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Coil parameters, etc.

$kJ := 1000J$ $\qquad \sigma_{SB} := 5.67 \cdot 10^{-8} \frac{W}{m^2 \cdot K^4}$

$cable_x := 1.25mm \qquad cable_y := 0.8mm$

$turnsPerLayer := 560 \qquad noOfLayers := 5$

$coil_{width} := 700mm \qquad coil_{thickness} := cable_y \cdot noOfLayers = 4 \cdot mm$

$coil_{ID} := 100mm \qquad coil_{OD} := coil_{ID} + 2 \cdot coil_{thickness} = 0.108\,m$

$$coil_{volume} := \frac{\pi}{4} \cdot \left(coil_{OD}^2 - coil_{ID}^2\right) \cdot coil_{width} = 9.148 \times 10^{-4}\,m^3$$

$coil_{mass.copper} := y_{copper} \cdot \rho_{copper} \cdot coil_{volume} = 6.558\,kg$

$coil_{mass.NbTi} := y_{NbTi} \cdot \rho_{NbTi} \cdot coil_{volume} = 1.098\,kg$

$coil_{mass} := coil_{mass.copper} + coil_{mass.NbTi} = 7.655\,kg$

$$coil_{surfaceArea} := \pi \cdot coil_{OD} \cdot coil_{width} + 2 \cdot \frac{\pi}{4} \cdot \left(coil_{OD}^2 - coil_{ID}^2\right) = 0.24\,m^2$$

$\rho_{copper} := 8960 \frac{kg}{m^3} \qquad \rho_{NbTi} := 6000 \frac{kg}{m^3}$

$y_{copper} := 0.8 \qquad y_{NbTi} := 1 - y_{copper} = 0.2$

y = volume fraction

epoxy not included

OFHC heat capacity data from NIST

4.2 K to 300 K

$a_{OFHC} := -1.91844 \qquad b_{OFHC} := -0.15973 \qquad c_{OFHC} := 8.61013 \qquad d_{OFHC} := -18.996$

$e_{OFHC} := 21.9661 \qquad f_{OFHC} := -12.7328 \qquad g_{OFHC} := 3.54322 \qquad h_{OFHC} := -0.3797 \qquad i_{OFHC} := 0$

$$cp_{OFHC}(T) := 10^{a_{OFHC}+b_{OFHC}\cdot\left(\log\left(\frac{T}{K}\right)\right)+c_{OFHC}\cdot\left(\log\left(\frac{T}{K}\right)\right)^2+d_{OFHC}\cdot\left(\log\left(\frac{T}{K}\right)\right)^3+e_{OFHC}\cdot\left(\log\left(\frac{T}{K}\right)\right)^4+f_{OFHC}\cdot\left(\log\left(\frac{T}{K}\right)\right)^5+g_{OFHC}\cdot\left(\log\left(\frac{T}{K}\right)\right)^6+h_{OFHC}\cdot\left(\log\left(\frac{T}{K}\right)\right)^7 \cdot}$$

NbTi heat capacity data from Andrew Davies (CERN) document, assuming a density of 6000 kg/m^3

$\text{magfield} := 0\text{G}$ assume no magnetic field

$$cp_{NbTi}(T) := \begin{cases} \left[0.0081834 \cdot \left(\frac{T}{K}\right)^3 + 0.010667 \cdot \left(\frac{T}{K}\right) \cdot \left(\frac{\text{magfield}}{G}\right)\right] \frac{J}{kg \cdot K} & \text{if } (T < 9.1K) \\ \left[0.1546667 \cdot \left(\frac{T}{K}\right) + 0.002706667 \cdot \left(\frac{T}{K}\right)^3\right] \frac{J}{kg \cdot K} & \text{if } (9.1K < T < 20K) \\ \left[6.9 - 1.307683333 \cdot \left(\frac{T}{K}\right) + 0.092285 \cdot \left(\frac{T}{K}\right)^2 + 1.996667 \cdot 10^{-3} \cdot \left(\frac{T}{K}\right)^3 - 3.63334 \cdot 10^{-5} \cdot \left(\frac{T}{K}\right)^4\right] \frac{J}{kg \cdot K} & \text{if } (20K < T < 50K) \\ \left[-255 + 13.837 \cdot \left(\frac{T}{K}\right) - 0.1193834 \cdot \left(\frac{T}{K}\right)^2 + 0.000496 \cdot \left(\frac{T}{K}\right)^3 - 8.034 \cdot 10^{-7} \cdot \left(\frac{T}{K}\right)^4\right] \frac{J}{kg \cdot K} & \text{if } (50K < T < 175K) \\ \left[206.67 + 2.28434 \cdot \left(\frac{T}{K}\right) - 0.00861 \cdot \left(\frac{T}{K}\right)^2 + 1.54934 \cdot 10^{-5} \cdot \left(\frac{T}{K}\right)^3 - 1.048334 \cdot 10^{-8} \cdot \left(\frac{T}{K}\right)^4\right] \frac{J}{kg \cdot K} & \text{if } (175K < T < 500K) \end{cases}$$

$$cp\_int_{copper}(T_c, T_h) := \int_{T_c}^{T_h} cp_{OFHC}(T)\, dT \qquad cp\_int_{NbTi}(T_c, T_h) := \int_{T_c}^{T_h} cp_{NbTi}(T)\, dT$$

$$coil_{heatCapacity}(T_c, T_h) := coil_{mass.copper} \cdot cp\_int_{copper}(T_c, T_h) + coil_{mass.NbTi} \cdot cp\_int_{NbTi}(T_c, T_h)$$

$$coil_{heatCapacity}(4.2K, 300K) = 638.247 \cdot kJ$$

Al6061-T6 heat capacity data from NIST

4 K to 300 K

$a_{Al6061} := 46.6467$ $b_{Al6061} := -314.292$ $c_{Al6061} := 866.662$ $d_{Al6061} := -1298.3$

$e_{Al6061} := 1162.27$ $f_{Al6061} := -637.795$ $g_{Al6061} := 210.351$ $h_{Al6061} := -38.3094$ $i_{Al6061} := 2.96344$

$$cp_{Al6061}(T) := 10^{a_{Al6061}+b_{Al6061}\cdot\left(\log\left(\frac{T}{K}\right)\right)+c_{Al6061}\cdot\left(\log\left(\frac{T}{K}\right)\right)^2+d_{Al6061}\cdot\left(\log\left(\frac{T}{K}\right)\right)^3+e_{Al6061}\cdot\left(\log\left(\frac{T}{K}\right)\right)^4+f_{Al6061}\cdot\left(\log\left(\frac{T}{K}\right)\right)^5+g_{Al6061}\cdot\left(\log\left(\frac{T}{K}\right)\right)^6+h_{Al6061}\cdot\left(\log\left(\frac{T}{K}\right.\right.}$$

$coilSupport_{OD} := coil_{ID} = 0.1\,m$ $coilSupport_{thickness} := 5mm$ $coilSupport_{length} := coil_{width} = 0.7\,m$ $\rho_{Al6061} := 2700\frac{kg}{m^3}$

$coilSupport_{ID} := coilSupport_{OD} - 2 \cdot coilSupport_{thickness} = 0.09\,m$

$coilSupport_{mass} := \rho_{Al6061} \cdot coilSupport_{length} \cdot \frac{\pi}{4} \cdot \left(coilSupport_{OD}^{\;2} - coilSupport_{ID}^{\;2}\right) = 2.82\,kg$

$$cp\_int_{coilSupport}(T_c, T_h) := \int_{T_c}^{T_h} cp_{Al6061}(T)\,dT$$

$coilSupport_{heatCapacity}(T_c, T_h) := coilSupport_{mass} \cdot cp\_int_{coilSupport}(T_c, T_h)$

$coilSupport_{heatCapacity}(4.2K, 300K) = 500.726 \cdot kJ$

Assume a thermal shield is made of OFHC copper

$$thermalShield_{ID} := coil_{OD} + 2 \cdot 50mm \qquad thermalShield_{thickness} := 2mm$$

$$thermalShield_{OD} := thermalShield_{ID} + 2 \cdot thermalShield_{thickness} = 0.212\,m$$

$$thermalShield_{length} := coil_{width} + 2 \cdot 100mm$$

$$thermalShield_{mass} := \rho_{copper} \cdot thermalShield_{length} \cdot \frac{\pi}{4} \cdot \left(thermalShield_{OD}^{\;2} - thermalShield_{ID}^{\;2}\right) = 10.64\,kg$$

$$thermalShield_{surfaceArea} := \pi \cdot thermalShield_{ID} \cdot thermalShield_{length} + 2 \cdot \frac{\pi}{4} \cdot thermalShield_{ID}^{\;2} = 0.656\,m^2$$

$$thermalShield_{heatCapacity}\left(T_c, T_h\right) := thermalShield_{mass} \cdot \int_{T_c}^{T_h} cp_{OFHC}(T)\,dT$$

$$thermalShield_{heatCapacity}(60K, 300K) = 827.812 \cdot kJ$$

Static heat loads on thermal shield

$$q_{rad.RTtoThermalShield} := 1.6\frac{W}{m^2}\cdot\left(2\cdot\frac{\pi}{4}\cdot thermalShield_{OD}{}^2 + \pi\cdot thermalShield_{OD}\cdot thermalShield_{length}\right) = 1.072\cdot W$$

MLI wrapped

$$q_{conduction.RTtoThermalShield} := 50W$$

conduction thru support structure and magnet current leads (assumed)

$$q_{static.RTtoThermalShield} := q_{rad.RTtoThermalShield} + q_{conduction.RTtoThermalShield} = 51.072\cdot W$$

Static heat load on the magnet coil

$$\varepsilon_{copper} := 1\% \qquad \varepsilon_{coil} := 100\%$$

$$q_{rad}\left(T_{coil}, T_{thermalShield}\right) := \frac{\left(coil_{surfaceArea}\right)\cdot\sigma_{SB}\cdot\left(T_{thermalShield}{}^4 - T_{coil}{}^4\right)}{\frac{1}{\varepsilon_{coil}} + \left(\frac{1}{\varepsilon_{copper}} - 1\right)\cdot\frac{coil_{surfaceArea}}{thermalShield_{surfaceArea}}}$$

radiation from thermal shield to magnet

$$q_{rad}(4.2K, 300K) = 2.962\cdot W$$

maximum value is when thermal shield is at 300 K and magnet coil is at 4.2 K.

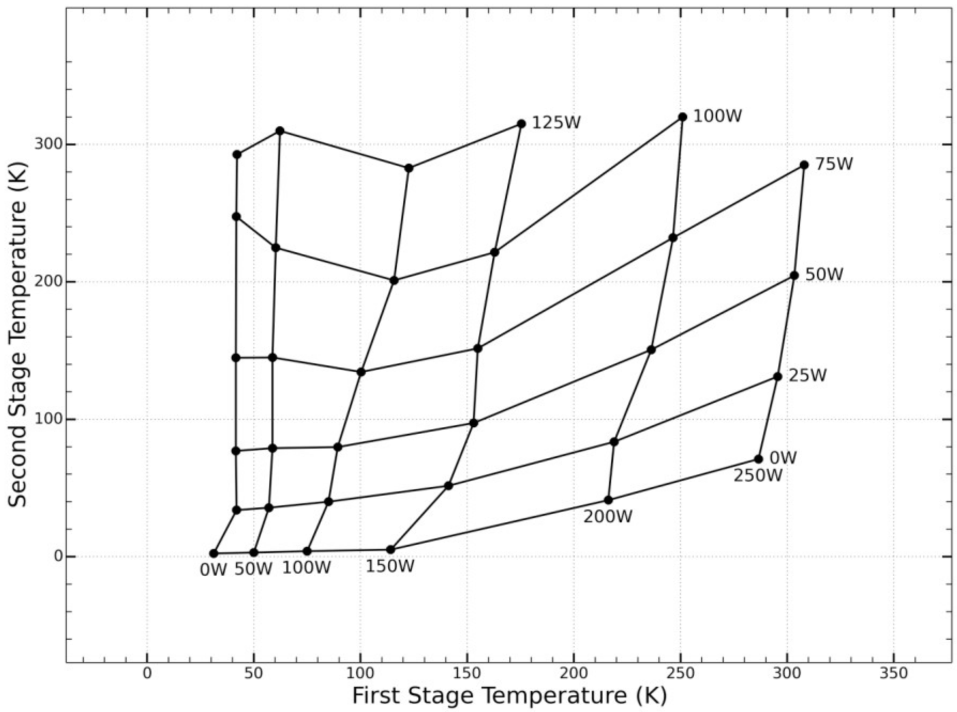
$$q_{rad}(4.2K, 60K) = 4.739\times 10^{-3}\cdot W$$

tiny, after the thermal shield has cooled to 60 K

$$q_{support.leak} := 1W$$

assumed

Cryocooler capacity (M A Green's paper): PT415



With 50 W on stage 1

stage2 :=

	0	1
0	4.2	1.5
1	27	25
2	80	50
3	145	75
4	205	100
5	307	125

With 0 W on stage 2

stage1 :=

	0	1
0	30	0
1	50	50
2	75	100
3	115	150
4	215	200
5	290	250

$stage2_{interpolation} := cspline\left(stage2^{\langle 0\rangle}, stage2^{\langle 1\rangle}\right)$

$stage2_{capacity}\left(stage2_{temp}\right) := interp\left(stage2_{interpolation}, stage2^{\langle 0\rangle}, stage2^{\langle 1\rangle}, \frac{stage2_{temp}}{K}\right) W$

$stage2_{capacity}(300K) = 124.64 \cdot W$

$$stage1_{interpolation} := cspline\left(stage1^{\langle 0 \rangle}, stage1^{\langle 1 \rangle}\right)$$

$$stage1_{capacity}\left(stage1_{temp}\right) := interp\left(stage1_{interpolation}, stage1^{\langle 0 \rangle}, stage1^{\langle 1 \rangle}, \frac{stage1_{temp}}{K}\right) W$$

$$stage1_{capacity}(300K) = 261.881 \cdot W$$

Cooldown time

$$\text{thermalShieldCooldown}_{rate}(T) := \frac{q_{static.RTtoThermalShield} - \text{stage1}_{capacity}(T)}{\text{thermalShield}_{mass} \cdot cp_{OFHC}(T)}$$

$$\text{thermalShieldCooldown}_{time} := \int_{300K}^{60K} \frac{1}{\text{thermalShieldCooldown}_{rate}(T)}\,dT = 1.919 \cdot hr$$

$$\text{magnetCooldown}_{rate}(T) := \frac{q_{support.leak} + q_{rad}(4.2K, 300K) - \text{stage2}_{capacity}(T)}{\text{coil}_{mass.copper} \cdot cp_{OFHC}(T) + \text{coil}_{mass.NbTi} \cdot cp_{NbTi}(T) + \text{coilSupport}_{mass} \cdot cp_{Al6061}(T)}$$

$$\text{magnetCooldown}_{time} := \int_{300K}^{4.2K} \frac{1}{\text{magnetCooldown}_{rate}(T)}\,dT = 4.058 \cdot hr$$

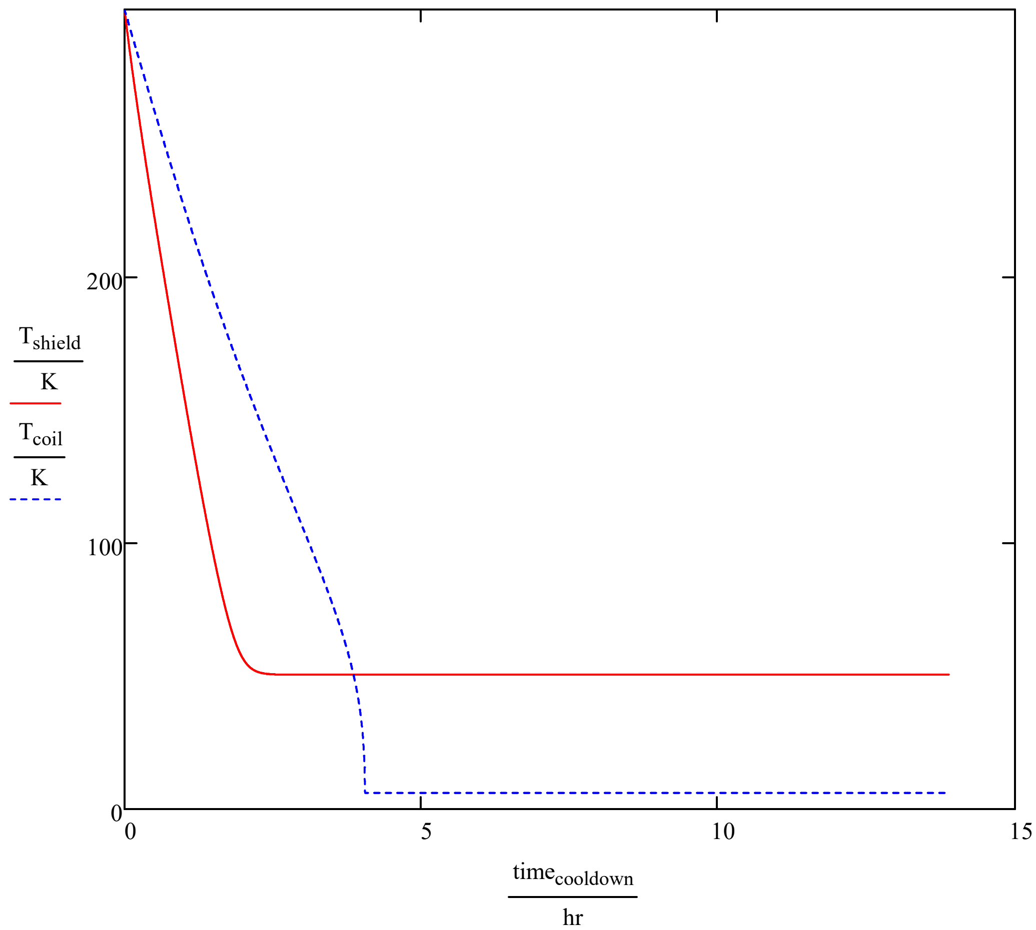
We have assumed perfect thermal conductance between the cryocooler and the coil, which is not the real case. The thermal link can have considerable thermal resistance that can prolong the cooldown time. Additionally, we have assumed the magnet coil + its aluminum support to be a 'lumped' element (i.e, all of it cools down with spatial uniformity). This is again not the real case. There will be spatial temperature gradients in the coil body. The two effects may cause the cooldown time to be as much as 2 or 3 times the one calculated above.

Temperature vs. time during cooldown

$\Delta t := 5s$ assume a time step << cooldown time

$$
\text{cooldown} := \left|
\begin{array}{l}
i \leftarrow 0 \\
T_{shield_i} \leftarrow 300K \\
T_{coil_i} \leftarrow 300K \\
time_{cooldown_i} \leftarrow 0s \\
\text{while } i < 10000 \\
\quad \left| \begin{array}{l}
T_{shield_{i+1}} \leftarrow T_{shield_i} + \Delta t \cdot thermalShieldCooldown_{rate}\left(T_{shield_i}\right) \\
T_{coil_{i+1}} \leftarrow T_{coil_i} + \Delta t \cdot magnetCooldown_{rate}\left(T_{coil_i}\right) \\
time_{cooldown_{i+1}} \leftarrow time_{cooldown_i} + \Delta t \\
i \leftarrow i + 1
\end{array} \right. \\
\begin{pmatrix} \dfrac{T_{shield}}{K} \\ \dfrac{T_{coil}}{K} \\ \dfrac{time_{cooldown}}{s} \end{pmatrix}
\end{array}
\right.
$$

$T_{shield} := cooldown_0 \cdot K$ $T_{coil} := cooldown_1 \cdot K$ $time_{cooldown} := cooldown_2 \cdot s$



Tshield / K
Tcoil / K
200
100
0
0
5
10
15
timecooldown / hr

Calculation of temperature rise following a quench

$coil_{current} := 160A \qquad coil_{inductance} := 0.44171H$

$$coil_{energy} := \frac{1}{2} coil_{inductance} \cdot coil_{current}^{2} = 5.654 \times 10^{3} \cdot J$$

a) assume that the ENTIRE COIL absorbes the stored energy (no heat transfer to coil support)

$T_{coil.cold} := 4.2K \qquad T_{coil.warm.guess} := 60K$

$$energy_{coil.zero}(y) := \int_{T_{coil.cold}}^{y} \left(coil_{mass.copper} \cdot cp_{OFHC}(T) + coil_{mass.NbTi} \cdot cp_{NbTi}(T)\right) dT - coil_{energy}$$

$$T_{coil.warm} := root\left(energy_{coil.zero}\left(T_{coil.warm.guess}\right), T_{coil.warm.guess}\right)$$

$T_{coil.warm} = 39.643\,K$

b) assume that only the quenching LAYER absorbes the stored energy (no heat transfer to coil support)

$T_{layer.cold} := 4.2K \qquad T_{layer.warm.guess} := 60K$

$$energy_{layer.zero}(y) := \int_{T_{layer.cold}}^{y} \left(\frac{coil_{mass.copper}}{noOfLayers} \cdot cp_{OFHC}(T) + \frac{coil_{mass.NbTi}}{noOfLayers} \cdot cp_{NbTi}(T)\right) dT - coil_{energy}$$

$$T_{layer.warm} := root\left(energy_{layer.zero}\left(T_{layer.warm.guess}\right), T_{layer.warm.guess}\right)$$

$T_{layer.warm} = 64.519\,K$

$$+i_{OFHC}\cdot\left(\log\left(\frac{T}{K}\right)\right)^{8}\cdot\frac{J}{kg\cdot K}$$

$$\left.\left.\frac{T}{K}\right)\right)^{7} + i_{Al6061}\cdot\left(\log\left(\frac{T}{K}\right)\right)^{8} \cdot \frac{J}{kg \cdot K}$$