hadronic contributions to the running of the electromagnetic coupling and the electroweak mixing angle

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Hadronic contributions to $(g-2)_\mu$
INT, Seattle, September 13, 2019

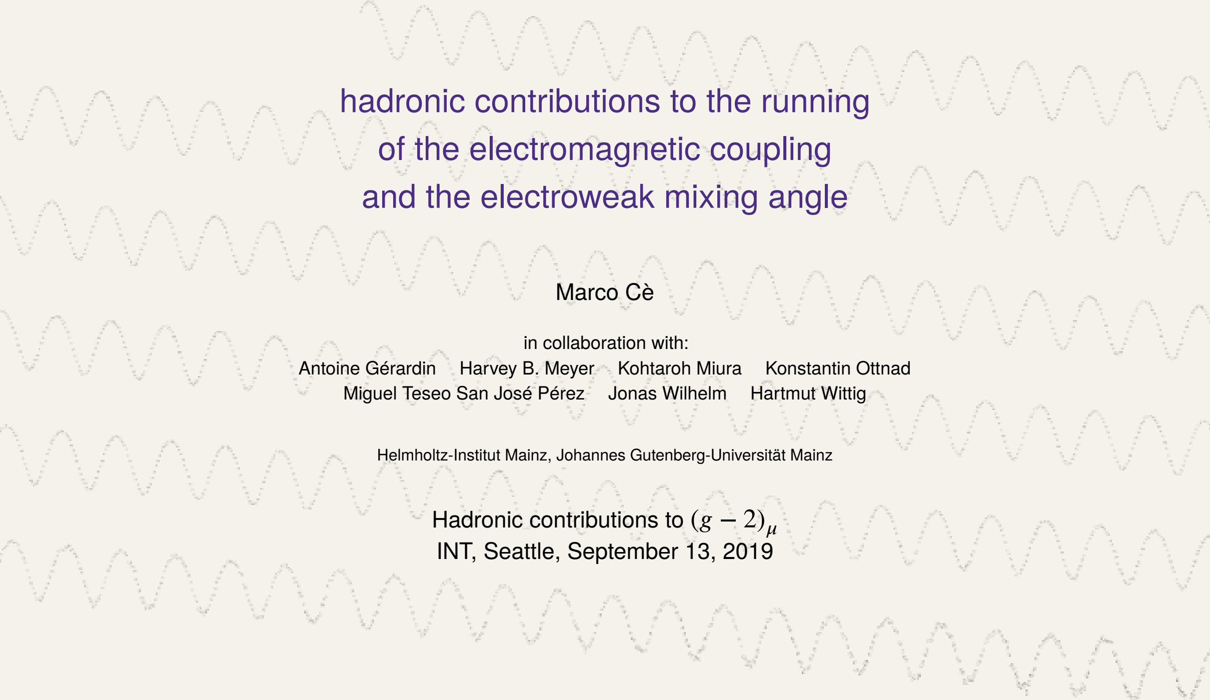


fig. 2 from Phys.Rev.D **73** (2006) 072003

the running of the electromagnetic coupling

the QED coupling $\alpha = \frac{g^2}{4\pi}$ runs with energy

$$\alpha(Q) = \frac{\alpha}{1 - \Delta\alpha(Q)}$$

- in the Thomson limit, the fine-structure constant is known at 0.23 ppb
  $\alpha^{-1} = \alpha(Q=0)^{-1} = 137.035\,999\,139(31)$ [PDG 2018, CODATA 2014]
- at the Z pole, in the $\overline{\text{MS}}$ scheme, $\alpha^{(5)}(M_Z)^{-1} = 127.955(10)$

main uncertainty to the running: the hadronic contribution

$$\Delta_{\text{had}}\alpha(Q) = 4\pi\alpha\hat{\Pi}(Q^2), \qquad \hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0)$$

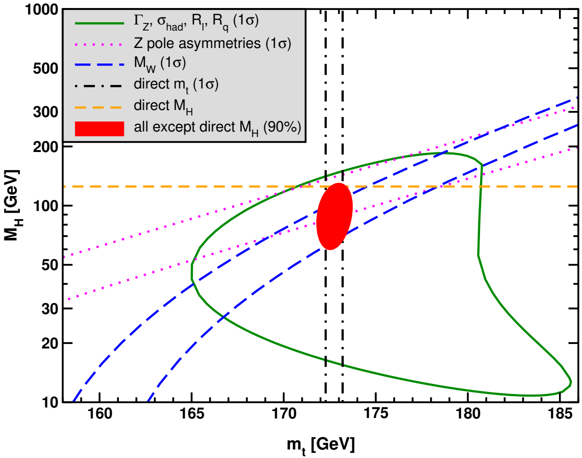
is proportional to the subtracted hadronic vacuum polarization

- extracted from hadronic cross-section data [Erler 1999; Davier *et al.* 2017; PDG 2018]

$$\Delta_{\text{had}}\alpha^{(3)}(2\,\text{GeV}) = 58.71(50)\times 10^{-4}, \qquad \Delta_{\text{had}}\alpha^{(5)}(M_Z) = 0.027\,64(7)$$

- or computed on the lattice [Burger *et al.* 2015; Francis *et al.* 2015; Borsanyi *et al.* 2018]

motivation – global Standard Model fits



excluding direct measurement, [PDG 2018]

$$M_H = 90^{+16}_{-17}\,\text{GeV}$$

the HVP contribution to the running

- is strongly correlated with a_μ^{HVP}
- is a main input in global SM fits

shift of $\pm 10^{-4}$ in $\varDelta_{\text{had}}\alpha^{(3)}(2\,\text{GeV})$

$\Rightarrow$ shift of $\mp 4.5\,\text{GeV}$ in M_H

if the $(g-2)_\mu$ discrepancy is solved by an increase of the SM determination of a_μ^{HVP}

$\Rightarrow$ correlated increase in $\varDelta_{\text{had}}\alpha$ [Passera, Marciano, Sirlin 2008]

$\Rightarrow$ lower M_H from global fits

motivation – t-channel scattering

the leading hadronic contribution to $(g-2)_\mu$ from the running of α [Lautrup, Peterman, de Rafael 1972]

$$a_\mu^{\mathrm{HVP}} = \frac{\alpha}{\pi} \int_0^1 \mathrm{d}x\,(1-x)\Delta\alpha_{\mathrm{had}}(Q), \qquad Q^2 = \frac{x^2 m_\mu^2}{1-x},$$

with the integrand peaked at $x \approx 0.914$, $Q^2 \approx 0.108\,\mathrm{GeV}^2$. [Carloni Calame *et al.* 2015]

MUonE experiment @ CERN: measure the Q^2 dependence of α [Abbiendi *et al.* 2017; and talks on Tuesday afternoon]

- in the range $0 < x < 0.932$, corresponding to $Q^2 \lesssim 0.14\,\mathrm{GeV}^2$
- $0.932 < x < 1$ or $Q^2 \gtrsim 0.14\,\mathrm{GeV}^2$ accounts for $13\,\%$ of a_μ^{HVP}

⇒ lattice input for the intermediate region $Q^2 = 0.14 - 4\,\mathrm{GeV}^2$

the running of the electroweak mixing angle

the electroweak mixing (Weinberg) angle θ_W parametrizes the mixing between the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ sectors of the Standard Model. At tree level,

$$\sin^2\theta_\mathrm{W} = \frac{g'^2}{g^2 + g'^2}, \quad \text{or} \quad \sin^2\theta_\mathrm{W} = 1 - \frac{M_W^2}{M_Z^2},$$

where g and g' are the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ coupling respectively

- Z vector coupling $v_f = T_f - 2Q_f \sin^2\theta_f^\mathrm{eff}$
- weak charge of the proton $Q_W(p) \sim 1 - 4\sin^2\theta_\mathrm{W}$

like the couplings, the mixing angle is renormalization scheme and energy dependent

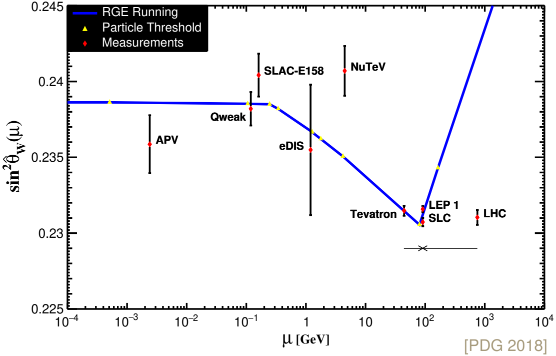
$$\sin^2\theta_\mathrm{W}(Q) = \sin^2\theta_0\left[1 + \Delta\sin^2\theta_\mathrm{W}(Q)\right],$$

and the leading hadronic contribution to the running [Jegerlehner 1986; 2011]

$$\Delta_\mathrm{had}\sin^2\theta_\mathrm{W}(Q) = -\frac{4\pi\alpha}{\sin^2\theta_\mathrm{W}}\hat{\Pi}^{Z\gamma}(Q^2), \qquad \hat{\Pi}^{Z\gamma}(Q^2) = \Pi^{Z\gamma}(Q^2) - \Pi^{Z\gamma}(0),$$

is proportional to the subtracted Z-γ hadronic vacuum polarization

the running of the electroweak mixing angle



[PDG 2018]

experimental values measured at colliders enter global SM fits [PDG 2016]

$$\sin^2\theta_W(M_Z) = 0.231\,29(5)$$

upcoming experiments at low Q^2

- MOLLER @ JLab [Benesch *et al.* 2014]
- P2 @ MESA, Mainz [Becker *et al.* 2018]

the running to the Thomson limit is affected by non-perturbative QCD physics that

- can be extracted from hadronic cross-section data [Erler, Ferro-Hernández 2017]

$$\sin^2\theta_W(0) = 0.238\,68(5)(2), \qquad (\overline{\text{MS}}\text{ scheme})$$

  with additional input for flavor separation
- or can be computed on the lattice [Burger *et al.* 2015; Francis *et al.* 2015; Gülpers *et al.* 2015]
  ⇒ lattice easily provides flavor separation

the hadronic vacuum polarization

we want to compute the subtracted hadronic vacuum polarization

$$\Delta_{\text{had}}\alpha(Q) = 4\pi\alpha \hat{\Pi}^{\gamma\gamma}(Q^2) \qquad \Delta_{\text{had}}\sin^2\theta_{\mathrm{W}}(Q) = -\frac{4\pi\alpha}{\sin^2\theta_{\mathrm{W}}}\hat{\Pi}^{Z\gamma}(Q^2)$$

$$(Q_\mu Q_\nu - \delta_{\mu\nu}Q^2)\Pi^{X\gamma}(Q^2) = \Pi^{X\gamma}_{\mu\nu}(Q^2) = \int \mathrm{d}^4x\, e^{\mathrm{i}Qx}\left\langle j^X_\mu(x) j^\gamma_\nu(0)\right\rangle$$

of the e.m. current and the vector part of the Z current

$$j^\gamma_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c,$$

$$j^{T_3}_\mu = \frac{1}{4}\bar{u}\gamma_\mu u - \frac{1}{4}\bar{d}\gamma_\mu d - \frac{1}{4}\bar{s}\gamma_\mu s + \frac{1}{4}\bar{c}\gamma_\mu c,$$

$$j^Z_\mu = j^{T_3}_\mu - \sin^2\theta_{\mathrm{W}} j^\gamma_\mu,$$

on the lattice [Burger *et al.* 2015; Francis *et al.* 2015; Gülpers *et al.* 2015]

the time-momentum representation (TMR) method

introduced for the HVP contribution to $(g-2)_\mu$ [Bernecker, Meyer 2011; Francis *et al.* 2013]

$$\hat{\Pi}(Q^2) = \int_0^\infty \mathrm{d}x_0\, G(x_0)\left[x_0^2 - \frac{4}{Q^2}\sin^2\left(\frac{Qx_0}{2}\right)\right], \qquad G(x_0) = -\frac{1}{3}\int \mathrm{d}^3x \sum_{k=1}^{3}\left\langle j_k^Z(x) j_k^\gamma(0)\right\rangle,$$

⇒ using correlators from $N_\mathrm{f} = 2+1$ Mainz effort in computing a_μ^HVP [Gérardin *et al.* 2019; status/plan Mainz by A. Gérardin]

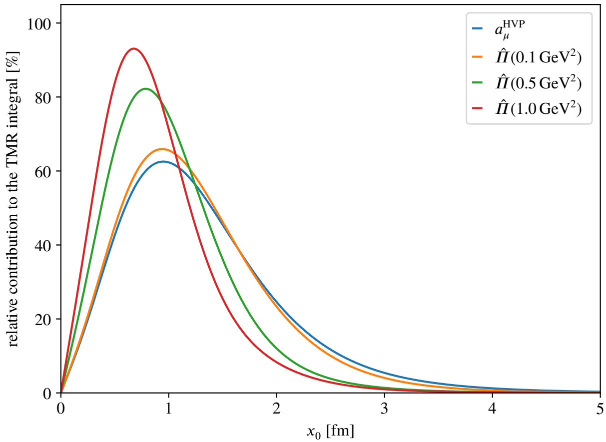
- CLS ensembles, four lattice spacings, one physical M_π, M_K ensemble [Bruno *et al.* 2015]
- non-perturbatively $\mathcal{O}(a)$-improved vector currents [Gérardin, Harris, Meyer 2018]
- two discretizations: local-local and local-conserved
- correction for finite lattice volume is included

in principle, any Q^2 can be input in the kernel

- at large Q^2, high sensitivity to cut-off effects
- w.r.t. the a_μ^HVP case, the kernel has a shorter range
- simpler large-distance systematic
  ⇒ no loss of signal in the tail of the connected correlator

the TMR method – contributions to the integrand

comparing $\hat{\Pi}(Q^2)$ at different Q^2 to a_μ^{HVP}

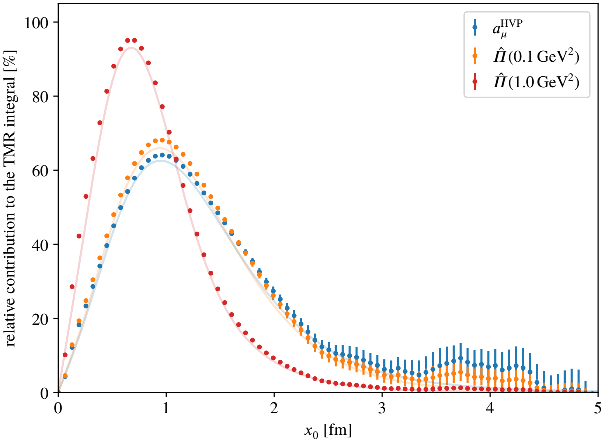


using a model for the Euclidean-time correlator

[Bernecker, Meyer 2011]

the TMR method – contributions to the integrand

comparing $\hat{\Pi}(Q^2)$ at different Q^2 to a_μ^{HVP}



using lattice data at physical M_π (E250) for the Euclidean-time correlator (connected contribution only)

lattice correlators

with $\mathrm{SU}(3)_F$ notation, in the isospin-symmetric limit (light quark ℓ: either u or d):

$$G^{33}_{\mu\nu}(x) = \frac{1}{2} C^{\ell,\ell}_{\mu\nu}(x),$$

$$G^{88}_{\mu\nu}(x) = \frac{1}{6}\Big[C^{\ell,\ell}_{\mu\nu}(x) + 2C^{s,s}_{\mu\nu}(x) + 2D^{\ell-s,\ell-s}_{\mu\nu}(x)\Big],$$

$$G^{08}_{\mu\nu}(x) = \frac{1}{2\sqrt{3}}\Big[C^{\ell,\ell}_{\mu\nu}(x) - C^{s,s}_{\mu\nu}(x) + D^{2\ell+s,\ell-s}_{\mu\nu}(x)\Big],$$

where the connected and disconnected Wick's contractions are

$$C^{f_1,f_2}_{\mu\nu}(x) = -\Big\langle \mathrm{Tr}\Big\{ D^{-1}_{f_1}(x,0)\gamma_\mu D^{-1}_{f_2}(0,x)\gamma_\nu \Big\}\Big\rangle,$$

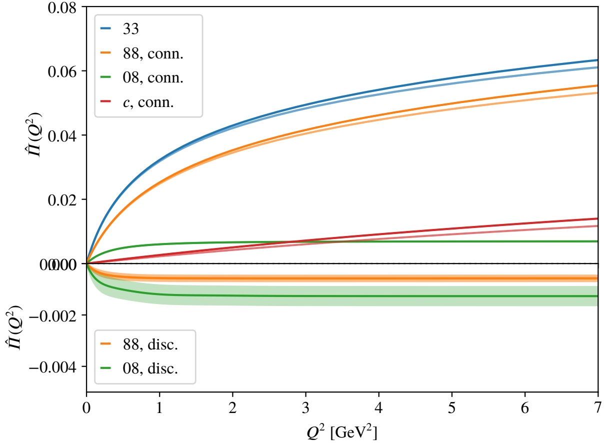
$$D^{f_1,f_2}_{\mu\nu}(x) = \Big\langle \mathrm{Tr}\Big\{ D^{-1}_{f_1}(x,x)\gamma_\mu \Big\} \mathrm{Tr}\Big\{ D^{-1}_{f_2}(0,0)\gamma_\nu \Big\}\Big\rangle,$$

and the relevant correlators are given by (note: $G^{\ell}_{\text{conn.}} = 2G^{33}$ and $G^{s}_{\text{conn.}} = 3G^{88}_{\text{conn.}} - G^{33}$)

$$G^{\gamma\gamma} = G^{33} + \frac{1}{3}G^{88} + \frac{4}{9}C^{c,c},$$

$$G^{Z\gamma} = \Big(\frac{1}{2} - \sin^2\theta_{\mathrm{W}}\Big)G^{\gamma\gamma} - \frac{1}{6\sqrt{3}}G^{08} + \frac{4}{9}\Big(\frac{3}{8} - \sin^2\theta_{\mathrm{W}}\Big)C^{c,c}.$$

preliminary results – running on the lattice



at $Q^2 = 1\,\mathrm{GeV}^2$

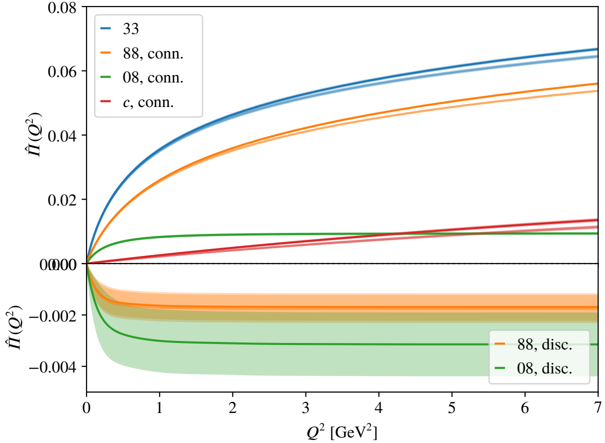
	l.c.	l.l.
33	0.032 26(13)	0.031 85(14)
88	0.025 26(5)	0.024 85(5)
08	0.006 00(8)	
c	0.002 664(6)	0.002 246(5)
88	−0.000 57(12)	−0.000 57(12)
08	−0.001 20(36)	

D200: $M_\pi \approx 200\,\mathrm{MeV}$, $a = 0.064\,26(74)\,\mathrm{fm}$

l.c. $\Delta_{\mathrm{had}}\alpha(1\,\mathrm{GeV}) = 0.003\,957(14)(4)(1)$, $\Delta_{\mathrm{had}}\sin^2\theta_{\mathrm{W}}(1\,\mathrm{GeV}) = -0.004\,026(12)(11)(0)$

l.l. $0.003\,869(14)(4)(1)$

preliminary results – running on the lattice



at $Q^2 = 1\,\mathrm{GeV}^2$

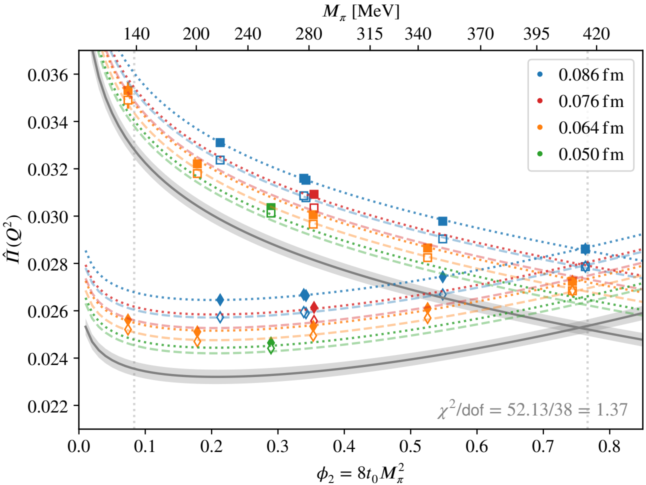
	l.c.	l.l.
33	0.035 52(36)	0.035 11(36)
88	0.025 94(12)	0.025 53(12)
08	0.008 26(21)	
c	0.002 59(8)	0.002 19(7)
88	−0.0016(5)	−0.0017(5)
08	−0.0030(12)	

E250: physical meson masses, $a = 0.064\,26(74)\,\mathrm{fm}$

l.c. $\Delta_{\text{had}}\alpha(1\,\mathrm{GeV}) = 0.004\,237(36)(15)(8)$, $\Delta_{\text{had}}\sin^2\theta_{\mathrm{W}}(1\,\mathrm{GeV}) = -0.004\,320(32)(33)(4)$

l.l. $0.004\,148(37)(16)(6)$

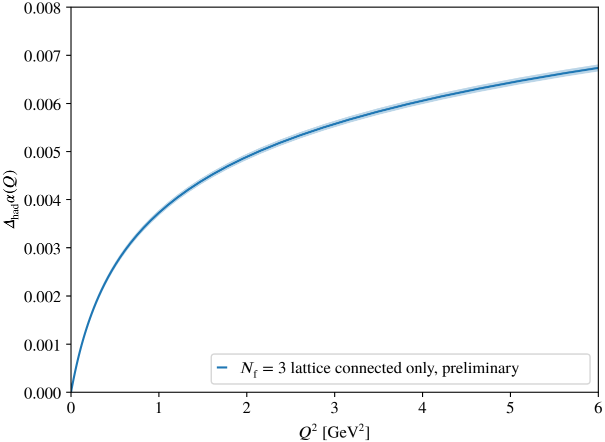
preliminary results – extrapolation to the physical point



combined fit of $\hat{\Pi}^{33}$ and $\hat{\Pi}^{88}_{\text{conn.}}$ at $Q^2 = 1\,\text{GeV}^2$, two discretizations each, and M_π, M_K, leads to

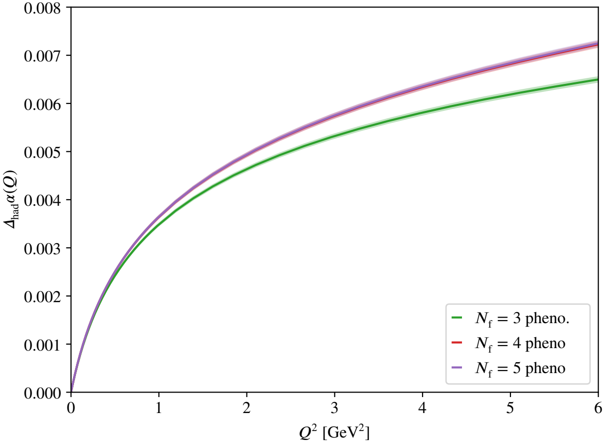
$$\hat{\Pi}^{33} = 0.0328(4), \qquad \hat{\Pi}^{88}_{\text{conn.}} = 0.023\,55(31) \quad \Rightarrow \quad \Delta_{\text{had,conn.}}\alpha(Q) = 0.003\,73(4)$$

preliminary results – α running at the physical point



at $Q^2 = 1\,\mathrm{GeV}^2$, the $N_{\mathrm{f}} = 3$ value is $\Delta_{\mathrm{had}}\alpha(Q) = 0.003\,73(4)$ (connected only)

preliminary results – α running at the physical point

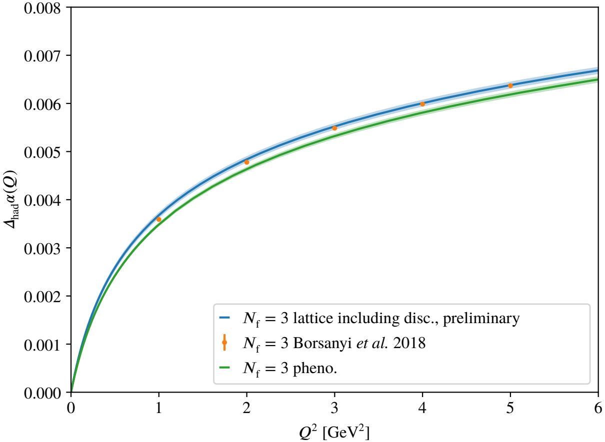


at $Q^2 = 1\,\mathrm{GeV}^2$, the $N_\mathrm{f} = 3$ value is $\Delta_{\mathrm{had}}\alpha(Q) = 0.003\,73(4)$ (connected only)

- phenomenology gives $\Delta_{\mathrm{had}}\alpha(Q) = 0.003\,49(2)$

[Jegerlehner and Miura]

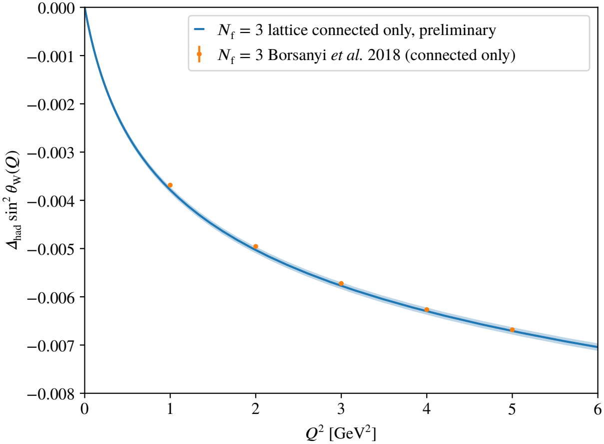
preliminary results – α running at the physical point



at $Q^2 = 1\,\mathrm{GeV}^2$, the $N_\mathrm{f} = 3$ value is $\mathit{\Delta}_{\mathrm{had}}\alpha(Q) = 0.003\,68(4)(2)$

- phenomenology gives $\mathit{\Delta}_{\mathrm{had}}\alpha(Q) = 0.003\,49(2)$ [Jegerlehner and Miura]
- disconnected contribution $-5.0(15) \times 10^{-5}$ estimated at $a \approx 0.064\,\mathrm{fm}$ and physical M_π
- independent lattice determination $\mathit{\Delta}_{\mathrm{had}}\alpha(Q) = 0.003\,59(1)(2)$ [Borsanyi *et al.* 2018]

preliminary results – $\sin^2\theta_\mathrm{W}$ running at the physical point



at $Q^2 = 1\,\mathrm{GeV}^2$, the $N_\mathrm{f} = 3$ value is $\varDelta_\mathrm{had}\sin^2\theta_\mathrm{W}(Q) = -0.003\,78(4)$ (connected only)

- independent connected-only lattice determination $\varDelta_\mathrm{had}\sin^2\theta_\mathrm{W}(Q) = -0.003\,69(1)(2)$ [Borsanyi *et al.* 2018]

conclusions & outlook

a lattice computation of the leading hadronic contribution to the running of α and $\sin^2\theta_\mathrm{W}$

- with $\mathcal{O}(1\,\%)$ errors
- comparable with the phenomenological estimate [comparison provided by K. Miura]
- including the disconnected contribution, with sub-percent determination [Harris *et al.* Lattice 2019; K. Ottnad]
- $\sin^2\theta_\mathrm{W}$: lattice provides flavor separation ⇒ input for the dispersive approach
- correction for finite-size effects is essential

- extrapolate the disconnected contribution to the physical point
- … and the charm contribution
- bounding method for small Q^2? [implemented in Gérardin *et al.* 2019]
- we have a new fine ($a \approx 0.050\,\mathrm{fm}$) and light ($M_\pi \approx 175\,\mathrm{MeV}$) ensemble
- add isospin breaking effects [work in progress: Risch, Wittig 2018]
- and other systematics (*e. g.* scale setting, physical-point extrapolation)
- …

thanks

for your attention!

questions?

backup slides

renormalization and $\mathcal{O}(a)$ improvement

for the local current [Bhattacharya *et al.* 2006, […], Gérardin, Harris, Meyer 2018]

$$V^3_{\mu,R} = Z_V\big(1 + 3\bar{b}_V a m_q^{\mathrm{av}} + b_V a m_{q,\ell}\big) V_\mu^{3,I} = Z_3 V_\mu^{3,I},$$

$$\begin{pmatrix} V^8_\mu \\ V^0_\mu \end{pmatrix}_R = Z_V \begin{pmatrix} 1 + 3\bar{b}_V a m_q^{\mathrm{av}} + b_V \frac{a(m_{q,\ell}+2m_{q,s})}{3} & \left(\frac{b_V}{3} + f_V\right)\frac{2a(m_{q,\ell}-m_{q,s})}{\sqrt{3}} \\ r_V d_V \frac{a(m_{q,\ell}-m_{q,s})}{\sqrt{3}} & r_V 1 + (3\bar{d}_V + d_V) a m_q^{\mathrm{av}} \end{pmatrix} \begin{pmatrix} V^8_\mu \\ V^0_\mu \end{pmatrix}^I = \begin{pmatrix} Z_8 & Z_{80} \\ Z_{08} & Z_0 \end{pmatrix} \begin{pmatrix} V^8_\mu \\ V^0_\mu \end{pmatrix}^I$$

where

$$V_\mu^{a,I} = V_\mu^a + a c_V \partial_0 T^a_{0\mu}, \qquad V_\mu^{0,I} = V_\mu^0 + a\bar{c}_V \partial_0 T^0_{0\mu}.$$

while for the conserved current

$$V^a_{\mu,R} = V^a_\mu + a c_V^{\mathrm{cs}} \partial_0 T^a_{0\mu}, \qquad V^0_{\mu,R} = V^0_\mu + a\bar{c}_V^{\mathrm{cs}} \partial_0 T^0_{0\mu}.$$

⇒ we use only the conserved vector current for the flavor-singlet component, and we set

$$f_V = 0, \qquad \bar{c}_V = c_V \qquad \bar{c}_V^{\mathrm{cs}} = c_V^{\mathrm{cs}}.$$

ensembles

from the CLS initiative

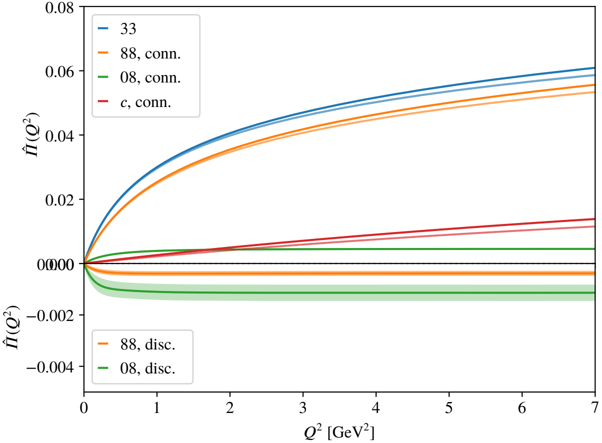
[Bruno *et al.* 2015, Bruno, Korzec, Schaefer 2017]

tree-level Lüscher-Weisz gauge action, non-perturbatively $\mathcal{O}(a)$-improved Wilson fermions, open BCs in time

	T/a	L/a	a [fm]	L [fm]	m_π, m_K [MeV]		$m_\pi L$
H101	96	32	0.086	2.8	415		5.8
H102	96	32		2.8	355	440	5.0
H105*	96	32		2.8	280	460	3.9
N101	128	48		4.1	280	460	5.8
C101*	96	48		4.1	220	470	4.6
B450§	64	32	0.076	2.4	415		5.1
S400	128	32		2.4	350	440	4.3
N401*	128	48		3.7	285	460	5.3
H200	96	32	0.064	2.1	420		4.4
N202	128	48		3.1	410		6.4
N203*	128	48		3.1	345	440	5.4
N200*	128	48		3.1	285	465	4.4
D200*	128	64		4.1	200	480	4.2
E250*§	192	96		6.2	130	490	4.1
N300	128	48	0.050	2.4	420		5.1
N302*	128	48		2.4	345	460	4.2
J303	192	64		3.2	260	475	4.2

* disconnected contribution available, § periodic BCs in time

preliminary results



at $Q^2 = 1\,\text{GeV}^2$

	l.c.	l.l.
33	0.030 02(11)	0.029 62(11)
88	0.025 37(5)	0.024 97(5)
08	0.003 93(6)	
c	0.002 629(5)	0.002 203(5)
88	−0.000 39(7)	−0.000 39(7)
08	−0.001 10(29)	

N200: $M_\pi \approx 285\,\text{MeV}$, $a = 0.064\,26(74)\,\text{fm}$

l.c. $\Delta_{\text{had}}\alpha(1\,\text{GeV}) = 0.003\,757(11)(2)(0)$, $\Delta_{\text{had}}\sin^2\theta_W(1\,\text{GeV}) = -0.003\,882(10)(9)(0)$

l.l. $0.003\,669(11)(2)(0)$

finite-size correction

added to the $I = 1$ correlator $G^{33}(t)$, with $t_i = (m_\pi L/4)^2/m_\pi$ [Gérardin *et al.* 2019]

$t < t_i$: correction from scalar QED / NLO χPT [Francis *et al.* 2013; Della Morte *et al.* 2017]

$$G^{33}(t,L) - G^{33}(t,\infty) = \frac{1}{3}\left(\frac{1}{L^3}\sum_{\vec{k}} - \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \right) \frac{\vec{k}^2 + m_\pi^2}{\vec{k}^2} \mathrm{e}^{-2t\sqrt{\vec{k}^2+m_\pi^2}}$$

$t > t_i$: correction from GS model of $F_\pi(\omega)$ [Gounaris, Sakurai 1968]

$$G^{33}(t,\infty) = \int_0^\infty \mathrm{d}\omega\, \omega^2 \rho(\omega^2) \mathrm{e}^{-\omega t} \qquad \rho(\omega^2) = \frac{1}{48\pi^2}\left(1 - \frac{4m_\pi^2}{\omega^2}\right)^{\frac{3}{2}} \left|F_\pi(\omega)\right|^2$$

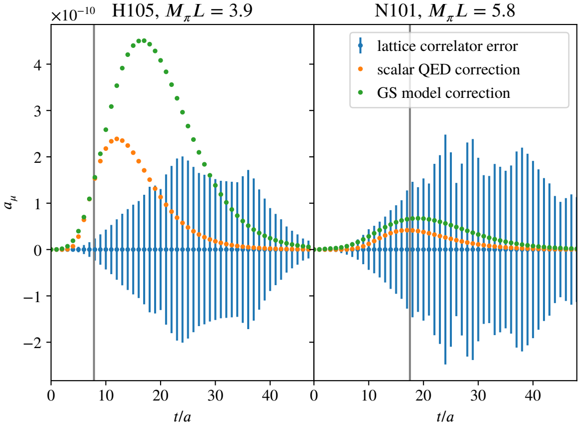
and the corresponding finite-volume correlator [Lüscher 1991; Lellouch, Lüscher 2000; Meyer 2011]

$$G^{33}(t,L) = \sum_n \left|A_n\right|^2 \mathrm{e}^{-\omega_n t} \qquad \text{with Lüscher's } \omega_n \text{ and LL's } A_n$$

note: the connected $I = 0$ correlator $G^{88}_{\text{conn.}}(t)$ receives a $I = 1$ finite-size correction $\propto G^{33}(t)/3$

finite-size correction – an example

using the TMR kernel to compute $(g-2)_\mu$



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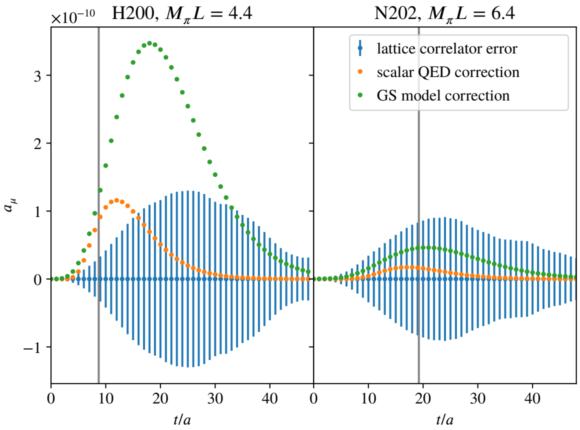


fit strategy

$$\chi^2 = \sum_{l \in \{\mathrm{CLS}\}} \left[v^l_{\text{model}} - v^l_{\text{data}}\right]^\top C^{-1}_{\text{data}} \left[v^l_{\text{model}} - v^l_{\text{data}}\right]$$

where

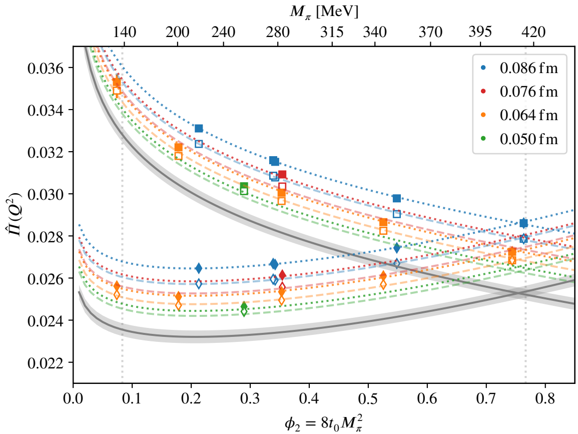
$$\left[v^l_{\text{model}} - v^l_{\text{data}}\right] = \begin{bmatrix} am_\pi^{\text{data}} - \frac{a}{4\sqrt{t_0}}\sqrt{2\phi_2^l} \\ \Pi^{\text{data}}_{33,c,l} - \Pi^{\text{model}}(a^2/t_0, \phi_2^l, \phi_4^l; \delta_i^{c,l}, \gamma_2^{33}, \dots) \\ \Pi^{\text{data}}_{33,l,l} - \Pi^{\text{model}}(a^2/t_0, \phi_2^l, \phi_4^l; \delta_i^{l,l}, \gamma_2^{33}, \dots) \\ am_K^{\text{data}} - \frac{a}{4\sqrt{t_0}}\sqrt{2\phi_4^l - \phi_2^l} \\ \Pi^{\text{data}}_{88,c,l} - \Pi^{\text{model}}(a^2/t_0, \phi_2^l, \phi_4^l; \delta_i^{c,l}, \gamma_2^{88}, \dots) \\ \Pi^{\text{data}}_{88,l,l} - \Pi^{\text{model}}(a^2/t_0, \phi_2^l, \phi_4^l; \delta_i^{l,l}, \gamma_2^{88}, \dots) \end{bmatrix}$$

where the general functional form of the model for Π is

$$\begin{aligned}\Pi^{\text{model}}\left(a^2/t_0, \phi_2^l, \phi_4^l; p_0, \delta_i, \gamma_j, \eta_k, \lambda_m, \phi_2^0, \phi_4^0\right) &= p_0 + \delta_i a/t_0^{i/2} \\ &+ \gamma_1(\phi_2^l - \phi_2^0) + \gamma_2(\log\phi_2^l - \log\phi_2^0) + \gamma_4(\phi_2^l - \phi_2^0)^2 + \eta_k(\phi_4^l - \phi_4^0)^k + \lambda_m(2\phi_4^l - 3\phi_2^l)^m\end{aligned}$$

note: on $\mathrm{SU}(3)$-symmetric ensembles $m_K^{\text{data}} \equiv m_\pi^{\text{data}}$, $\phi_4^l = 1.5\phi_2^l$, $\Pi_{88}^{\text{data}} = \Pi_{33}^{\text{data}}$

combined fit at $Q^2 = 1\,\mathrm{GeV}^2$, excluding ensembles with $L < 2.5\,\mathrm{fm}$



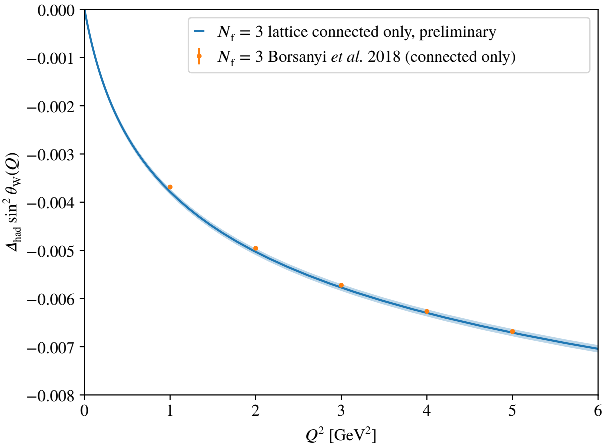
l	χ^2_l
H101	2.597
H102	2.239
H105	1.066
N101	1.145
C101	0.643
B450	-
S400	-
N401	6.108
H200	-
N202	1.291
N203	5.898
N200	2.266
D200	8.612
E250	8.150
N300	-
N302	-
J303	12.112

$\chi^2/\text{dof} = 52.13/38 = 1.37$, p-value $= 0.0631$, $\Pi^{33}_{\text{phys.}} = 0.0328(4)$, $\Pi^{88}_{\text{phys.}} = 0.023\,55(31) \Rightarrow \Delta\alpha = 0.003\,73(4)$

$\{\phi_2^l, \phi_4^l\}, \phi_2^0 = 0.754(5), p_0 = 0.025\,16(29), \delta_2^{c,l} = 0.0126(31), \delta_2^{l,l} = 0.0106(31), \delta_3^{c,l} = -0.005(4), \delta_3^{l,l} = -0.006(4), \beta_{2,1} = 0.0004(23),$

$\gamma_1^{33} = -0.0017(9), \quad \gamma_1^{88} = 0.003\,63(33), \quad \gamma_2^{33} = -0.002\,91(28), \quad \gamma_2^{88} = \gamma_2^{33}/3, \quad \gamma_4^{88} = 0.0026(5), \quad \eta_1 = -0.011(4)$

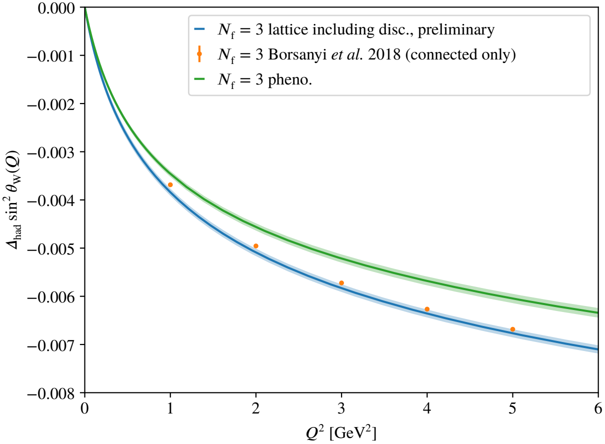
preliminary results – $\sin^2\theta_{\mathrm{W}}$ running at the physical point



at $Q^2 = 1\,\mathrm{GeV}^2$, the $N_{\mathrm{f}} = 3$ value is $\varDelta_{\mathrm{had}}\sin^2\theta_{\mathrm{W}}(Q) = -0.003\,78(4)$ (connected only)

- independent connected-only lattice determination $\varDelta_{\mathrm{had}}\sin^2\theta_{\mathrm{W}}(Q) = -0.003\,65(1)(2)$ [Borsanyi *et al.* 2018]

preliminary results – $\sin^2\theta_W$ running at the physical point



at $Q^2 = 1\,\mathrm{GeV}^2$, the $N_f = 3$ value is $\Delta_{had}\sin^2\theta_W(Q) = -0.003\,84(4)(3)$

- phenomenology gives $\Delta_{had}\sin^2\theta_W(Q) = -0.003\,46(4)$ [Jegerlehner and Miura]
- disconnected contribution $-5.6(33)\times 10^{-5}$ estimated at $a \approx 0.064\,\mathrm{fm}$ and physical M_π
- independent connected-only lattice determination $\Delta_{had}\sin^2\theta_W(Q) = -0.003\,65(1)(2)$ [Borsanyi *et al.* 2018]