Solenoid Focusing in an Ion Linac

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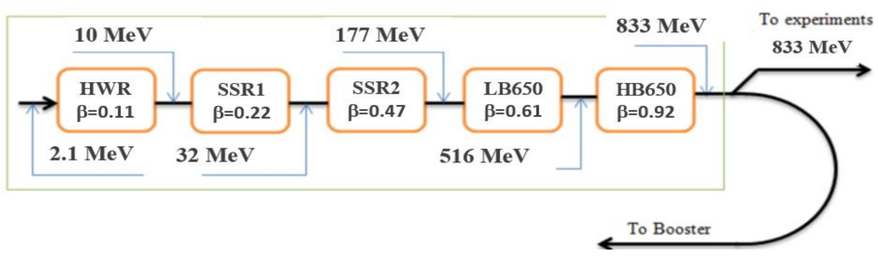
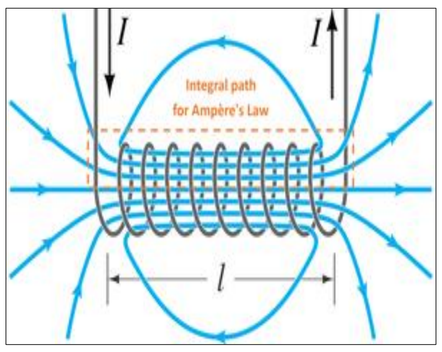
PIP-II Superconducting RF Linac

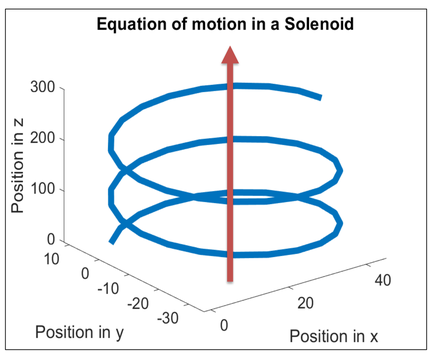
- PIP-II is a linear accelerator that encompasses a set of upgrades and improvements to Fermilab accelerator complex
- The central element of PIP-II is a new 800 MeV superconducting Linac accelerating H^- ions and located in close proximity to the existing Booster
- It accelerates the beam from 2.1 MeV to 800 MeV and includes five types of SC cavities to cover the entire velocity range required for acceleration of H^- (or protons)



In the first three sections, HWR, SSR1 and SSR2, superconducting solenoids were chosen to provide transverse focusing. In the last two sections, LB650 and HB650, quadrupoles were selected

Solenoids Focusing





Charged particle trajectory in a solenoid using Matlab in terms of time. As expected, it has a helical motion

- Solenoids are a long straight coil of wire that can be used to generate a nearly uniform magnetic field
- The magnetic field is concentrated in the center which produces longitudinal fields
- Used to contain and transport low momentum particles in helical trajectories along the field lines and are utilized to focus low energy beam
- Using the force equation **F** = q**v** x **B**, we can find the trajectory for a charged particle in a solenoid

$$x = \frac{v_{x0}\sin(\omega t) - v_{y0}\cos(\omega t) + v_{y0}}{\omega} + x_0$$

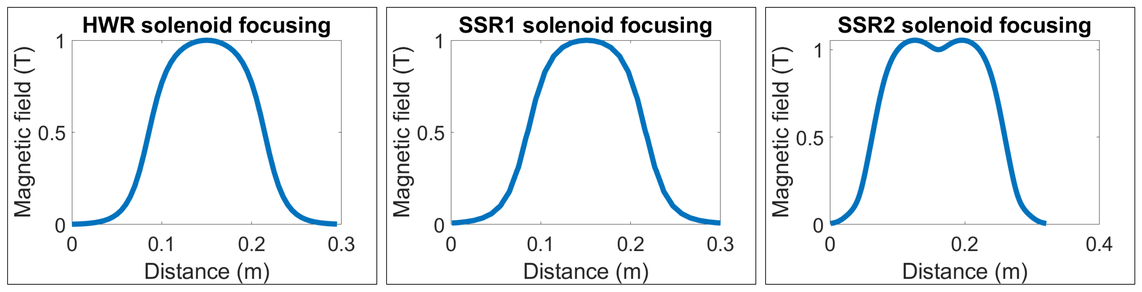
$$y = \frac{v_{y0}\sin(\omega t) + v_{x0}\cos(\omega t) - v_{x0}}{\omega} + y_0$$

Objective

Solenoids provide axial symmetric focusing and are compact relative to quadrupole magnets. Quadrupoles focus beam in one plane while defocusing in the other plane. HWR, SSR1 and SSR2 cavities result in a weak quadrupole defocusing. Solenoids could compensate this weak asymmetric focusing. The objective of this work is to develop a theory to understand and explain this effect.

PIP-II SRF Solenoids

In PIP-II, there are parameters that are calculated to understand the particle beam dynamics and to verify the charged particle path. Data was collected from simulations of the magnetic fields for HWR, SSR1, and SSR2. Using the data, we calculated solenoid parameters used for PIP-II Linac in order to keep the particles on a designed path.

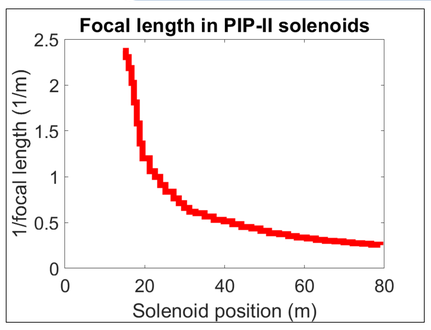


Magnetic fields in HWR, SSR1 and SSR2 solenoids, normalized for 1 T

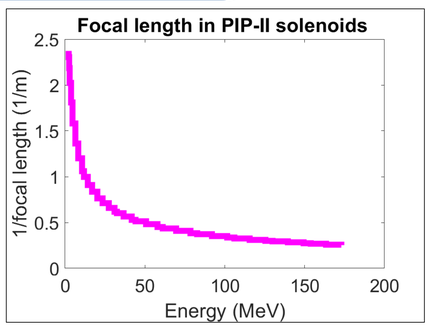
	Effective field (T)	Effective length (m)	Total Integration field (T)
HWR	0.8124	0.1590	0.1292
SSR1	0.8021	0.1674	0.1342
SSR2	0.91	0.2230	0.2030

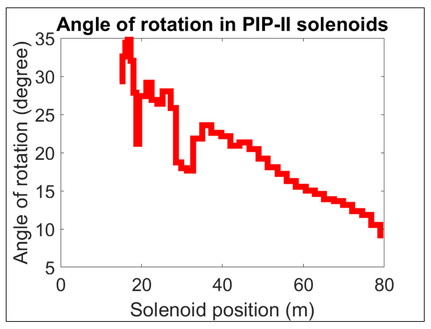
- Parameters were calculated using $B_{axis} = 1\,T$

Focal Strength and Angle in PIP-II Linac

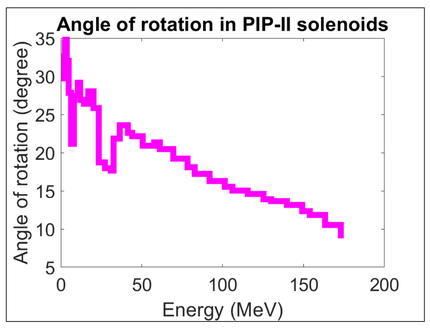


$$\frac{1}{f} = k_s^{\,4}\int_{z1}^{z2} B^2(z)\,dz$$





$$\theta = -\int_{z1}^{z2} k^2\,dz$$



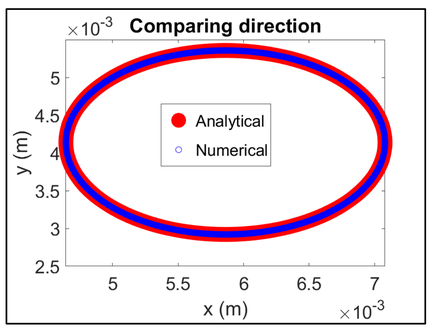
Implication of Cavity Quadrupole Defocusing In Solenoidal Focusing Channel

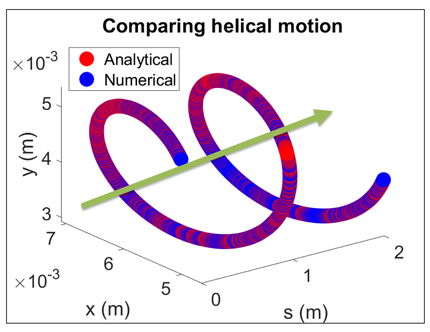
$$x'' - k_s^{\,2}y' + k_q^{\,2}x = 0$$
$$y'' + k_s^{\,2}x' - k_q^{\,2}y = 0$$

Where

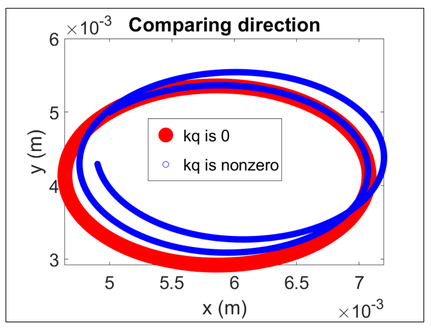
$$k_s = \sqrt{\frac{qB}{2mc\gamma\beta}} \qquad k_q = \sqrt{\frac{qG}{mc\gamma\beta}}$$

- Two scenarios, one with the equation of motion in a solenoid from a relativistic perspective, in terms of position, where $t = \frac{s}{v_z}$ (analytical), and one with a system of coupled second order differential equations solved numerically using Matlab

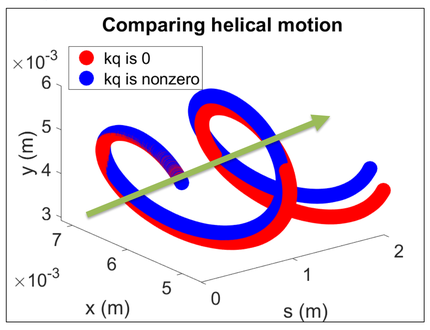




Both plots above are with the case of $k_q = 0$



- Compared both solutions and observed the charged particle's trajectory from the data collected



Both plots above are with the case of $k_q \neq 0$

Summary

A study was performed to understand charge particle motion in the solenoid. Equation of motion was derived analytically as well as solved numerically. Results were benchmarked for two cases. Solenoidal focal lengths and beam rotation angles in the PIP-II SRF linac were evaluated. Equation of motion in a solenoid focusing channel with a quadrupole field component is solved and analyzed. The theory is still being developed. This work will contribute to further research on applications of solenoid focusing.