



Muon ($g-2$): the view from the lattice

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Outline

Goal: clarify where the SM theory prediction comes from, focusing on the QCD contributions.

1. Muon (g-2) and QCD

2. Light-by-light scattering

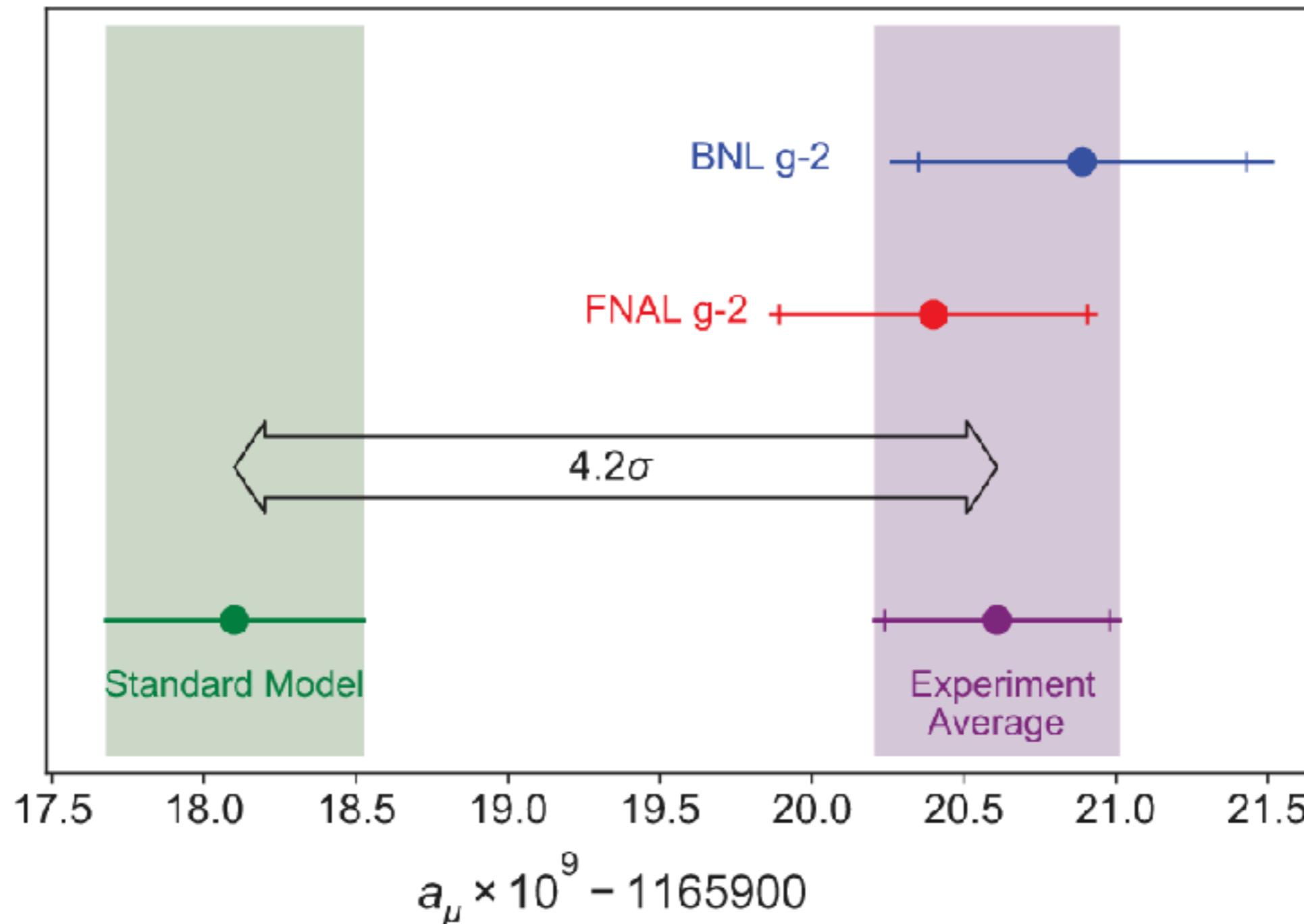
3. Hadronic vacuum polarization

- Muon (g-2) Theory Initiative whitepaper: T. Aoyama et al,
[arXiv:2006.04822](https://arxiv.org/abs/2006.04822)
- Snowmass update Theory Initiative whitepaper: G. Colangelo et al.,
[arXiv:2203.15810](https://arxiv.org/abs/2203.15810)
- 5th plenary workshop of the Theory Initiative, Higgs Center, University of Edinburgh, 09/22: <https://indico.ph.ed.ac.uk/event/112/>

Useful refs:

Fine print: I am involved with the Muon (g-2) Theory Initiative, but I am not giving this talk on behalf of the Initiative; opinions expressed are my own.

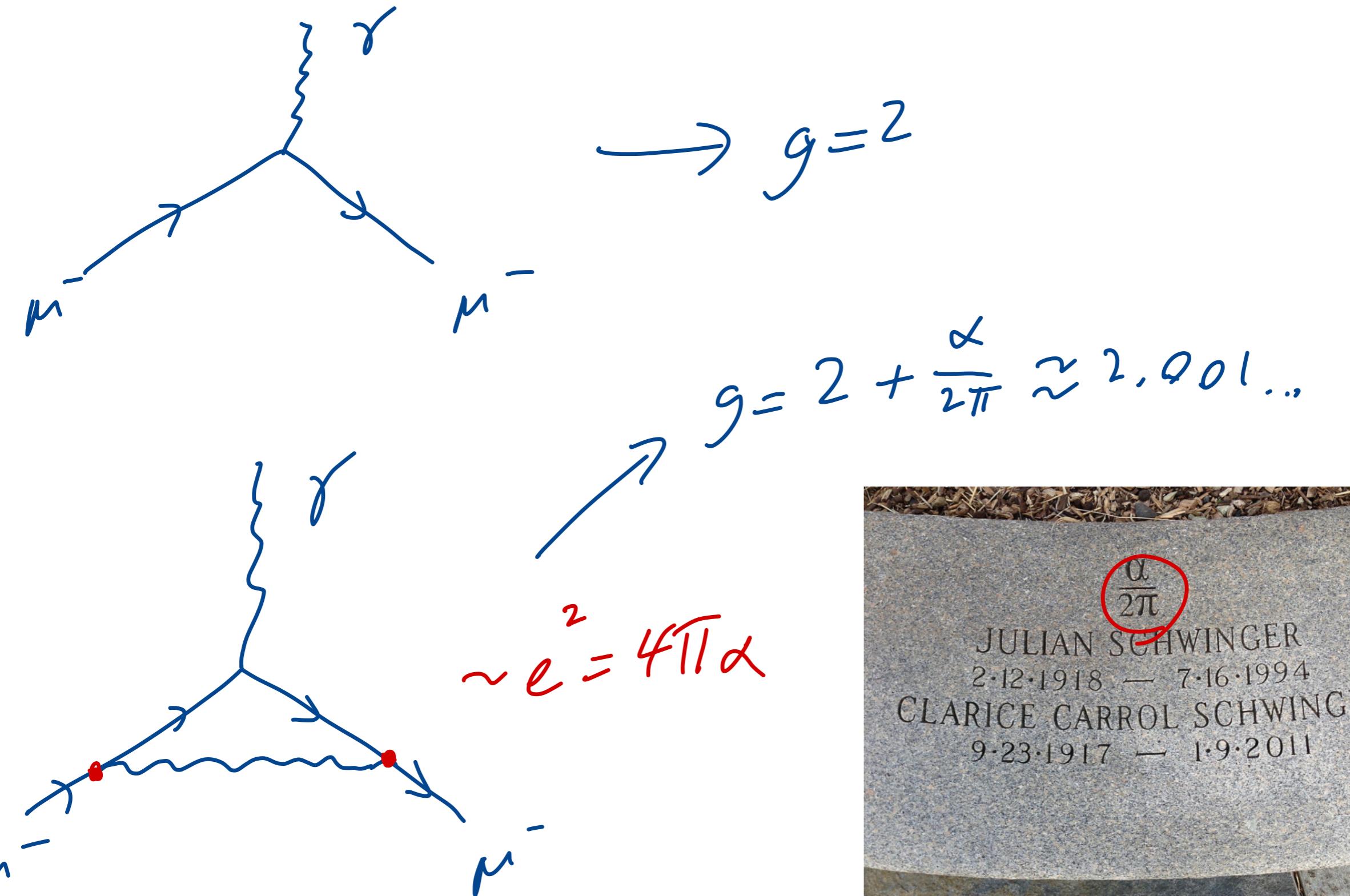
Summary of current results



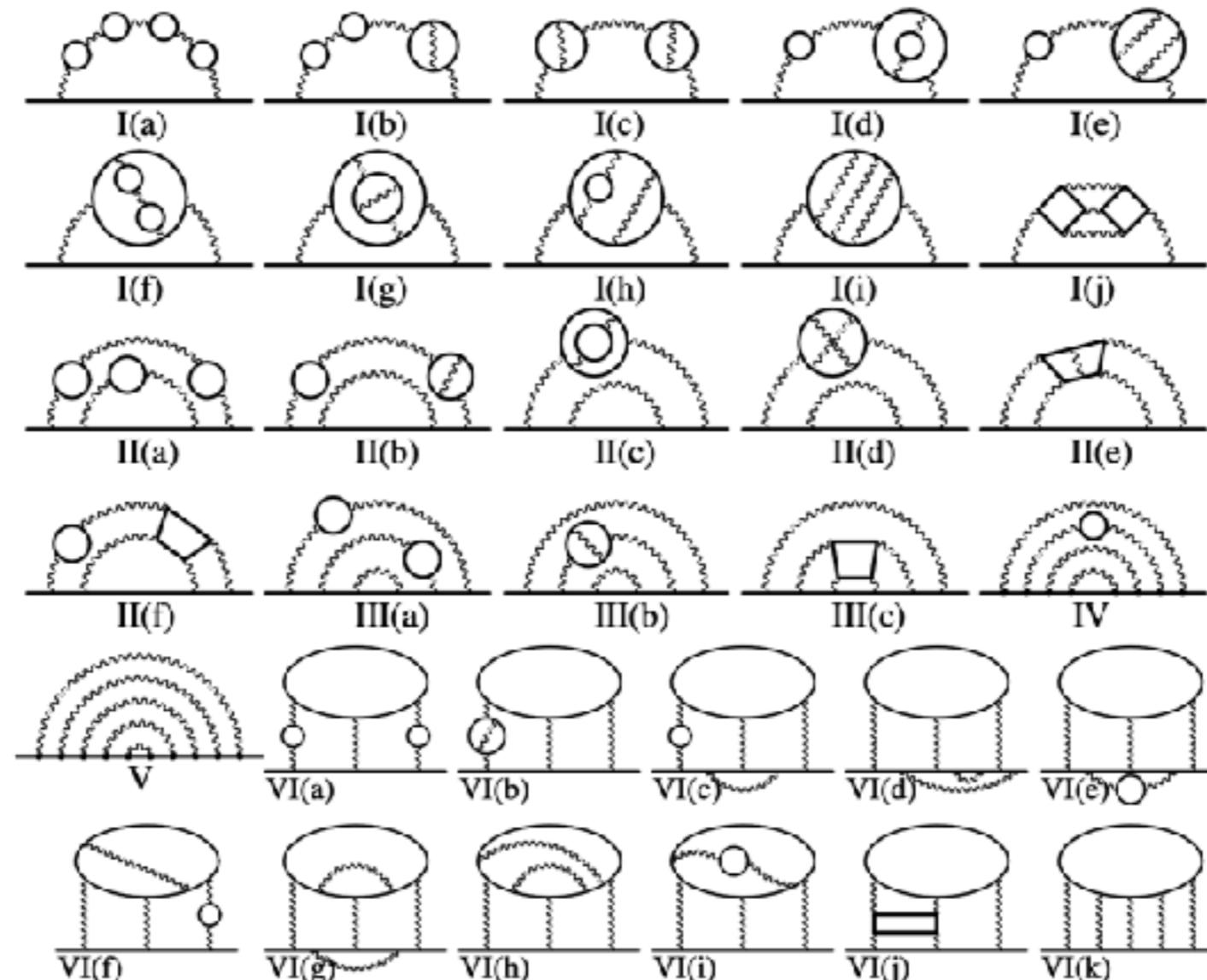
(image credit: Fermilab Muon (g-2) Experiment)

1. QCD and muon ($g-2$)

- Standard Model theory prediction of muon ($g-2$) is a story of Feynman diagrams, starting with the simplest QED correction:



- The first correction is very important, but far from the end of the story, even if we stick to QED only:

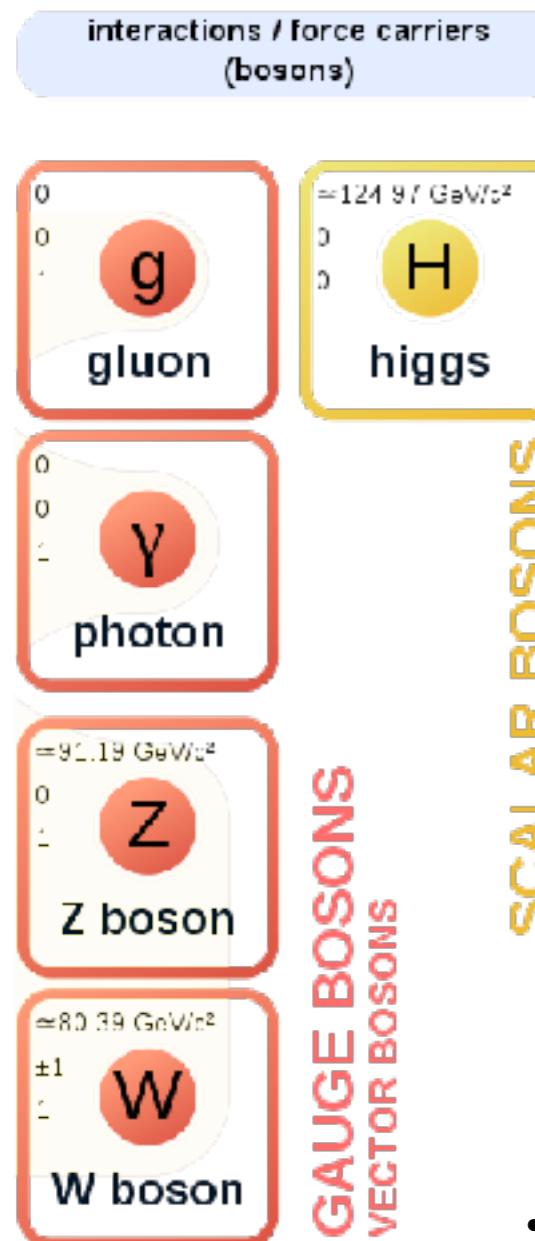


(from the Muon g-2 Theory Initiative, arXiv:2006.04822)

- “Tenth order” corrections (ten interactions → ten powers of e).
12,672 unique diagrams represented in classes above!

- We can break apart all Standard Model contributions to (g-2) into two more parts, on top of QED. EW involves W/Z/h; QCD involves gluons.

reminder: $a_\mu \equiv \frac{(g-2)_\mu}{2}$



$$a_\mu(\text{QED}) = 1165847.189(1)$$

$$a_\mu(\text{EW}) = 1.536(10)$$

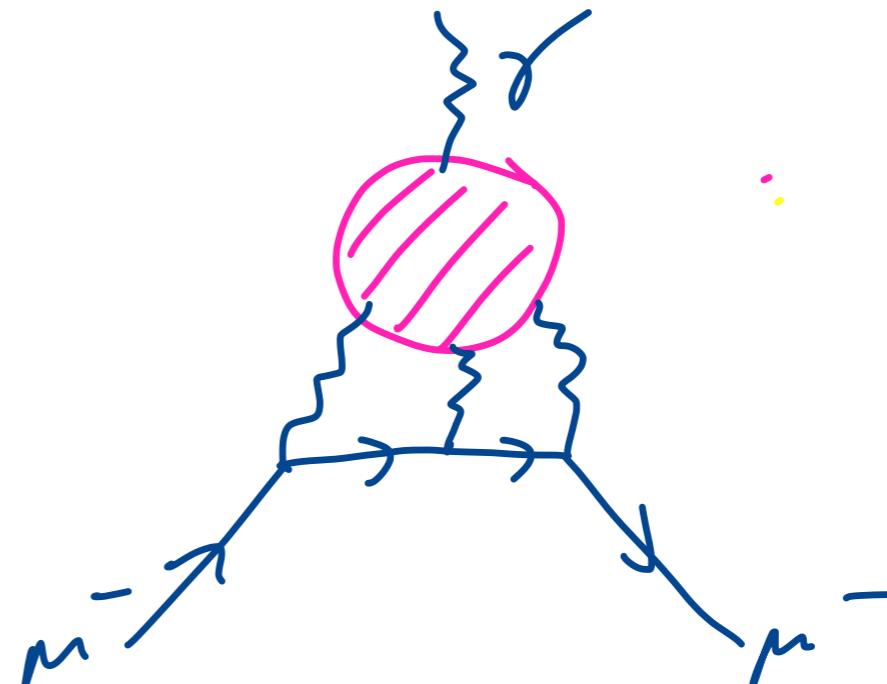
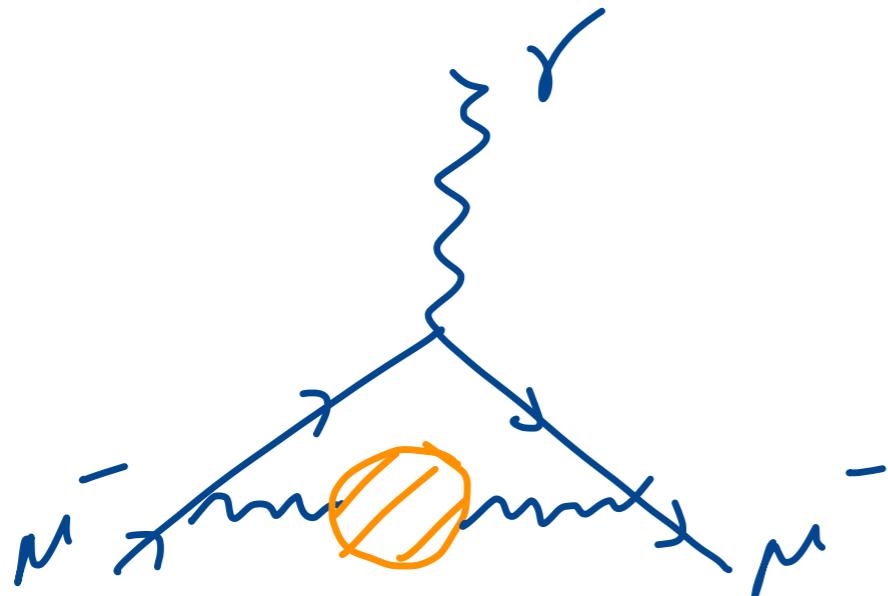
$$a_\mu(\text{QCD}) = 69.370(440)$$

$$a_\mu(\text{SM}) = 1165918.10(43)$$

$$a_\mu(\text{exp.}) = 1165920.61(41)$$

- Precision goal of experiment is (41) \rightarrow (16). Commensurate precision from QCD would be $0.16/69.37 \sim 0.2\%$.

- Describe QCD contribution using “QCD blobs”. Two meaningful diagrams: **hadronic vacuum polarization (HVP)** and **hadronic light-by-light scattering (HLbL)**.



- Relative size of contributions: $a_\mu^{\text{HVP}} \sim 70 \times 10^9$; $a_\mu^{\text{HLbL}} \sim 1 \times 10^9$.
- Overall precision goal of 0.2% for QCD overall \rightarrow 0.2% precision for HVP, and 10% precision for HLbL.
- Both terms can be estimated both by leveraging experimental input (dispersive analysis), or *ab initio* by lattice QCD calculation. Improvements are in the works for both approaches; the more cross-checks, the better!

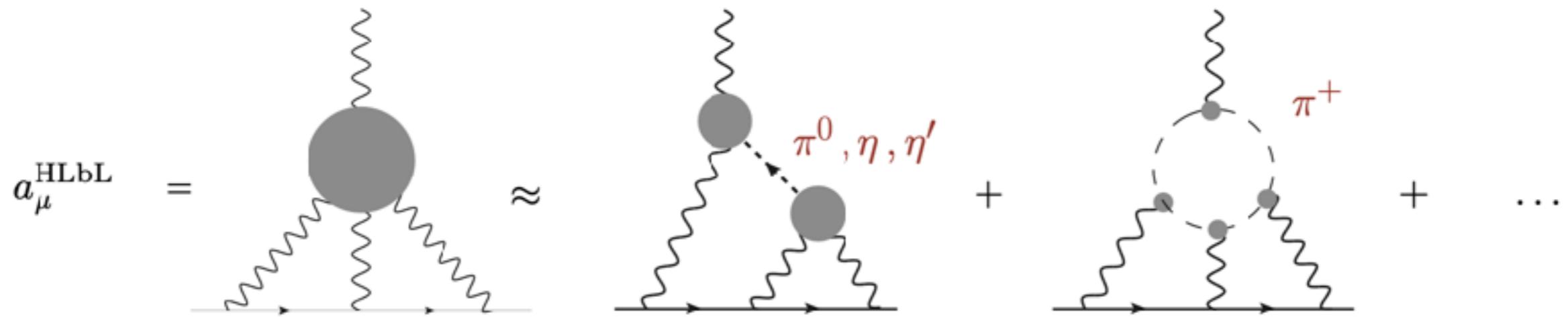
- The lattice QCD “world” in which HVP/HLbL are calculated is necessarily distorted: various effects must be removed to get the “real world” answer.
- **Discretization** (grid size) and **finite volume** (box size) must be **extrapolated** or otherwise removed
- **Physical scales** (quark masses and overall energy) must be **tuned**
- Also **QED** effects and **$(m_u - m_d)$** corrections. For HVP, these are small and can be studied separately. For HLbL, QED is more important and getting it right is challenging.



(image credits: [amazon.com](#))

2. Light-by-light scattering

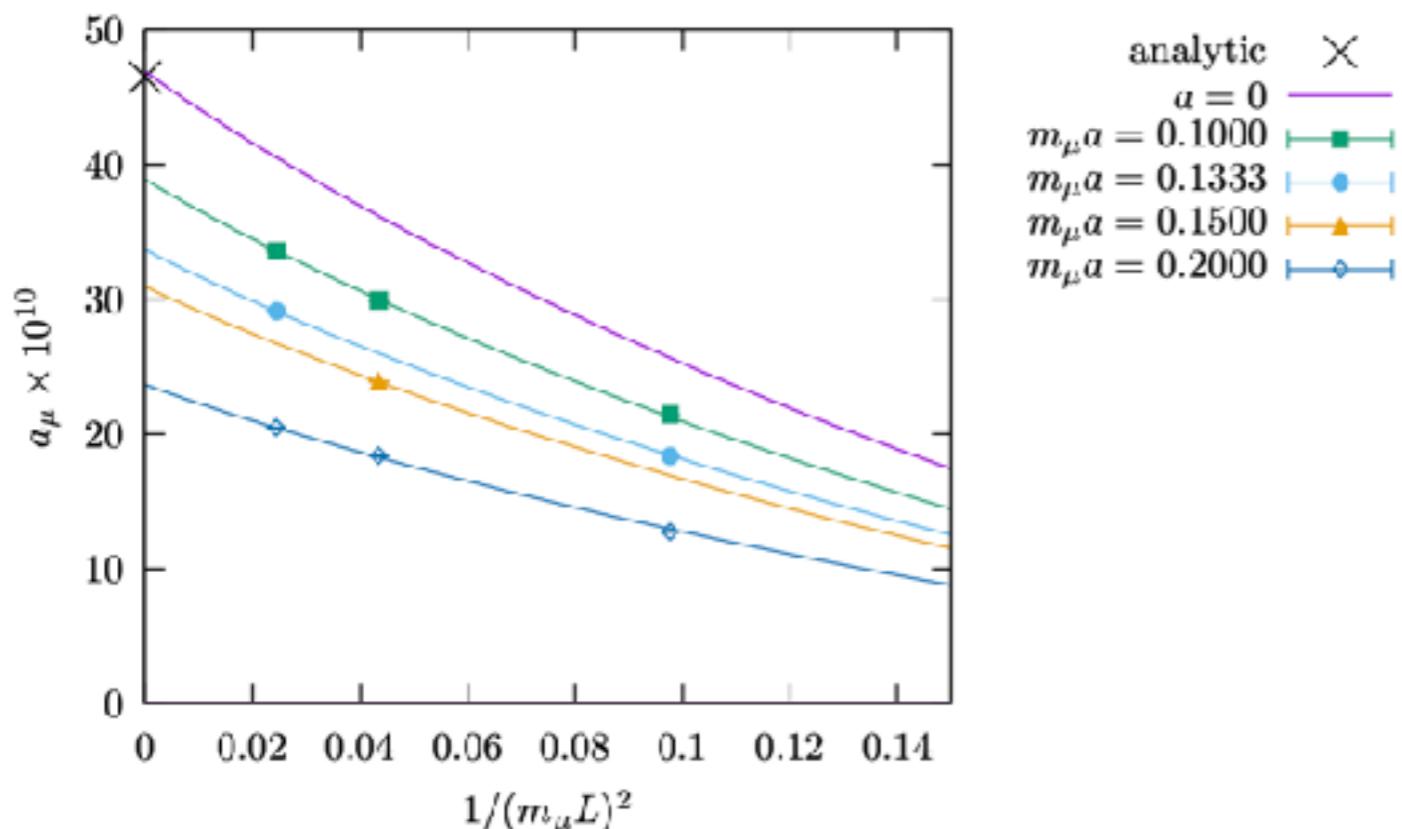
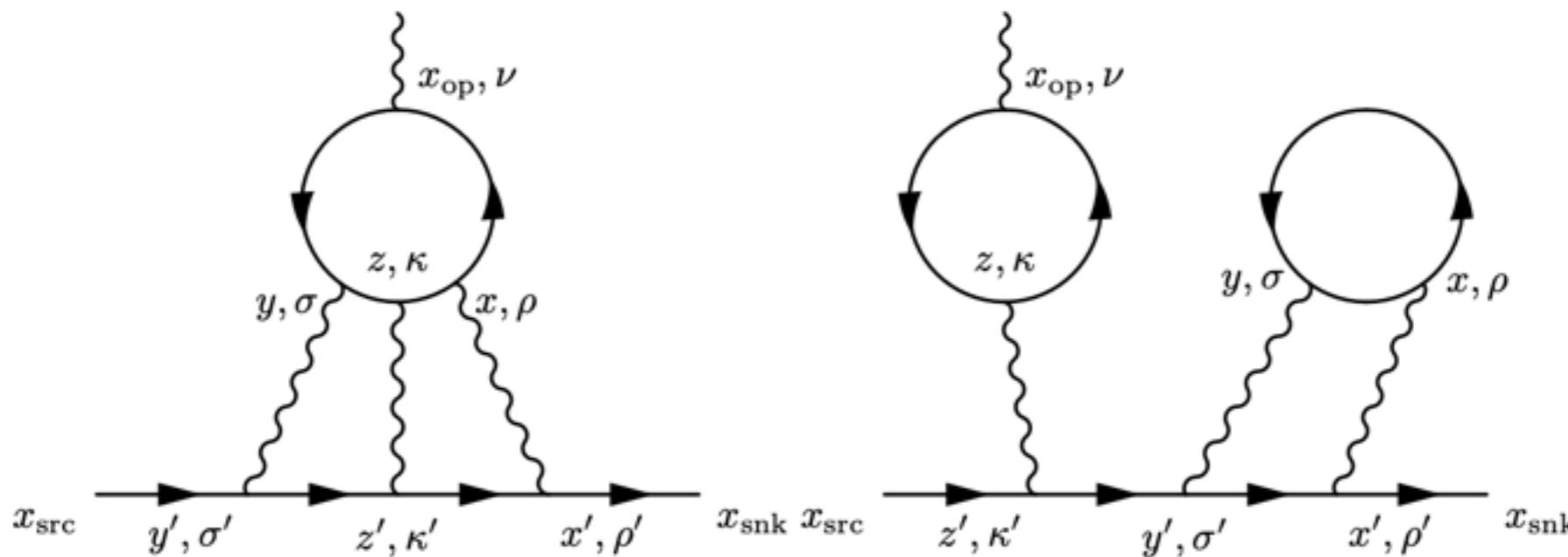
- Dispersive analysis uses experimental input; pseudoscalar form factors, scattering parameters, etc. are most significant.



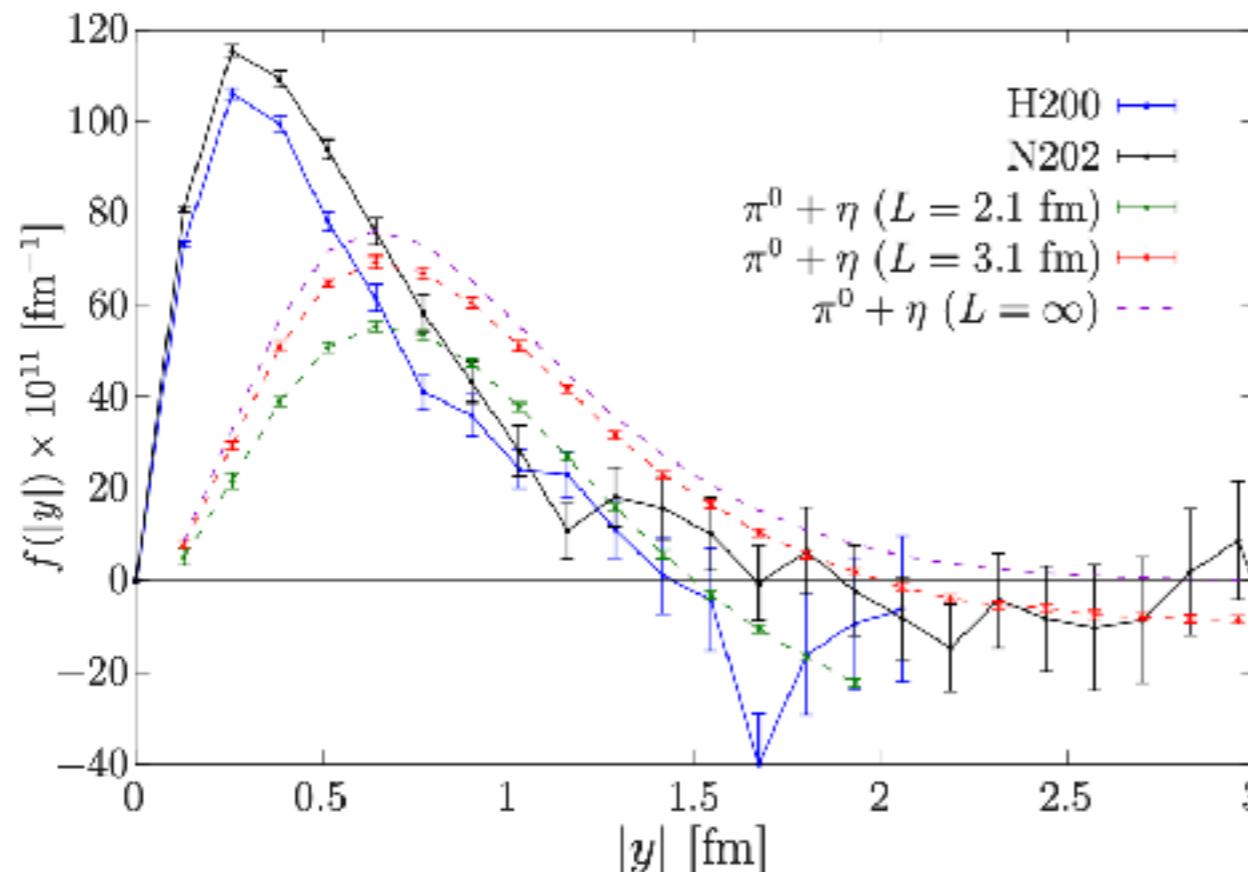
- Dominant contributions from $\pi^0/\eta/\eta'$ well-understood. Larger uncertainty from higher states and “short distance” contributions.

Dispersive framework ('21)	$a_\mu \times 10^{11}$
π^0, η, η'	93.8 ± 4
pion/kaon loops	-16.4 ± 0.2
S-wave $\pi\pi$	-8 ± 1
axial vector	6 ± 6
scalar + tensor	-1 ± 3
q-loops / short. dist. cstr	15 ± 10
charm + heavy q	3 ± 1
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HLbL (dispersive)	92 ± 19

(from talk by A. Gerardin, Muon g-2 Theory Initiative workshop)

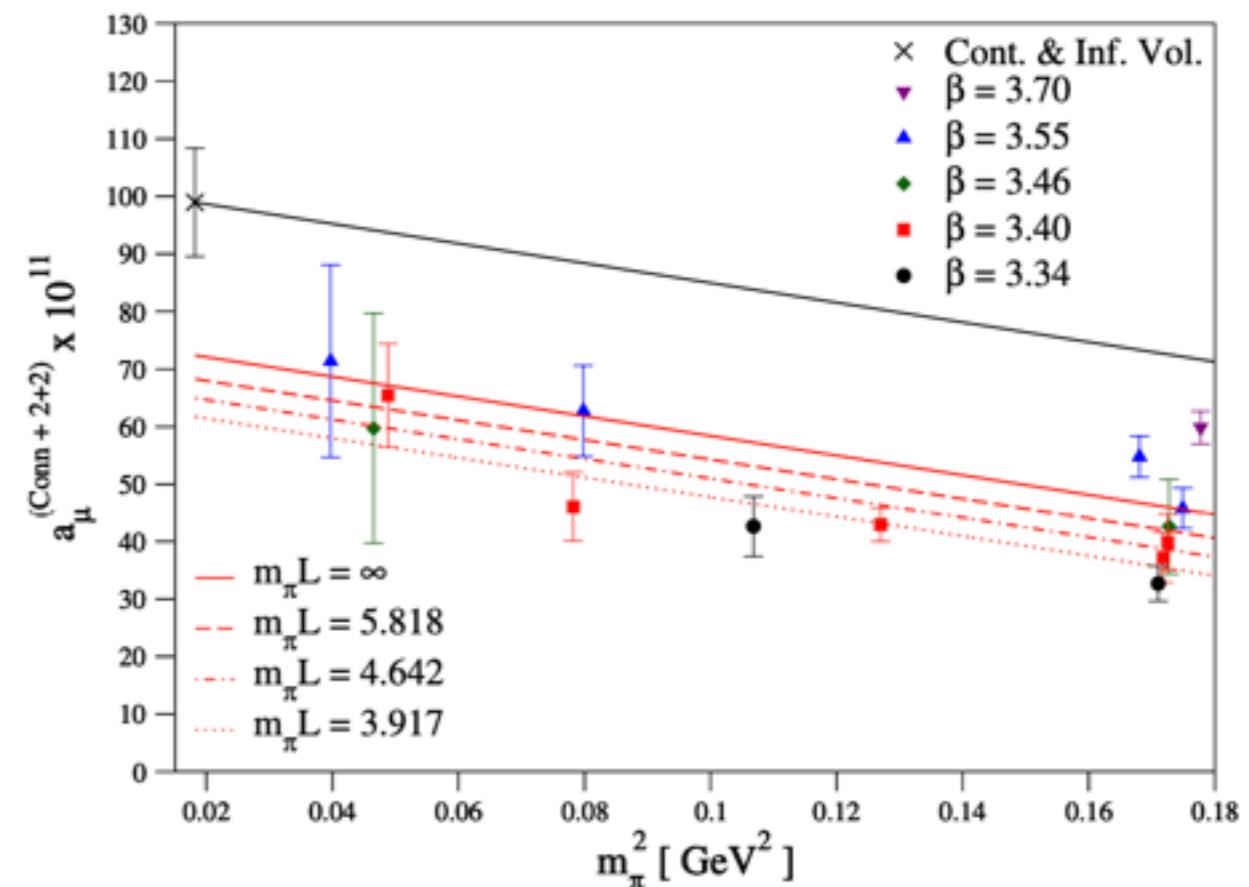


- Lattice QCD can calculate HLbL directly; correlation between four insertions of EM current.
- Dealing with “QED in a box” is hard! Either work directly in finite box, or separate out QED as much as possible and treat in infinite volume perturbation theory.
- Left: testing QED FV method with QCD turned off, against exactly calculable “muon loop”.

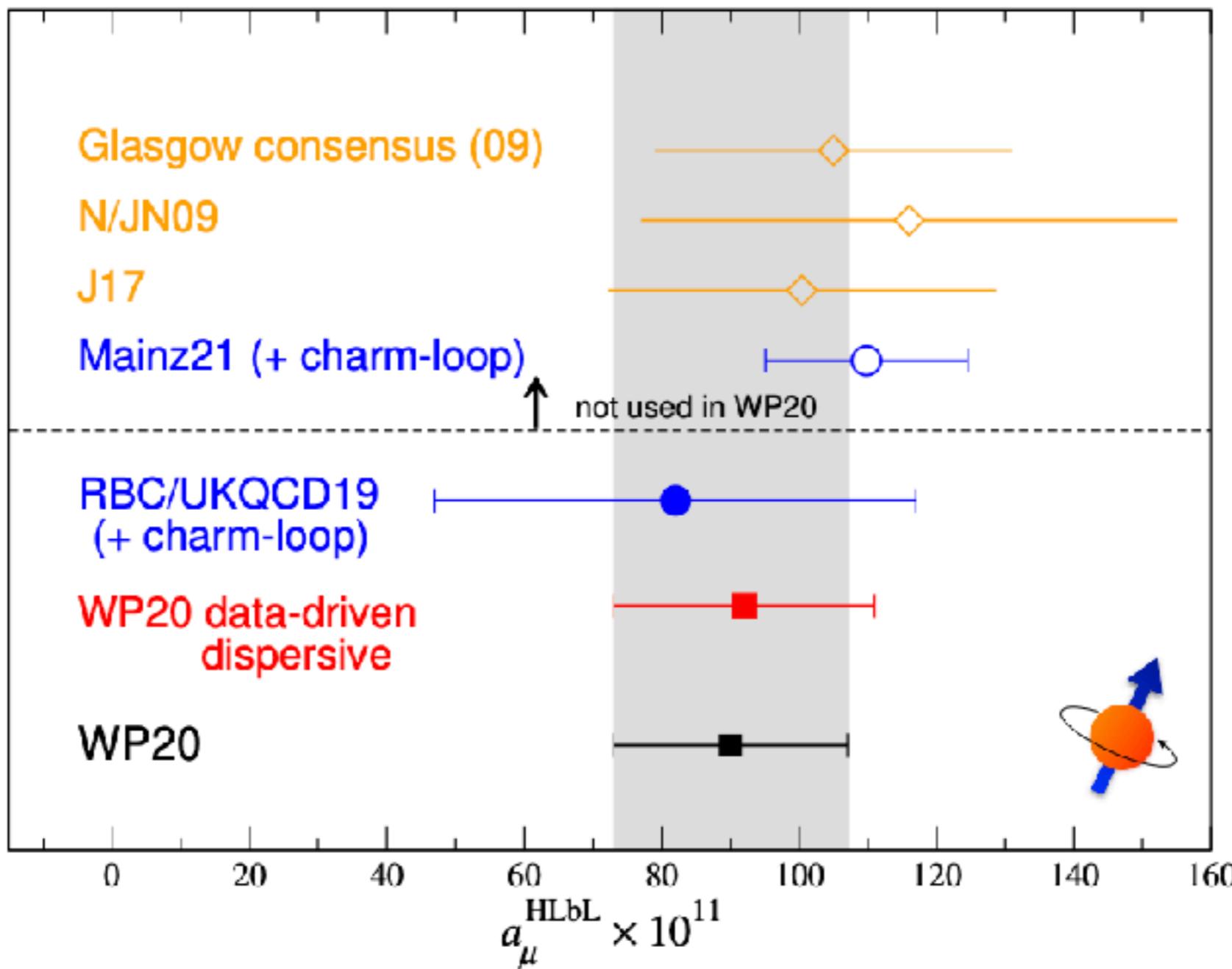


- **Left:** Integrate over one current position, show remaining integrand $f(y)$ to get a sense for energy/distance dependence.
- (Solid lines: lattice complete; dashed lines: π^0/η form factor only. “SU(3) point”.)

- **Right:** Continuum and infinite-volume extrapolation of (physical, connected) light-quark HLbL. Other terms (heavy quarks, disconnected) also calculated



(from talk by A. Gerardin, Mainz group, Muon g-2 Theory Initiative workshop)

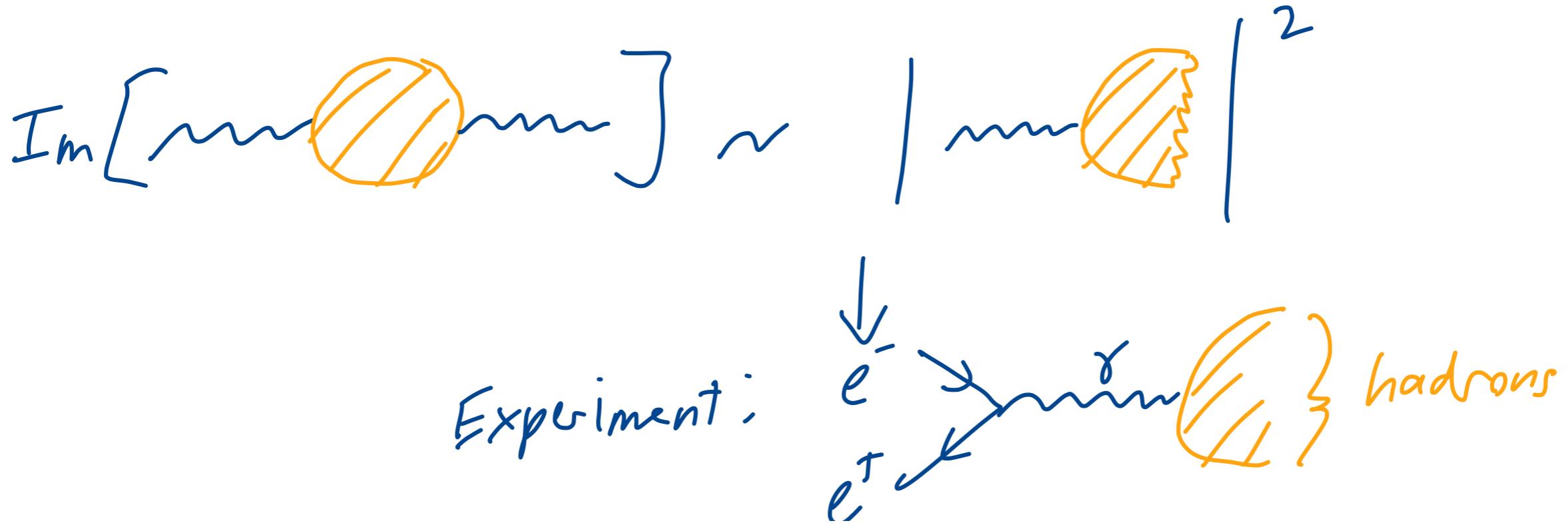


(from G. Colangelo et al, arXiv:2203.15810)

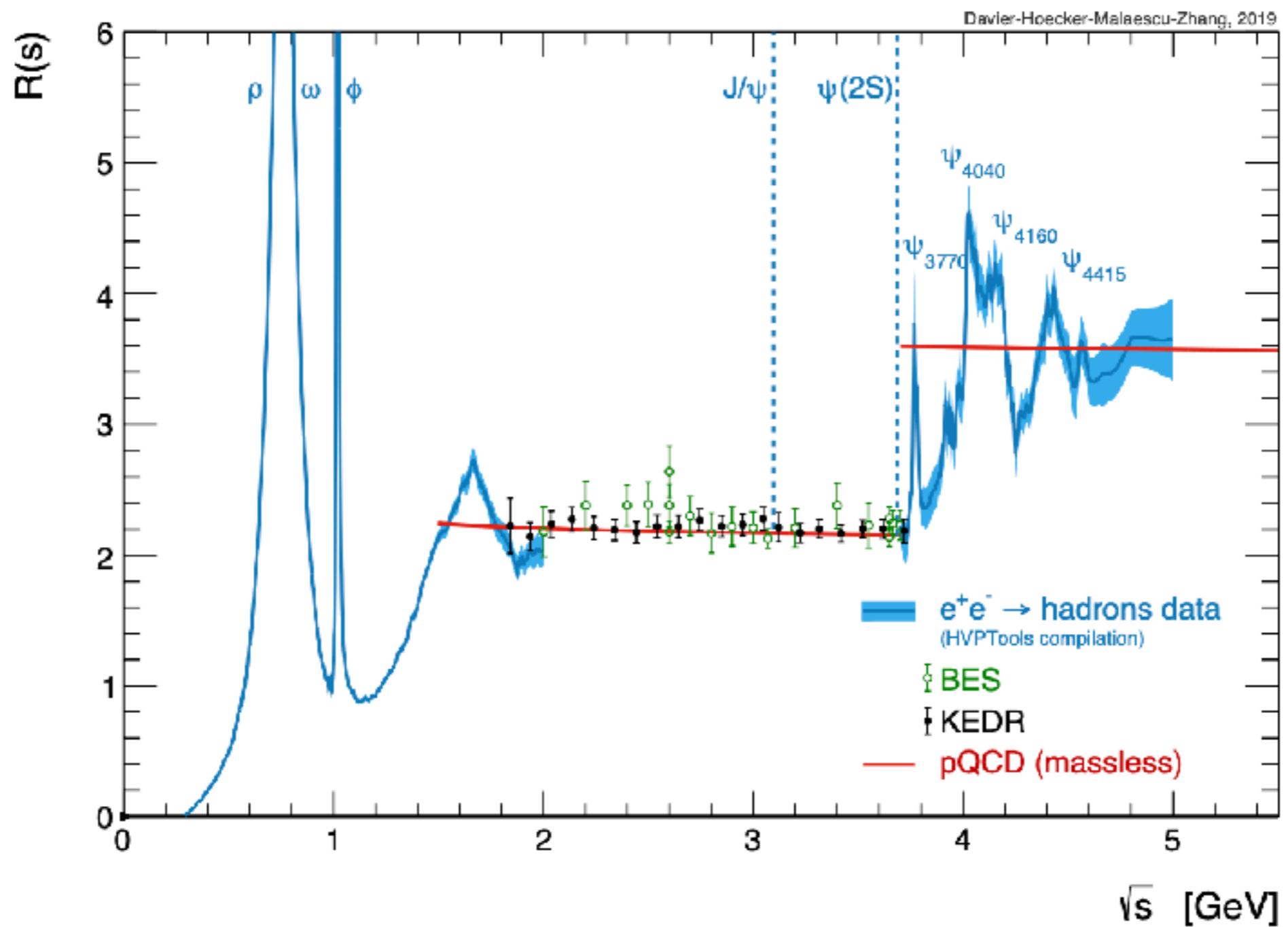
- Summary: direct lattice and dispersive approaches agree well; lattice input on form factors —> dispersive can also give further checks. Precision is getting close to 10% goal already!

3. Hadronic vacuum polarization

- HVP can be estimated from experiment using a dispersion relation:

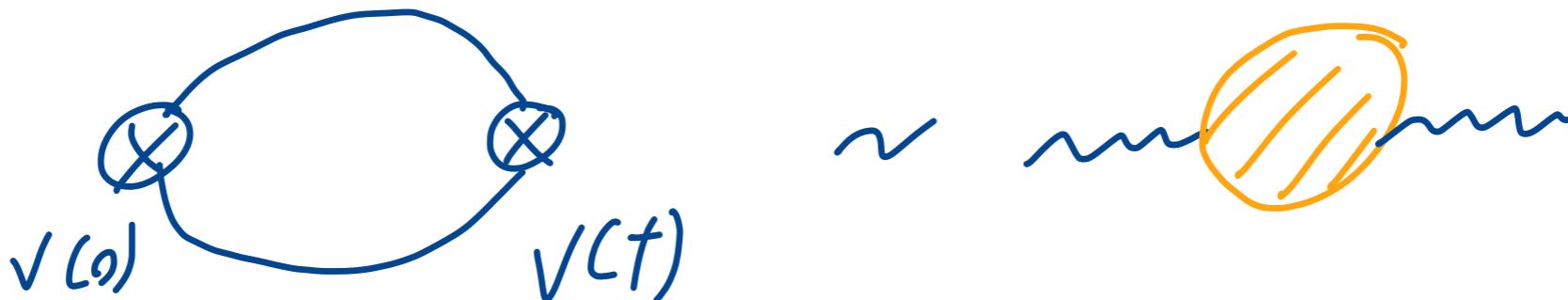


- In principle, this is a simple integral over the ratio:
$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$
- In practice, combination of results from multiple experiments is intricate; multiple groups do the extraction independently.

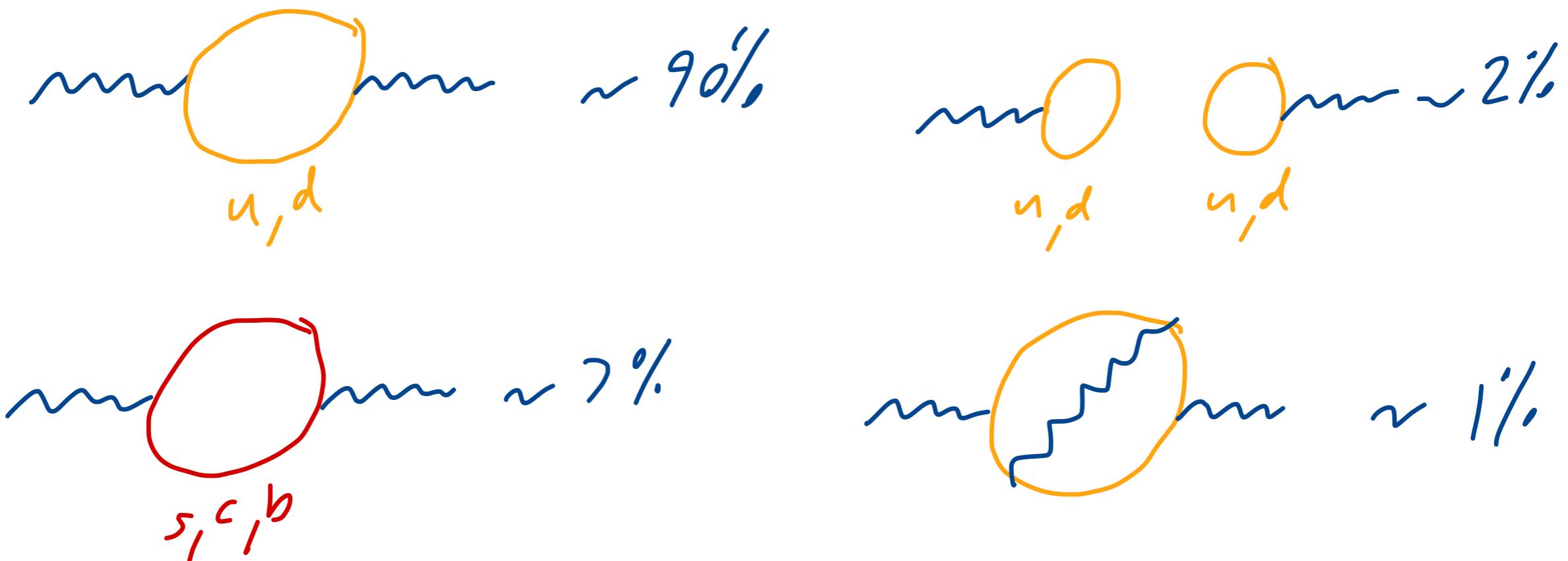


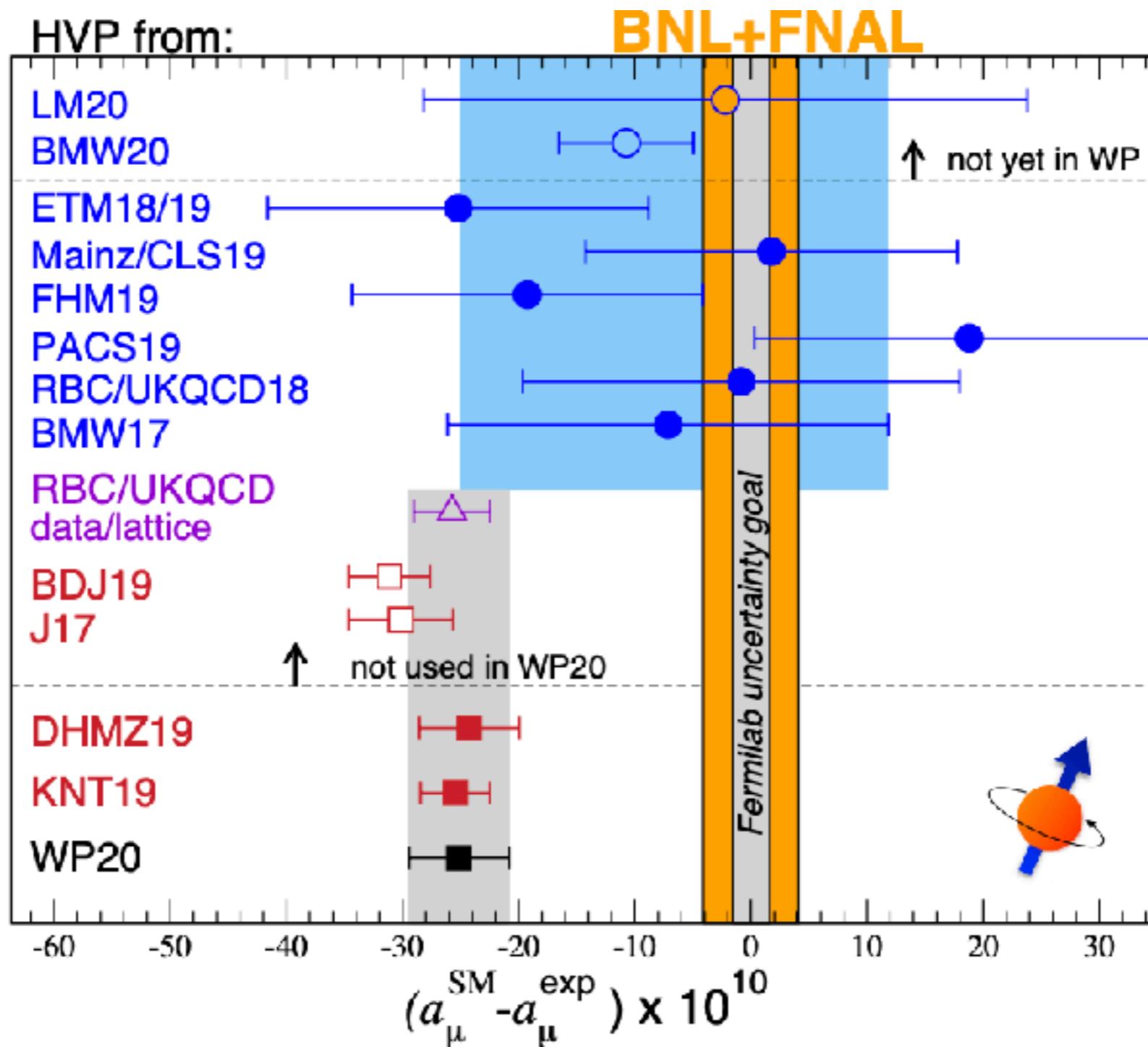
- Some tension between experiments in places, particularly two-pion data from 600-900 MeV (new CMD-3 result, arXiv:2302.08834, talk earlier this morning!)

- To compute HVP on lattice, we compute *correlation* between two “vector currents”:



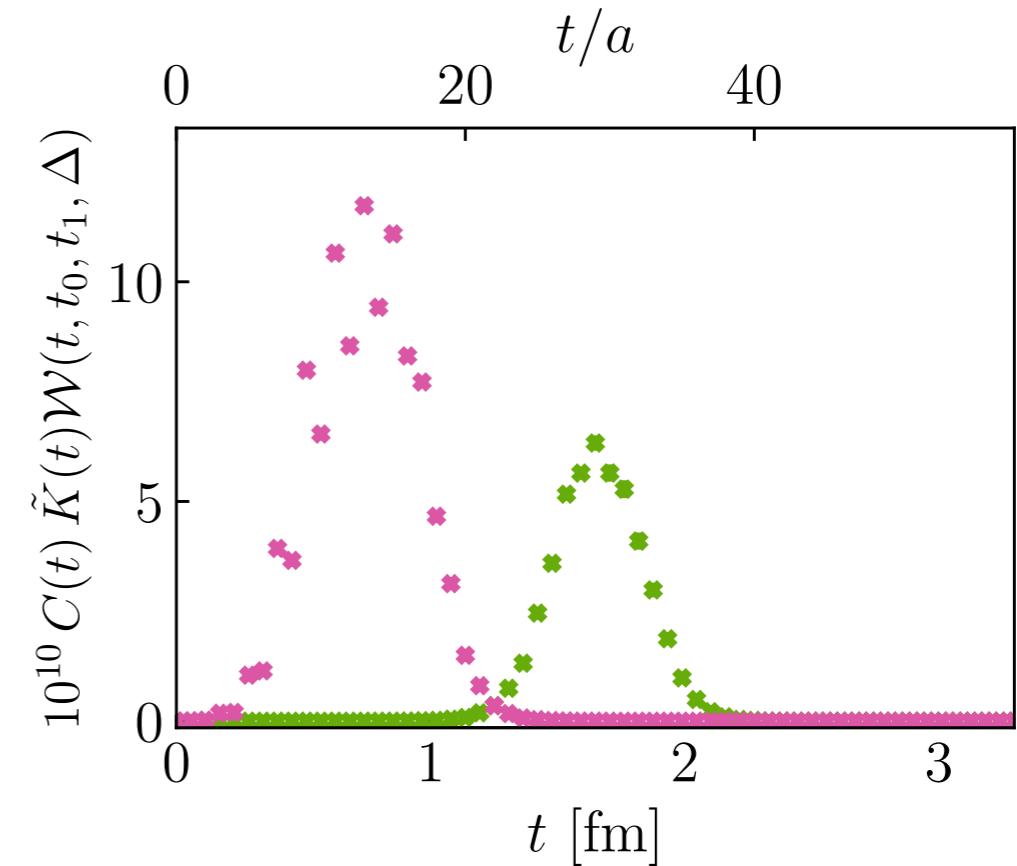
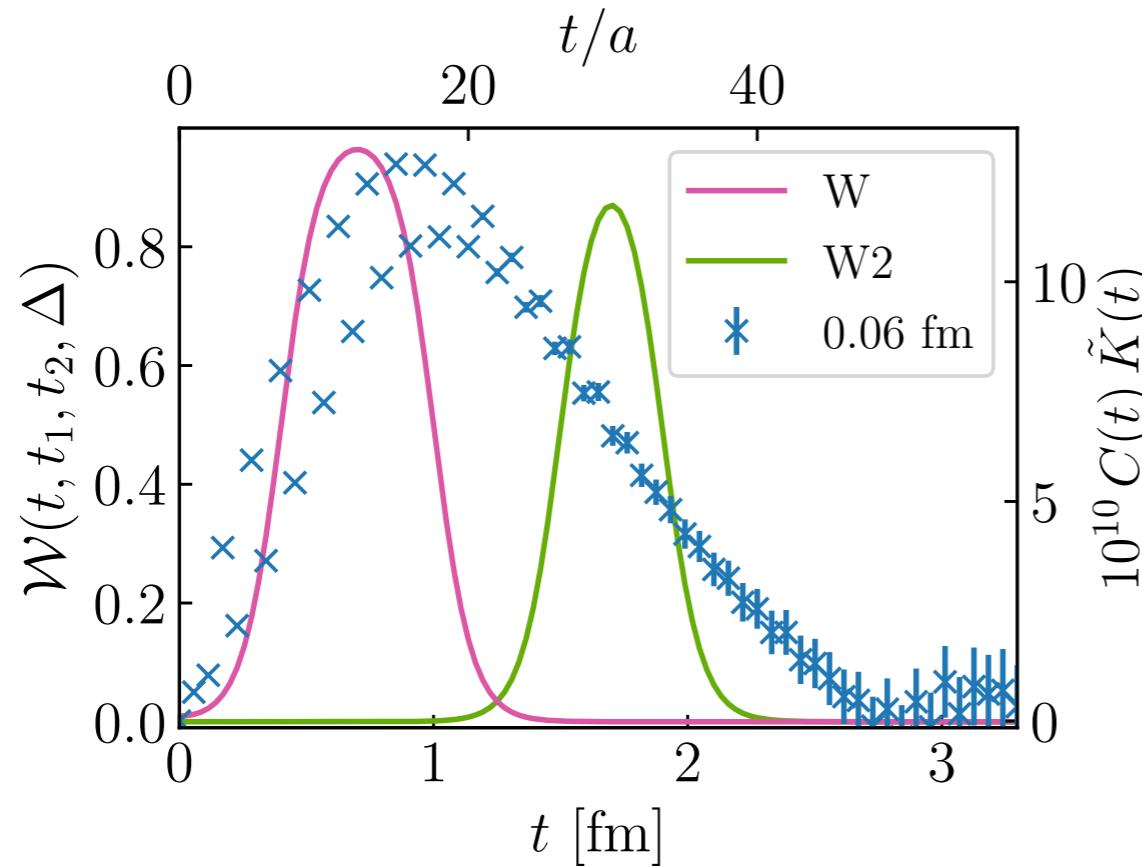
- Path integration automatically “sums over” all gluon configurations, but fermions treated more explicitly:



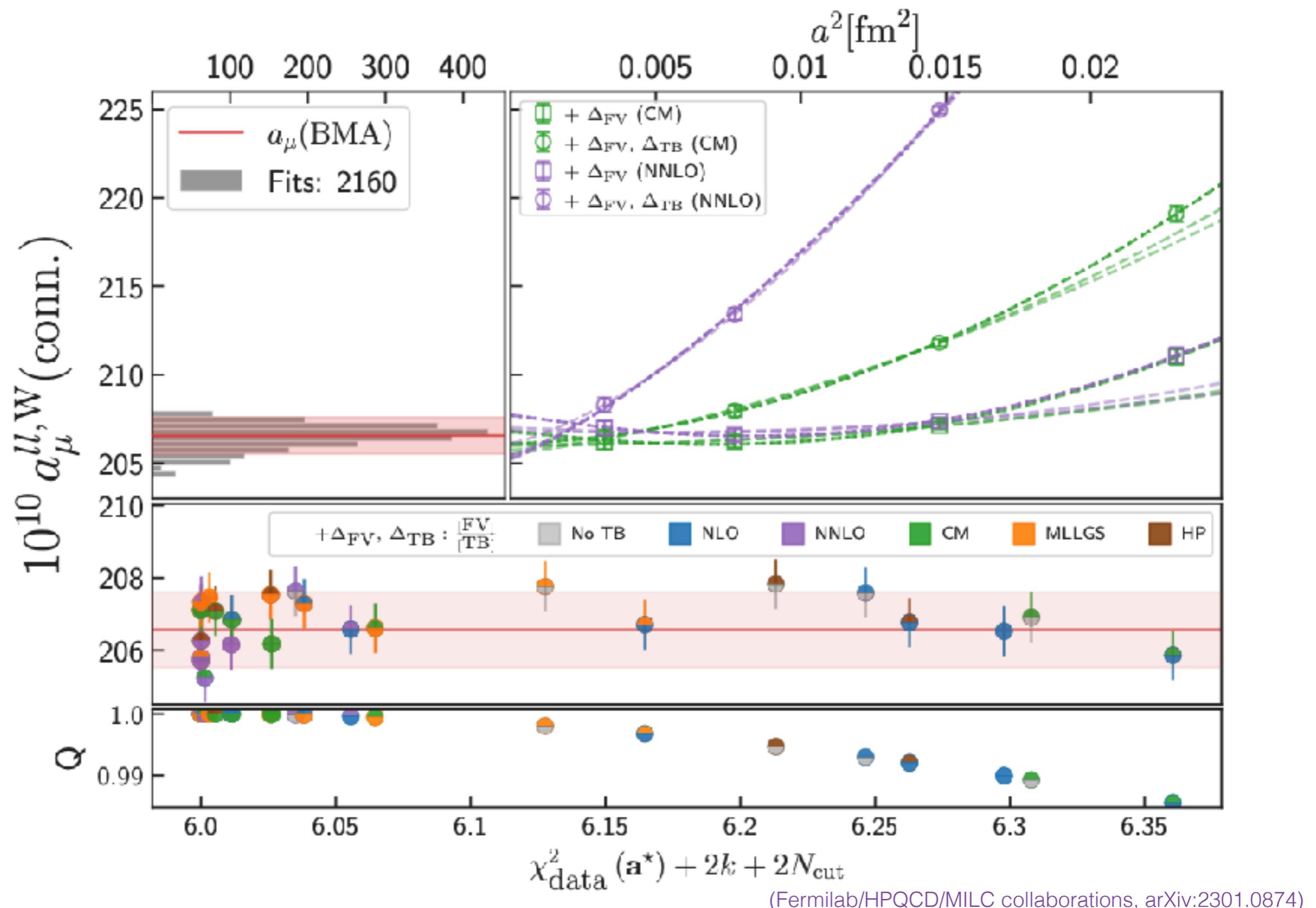


- Current theory value for HVP dominated by dispersive results; lattice determinations not precise enough (yet!) Lattice “world average” agrees statistically with both experiment and dispersive theory.

(Fermilab/HPQCD/MILC collaborations, arXiv:2301.08274)



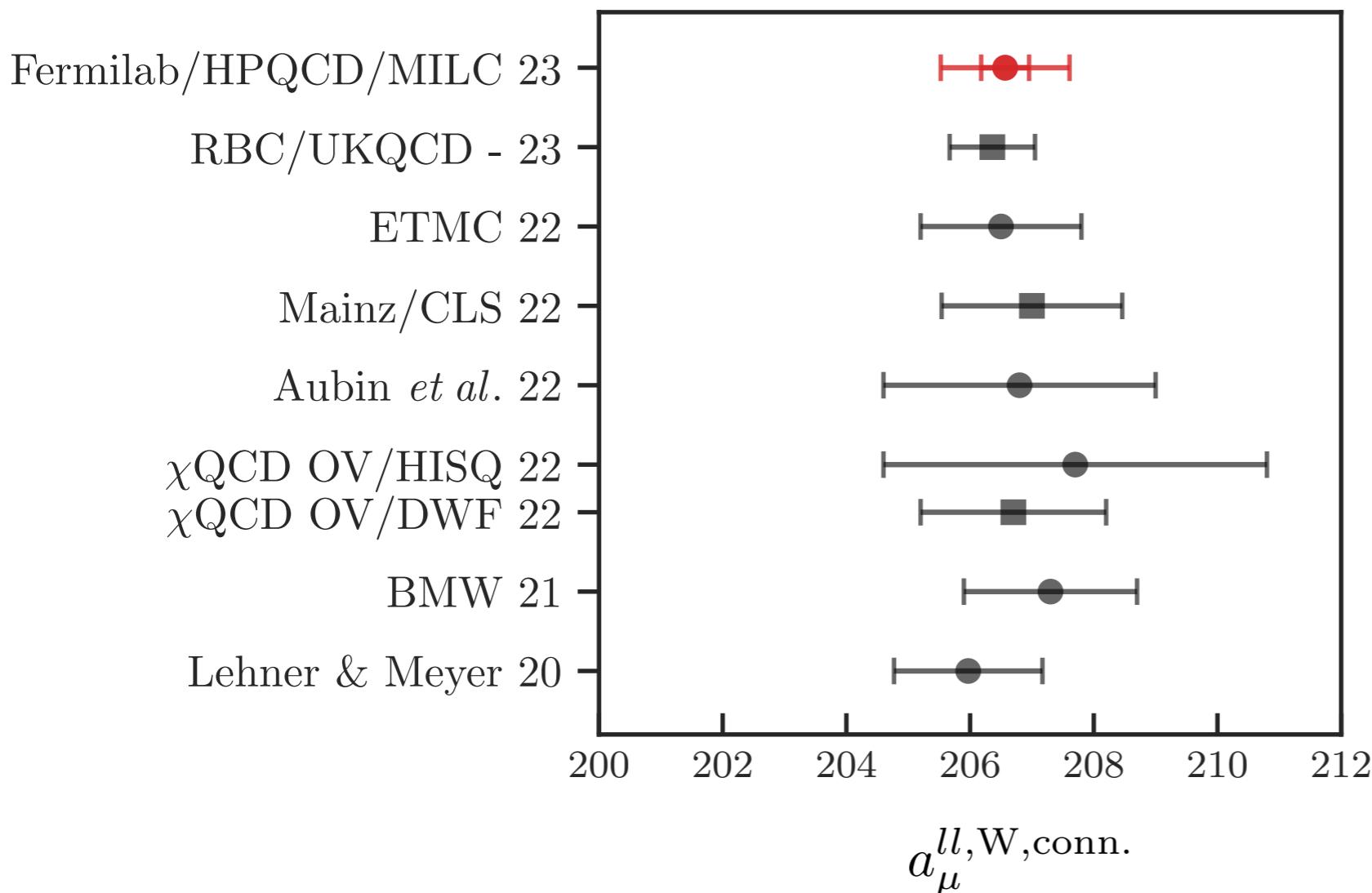
- To validate and cross-check lattice results, calculation can be restricted to an “**intermediate window**”; contributions in $[0.4, 1.0]$ fm. (Other windows have been considered, but this is the most common choice.)
- No long-distance or short-distance contributions makes this piece of HVP **relatively clean** and **highly precise**. Cross-check lattice results in idealized world (no QED, no disconnected, two degenerate light quarks only.)



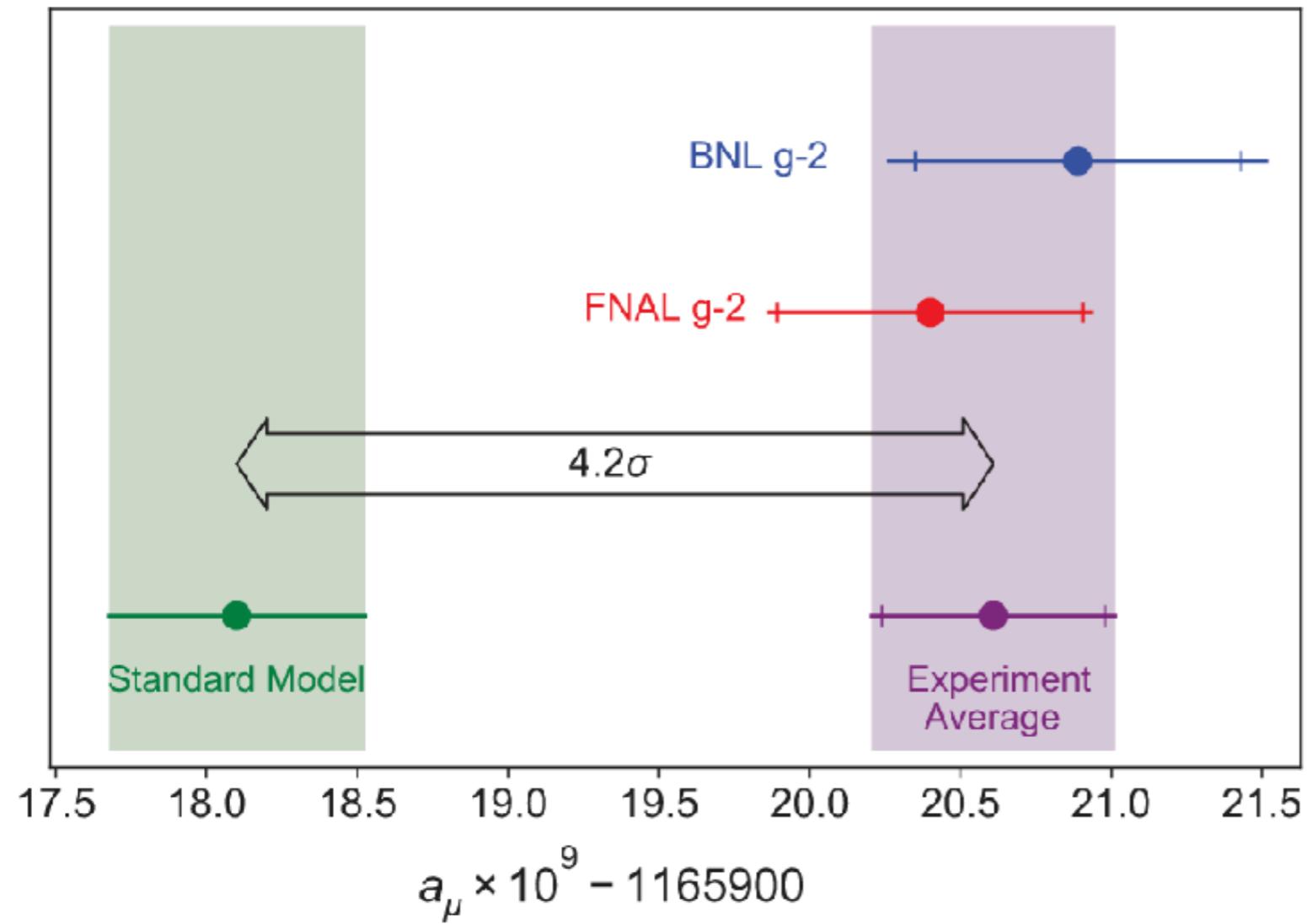
- **2160** fit variations - dealing with discretization, finite volume, mass corrections, and other effects! Variations combined using **Bayesian model averaging**.

- Our collaboration's new result (top) compared to other recent lattice determinations for HVP "LQ connected" intermediate window
- Excellent agreement overall between many groups, including BMW '21 (whose total HVP is closer to expt)
- Only one piece of the HVP; other contributions yet to be cross-checked thoroughly!

(Fermilab/HPQCD/MILC collaborations, arXiv:2301.08274)



Summary

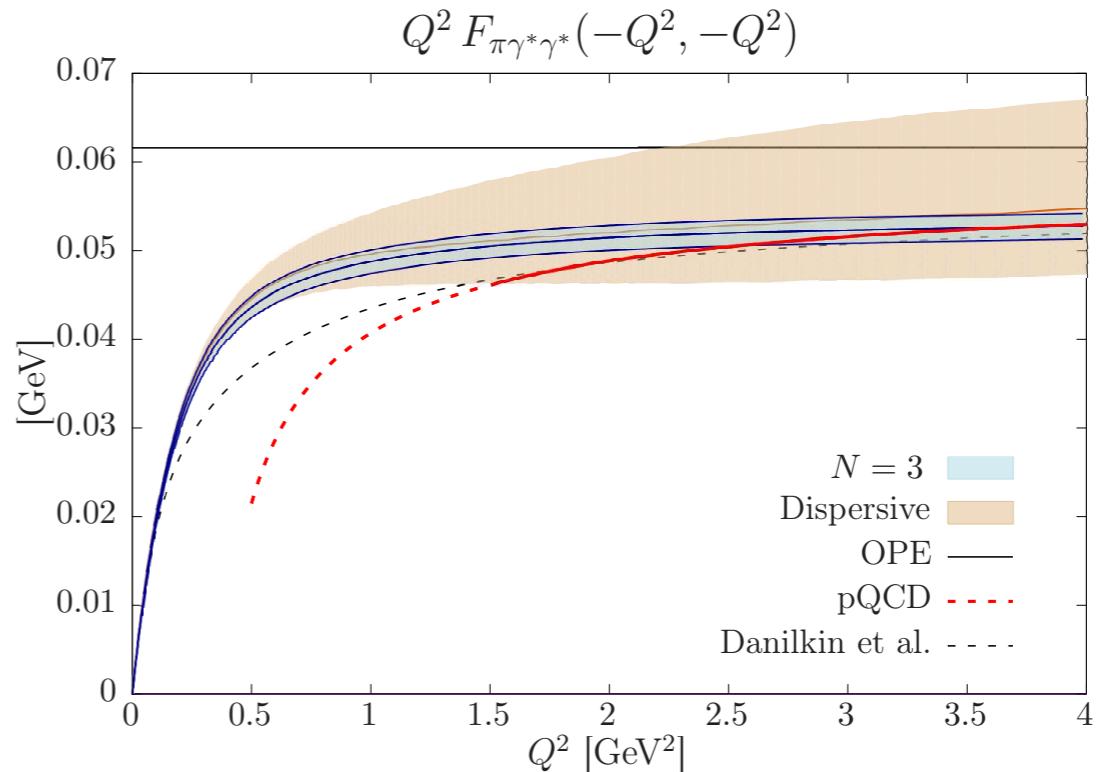
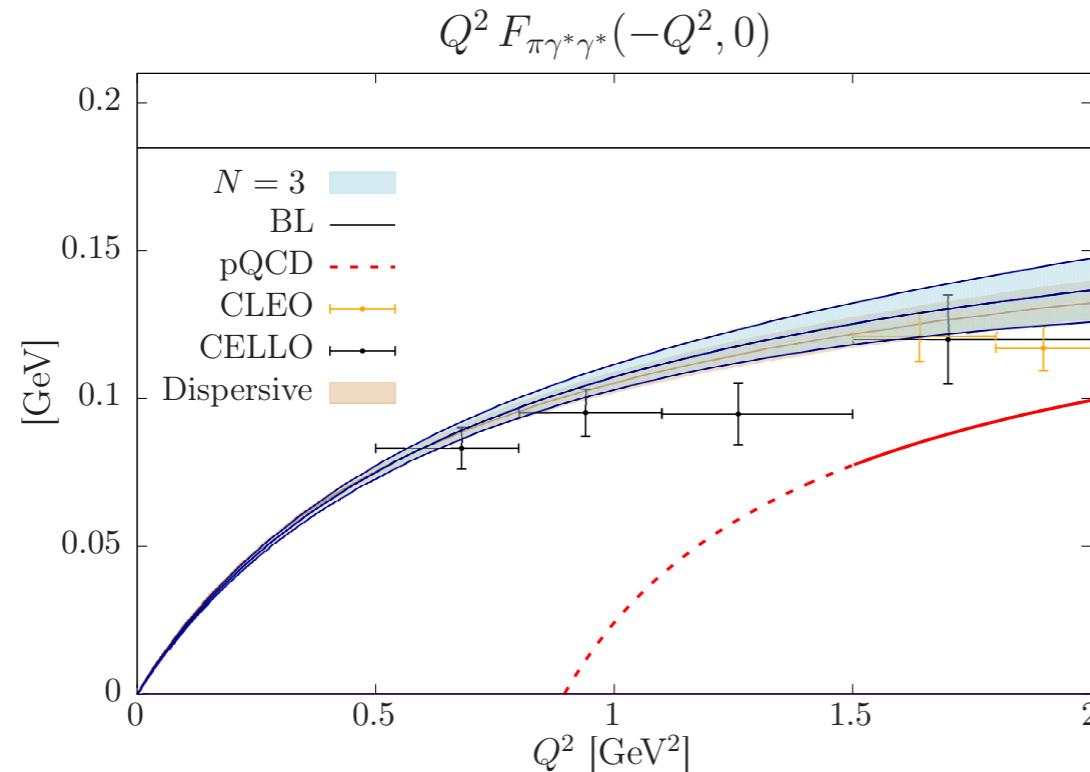


- The final story on muon (g-2) depends on improving our knowledge of the QCD contributions
- Hadronic light-by-light seems under control, uncertainty is near the precision goal
- Reaching the much tighter precision goal for HVP will take longer, and important to have many cross-checks (group vs. group, lattice vs. dispersive, “windows”)
- Stay tuned over the next few years!

Backup

The pion-pole contribution

- Pion transition form factor at the physical point (model independent parametrization)



$$a_\mu^{\text{HLbL};\pi^0} = (59.9 \pm 3.6) \times 10^{-11}$$

[A. G et al, Phys.Rev. D100 (2019)]

→ Compatible with data-driven result

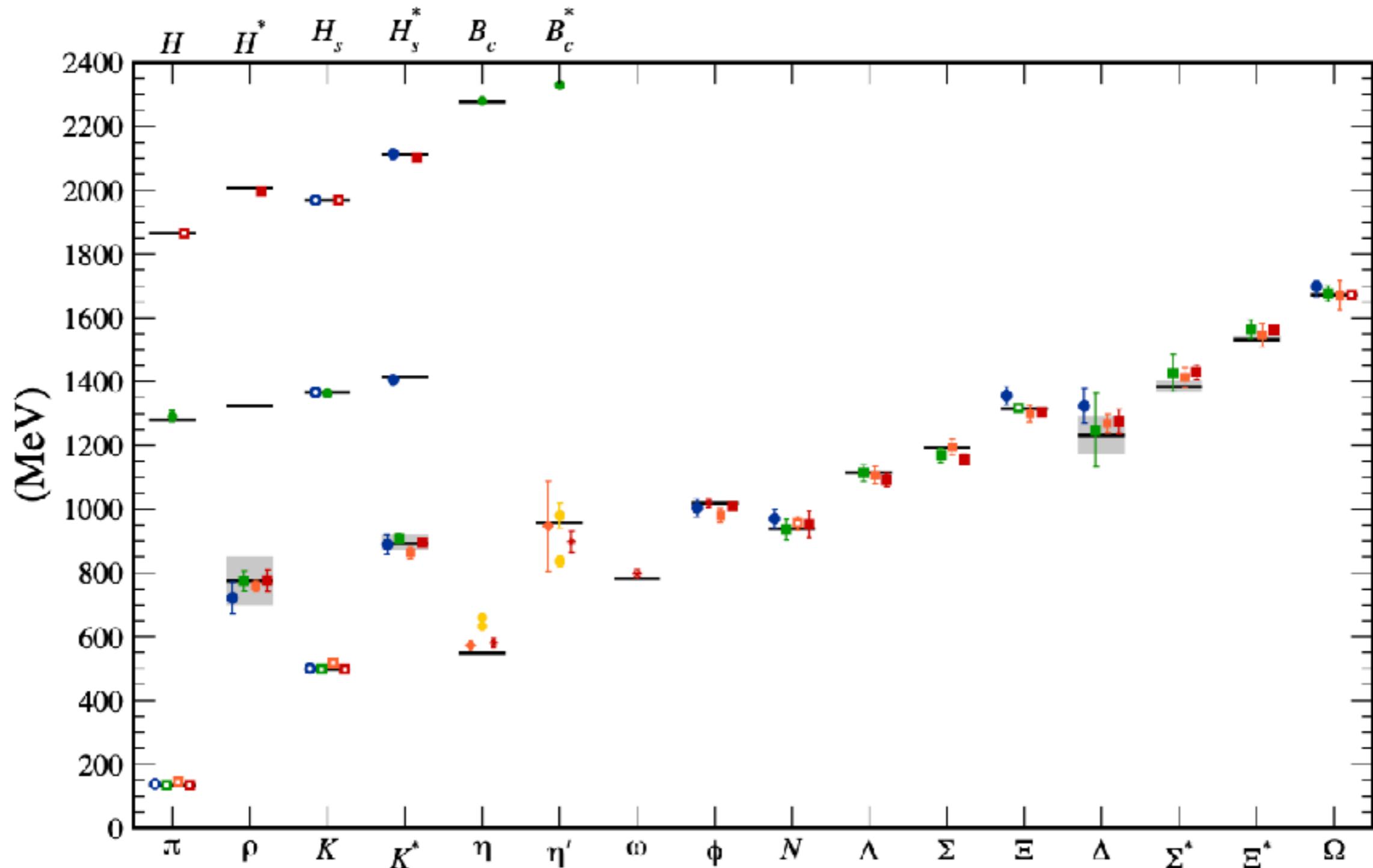
$$a_\mu^{\text{HLbL};\pi^0} = 62.6^{+3.0}_{-2.5} \times 10^{-11}$$

[Hoferichter et al. '18]

- Also useful for the direct lattice calculation of HLbL

→ study of finite-size effects

→ chiral extrapolation ($m_\pi \rightarrow m_\pi^{\text{phys}}$)



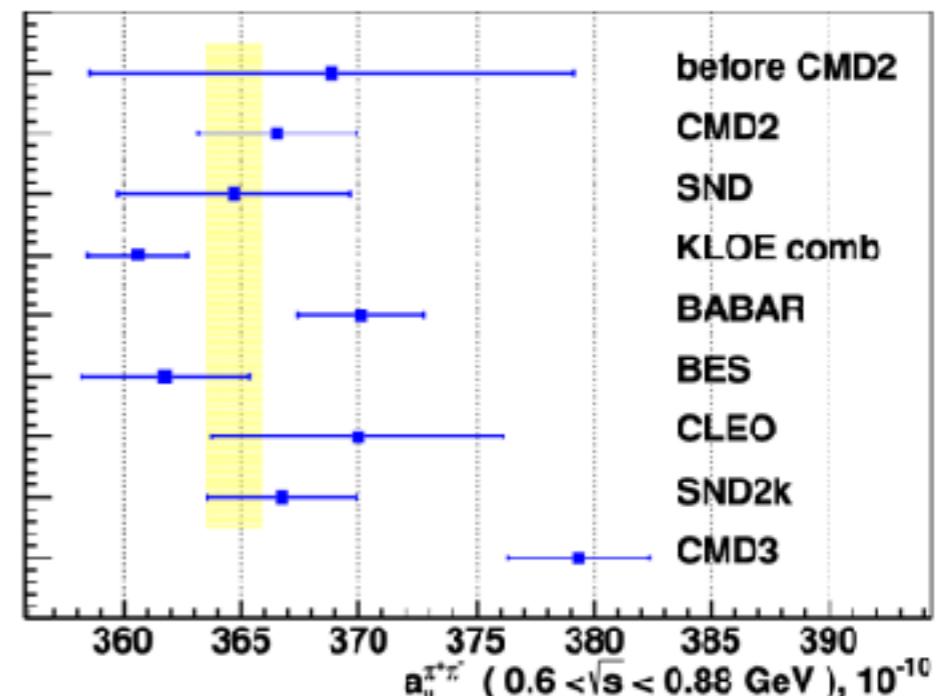
- Modern lattice QCD is well-tested and reliable, for a wide variety of *ab initio* predictions - like the hadron spectrum above!

- Some tensions in the input data exist, particularly in pi-pi between 600-900 MeV. The final theory average assigns a systematic error to cover this spread.

(arXiv:2302.08834)

Measurement of the $e^+e^- \rightarrow \pi^+\pi^-$ cross section from threshold to 1.2 GeV with the CMD-3 detector

- New result from CMD-3 experiment last month! Taken alone, would shift the SM HVP estimate by about 1.5×10^{-9} ; current tension is 2.5. (But, I don't know of any reason to take it alone! Further study is needed...)



Source	$\delta a_{\mu}^{ll,W}(\text{conn.}) (\%)$
Monte Carlo statistics	0.19
Continuum extrapolation ($a \rightarrow 0$, Δ_{TB})	0.34
Finite-volume correction (Δ_{FV})	0.16
Pion-mass adjustment ($\Delta_{M_{\pi}}$)	0.06
Scale setting (w_0 (fm), w_0/a)	0.24
Current renormalization (Z_V)	0.17
Total	0.50%

- “Error budget” shows the contribution to uncertainty from various sources
- Continuum extrapolation is currently dominant; scale setting error is larger for total HVP, but sub-leading for the window.

Lattice Ensembles

$\approx a[\text{fm}]$	$N_s^3 \times N_t$	$am_l^{\text{sea}} / am_s^{\text{sea}} / am_c^{\text{sea}}$	w_0/a	$M_{\pi_5}(\text{MeV})$	$N_{\text{conf.}}$	N_{wall}
0.15	$32^3 \times 48$	0.002426 / 0.0673 / 0.8447	1.13215(35)	134.73(71)	9362	48
0.12	$48^3 \times 64$	0.001907 / 0.05252 / 0.6382	1.41110(59)	134.86(71)	9011	64
0.09	$64^3 \times 96$	0.00120 / 0.0363 / 0.432	1.95180(70)	128.34(68)	5384	48
0.06	$96^3 \times 128$	0.0008 / 0.022 / 0.260	3.0170(23)	134.95(72)	2621	24

- **HISQ action.** 2+1+1 flavor. Four lattice spacings for continuum extrapolation. Non-perturbative current renormalization - Z_V from η_s correlation functions. Scale setting with w_0 . Note *pion mass mistuning at 0.09fm*.
- Compared to [previous publication*](#), statistics greatly increased at 0.12fm (**x9**), 0.09fm (**x3.5**), 0.06fm (**x2**)
- Data also available at [two other valence quark masses](#) at 0.15fm, 0.12fm - estimate SIB (future work) and data-driven pion mass correction.

