Magnetic Field Measurement in Undulators

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Introduction

- When electrons travelling at a speed close to the speed of light, *c*, are radially accelerated, they lose a large amount of energy by *Synchrotron Radiation* (SR).
- Synchrotron Radiation is used in various fields such as solid-state physics, atomic physics, medicine and so on. Many electron storage rings, including the Advanced Photon Source (APS) at Argonne, are installed to produce Synchrotron Radiation.
- *Undulator*, a device allowing electrons to follow a periodic and undulating trajectory in which SR waves interfere, is used in APS to create a high luminosity of radiation. Consequently, the measurement of magnetic field in undulators, from which we can determine electron trajectory and hence several quality factors of undulators such as slippage and phase error, is important for improving the luminosity of Synchrotron Radiation.
- In an undulator, the electrons cannot keep up with SR wave, which travels at *c*. Also, alternating magnetic field in an undulator causes electron oscillation, which adds path length to electron trajectory. These two facts cause the electron to lag behind the SR wave. We define *slippage* to be the distance between a point on SR wave and the electron. Slippage is thus a factor that determines the quality and efficiency of an undulator.

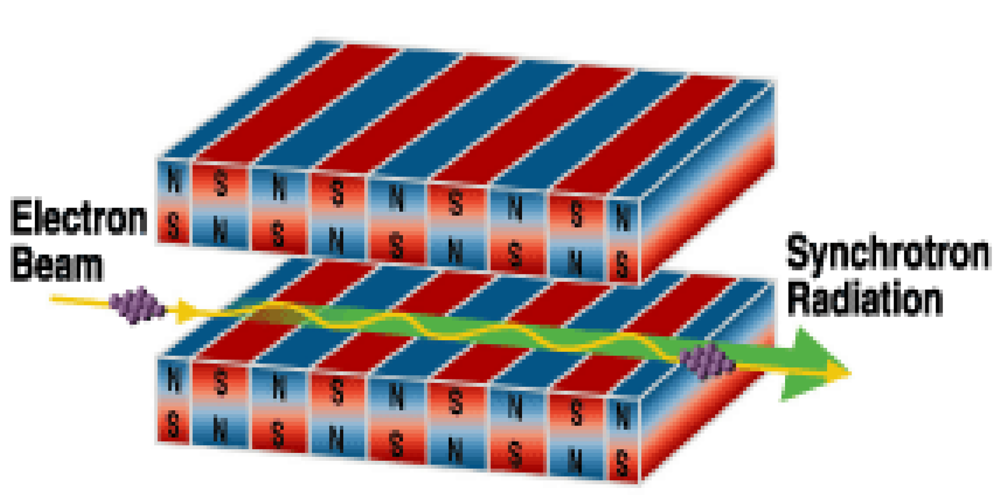

Figure 1. A Model of Undulator.
Periodic combination of magnets creates a strong magnetic field in which an electron beam oscillates with a small deviation angle. Thus, the interference effect associated with oscillation produces synchrotron radiation.

Purpose of this Project

- Our lab is currently developing superconducting undulators (SCUs), a kind of undulators that produce high magnetic field thanks to the superconducting property, which can be later inserted into the updated APS storage ring.
- This project aims to use the data of magnetic field measurement acquired in the superconducting undulators to generate an algorithm to find electron trajectory. After developing the integration routine to find electron trajectory, we plan to generate further routines to find slippage and phase error of SCUs, the quality factors that determine the luminosity of Synchrotron Radiation.

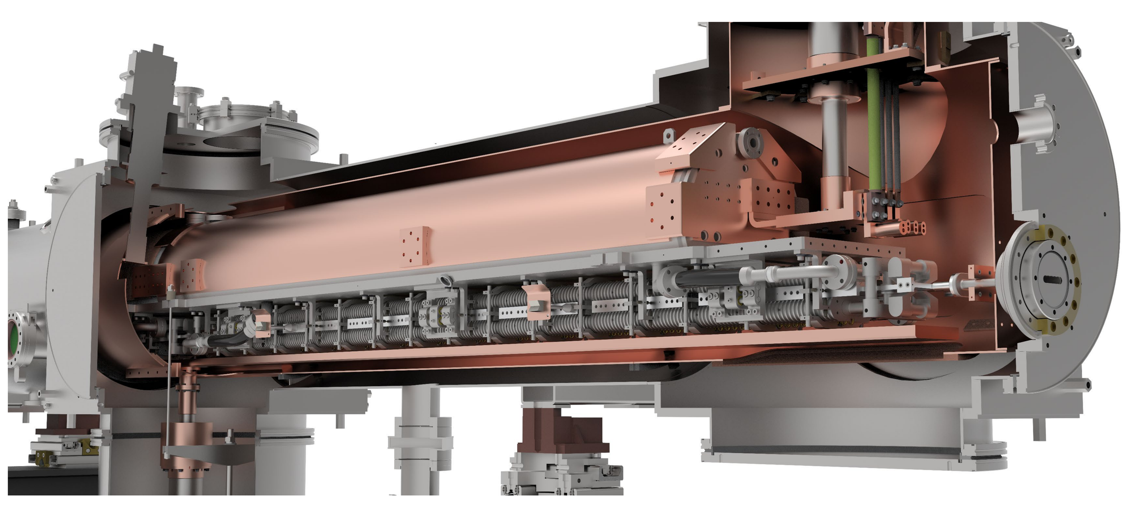

Figure 2. Design Picture of SCU in APS at Argonne National Laboratory.
Superconducting magnets are installed in a cryostat. Liquid Helium is injected to create low temperature.

Methods

- The theoretical basis for this project is as follows. Suppose that *z* is the direction of electron beam along the undulator, *y* is the upward direction, and *x* is determined by the right-hand coordinate system. Lorentz equation tells $\frac{d}{dt}v = \frac{q}{\gamma m} v \times B$, which can be decomposed in this coordinate system as

$$\frac{d}{dt}v_x = -\frac{q}{\gamma m}v_z B_y \quad ; \quad \frac{d}{dt}v_y = \frac{q}{\gamma m}v_z B_x \qquad (1)$$

- Since the electron is accelerated to a speed comparable to *c*, we have $v_z \gg v_x$ and $v_z \gg v_y$. Thus, we have $v_z = \frac{dz}{dt}$, so $\frac{d}{dt} = \frac{dz}{dt}\frac{d}{dz} = v_z\frac{d}{dz}$. Let the prime notation be the derivative with respect to *z*. Then (1) can be written as

$$x'' = -\frac{q}{\gamma m v_z}B_y \quad ; \quad y'' = \frac{q}{\gamma m v_z}B_x \qquad (2)$$

- Integrating (2) gives the **slope** of the electron trajectory:

$$x'(z) = -\frac{q}{\gamma m v_z}\int_{z_0}^{z} B_y(z_1)\,dz_1 \;;\; y'(z) = \frac{q}{\gamma m v_z}\int_{z_0}^{z} B_x(z_1)\,dz_1 \qquad (3)$$

- Integrating (3) gives the **position**:

$$x(z) = -\frac{q}{\gamma m v_z}\int_{z_0}^{z}\int_{z_0}^{z_2} B_y(z_1)\,dz_1 dz_2 \;;\, y(z) = \frac{q}{\gamma m v_z}\int_{z_0}^{z}\int_{z_0}^{z_2} B_x(z_1)\,dz_1 dz_2 \qquad (4)$$

- We can also find the **slippage**:

$$S(z) = \int_{z_0}^{z}\left(\frac{1}{2\gamma^2} + \frac{1}{2}x'^2 + \frac{1}{2}y'^2\right)dz_1 \qquad (5)$$

- We use LabVIEW as coding environment to render (3), (4), and (5).

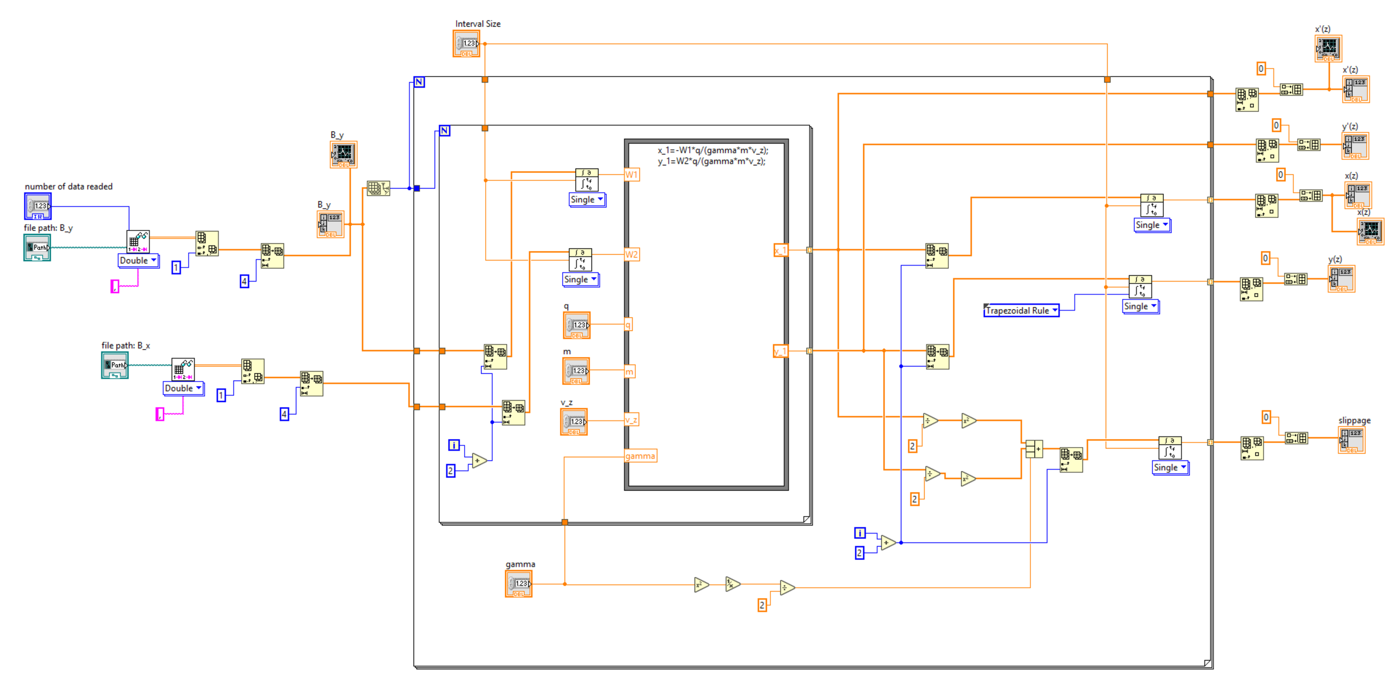

Figure 3. A part of LabVIEW VI to generate the slope of electron trajectory, position, and slippage.
The program reads the CSV files which contain the data of magnetic field measurement in an SCU. It then utilizes numeric integration to realize the desired algorithm.

Results and Future Plan

- This LabVIEW integration routine generates desired parameters. The results correspond with the expected features of an SCU.
- Further algebraic manipulations will be added to this VI to generate other quality factors of an SCU, including K_{eff}, B_{eff}, and phase error.

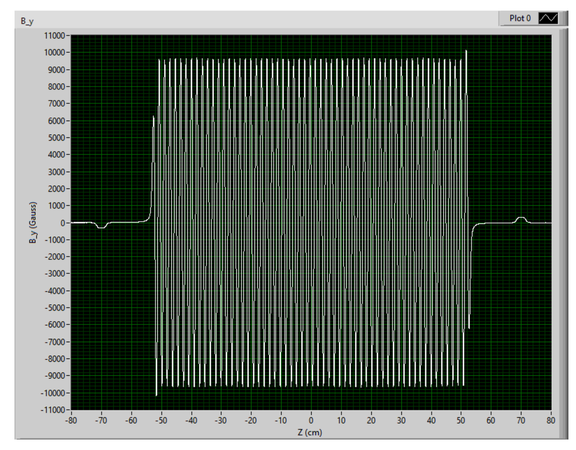

Figure 4. A plot of B_y.
This plot is generated by the data of one scanning of an SCU in APS at Argonne.

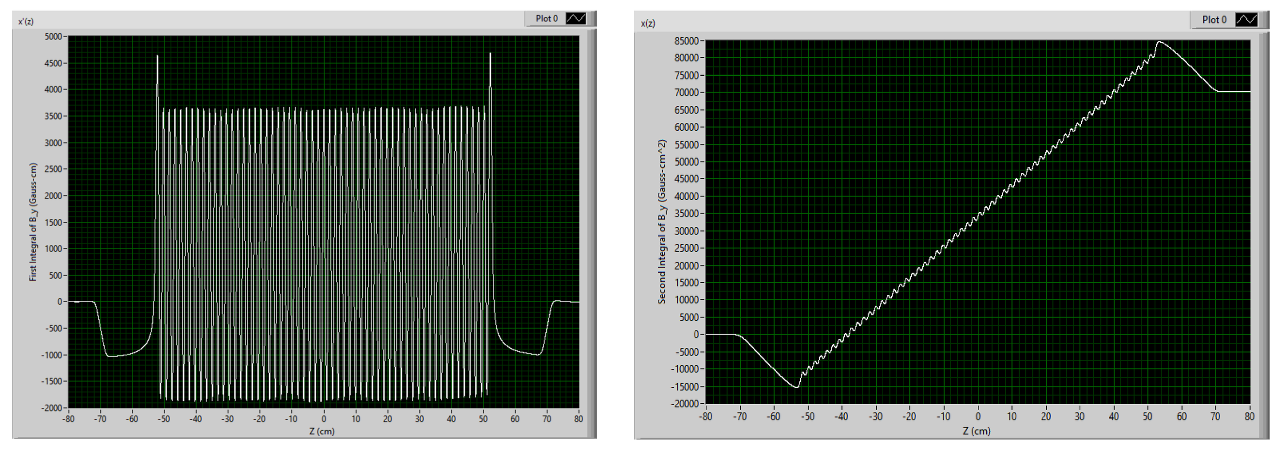

Figure 5. A plot of the first and second integrals of B_y.
The two plots are generated by the above LabVIEW VI. These results can be multiplied with the scaling factor $-\frac{q}{\gamma m v_z}$ to obtain $x'(z)$ and $x(z)$.

References

- *Figure 1*: "Synchrotron radiation produced at an undulator", RIKEN and Japan Synchrotron Radiation Research Institute.
- *Figure 2*: "Design Picture of SCU", ASD Division, Argonne National Laboratory.
- Clarke, J.A. *The Science and Technology of Undulators and Wigglers*; Oxford University Press: New York, 2004.
- Wolf, Z. *Introduction to LCLS Undulator Tuning*. No. LCLS-TN-04-7. 2004.

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