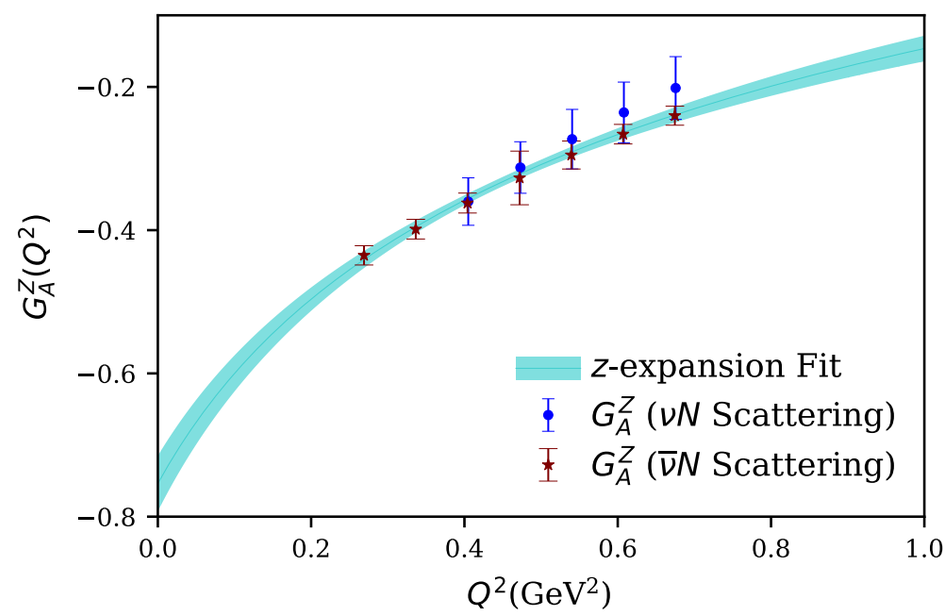
$$f\left(t(z)\right) \propto \sum_n a_n z^n$$

dipole underestimates uncertainty!

Neutrino Phenomenology

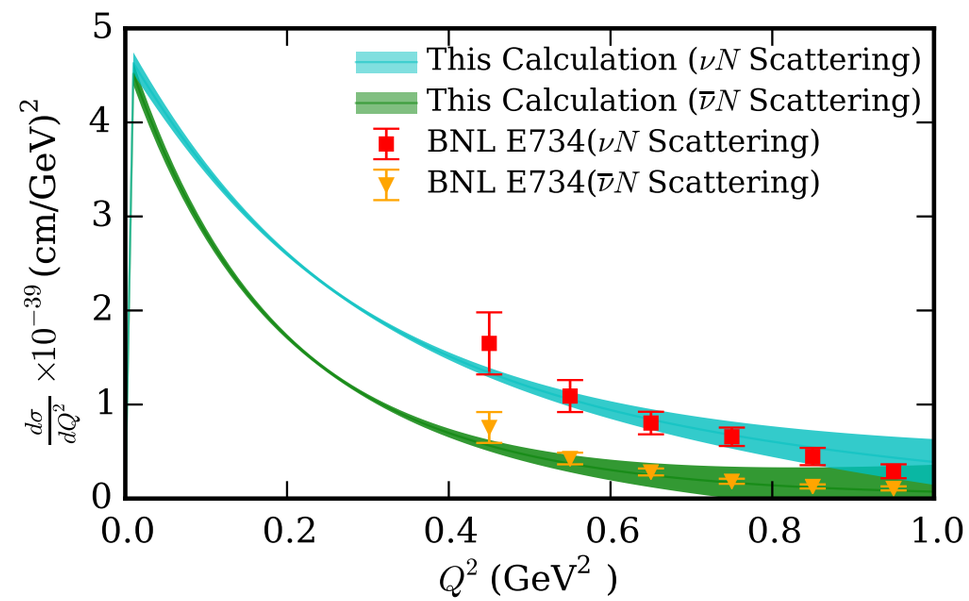
- Sufian, Liu, Richards [arXiv:1809.03509].
- Pragmatic inputs:
  - vector form factors from ep scattering (z parametrization) [Ye *et al.*, arXiv:1707.09063];
  - $s\bar{s}$ form factor from lattice QCD [χQCD, arXiv:1705.05849];
  - extract $G_A(Q^2(z))$ from MiniBooNE;
  - compare with BNL E734.

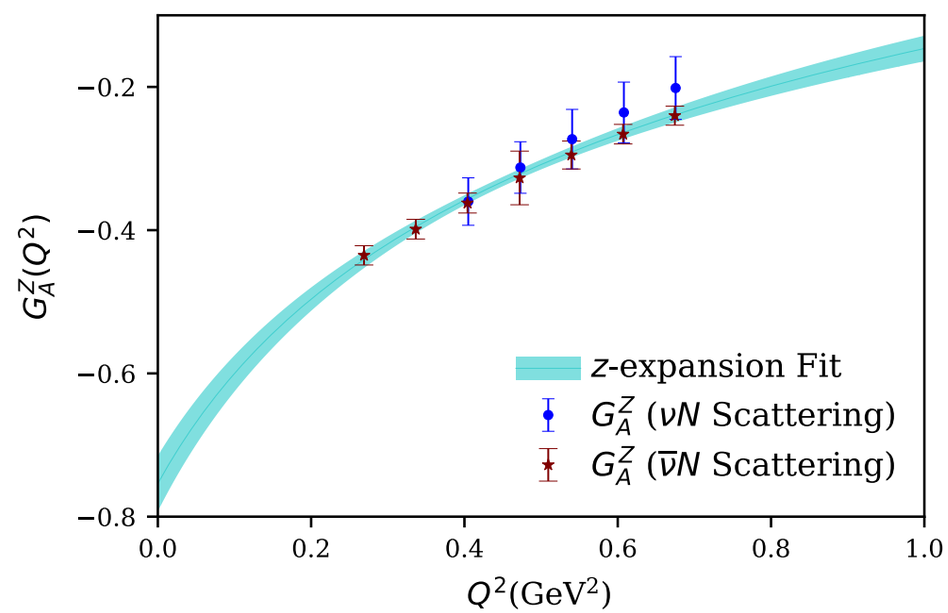




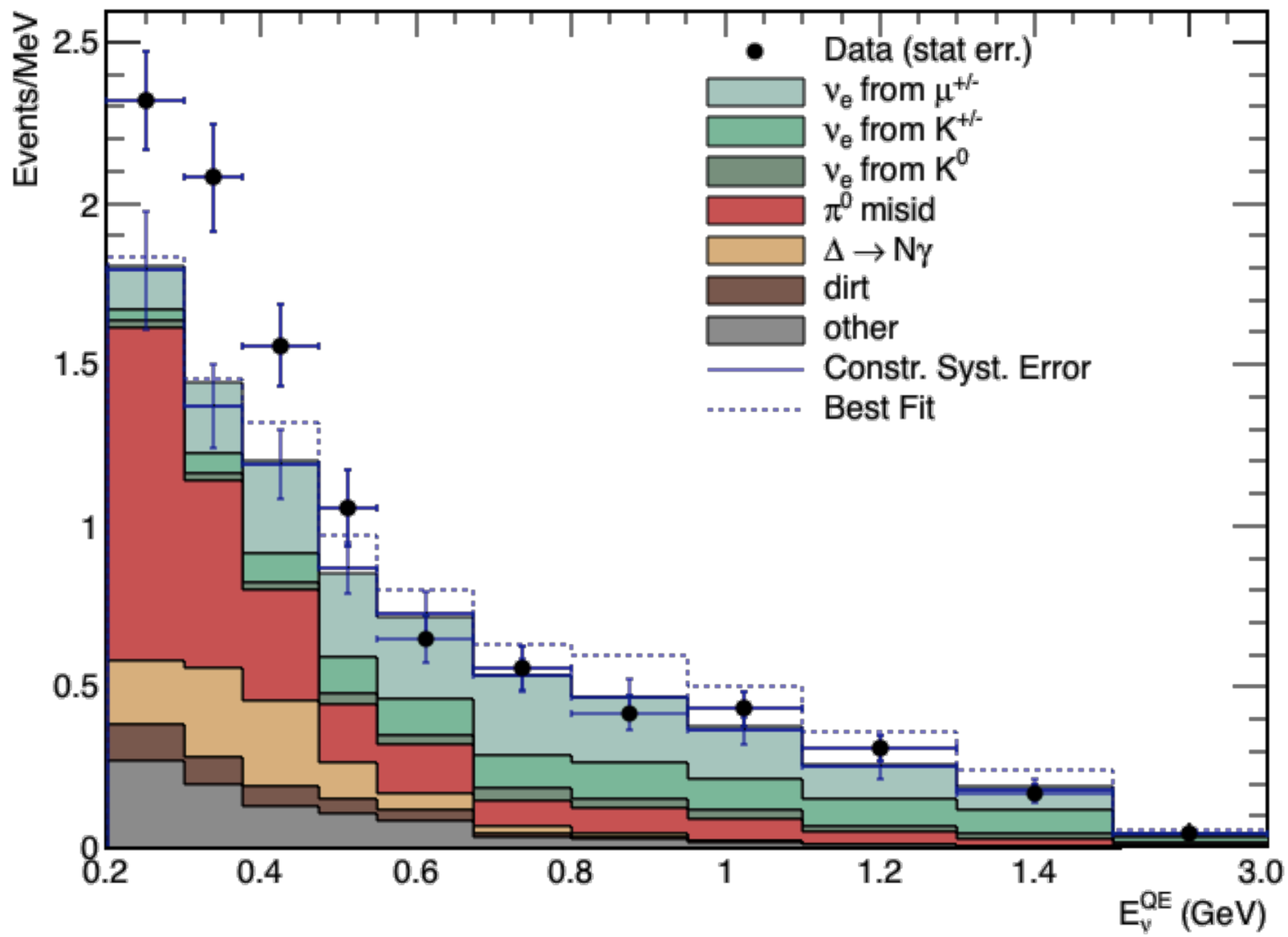
Neutrino Phenomenology

- Sufian, Liu, Richards [arXiv:1809.03509].
- Pragmatic inputs:
  - vector form factors from ep scattering (z parametrization) [Ye *et al.*, arXiv:1707.09063];
  - $s\bar{s}$ form factor from lattice QCD [χQCD, arXiv:1705.05849];
  - extract $G_A(Q^2(z))$ from MiniBooNE;
  - compare with BNL E734.





MiniBooNE Excess

- What do lattice-QCD form factors say about this?

- Aguilar-Arevalo *et al.* (MiniBooNE) [arXiv:1805.12028].