Isospin breaking in tau decays and $(g-2)_\mu$

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in collaboration with
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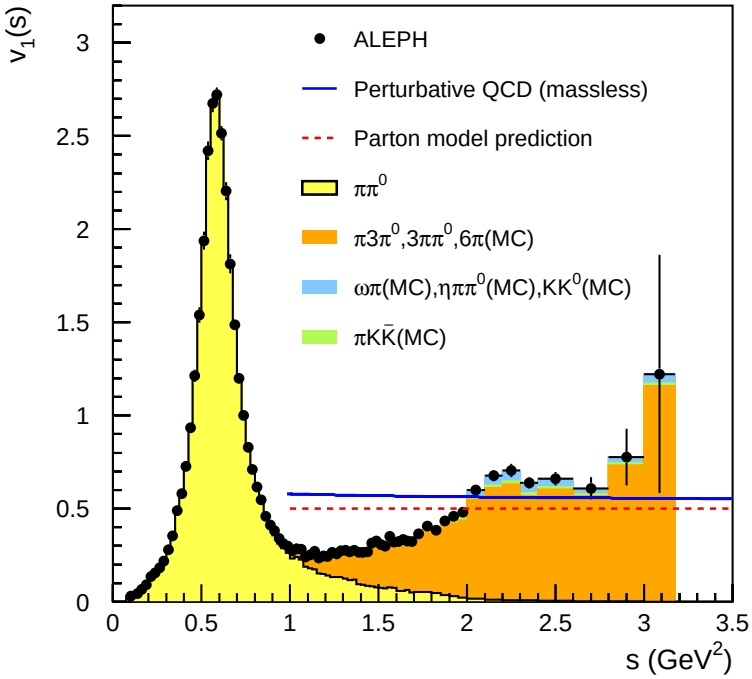
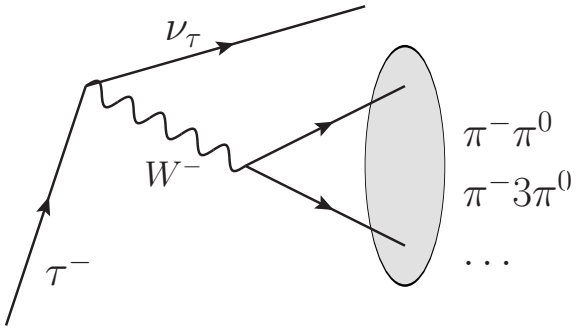
for the RBC/UKQCD Collaboration

3rd Plenary Workshop of the Muon g-2 Theory Initiative, Seattle
September 13th, 2019

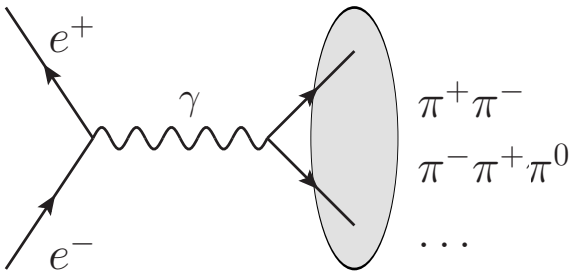
Motivations for τ





$V - A$ current

Final states $I = 1$ charged



EM current

Final states $I = 0, 1$ neutral

τ data can improve $a_\mu[\pi\pi]$

$\rightarrow 72\%$ of total Hadronic LO

or $a_\mu^{ee} \neq a^\tau \rightarrow$ NP [Cirigliano et al '18]

Isospin Corrections

Restriction to $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^-\pi^0\,\nu_\tau$

$$v_0(s) = \frac{s}{4\pi\alpha^2}\sigma_{\pi^+\pi^-(\gamma)}(s)$$

$$v_-(s) = \frac{m_\tau^2}{6|V_{ud}|^2}\frac{\mathcal{B}_{\pi\pi^0}}{\mathcal{B}_e}\frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds}\left(1-\frac{s}{m_\tau^2}\right)^{-1}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\frac{1}{S_{\rm EW}}$$

Isospin correction $v_0 = R_{\rm IB}v_-$ $\quad R_{\rm IB} = \frac{\rm FSR}{G_{\rm EM}}\frac{\beta_0^3|F_\pi^0|^2}{\beta_-^3|F_\pi^-|^2}$ [Alemani et al. '98]

0. $S_{\rm EW}$ electro-weak radiative correct. [Marciano, Sirlin '88][Braaten, Li '90]

1. Final State Radiation of $\pi^+\pi^-$ system [Schwinger '89][Drees, Hikasa '90]

2. $G_{\rm EM}$ (long distance) radiative corrections in τ decays
- Chiral Resonance Theory [Cirigliano et al. '01, '02]
- Meson Dominance [Flores-Talpa et al. '06, '07]

3. Phase Space $(\beta_{0,-})$ due to $(m_{\pi^\pm} - m_{\pi^0})$

Long distance QED - I

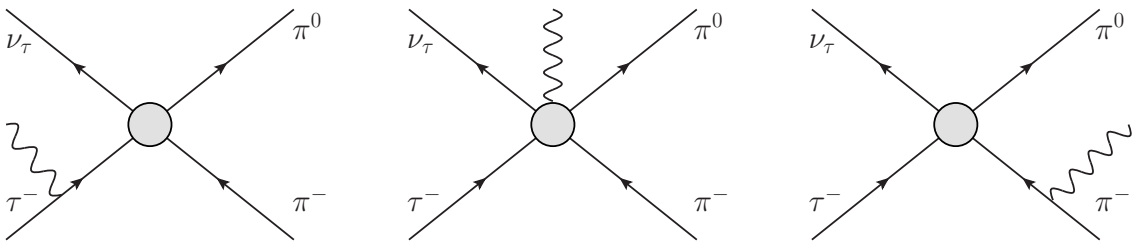
At low energies relevant degrees of freedom are mesons

Chiral Perturbation Theory [Cirigliano et al. '01, '02]

Meson dominance model [Flores-Talpa et al. '06, '07]

Corrections casted in one function $v_-(s) \to v_-(s) G_{\mathrm{EM}}(s)$

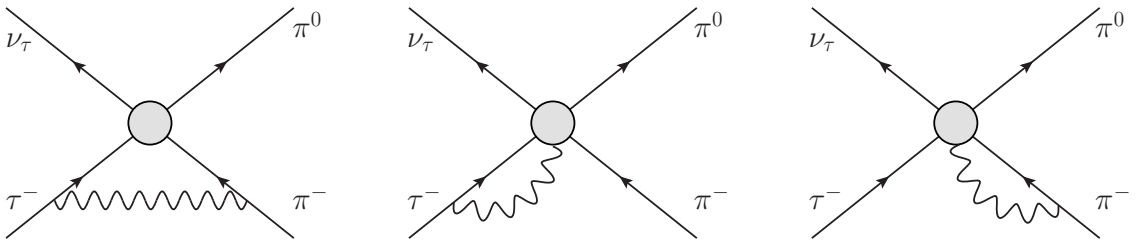
Real photon corrections



Real + virtual

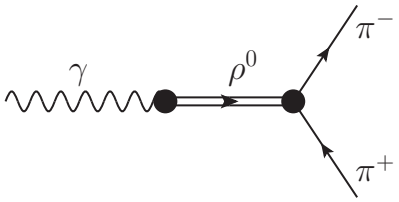
$\to$ IR divergences cancel

Virtual photon corrections



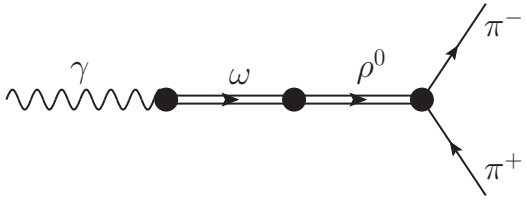
Pion form factors

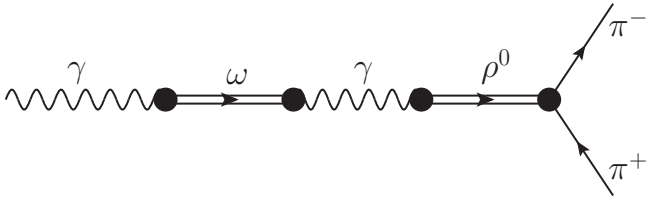
$$F_\pi^0(s) \propto \frac{m_\rho^2}{D_\rho(s)}$$



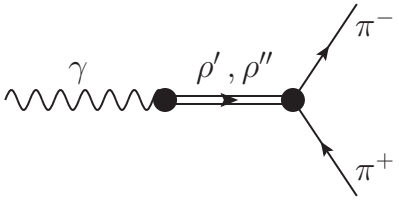
[Gounaris, Sakurai '68]
[Kühn, Santamaria '90]

$$\times\left[1 + \delta_{\rho\omega}\frac{s}{D_\omega(s)}\right]$$

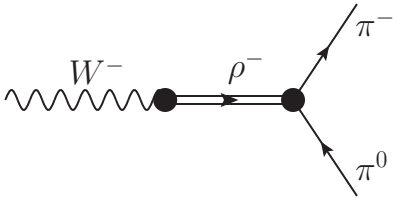




$$+\frac{m_X^2}{D_X(s)} \quad X = \rho' , \rho''$$



$$F_\pi^-(s) \propto \frac{m_{\rho^-}^2}{D_{\rho^-}(s)} + (\rho' , \rho'')$$



Sources of IB breaking in phenomenological models

$m_{\rho^0} \neq m_{\rho^\pm}$, $\Gamma_{\rho^0} \neq \Gamma_{\rho^\pm}$, $m_{\pi^0} \neq m_{\pi^\pm}$

$\rho - \omega$ mixing $\delta_{\rho\omega} \simeq O(m_{\rm u} - m_{\rm d}) + O(e^2)$

Status

$a_\mu^{\mathrm{HVP,LO}}[\pi\pi, ee] = 503.51(3.5) \times 10^{-10}$ with $E \in [2m_\pi, 1.8\ \mathrm{GeV}]$

$a_\mu^{\mathrm{HVP,LO}}[\pi\pi, \tau] \ = 531.3(3.3) \times 10^{-10}$

$a_\mu[\pi\pi, ee] - a_\mu[\pi\pi, \tau] = -12.0(2.6)$ [Cirigliano et al.]

$a_\mu[\pi\pi, ee] - a_\mu[\pi\pi, \tau] = -16.1(1.8)$ [Davier et al. '09]

(≈ -12 due to S_{EW}, rest R_{IB})

$a_\mu[\tau]$: { model dependence / e^+e^- data more precise } = abandoned

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Additional $\rho\gamma$ mixing correction [Jegerlehner, Szafron '11]

partly accounted in $m_{\rho^0} - m_{\rho^-}$ in [Davier et al. '09]

$a_\mu[\pi\pi, ee] = 385.2(1.5)$ with $E \in [0.582 - 0.975]$ GeV

$a_\mu[\pi\pi, \tau] \ = 386.0(2.4)$ after R_{IB}

Contribution to a_μ

Time-momentum representation [Bernecker, Meyer, '11]

$$G^\gamma(t) = \frac{1}{3}\sum_k \int d\vec{x}\ \langle j_k^\gamma(x) j_k^\gamma(0)\rangle \quad \rightarrow \quad a_\mu = 4\alpha^2 \sum_t w_t G^\gamma(t)$$

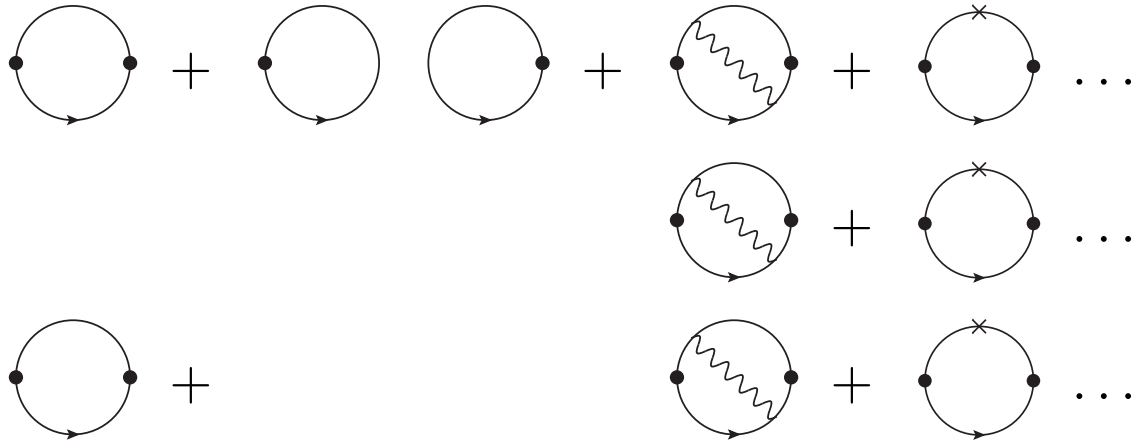
Isospin decomposition of u, d current

$$j_\mu^\gamma = \frac{i}{6}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right) + \frac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right) = j_\mu^{(0)} + j_\mu^{(1)}$$

$G_{00}^\gamma \leftarrow \langle j_k^{(0)}(x) j_k^{(0)}(0)\rangle =$

$G_{01}^\gamma \leftarrow \langle j_k^{(0)}(x) j_k^{(1)}(0)\rangle =$

$G_{11}^\gamma \leftarrow \langle j_k^{(1)}(x) j_k^{(1)}(0)\rangle =$



Decompose $a_\mu = a_\mu^{(0,0)} + a_\mu^{(0,1)} + a_\mu^{(1,1)}$

Neutral vs Charged

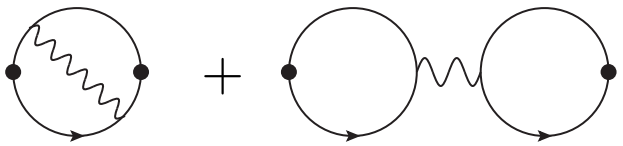
$$\frac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right), \left[\begin{array}{l} I=1 \\ I_3=0 \end{array}\right] \quad \rightarrow \quad j_\mu^{(1,-)} = \frac{i}{\sqrt{2}}\left(\bar{u}\gamma_\mu d\right), \left[\begin{array}{l} I=1 \\ I_3=-1 \end{array}\right]$$

Isospin 1 charged correlator $G_{11}^W = \frac{1}{3}\sum_k \int d\vec{x}\, \langle j_k^{(1,+)}(x) j_k^{(1,-)}(0)\rangle$

$$\delta G^{(1,1)} \equiv G_{11}^\gamma - G_{11}^W$$

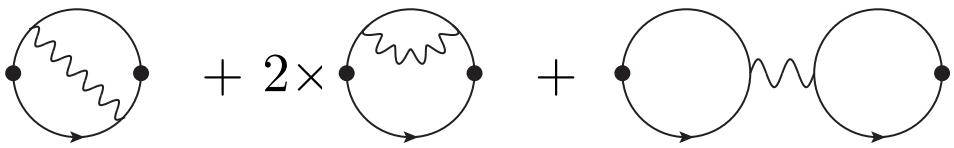
[MB et al.' Latt18]

$$= Z_V^4 (4\pi\alpha) \frac{(Q_\mathrm{u} - Q_\mathrm{d})^4}{4} \Big[$$



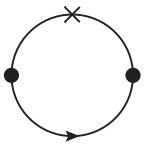
$$\Big]$$

$$G_{01}^\gamma = Z_V^4 \frac{(Q_u^2 - Q_d^2)^2}{2} (4\pi\alpha) \Big[$$



$$+ \ldots \Big]$$

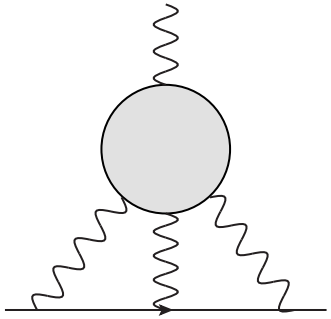
$$+ Z_V^2 \frac{Q_u^2 - Q_d^2}{2} (m_u - m_d) \Big[2\times$$



$$+ \ldots \Big]$$

$\ldots$ = subleading diagrams currently not included

Synergy - I



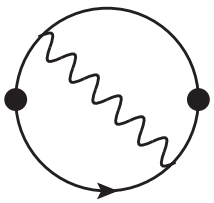
from QCD we need a 4-point function $f(x, y, z, t)$:

known kernel with details of photons and muon line

1 pair of point sources (x, y), sum over z, t exact at sink

stochastic sampling over (x, y) (based on $|x - y|$)

Successfull strategy: x10 error reduction [RBC '16]



from QCD we need a 4-point function $f(x, y, z, t)$:

$(g-2)_\mu$ kernel + photon propagator

Similar problem $\rightarrow$ re-use HLbL point sources!

The RBC & UKQCD collaborations

BNL and BNL/RBRC

Yasumichi Aoki (KEK)
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Meifeng Lin
Aaron Meyer
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

UC Boulder

Oliver Witzel

CERN

Mattia Bruno

Columbia University

Ryan Abbot
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
Masaaki Tomii
Jiqun Tu
Bigeng Wang
Tianle Wang
Yidi Zhao

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Tom Blum
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Michael Marshall
Fionn Ó hÓgáin
Antonin Portelli
Tobias Tsang
Andrew Yong
Azusa Yamaguchi

KEK

Julien Frison

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Nicolas Garron

MIT

David Murphy

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Xu Feng

University of Regensburg

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University of Southampton

Nils Asmussen
Jonathan Flynn
Ryan Hill
Andreas Jüttner
James Richings
Chris Sachrajda

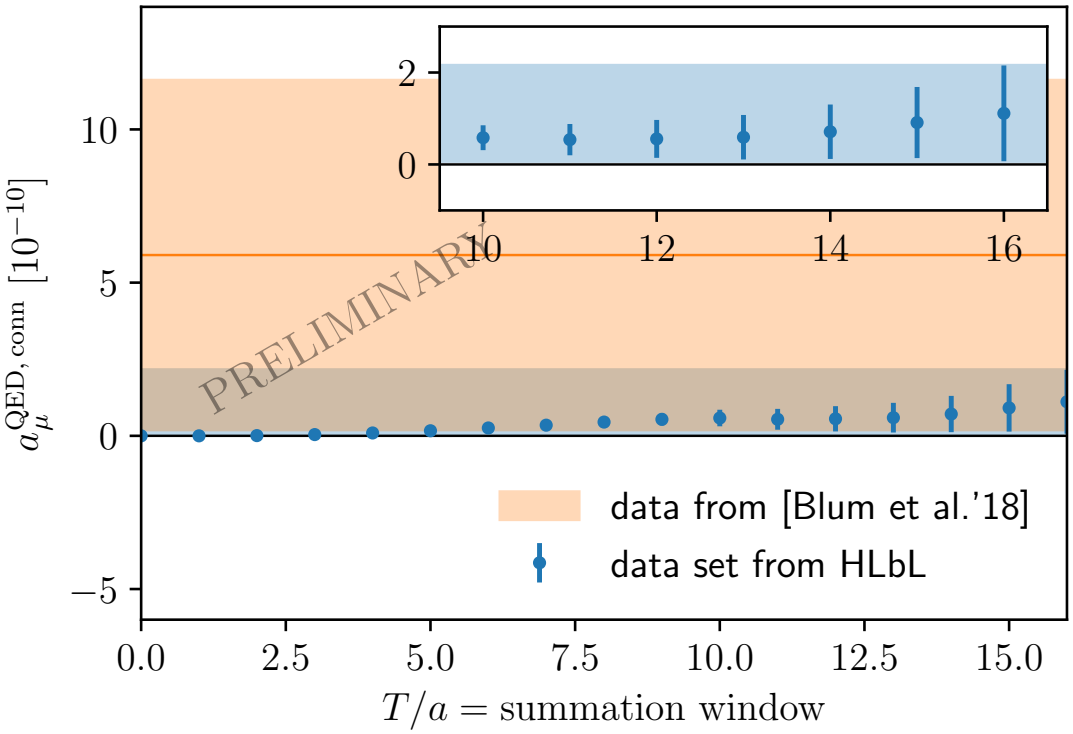
Stony Brook University

Jun-Sik Yoo
Sergey Syritsyn (RBRC)

Synergy - II

Stat. improvements from data of HLbL project [Phys.Rev.Lett. 118 (2017)]

contribution of diagrams V, S to a_μ



$O(2000)$ point-src per conf.
~ 3000 combinations
$O(10)$ configurations

$\times 4$ reduction in error

expected QED conn. error $\leq 3 \times 10^{-10} \rightarrow$ matches target

Last slide, then Plots!

Restriction to $2\pi \to$ neglect pure $I = 0$ part $a_\mu^{(0,0)}[\pi^0\gamma, 3\pi\,, \ldots]$

Lattice: $\Delta a_\mu[\pi\pi, \tau] = 4\alpha^2 \sum_t w_t \;\times\; \left[G_{01}^\gamma(t) + G_{11}^\gamma(t) - G_{11}^W(t)\right]$

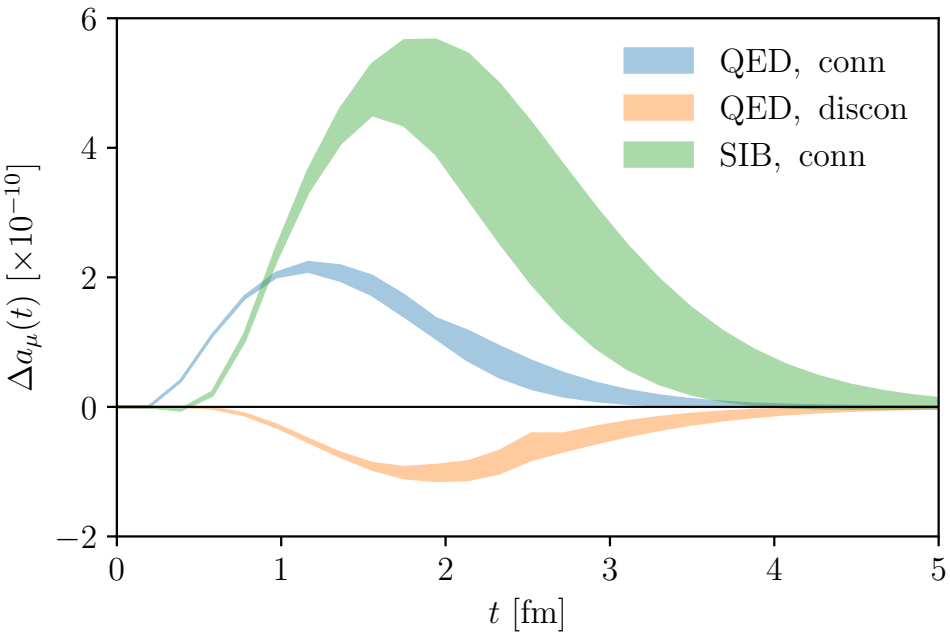
Pheno: $\Delta a_\mu[\pi\pi, \tau] = \int_{4m_\pi^2}^{m_\tau^2} ds K(s) \;\left[\qquad v_0(s) \qquad - v_-(s)\right]$

Conversion to Euclidean time for direct comparison

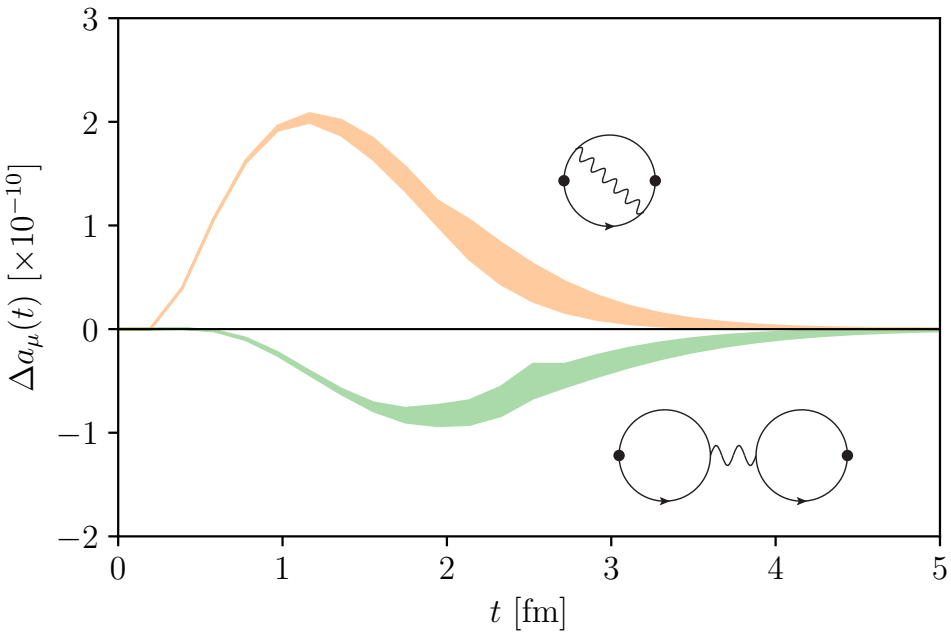
$$\Delta a_\mu[\pi\pi, \tau] = 4\alpha^2 \sum_t w_t \times \left\{ \frac{1}{12\pi^2} \int d\omega\; \omega^2 e^{-\omega t} \left[R_{\rm IB}(\omega^2) - 1\right] v_-(\omega^2) \right\}$$

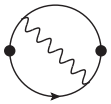
Lattice: Preliminary results - I

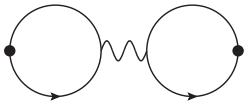
$\Delta a_\mu \to G_{01} + \delta G_{11}$:



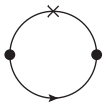
Pure $I = 1$ only $O(\alpha)$ terms:

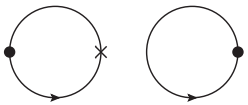


$V =$ 

$F =$ 

$S =$

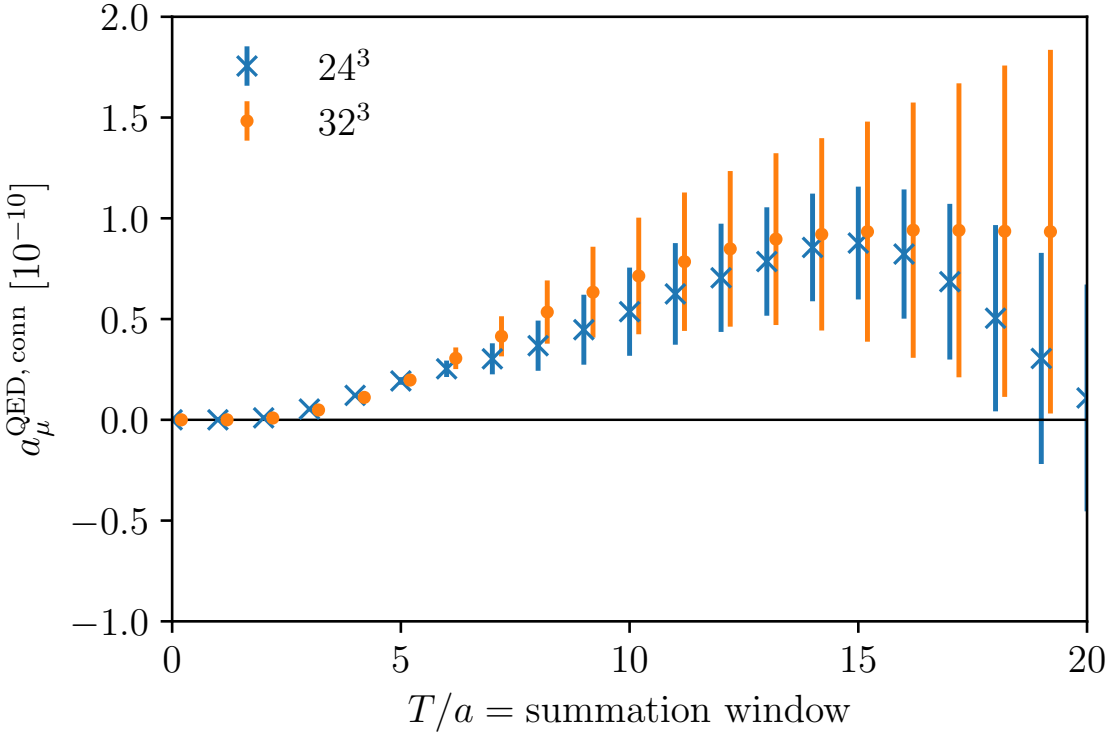
$M =$ 

$O =$  relevant, negative, not-included

Systematic errors

$$a_\mu^{\mathrm{QED,conn}} = V + 2S$$

FV study at coarse
$a^{-1} \sim 1$ GeV



Finite volume errors

empirical observation: diagrams may have largish FV errors

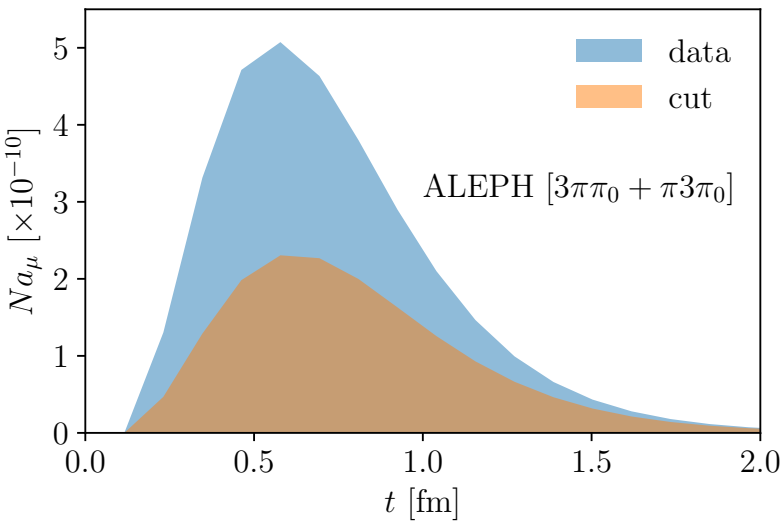
cancellation of FV effects in physical combinations

Systematic Errors

Lattice fully inclusive

1. cut $E < m_\tau$
2. higher multiplicity channels, 4π

effects above ~ 1.8 GeV suppressed by (muon) kernel



$E_0 = 2$ GeV, $1/(2k) = 0.25$ GeV

Smeared step function
$\Theta(s) = (1 + e^{2k(E-E_0)})^{-1}$
$\rightarrow$ control syst. effect of 4π

manipulate $G(t)$ (e.g. Backus-Gilbert)

preliminary: w/o cut $\Delta a_\mu[4\pi] \approx 2(1)$

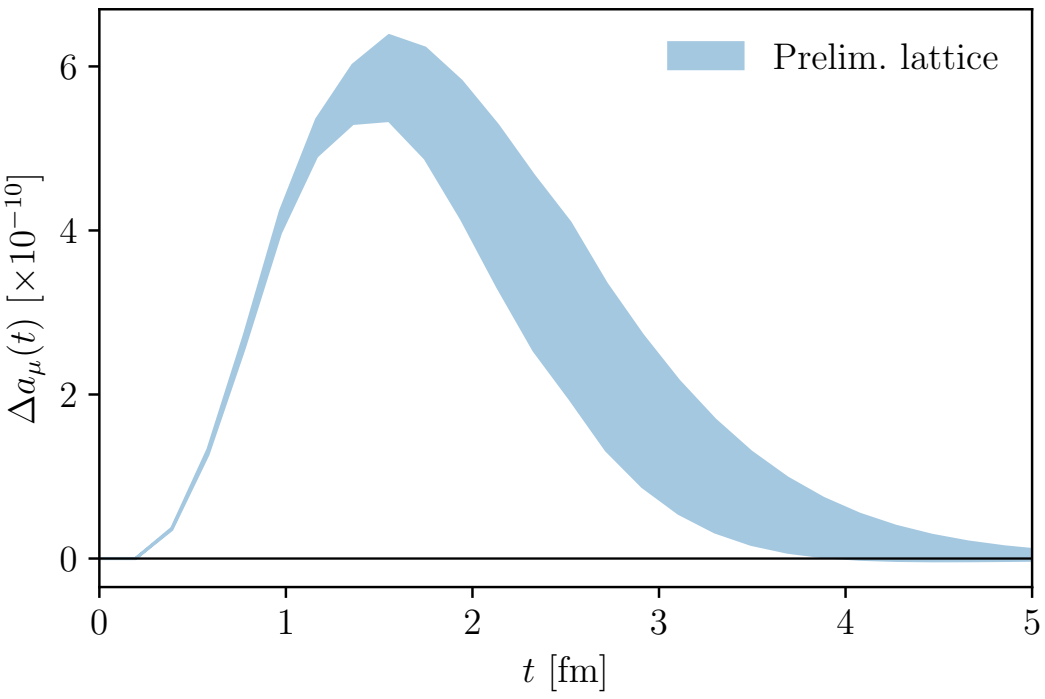
Lattice: Preliminary results - II

Study integrand in euclidean time $\rightarrow$ as important as integral

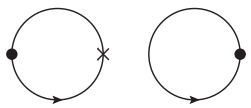
direct comparison
Lattice vs. EFT+Pheno

1. validate previous estimates of R_{IB}
2. study neutral/charged ρ and ω properties

Preliminary lattice (full) calculation: $G^{\gamma}_{01} + \delta G$



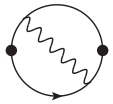
Not included:

1.  relevant
2. sub-leading $1/N_c$, $SU(N_f)$

Under consideration:

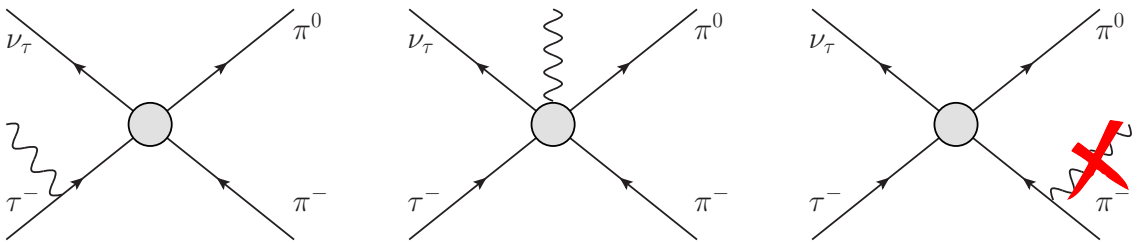
3. finite-volume errors
4. discretization errors

Towards a comparison

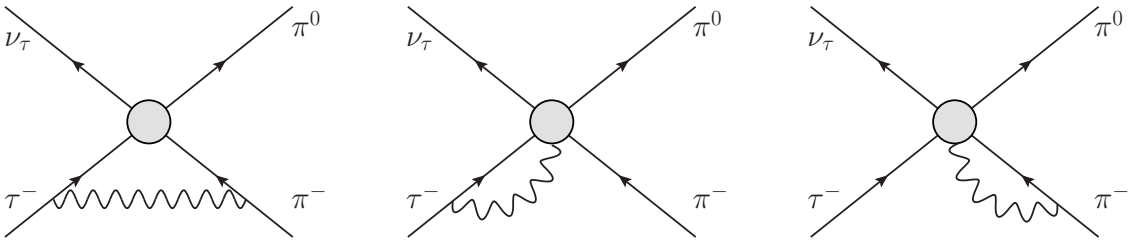
Lattice contains $\pi^0\pi^-\gamma$ states $\rightarrow$ 

Re-evaluation of $G_{\rm EM} \rightarrow G^{\pi}_{\rm EM}$ [in collab. with Cirigliano]

Real photon corrections



Virtual photon corrections



$G^{\pi}_{\rm EM}$ w/o $\pi^0\pi^-\gamma$ FSR

$\dfrac{v_-}{G^{\pi}_{\rm EM}}$ w $\pi^0\pi^-\gamma$ FSR

Conclusions

These are exciting times for $(g-2)_\mu$:

1% goal for lattice results to be expected soon

QED+SIB crucial to reach target uncertainty

As a bi-product we get $\Delta a_\mu[\tau]$:

1. first lattice calculation of $\Delta a_\mu[\tau]$ almost complete
2. tests/checks previous calculations

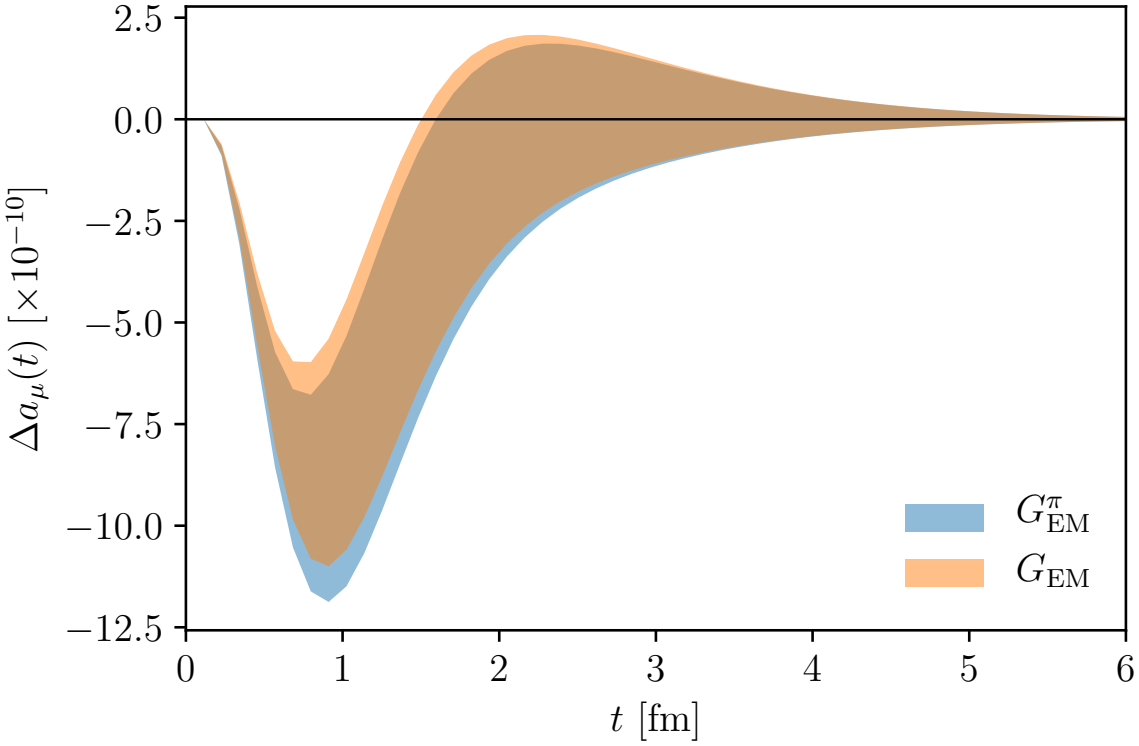
   comparing v_- with experiment requires $G^\pi_{\rm EM}$

   study G^γ_{01} alone $\to \rho\omega$ mixing; $\delta G^{(1,1)}$ alone $\to \rho^0$ vs ρ^-
3. possibly bound new physics

Thanks for your attention

Experimental results

$$\Delta a_\mu(t) = 4\alpha^2 \sum_t w_t \Big\{ \int ds\, h(s,t) \Big[v_0(s) - \frac{v_1(s)}{G_{\mathrm{EM}}(s)} \Big] \Big\}$$



v_0 BaBar, v_1 Aleph

preliminary

$\rho\gamma$ MIXING - I

Gounaris-Sakurai based on VMD model w/o EM gauge invariance

- generation of a photon mass

+ based on phase shift (proper pion rescattering behavior)

widely used: e.g. PDG estimates of m_ρ, Γ_ρ

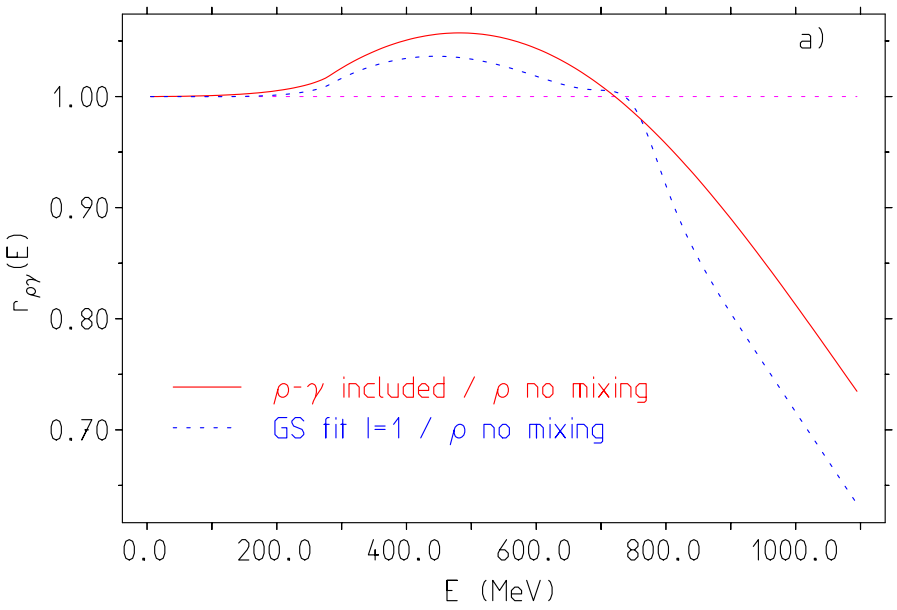
VMD model with gauge-invariance [Kroll, Lee, Zumino '67]

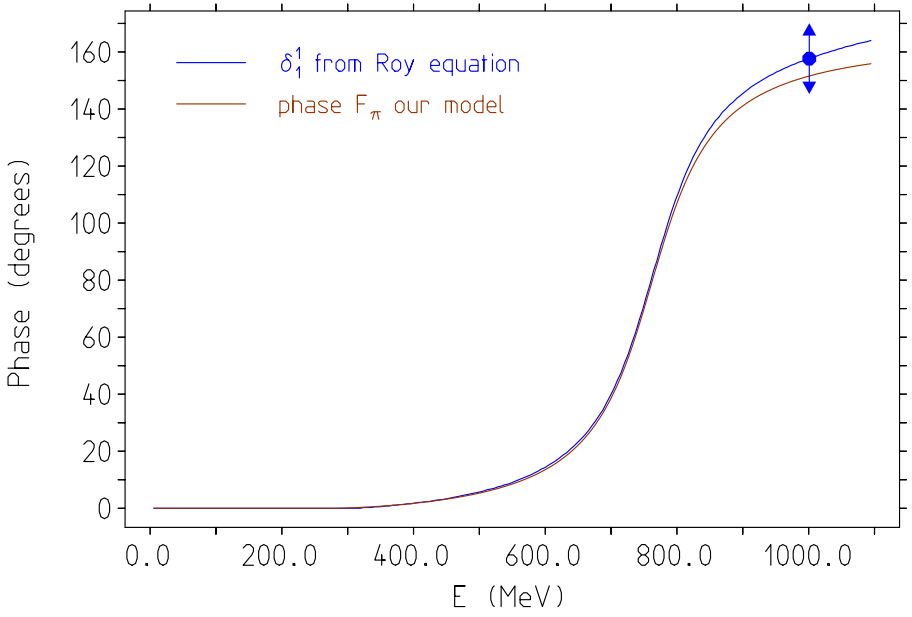
at 1-loop s-dependent mass matrix [Jegerlehner, Szafron '11]

limits of validity pion-loop? high enough energy must break down

$\rho\gamma$ MIXING - II

[Jegerlehner, Szafron '11]





30% correction at 1 GeV

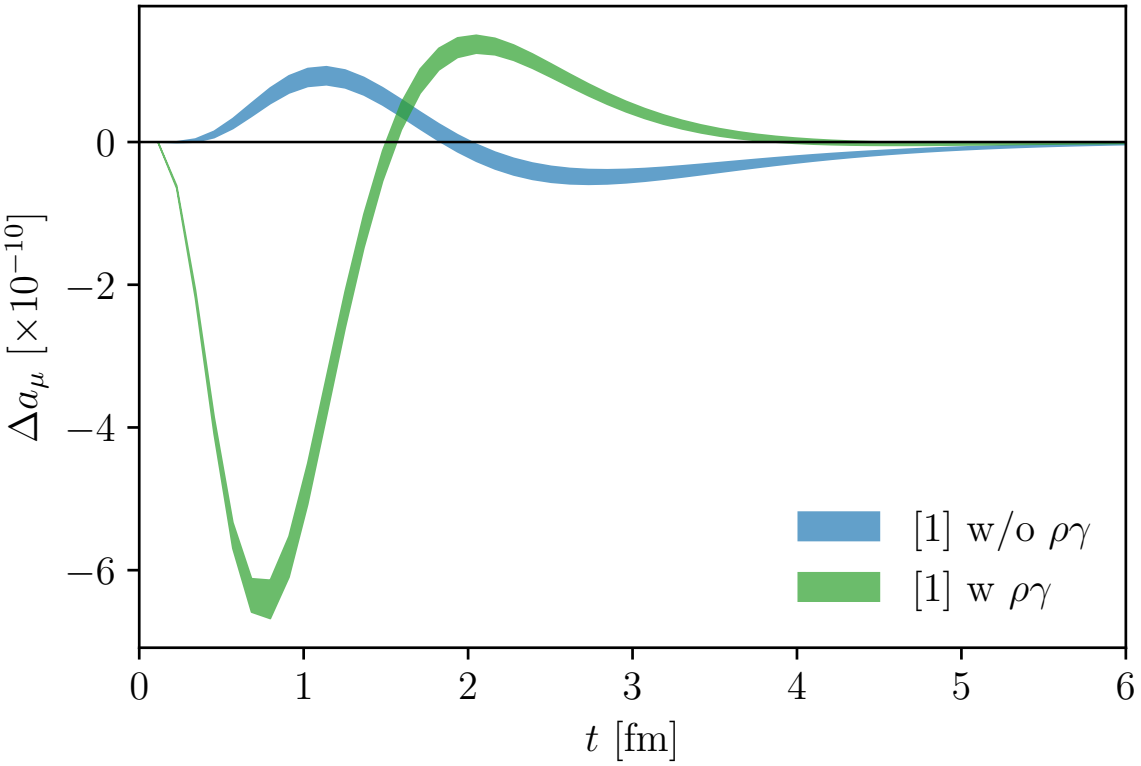
δ_1^1 in good agreement $E < 800$ MeV

perhaps restrict the $\rho\gamma$ below 800 MeV?

$\rho\gamma$ MIXING-III

$[1]$ = [Jegelehner, Szafron '17]

modified $\rho\gamma$ coupling
large negative Δa_μ



Modified $\rho\gamma$ suggests different behavior from lattice data

direct comparison with lattice not possible $\rightarrow$ hard cut at 1 GeV