Status and Plans for HVP contribution to the muon anomalous magnetic moment from FNAL/HPQCD/MILC

C.T.H. Davies, C. DeTar, A.X. El-Khadra, E. Gámiz, Steven Gottlieb*, D. Hatton, A.S. Kronfeld, J. Laiho, G.P. Lepage, Yuzhi Liu, P.B. Mackenzie, C. McNeile, E.T. Neil, T. Primer, J.N. Simone, D. Toussaint, R.S. Van de Water, and A. Vaquero
(Fermilab Lattice/HPQCD/MILC Collaborations)

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Institute of Nuclear Theory
University of Washington
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Current Status

- Light quarks make by far the largest contribution compared with s, c, and b quarks.

light	$623.1 \pm 8.3 \pm 13.0$
strange	53.40 ± 0.60
charm	14.40 ± 0.40
bottom	0.270 ± 0.040
$a_\mu^{\mathrm{HVP-LO}}$	691 ± 15

- Second error of first line is from estimate of strong iso-spin breaking, electromagnetism, and quark-disconnected diagrams. (See 1902.04223 for details.)

Outline

- ✦ Start with connected light quark contribution
  - last publication
  - current status
  - future running
- ✦ Scale setting
- ✦ Disconnected contribution
  - current and future running
- ✦ Potential improvements

Lattice Ensembles

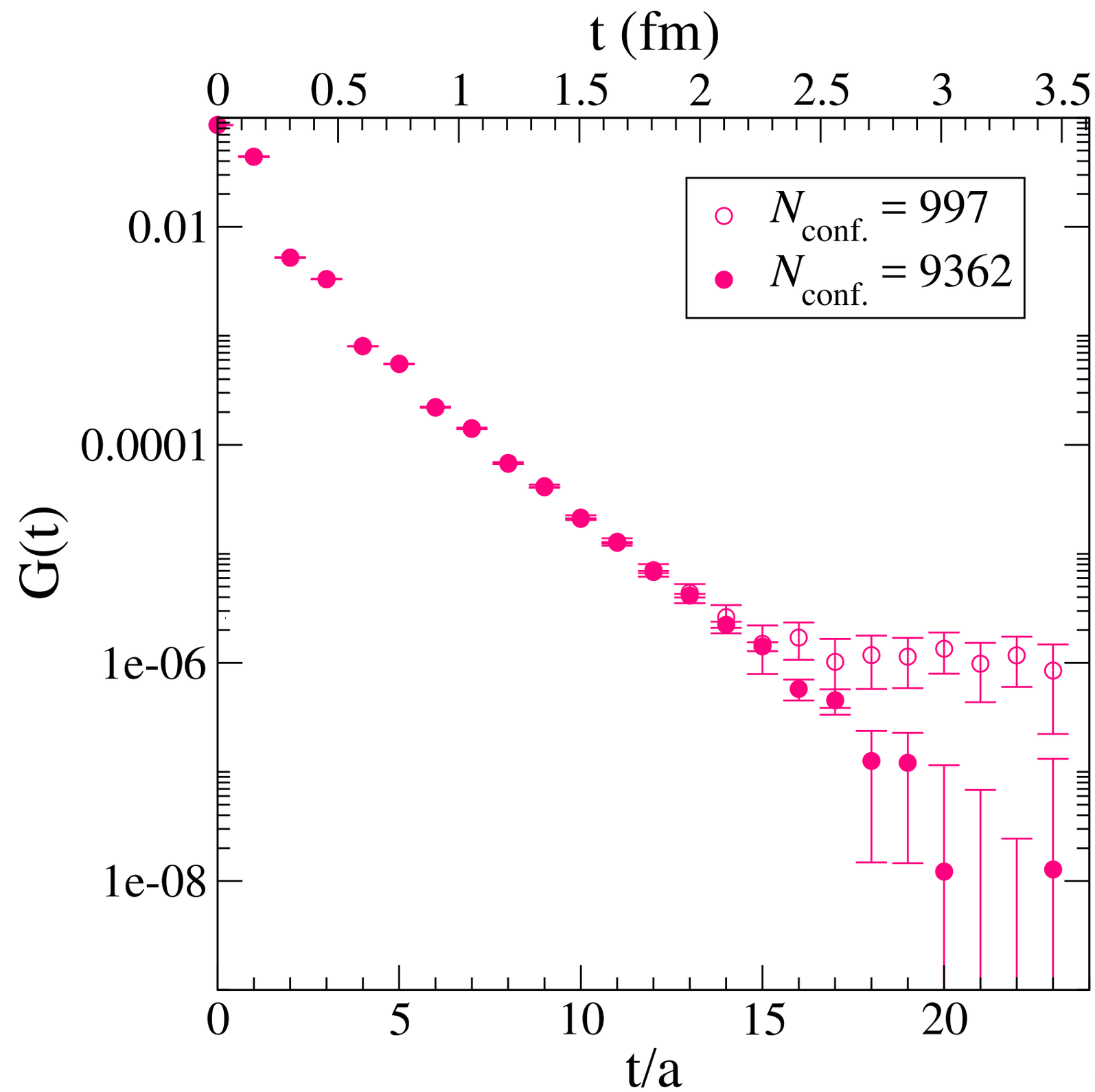
- We use N_f=2+1+1 HISQ ensembles from the MILC collaboration with physical light quark masses

$\approx a$ (fm)	$am_l^{\rm sea}/am_s^{\rm sea}/am_c^{\rm sea}$	w_0/a	M_{π_5} (MeV)	$(L/a)^3 \times (T/a)$	$N_{\rm conf.}$
0.15	0.00235/0.0647/0.831	1.13670(50)	133.04(70)	$32^3 \times 48$	997
0.15	0.002426/0.0673/0.8447	1.13215(35)	134.73(71)	$32^3 \times 48$	9362
0.12	0.00184/0.0507/0.628	1.41490(60)	132.73(70)	$48^3 \times 64$	998
0.09	0.00120/0.0363/0.432	1.95180(70)	128.34(68)	$64^3 \times 96$	1557
0.06	0.0008/0.022/0.260	3.0170(23)	134.95(72)	$96^3 \times 192$	1230

- Results from above ensembles appeared in arXiv: 1902.04223
- Challenging to calculate current-current correlator at large t, but higher statistics do help.

a≈0.15 fm Correlator

- Higher statistics help, but beyond about 2 fm, difficult to get accurate results
- Can fit the propagator at shorter distance and use fit to extend to t>t*
- Bounding method replaces correlator by (a) expected 2-pion contribution for $t>t_c$, or (b) zero.

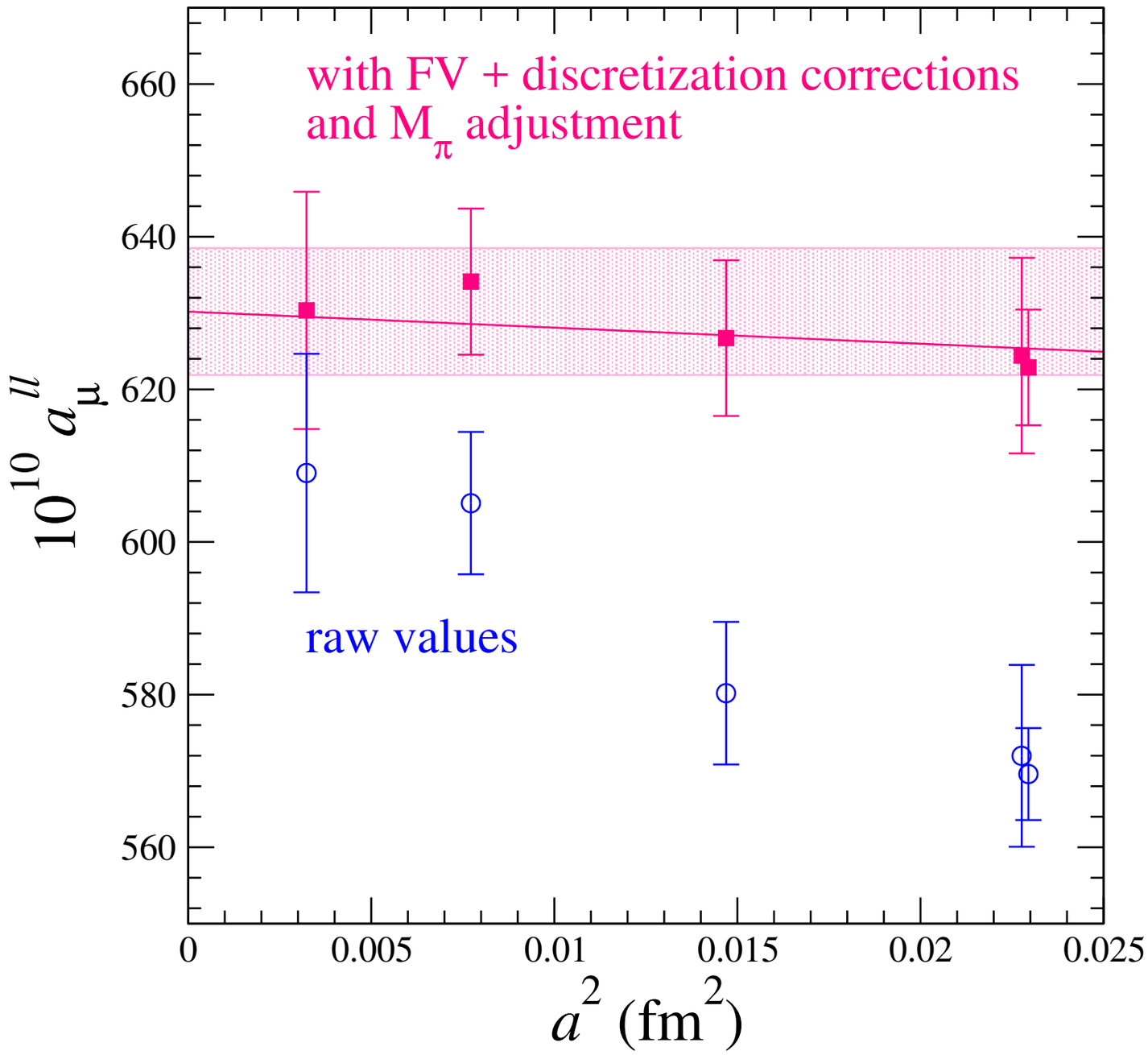


Continuum Limit (Feb. 2019)

- Must fit results as a function of a^2, taking mistuning into account

$$a_\mu^{ll}(\text{latt.}) = a_\mu^{ll}(\text{conn.})\left(1 + c_s \sum_{f=l,l,s,c} \frac{\delta m_f}{\Lambda} + c_{a^2}\frac{(a\Lambda)^2}{\pi^2}\right)$$

- Λ=500 MeV
- Priors: c_s=0.0(3); c_{a^2}=0(1)
- $a_\mu^{\ell\ell}(\text{conn}) = 630.1(8.3)$
- This is only light-quark connected contribution



Connected light-quark error budget

- Error budget in percent for anomalous magnetic moment and two Taylor coefficients
  - top three contributions are dominant, but finite volume effects become important for larger moments

Source	a_μ^{ll}(conn.) (%)	Π_1^{ll}(conn.) (%)	Π_2^{ll}(conn.) (%)
Lattice-spacing (a^{-1}) uncertainty	0.8	0.9	1.6
Monte Carlo statistics	0.7	0.7	1.1
Continuum ($a \to 0$) extrapolation	0.7	0.7	0.8
Finite-volume & discretization corrections	0.3	0.4	1.7
Current renormalization (Z_V)	0.1	0.1	0.1
Chiral (m_l) interpolation	0.1	0.1	0.0
Sea (m_s) adjustment	0.1	0.1	0.1
Pion mass ($M_{\pi,5}$) uncertainty	0.0	0.0	0.1
Total	1.3%	1.4%	2.7%

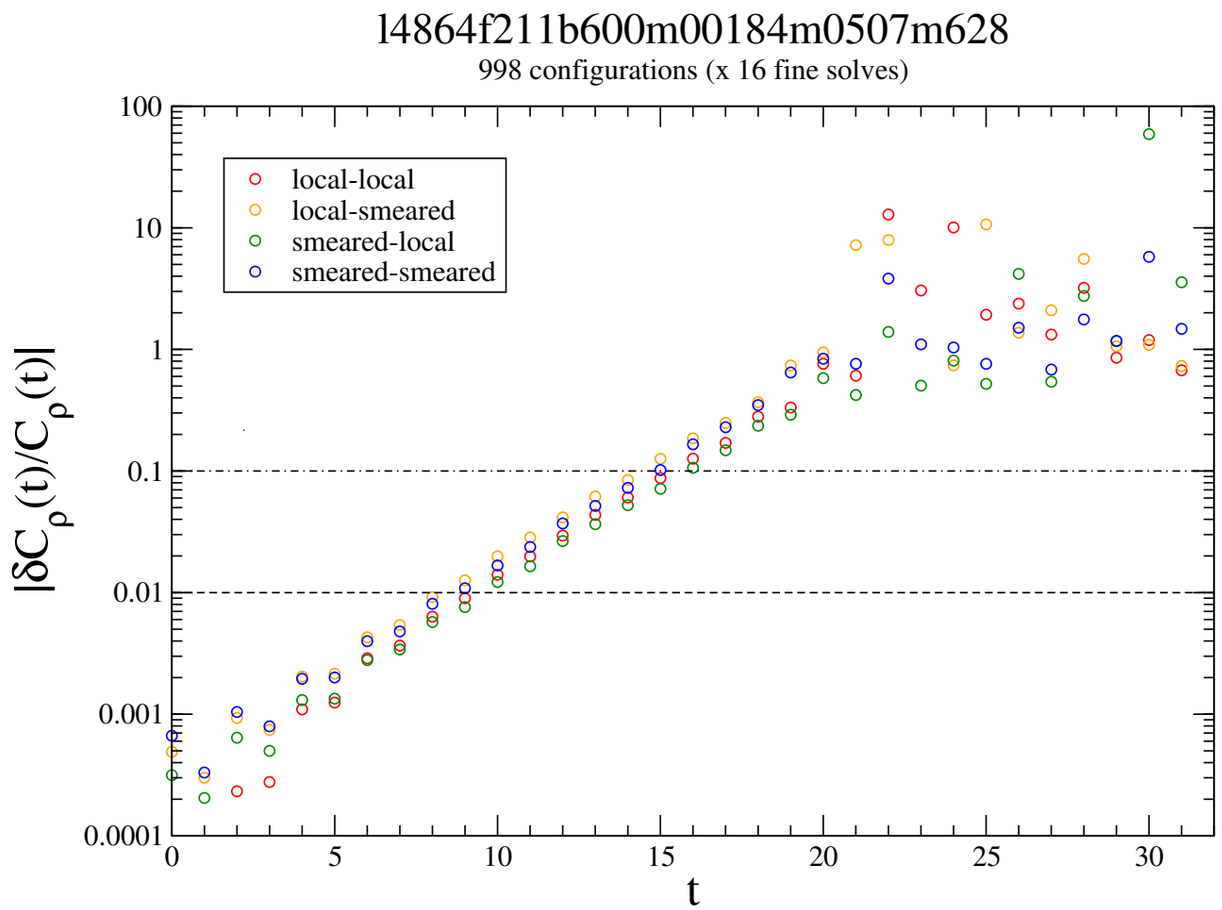
What Needs to be Improved?

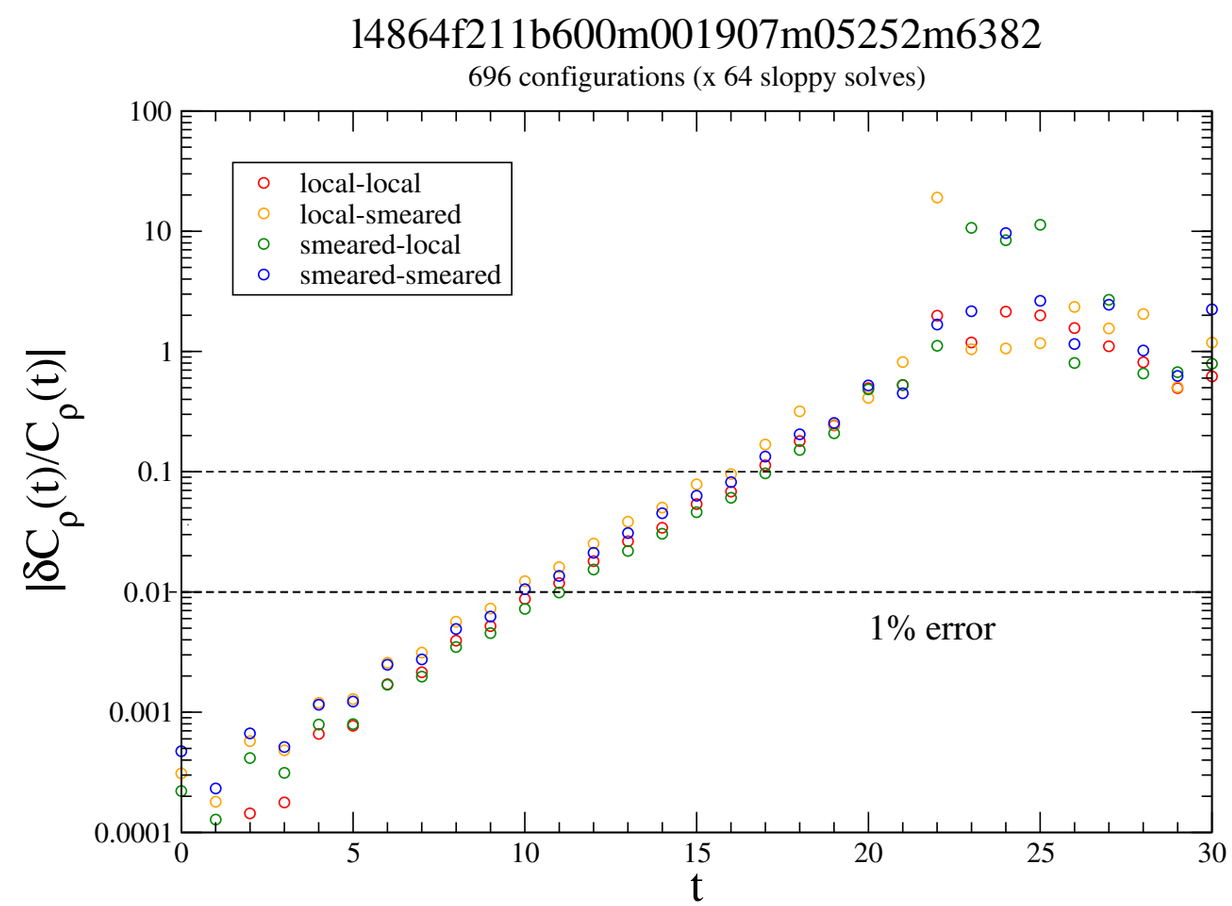
- ✦ Biggest error comes from scale setting
  - We used ω_0 to set the relative scale, using f_π for absolute scale
  - We can improve our own calculation of ω_0 by increasing statistics of our Wilson flow calculation
  - Need to consider other ways of setting scale
- ✦ Next largest errors are from statistics and continuum extrapolation
  - They can both be improved by reducing errors on our physical mass ensembles
  - 0.12 fm ensemble is being replaced with one with better tuned quark masses and much higher statistics

Current Status

- ✦ 0.12 fm
  - previously 998 configurations with 16 fine solves
  - new ensemble with better tuned quark masses
  - 64 sloppy solves, 1 fine; currently ~700 analyzed
  - plan for 10,000 configs.; currently >6600
- ✦ 0.09 fm
  - previously 1557 configs, 16 sloppy solves
  - increased to 48 sloppy solves
- ✦ 0.06 fm
  - slowly increasing number of configs in backfill; 1230→1364

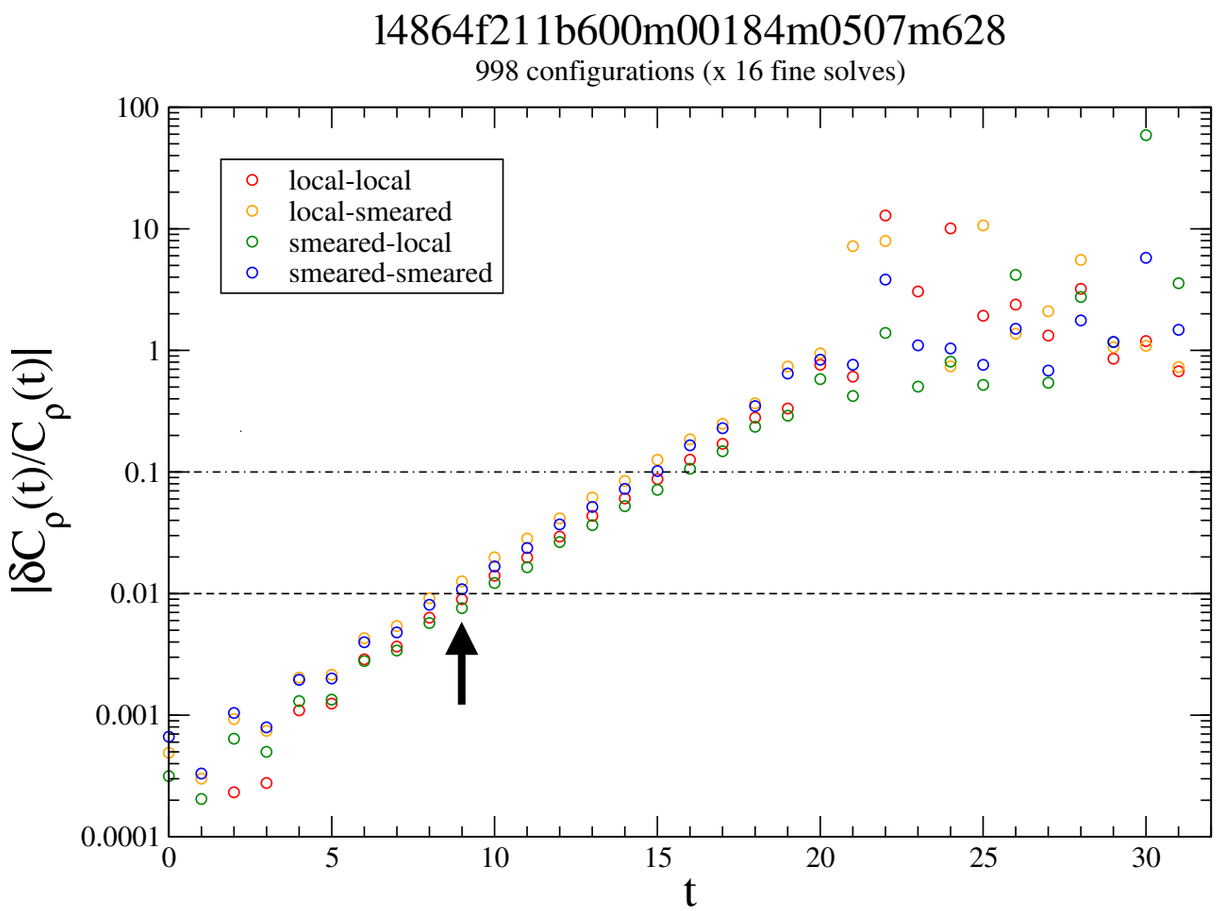
0.12 fm

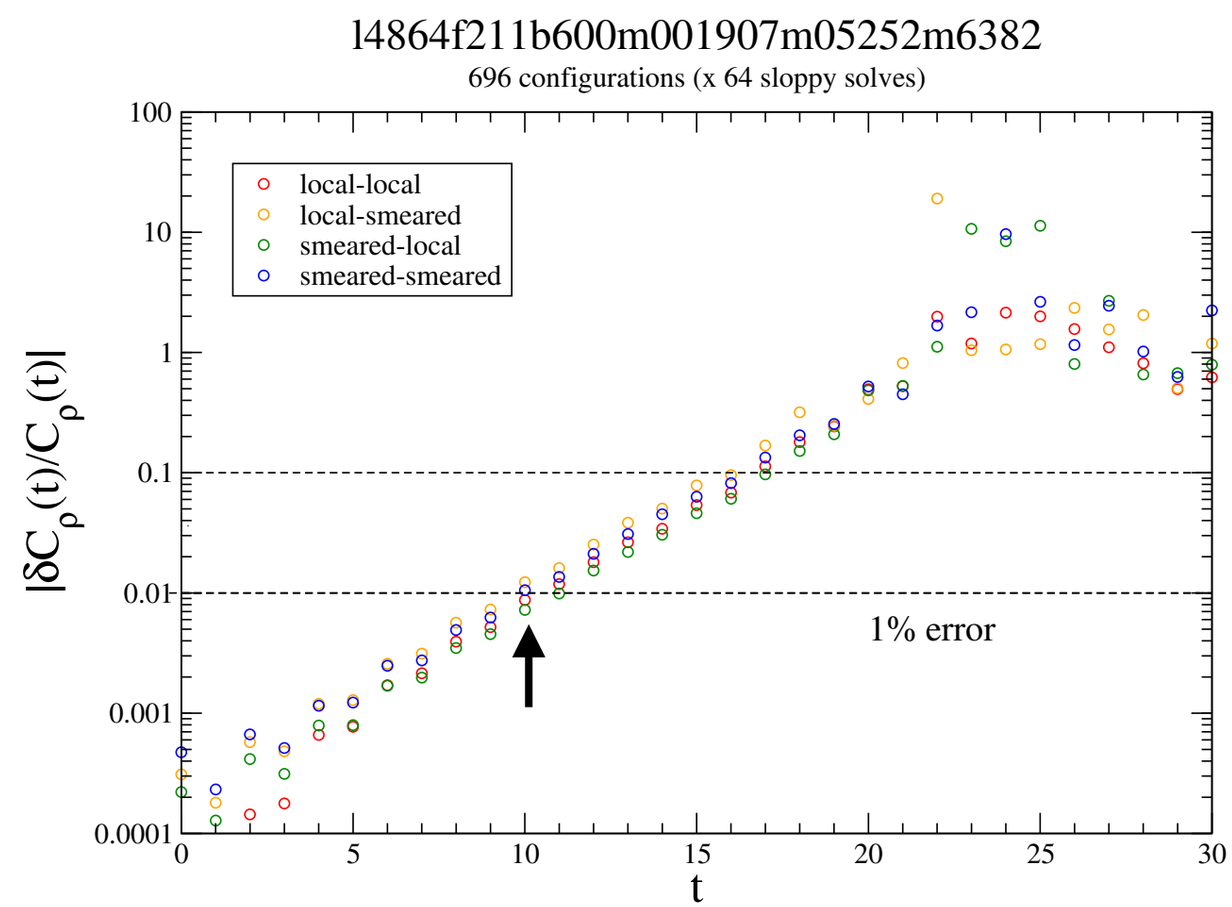




- Left: Relative error in vector propagator using 16 fine solves
- Right: Relative error using 64 sloppy and 1 fine solves.
  - Note smaller number of configurations and retuned quark masses
  - Planning for 10,000 configurations.

0.12 fm





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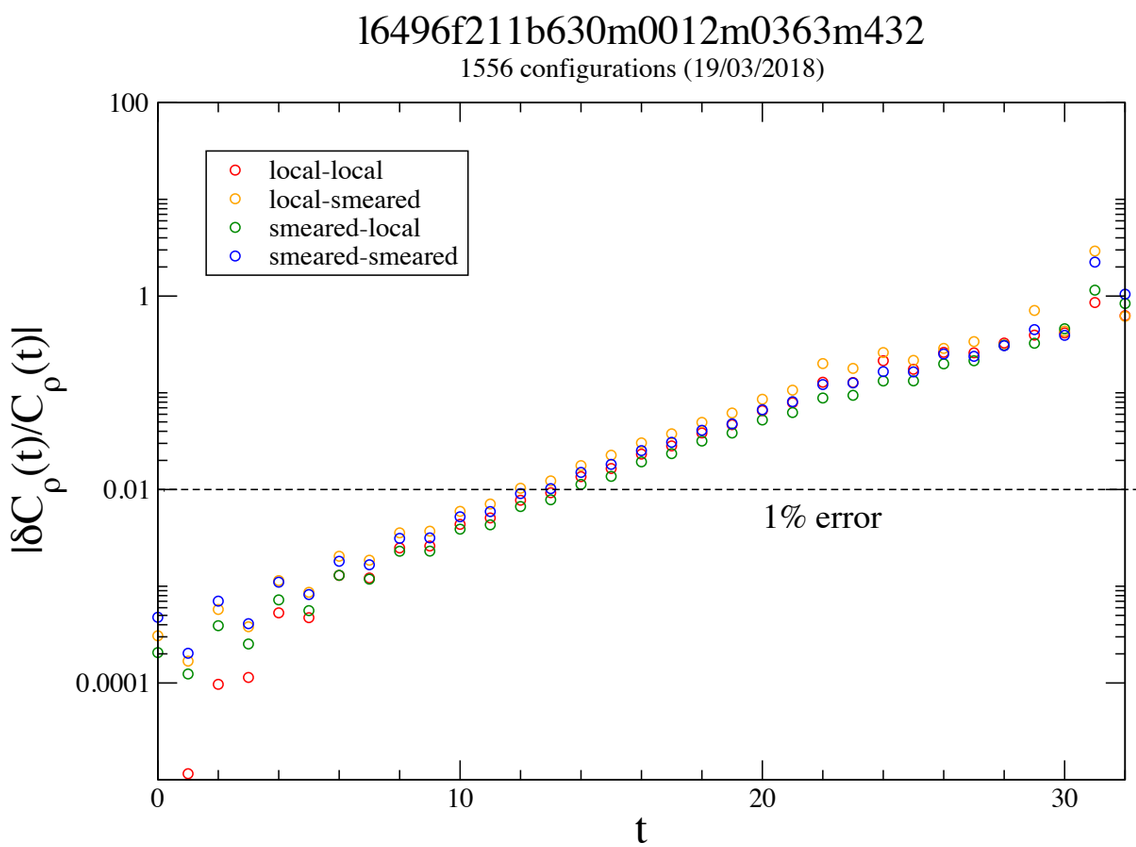
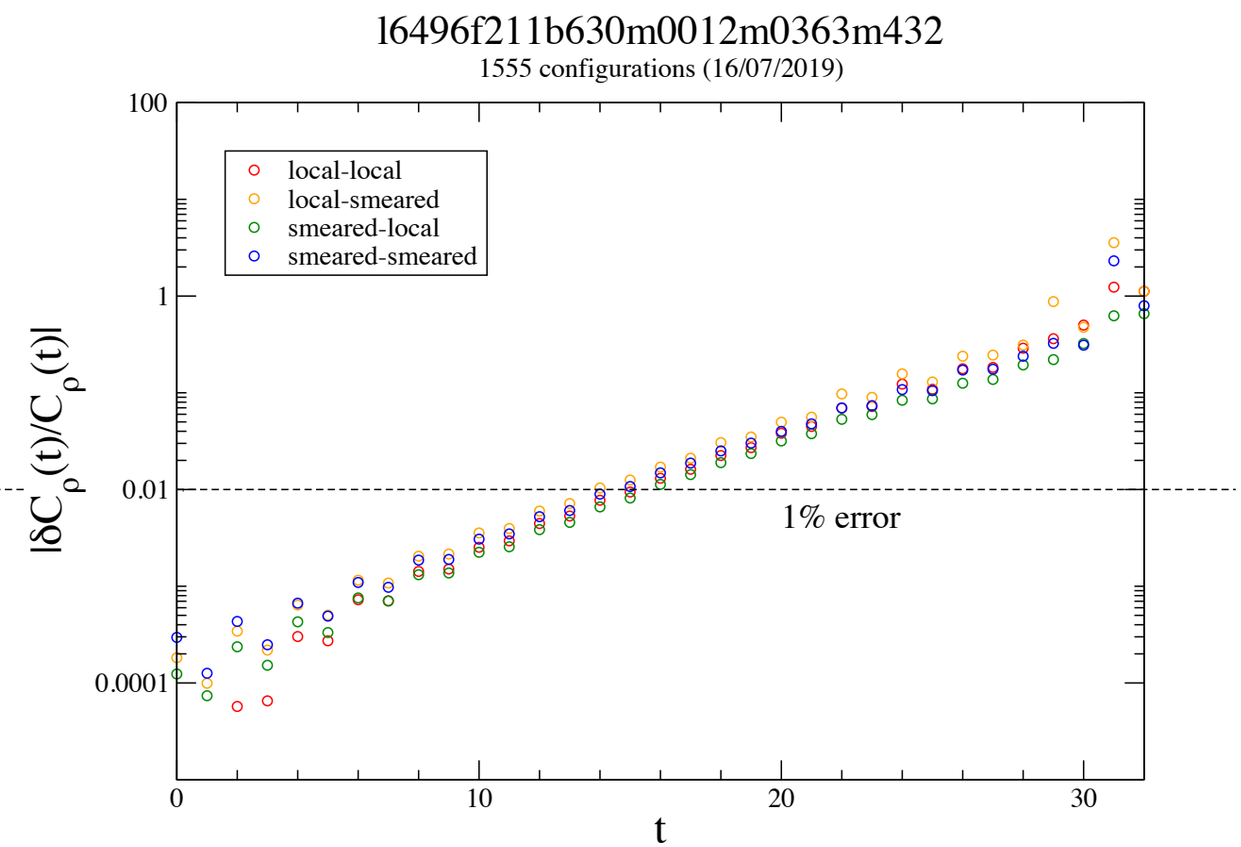
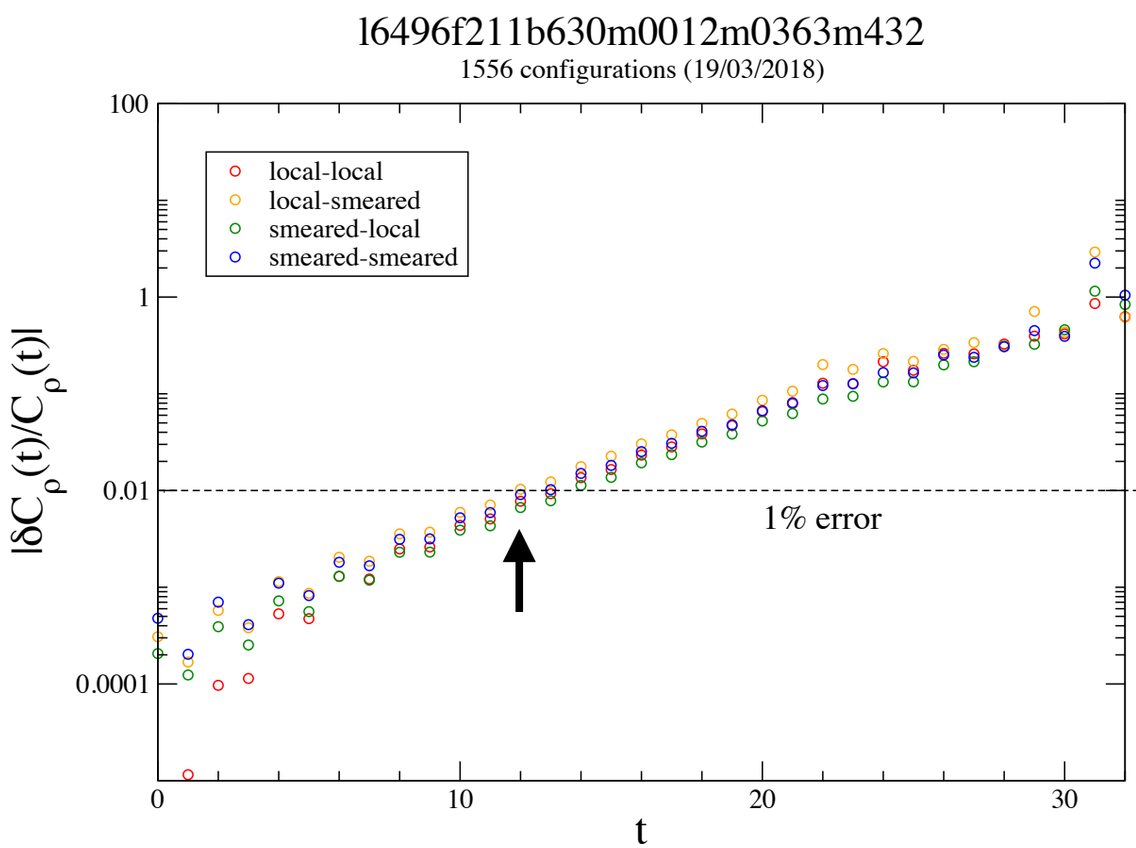
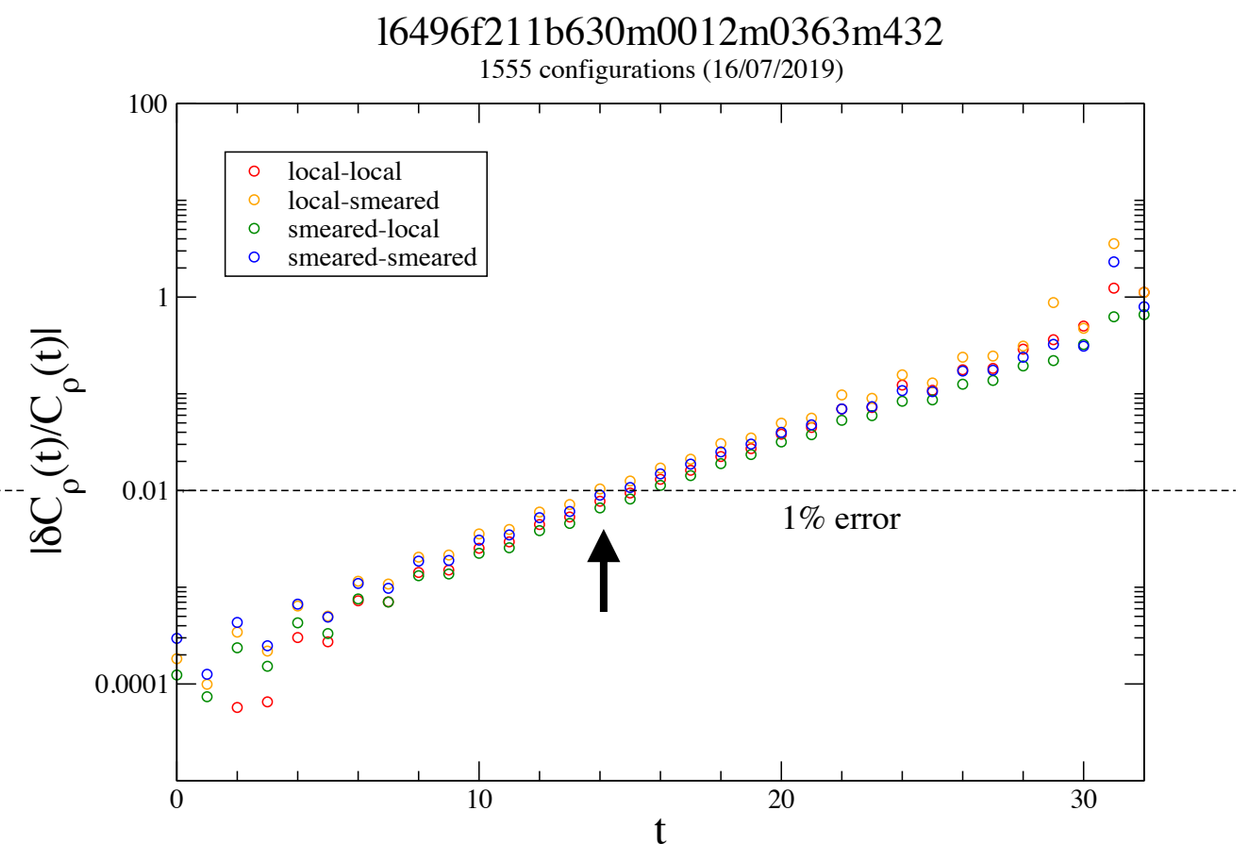




- On the left we have only 16 sloppy solves per configuration, and on the right we have 48 sloppy solves.
- Note that 1% error goes from t=12 to t=14.

0.09 fm





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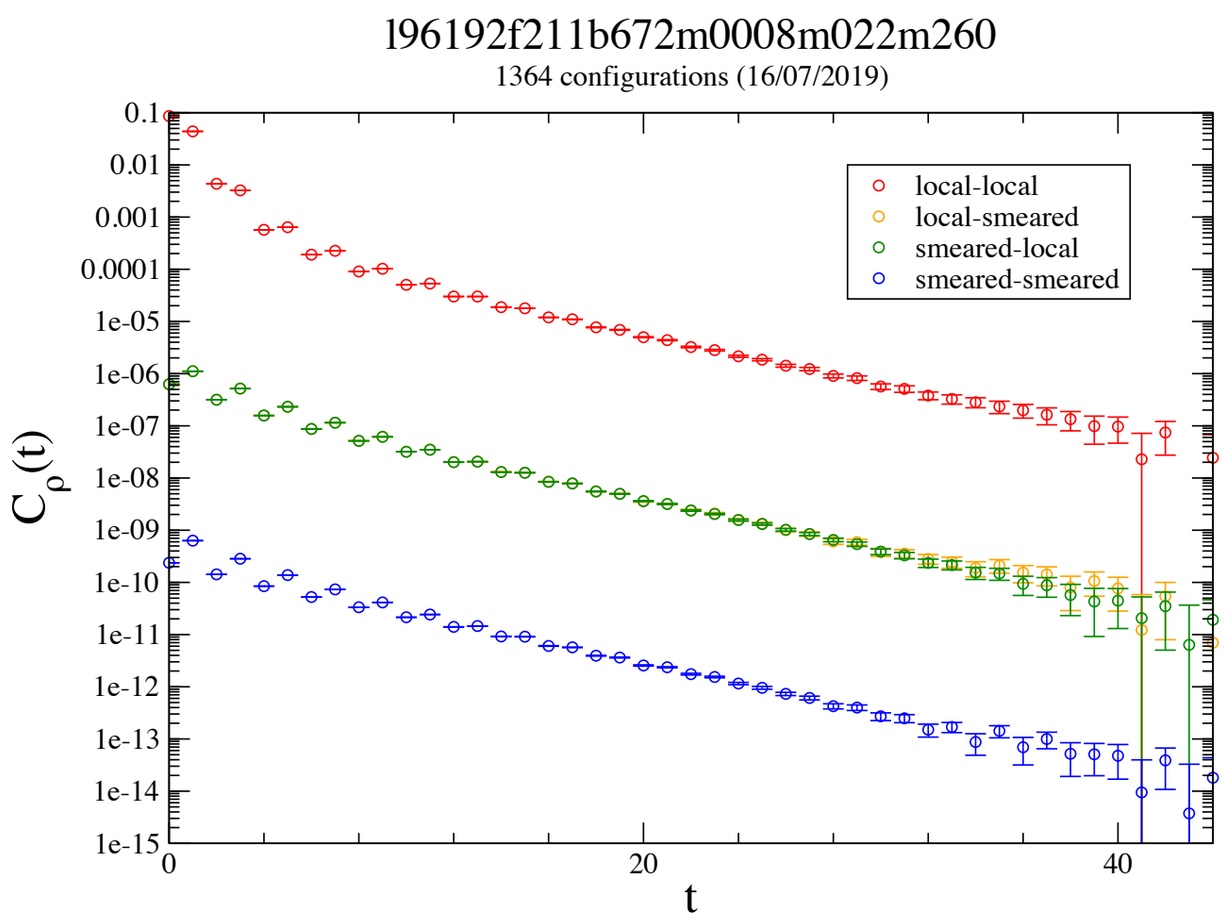
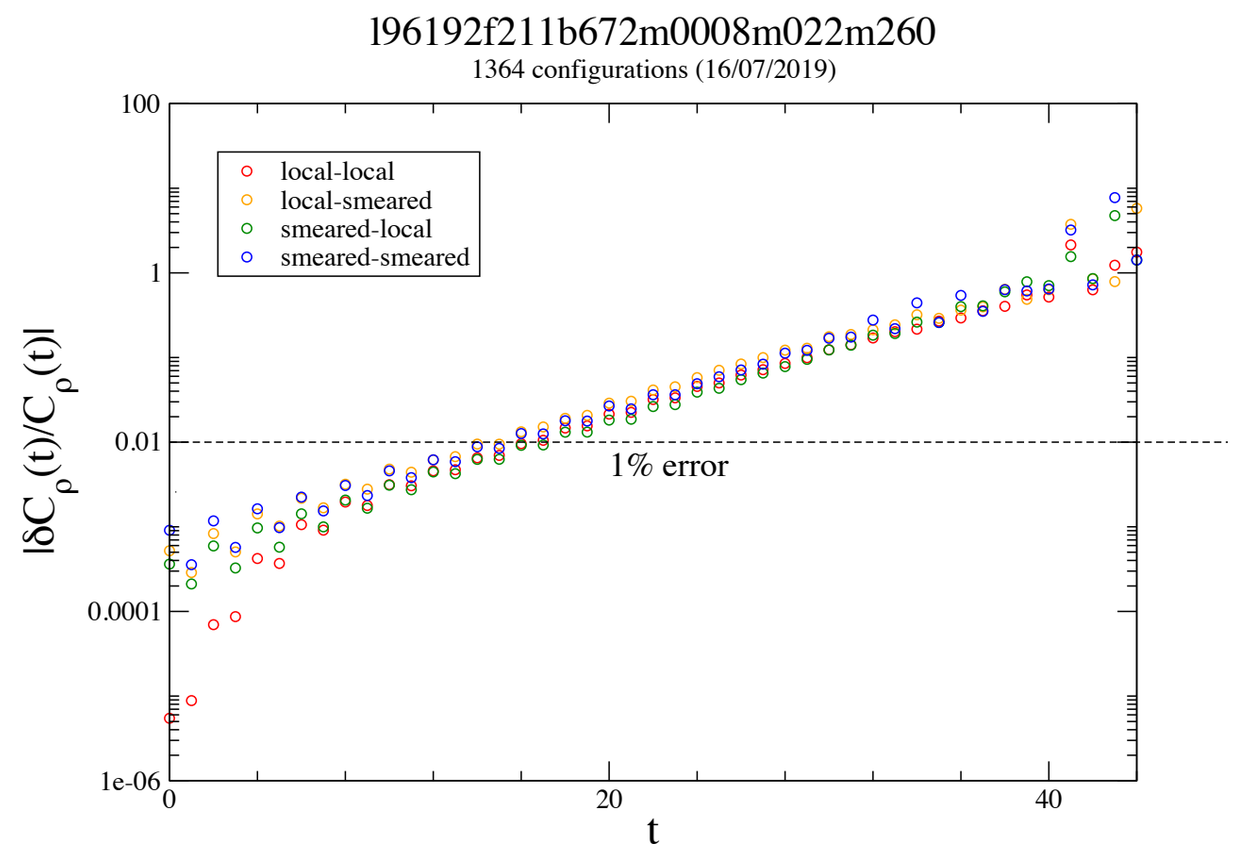
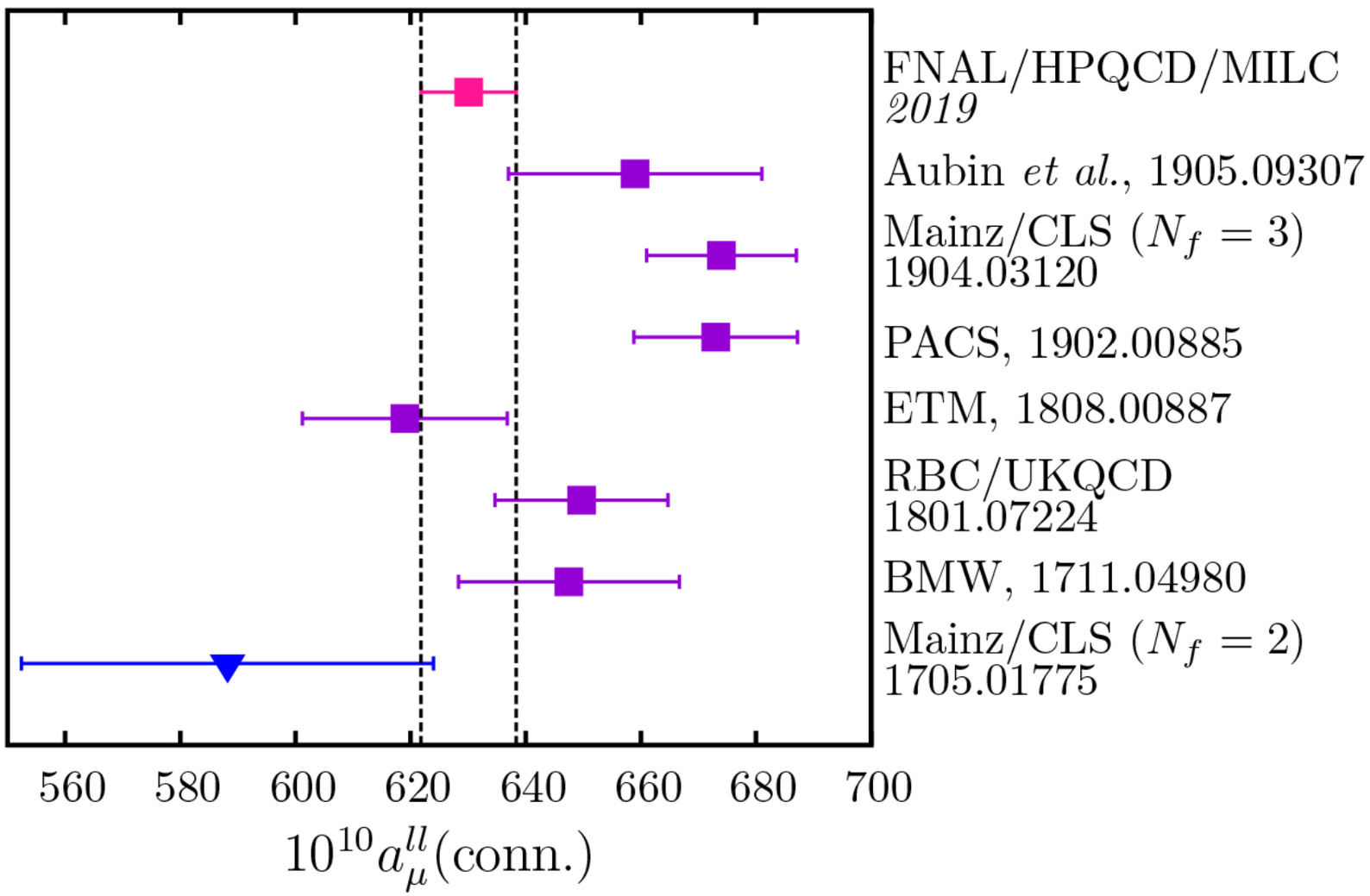




- Here we show vector current correlator on left and relative error on right.
- Statistics are only marginally better than in arXiv:1902.04223

Connected light-quark contribution comparison



- Comparison of our result and seven other recent lattice QCD calculations
- Small overall error results from small error in fit-method extrapolation and finite-volume correction

Scale Setting

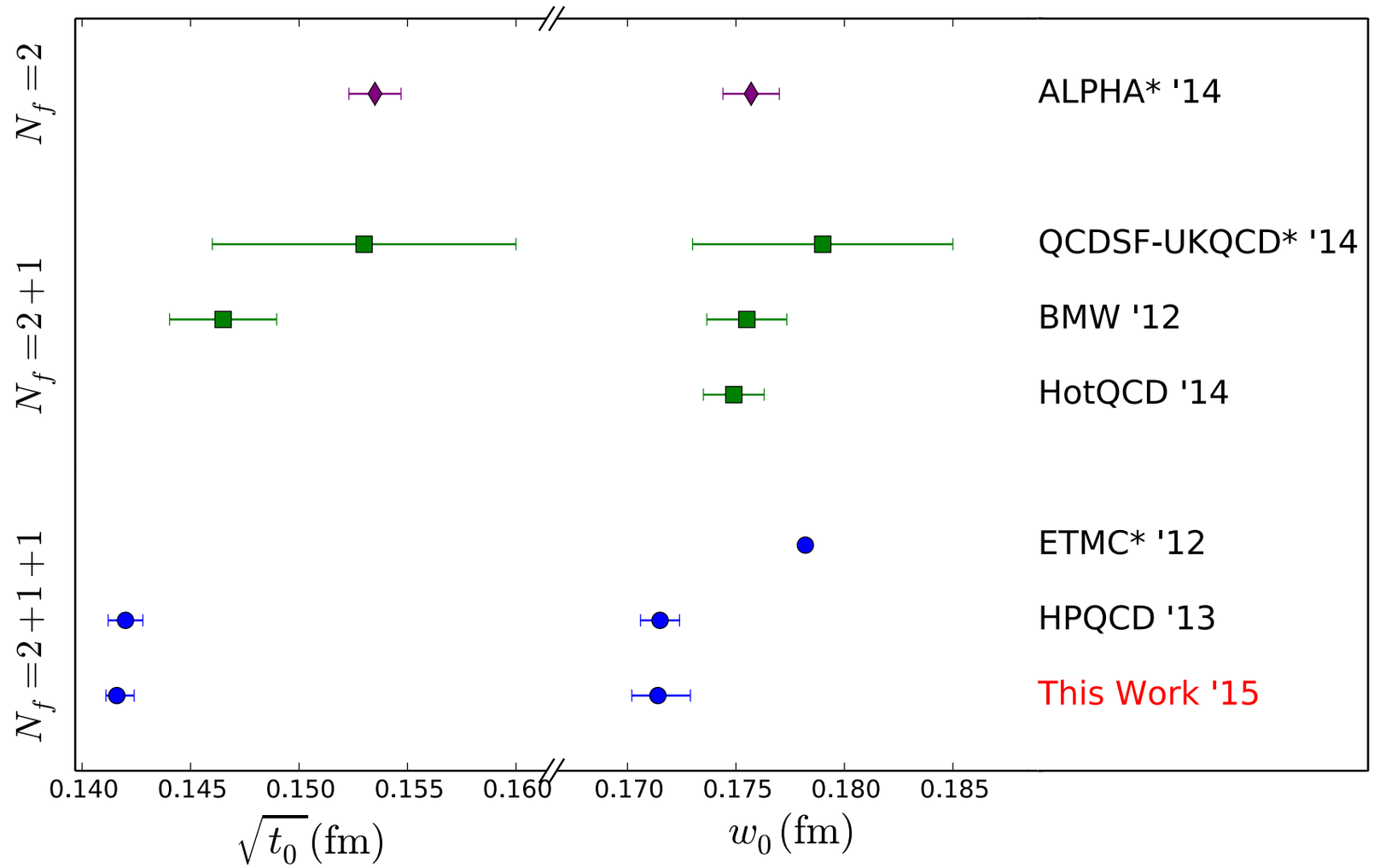
- Scale setting accounts for largest error
- Currently using Wilson flow parameter w_0

$\approx a(\mathrm{fm})$	Statistical Error (%)				
	r_1/a	af_π	aF_{p4s}	$\sqrt{t_0}/a$	w_0/a
0.15	0.36	0.11	0.06	0.02	0.03
0.12	0.25	0.08	0.06	0.04	0.07
0.09	0.33	0.09	0.06	0.03	0.07
0.06	0.12	0.11	0.06	0.03	0.06

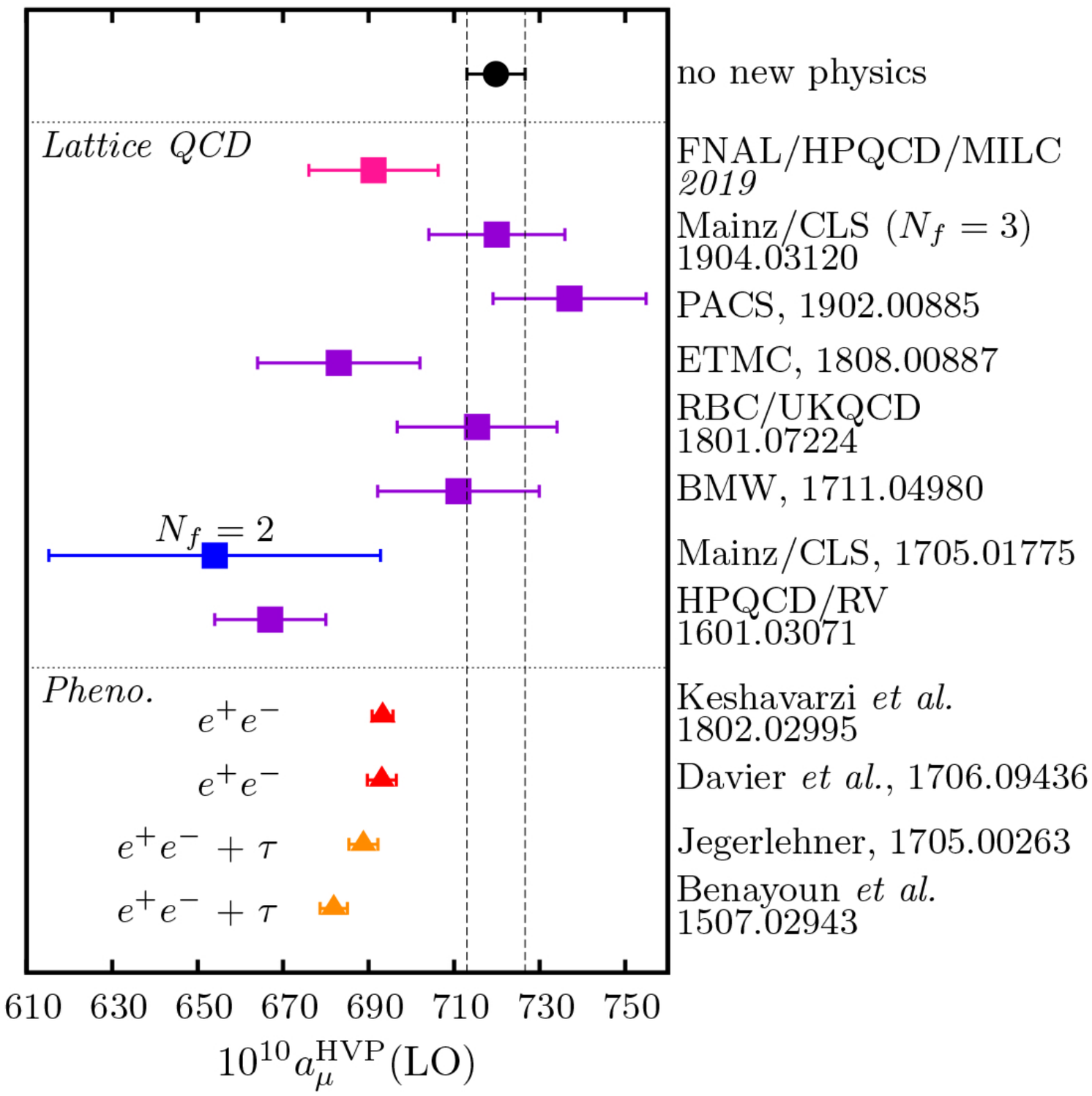
- It has excellent statistical precision, which we could easily improve with more running
- Does not depend on valence quark masses
- However, need to know continuum value for absolute scale

Comparison of w_0 Results

- From Bazavov et al. [MILC], PRD93, 094510 (2016)
- We are planning to improve this analysis with more statistics and finer lattice spacing
- Using HPQCD result
- Also looking at Ω baryon mass, and other possibilities



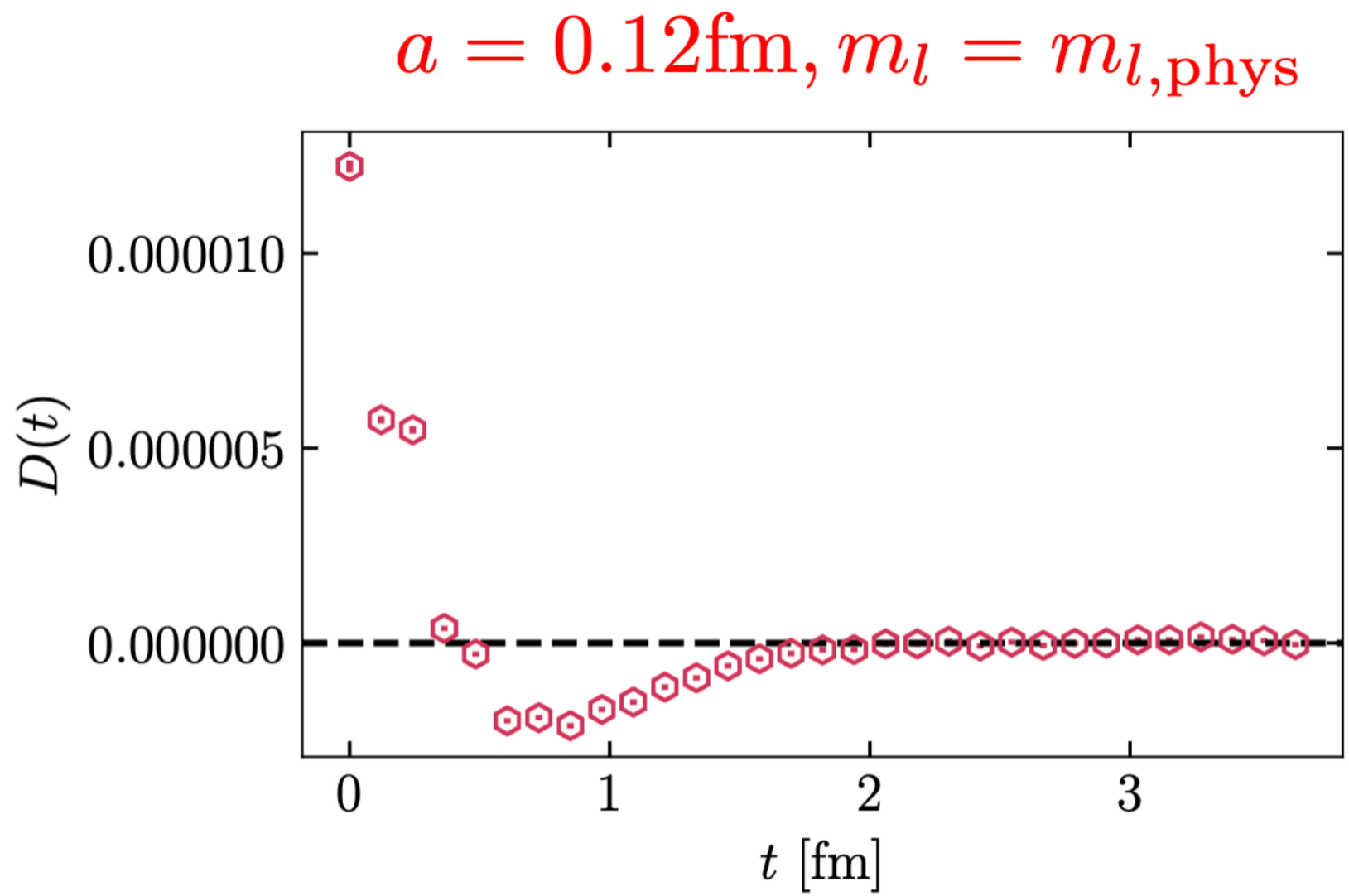
Leading Order HVP Comparison



- No new physics range shown at top
- Our result is next
- Seven recent LQCD calculations
- Four results based on R method shown under label Pheno.

Disconnected HVP

- Disconnected contribution is challenging to calculate, but not that large
- Beyond 2 fm, use fit to replace data to reduce noise
- Fit includes omega-like and rho-like states



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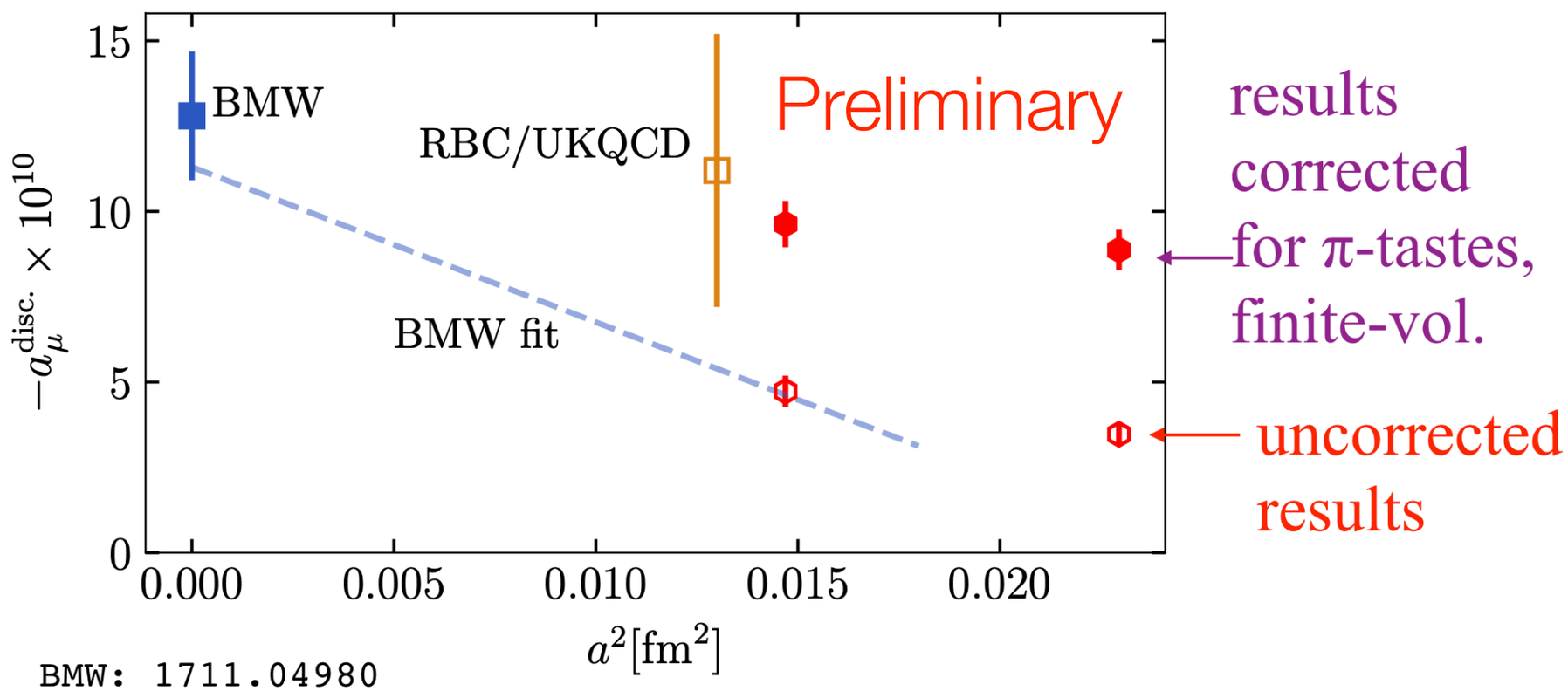

BMW: 1711.04980
RBC/UKQCD:1801.07224

- We find relatively large isospin breaking effects
- Plan to add another point with a≈0.09 fm
- BMW slope larger than what we are finding—actions are different

Other Important Corrections

Contribution	$10^{10} a_\mu^{ll}$(conn.)
$M_{\pi^0} \to M_{\pi^+}$	-4..3
$\pi\pi$ disconnected	-7..9
Total $\pi\pi$	-12.(3)
ρ, ω disconnected	-5.(5)
Strong-isospin breaking	10.(10)
Electromagnetism	0.(5)
Total correction	-7.(13)

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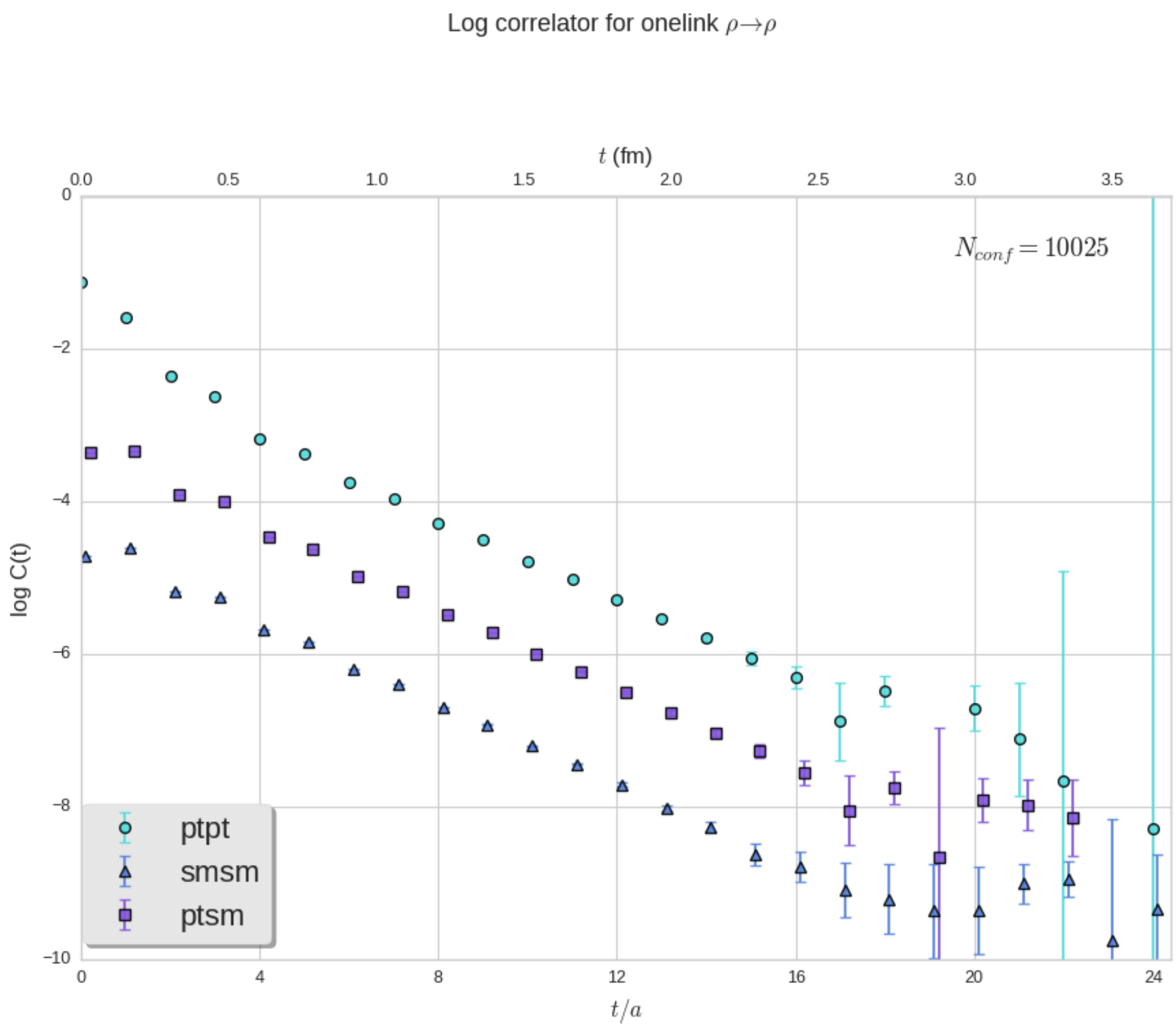
- Adding additional lattice spacings for disconnected contribution
- Including valence isospin breaking in 0.12 fm connected calculation
- Have one ensemble with dynamical QED

Exploring Additional Operators



- Shaun Lahert (UIUC) on Blue Waters
- one-link ρ source
- also use 2-π ops as source/sink to reconstruct long-distance tail
- expect it will reduce statistical errors
- also provides additional test of fit method

Conclusions & Prospects

✦We have calculated the LO-HVP contribution with a precision of 2.2%.

✦To achieve a precision of 0.5% within the next few years, we need to:

- Improve determination of lattice spacing (used to set scale)
- Improve statistics from 0.7% by a significant factor (investigating low-mode averaging for the connected correlator)
- Improve the continuum extrapolation error from 0.7% by analyzing finer lattice spacing ensembles we already have
- Include electromagnetic effects
- Continue to study isospin breaking

✦We look forward to the challenge!