Aubin, Blum, Golterman, Jung, Peris, Tu: Summary/Plans

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Muon g-2 Theory Initiative INT Workshop

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1. Summary
2. Future plans
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4. Appendix

Setup for the HVP calculation [Aubin et al., 2019]

- 2+1+1 flavors of HISQ fermions MILC [Bazavov et al., 2017]
- Three lattice spacings, $\sim$ 0.06, 0.09, 0.12 fm, roughly same volume
- Connected HVP computed only for degenerate light quarks, physical pion mass
- HVP constructed from conserved vector currents (no renormalization)
- Partially quenched: Naik term is omitted in the current
- Time momentum representation Bernecker-Meyer 2011
- AMA and full volume LMA RBC/UKQCD

  [Blum et al., 2018, Blum et al., 2013, Bali et al., 2010, Giusti et al., 2004, DeGrand and Schaefer, 2005]
- Finite volume corrections to NNLO in ChiPT in coordinate space

  c.f. [Bijnens and Relefors, 2017] in momentum space

HISQ 2+1+1 physical point ensembles MILC [Bazavov et al., 2017]

m_π (MeV)	a (fm)	size	L (fm)	$m_\pi L$	LM	AMA srcs	measurements (approx-exact-LMA)
133	0.12121(64)	$48^3 \times 64$	5.82	3.91	3000	$4^3 \times 4$	26-26-26
130	0.08787(46)	$64^3 \times 96$	5.62	3.66	3000	$4^3 \times 4$	36-36-40
134	0.05684(30)	$96^3 \times 192$	5.46	3.73	2000	$3^3 \times 8$	21-21-22

AMA+LMA:

- 3000 and 2000 (96^3) exact low modes of preconditioned Dirac op, $M^\dagger M$
- Eigenvectors of staggered Dirac op M come in pairs, $\pm i\lambda$
- Reconstruct eigenvector of M on all sites, (n_e, n_o), $i\lambda_n$
- Get second eignvector for free: $(n_e, -n_o)$, $-i\lambda_n$
- Use for full volume LMA, deflation of CG, improved approximation in AMA

Noise reduction: AMA+LMA RBC/UKQCD [Blum et al., 2013, Giusti et al., 2004, DeGrand and Schaefer, 2005]

All mode averaging (AMA) combined with full volume low mode averaging (LMA) can be very effective in reducing statistical errors for HVP (C. Lehner RBC/UKQCD [Blum et al., 2018])

$$\textbf{AMA} \qquad \langle O \rangle \;=\; \langle O \rangle_{\mathrm{exact}} - \langle O \rangle_{\mathrm{approx}} + \frac{1}{N} \sum_i \langle O_i \rangle_{\mathrm{approx}}$$

$\langle O_i \rangle_{\mathrm{approx}}$: props with N_{low} exact low modes, sloppy (relaxed stopping condition) CG

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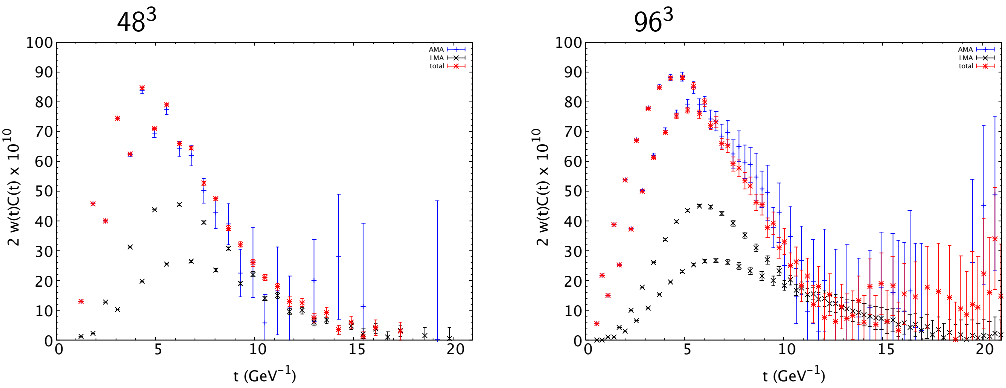
$$\begin{aligned}\textbf{AMA} \qquad \langle O \rangle &= \langle O \rangle_{\rm exact} - \langle O \rangle_{\rm approx} + \frac{1}{N}\sum_i \langle O_i \rangle_{\rm approx} \\ +\textbf{LMA} \qquad &- \frac{1}{N}\sum_i \langle O_i \rangle_{\rm LM} + \frac{1}{V}\sum_i \langle O_i \rangle_{\rm LM}\end{aligned}$$

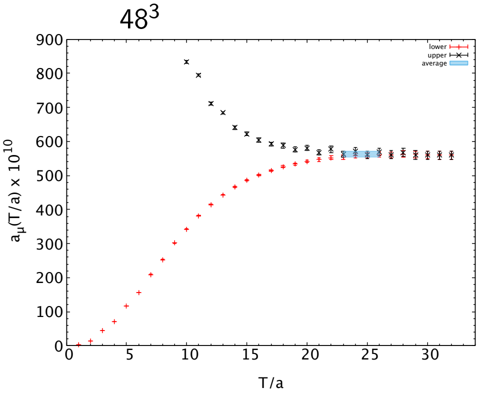
$\langle O_i \rangle_{\rm approx}$: props with $N_{\rm low}$ exact low modes, sloppy (relaxed stopping condition) CG

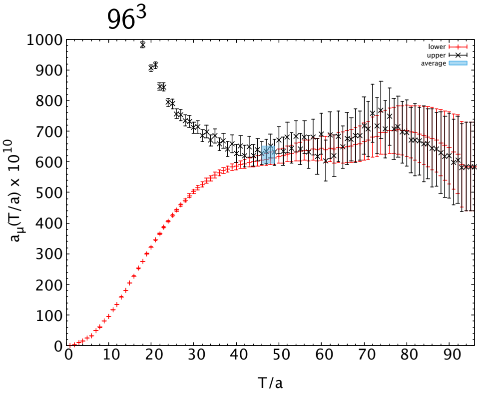
Noise reduction: AMA+LMA [Blum et al., 2013, Giusti et al., 2004, DeGrand and Schaefer, 2005]



- AMA: $4^3 \times 4 = 256$ ($3^3 \times 8 = 216$, 96^3) approx props, 8 exact props
- LMA: 3000($\times$2) Low modes (2000($\times$2) for 96^3)
- Huge reduction in statistical error at long distance from full volume low mode average *c.f.* RBC/UKQCD [Blum et al., 2018]

Bounding method RBC/UKQCD [Blum et al., 2018], BMW [Borsanyi et al., 2018]





(total a_μ for choice of T is plotted)

- Lower bound: $C(t) = 0, \; t > T$ (BMW choice)
- Upper bound: $C(t) = C(T)e^{-E_0(t-T)}$, $E_0 = 2\sqrt{m_\pi^2 + (2\pi/L)^2}$
- averages: 2.7-3.2 fm and 2.6-2.8 fm (96^3)

Finite Volume Corrections

Use chiral perturbation theory (ChiPT) in configuration space to compute finite volume corrections to $C(t)$
(Bijnens and Relefors computed NNLO corrections in momentum space)

- NLO including leading discretization (taste symmetry breaking) effects
- NNLO in continuum. Big!

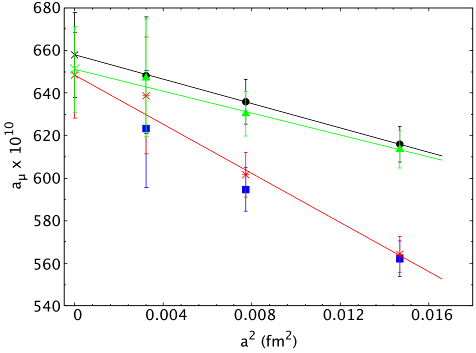
	NLO+taste ($a \neq 0$)	NLO ($a = 0$)	NNLO ($a = 0$)
48^3, 0.12 fm	51.6	18.1	7.4
64^3, 0.09 fm	34.2	21.6	9.0
96^3, 0.06 fm	9.5	20.6	9.1

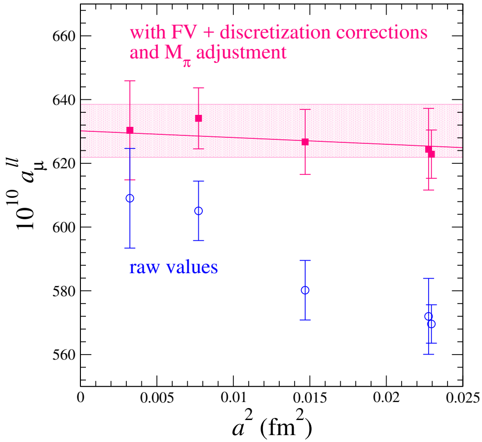
Corrections to a_μ given in units of 10^{-10}

- Procedure: correct $a \neq 0$ to NLO, $a \to 0$, add average NNLO correction

Continuum limit

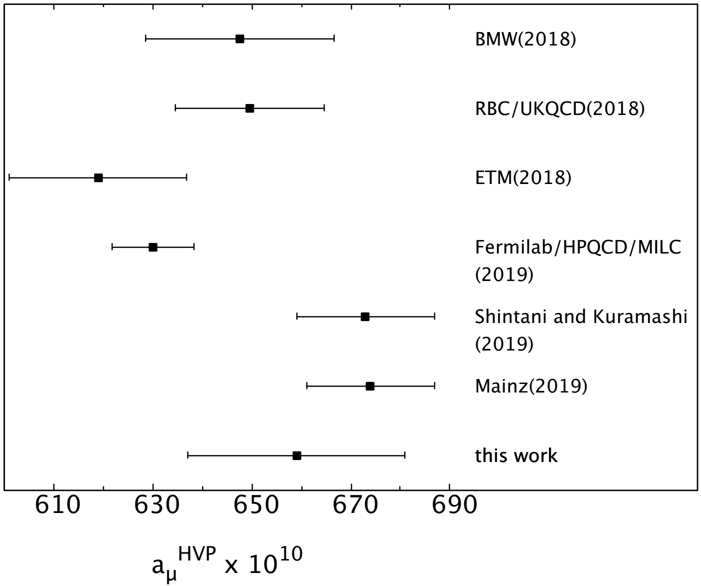
Same ensembles as FHM, different analysis method (FHM [Davies et al., 2019])





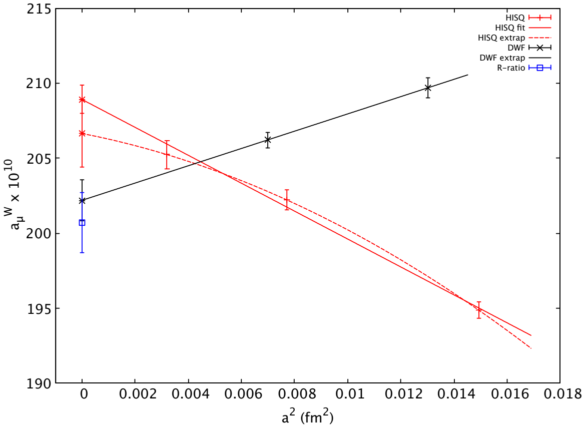
Difference: mostly NNLO ChiPT (us) and model (them) which have opposite signs

Comparison with other recent results



Window method RBC/UKQCD [Blum et al., 2018] comparison with DWF and R-ratio

$$a_\mu^W = \sum C(t)w(t)(\Theta(t,t_0,\Delta) - \Theta(t,t_1,\Delta)), \quad \Theta(t,t',\Delta) = 0.5(1+\tanh((t-t')/\Delta))$$



- allows precise comparison of continuum limit and
- combine lattice and dispersive results
- all points physical
- all $L \approx 5.5$ fm
- no ISB or FV corrections
- difference is 2-3 σ: lattice spacing, statistics may be responsible

window parameters: $t_0 = 0.4$, $t_1 = 1.0$, $\Delta = 0.15$ fm

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Immediate plan

1. Extend LMA to 3000 ($\times 2$) eigenvectors on 96^3 to match 48^3, 64^3
   - number of needed LM depends on physical volume
   - If 96^3 error $\approx 64^3$ error, $\rightarrow$ stat error in continuum limit $\times 1/2$
2. New ensemble, 0.15 fm
   - Improve continuum limit
   - test LMA on smaller lattice, but same physical volume, mass
3. check NNLO ChiPT against momentum space calculation (Bijnens and Relefors)
4. extend NNLO ChiPT to finite a (taste breaking)

Longer term plans

Improving long distance contributions

- GEVP, improved bounding method (BMW, Mainz, RBC/UKQCD)
- Model independent fit using Padé approximants

Improve statistics by $\sim \times 10$, competitive with RBC/UKQCD, FHM, ...

- implement GEVP, local currents, ...

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Time-momentum representation Bernecker-Meyer 2011

Interchange order of FT and momentum integrals

$$\begin{aligned} \Pi(q^2) - \Pi(0) &= \sum_t \left(\frac{\cos qt - 1}{q^2} + \frac{1}{2} t^2 \right) C(t) \\ C(t) &= \frac{1}{3} \sum_{x,i} \langle j_i(x) j_i(0) \rangle \\ w(t) &= 2\alpha^2 \int_0^\infty \frac{d\omega}{\omega} f(\omega^2) \left[\frac{\cos \omega t - 1}{\omega^2} + \frac{t^2}{2} \right] \end{aligned}$$

$$a_\mu^{\mathrm{HVP}} = \sum_t w(t) C(t)$$

(note double subtraction)

Staggered Dirac operator M

Sum of hermitian and anti-hermitian parts, so it satisfies (even-odd ordering)

$$M \begin{pmatrix} n_o \\ n_e \end{pmatrix} = \begin{pmatrix} m & M_{oe} \\ M_{eo} & m \end{pmatrix} \begin{pmatrix} n_o \\ n_e \end{pmatrix} = (m + i\lambda_n) \begin{pmatrix} n_o \\ n_e \end{pmatrix} \tag{1}$$

and

$$\begin{pmatrix} m & -M_{oe} \\ -M_{eo} & m \end{pmatrix} \begin{pmatrix} m & M_{oe} \\ M_{eo} & m \end{pmatrix} \begin{pmatrix} n_o \\ n_e \end{pmatrix} = \tag{2}$$

$$\begin{pmatrix} m^2 - M_{oe}M_{eo} & 0 \\ 0 & m^2 - M_{eo}M_{oe} \end{pmatrix} \begin{pmatrix} n_o \\ n_e \end{pmatrix} = (m^2 + \lambda_n^2) \begin{pmatrix} n_o \\ n_e \end{pmatrix} \tag{3}$$

Compute eigenvectors $n_{o(e)}$, $m^2 + \lambda^2$ of preconditioned Dirac operator

Staggered Dirac operator M

Eigenvectors of preconditioned operator are eigenvectors of M with squared magnitude eigenvalues, construct the even part from odd,

$$n_e = \frac{-i}{\lambda_n} M_{eo} n_o.$$

eigenvalues come in $\pm$ pairs: If (n_o, n_e) is an eigenvector with eigenvalue λ, then

$$(-1)^x \psi(x) = (-n_o, n_e)$$

is also an eigenvector with eigenvalue $-\lambda$.

$$\begin{pmatrix} m & M_{oe} \\ M_{eo} & m \end{pmatrix} \begin{pmatrix} -n_o \\ n_e \end{pmatrix} = (m - i\lambda_n) \begin{pmatrix} -n_o \\ n_e \end{pmatrix}, \tag{4}$$

Thus we can construct pairs of eigenvectors with $\pm i\lambda$ for each λ^2, n_o!

HVP using spectral decomposition of M^{-1}

Use conserved current

$$J^{\mu}(x) \quad = \quad -\frac{1}{2}\eta_{\mu}(x)\left(\bar{\chi}(x+\hat{\mu})U_{\mu}^{\dagger}(x)\chi(x) \;+\; \bar{\chi}(x)U_{\mu}(x)\chi(x+\hat{\mu})\right)$$

and spectral decomposition of propagator

$$M^{-1}_{x,y} \quad = \quad \sum_{n}^{N_{(\mathrm{low})}} \left(\frac{\langle x|n\rangle\langle n|y\rangle}{m+i\lambda_n} + \frac{\langle x|n_-\rangle\langle n_-|y\rangle}{m-i\lambda_n} \right) \tag{5}$$

HVP using spectral decomposition of M^{-1}

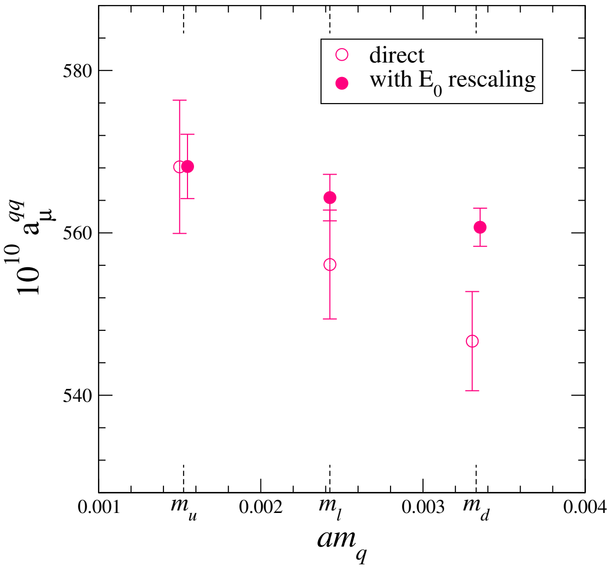
$$
\begin{aligned}
4J_\mu(t_x)J_\nu(t_y) &= \sum_{m,n}\sum_{\vec{x}} \frac{\langle m|x+\mu\rangle U_\mu^\dagger(x)\langle x|n\rangle}{\lambda_m} \sum_{\vec{y}} \frac{\langle n|y\rangle U_\nu(y)\langle y+\nu|m\rangle}{\lambda_n} \\
&+ \sum_{\vec{x}} \frac{\langle m|x\rangle U_\mu(x)\langle x+\mu|n\rangle}{\lambda_m} \sum_{\vec{y}} \frac{\langle n|y\rangle U_\nu(y)\langle y+\nu|m\rangle}{\lambda_n} \\
&+ \sum_{\vec{x}} \frac{\langle m|x+\mu\rangle U_\mu^\dagger(x)\langle x|n\rangle}{\lambda_m} \sum_{\vec{y}} \frac{\langle n|y+\nu\rangle U_\nu^\dagger(y)\langle y|m\rangle}{\lambda_n} \\
&+ \sum_{\vec{x}} \frac{\langle m|x\rangle U_\mu(x)\langle x+\mu|n\rangle}{\lambda_m} \sum_{\vec{y}} \frac{\langle n|y+\nu\rangle U_\nu^\dagger(y)\langle y|m\rangle}{\lambda_n}
\end{aligned}
$$

λ_n shorthand for $m \pm i\lambda_n$, need to construct the matrices (meson fields)

$$
(\Lambda_\mu(t))_{n,m} = \sum_{\vec{x}} \langle n|x\rangle U_\mu(x)\langle x+\mu|m\rangle(-1)^{(m+n)x+m}
$$

(order eigenvectors $\lambda_0, -\lambda_0, \lambda_1, -\lambda_1, \ldots, -\lambda_{2N_{\mathrm{low}}}$)

Light quark mass dependence of a_μ FnalHpqcdMilc[Chakraborty et al., 2018]



- strong isospin breaking study
- $m_\pi = 135$ MeV
- $a = 0.15$ fm
- change in a_μ for 130 MeV pion is negligible $\sim -2 \times 10^{-10}$