JLU
NEUE WEGE. SEIT 1607.

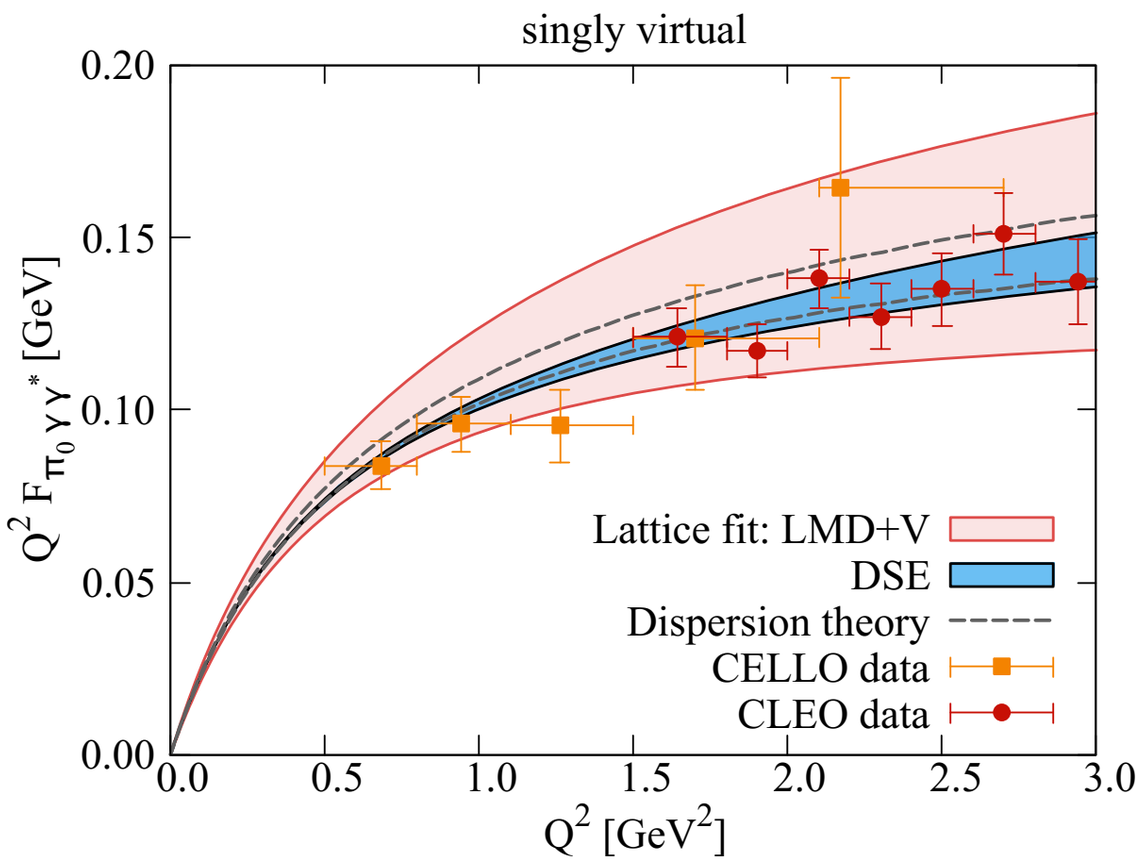
JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

HLBL from Dyson-Schwinger equations

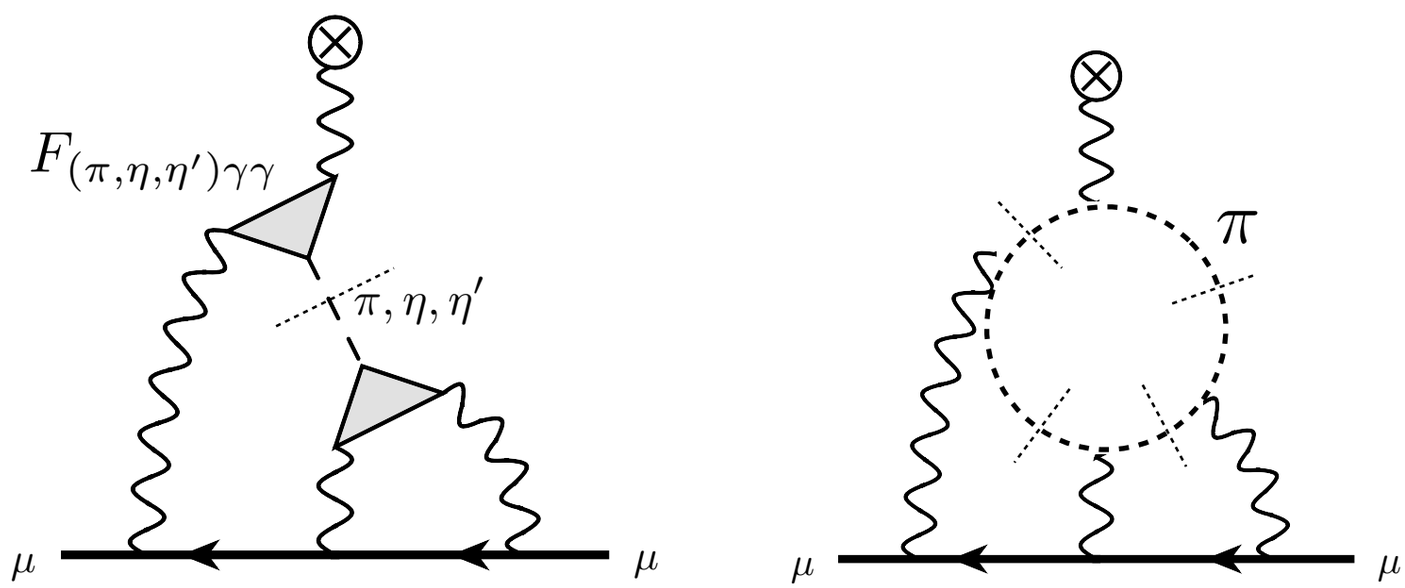
Eichmann, CF, Weil and Williams, PLB 797 (2019), 134855 [arXiv:1903.10844]

Overview

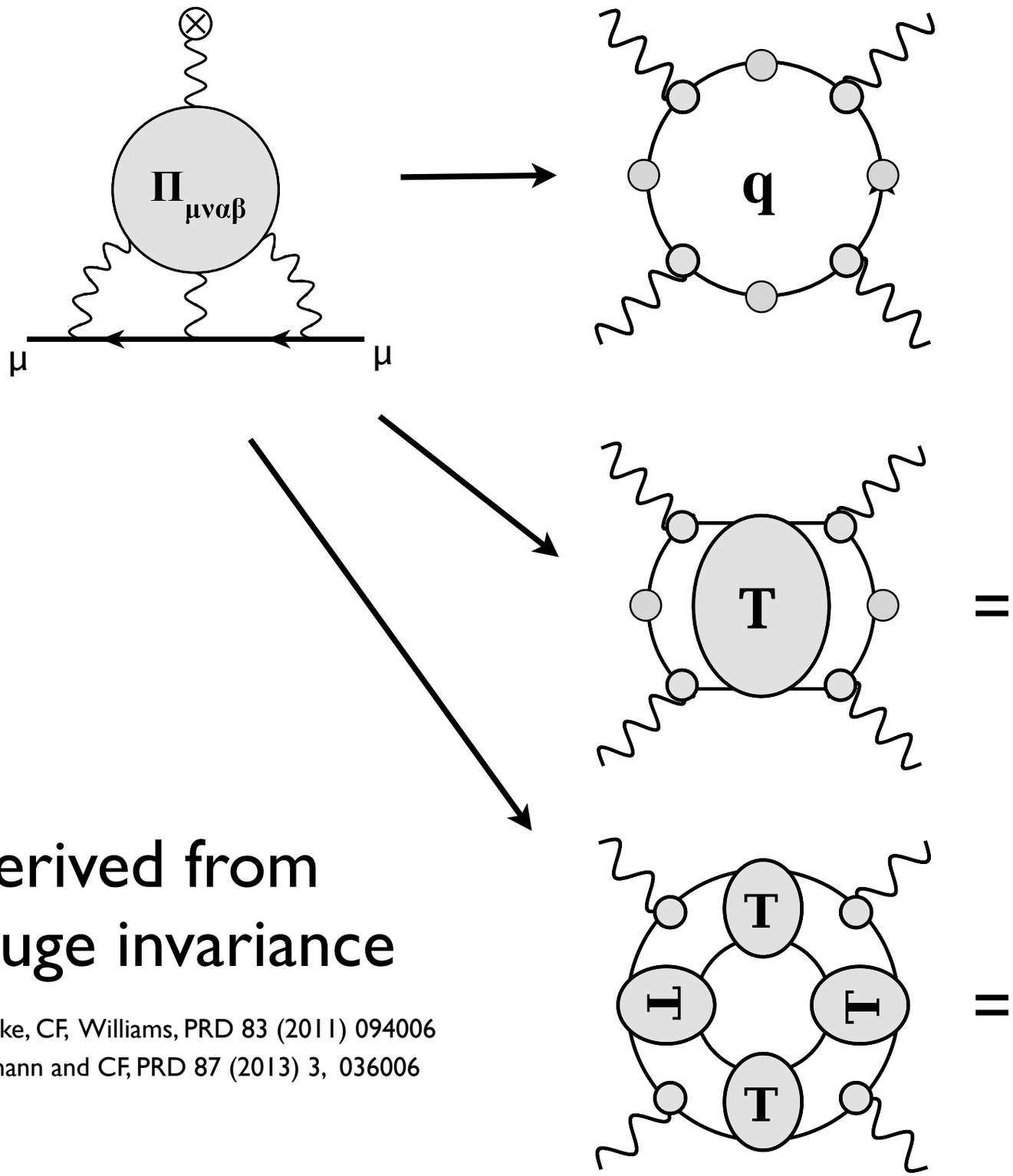
1.(Transition) Form Factors



2.HLBL



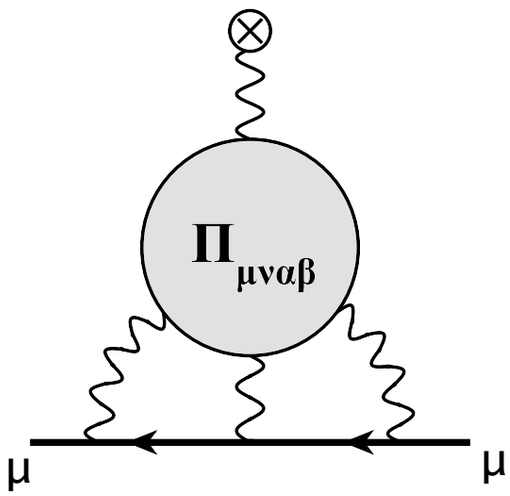
HLBL: diagrammatic approach

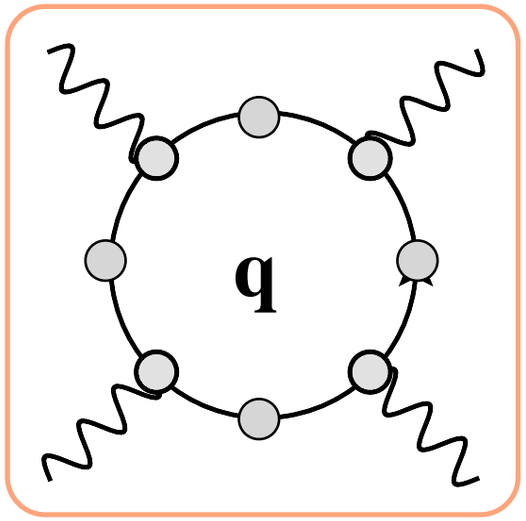


Derived from gauge invariance

Goecke, CF, Williams, PRD 83 (2011) 094006
Eichmann and CF, PRD 87 (2013) 3, 036006

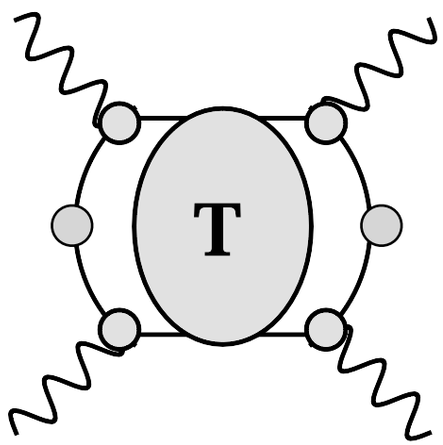
HLBL: diagrammatic approach



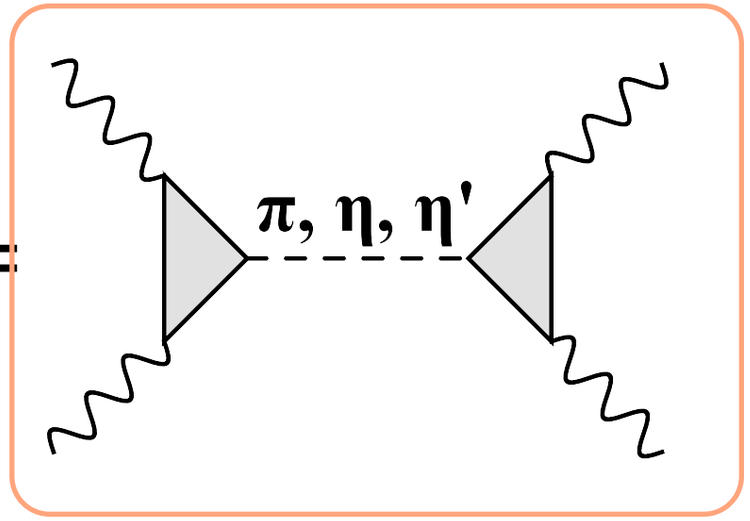


Quark-loop

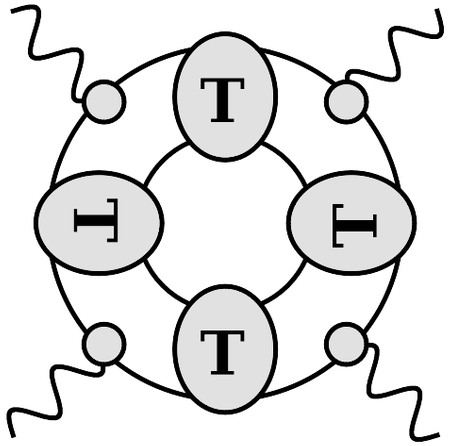
Off-shell meson-exchange



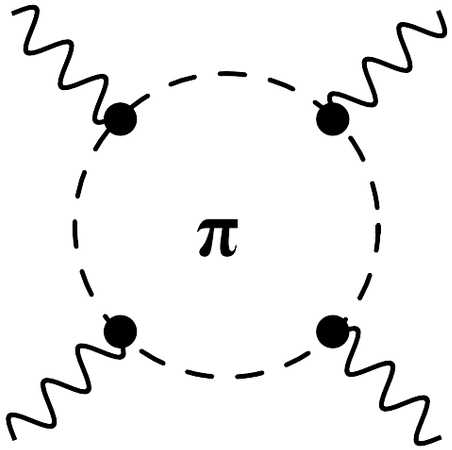
=



+...



=

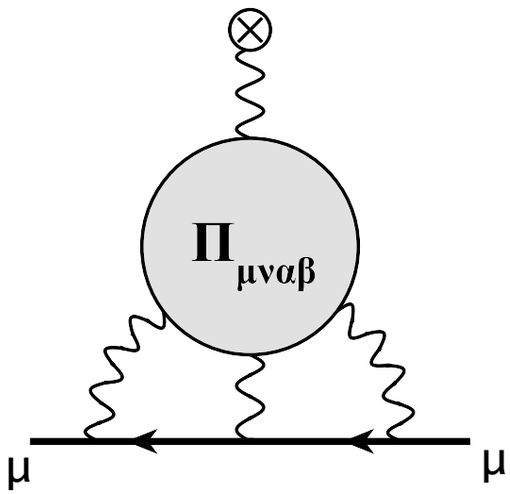


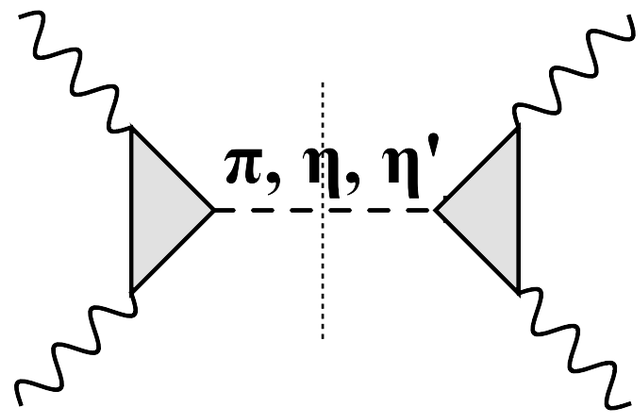
+...

Derived from gauge invariance

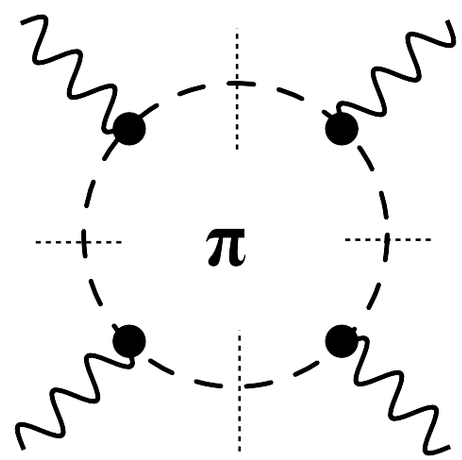
Goecke, CF, Williams, PRD 83 (2011) 094006
Eichmann and CF, PRD 87 (2013) 3, 036006

HLBL: dispersive approach





On-shell meson-exchange



On-shell meson loop

higher contributions

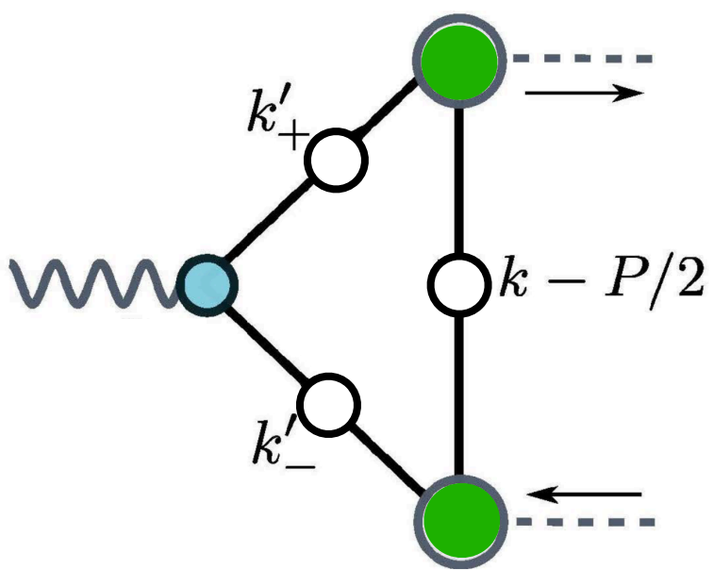
Need on-shell (transition) form factors

Colangelo, Hoferichter, Procura and Stoffer, JHEP 1409 (2014) 091, JHEP 1509 (2015) 074
Pauk and Vanderhaeghen, PRD 90 (2014) no.11, 113012

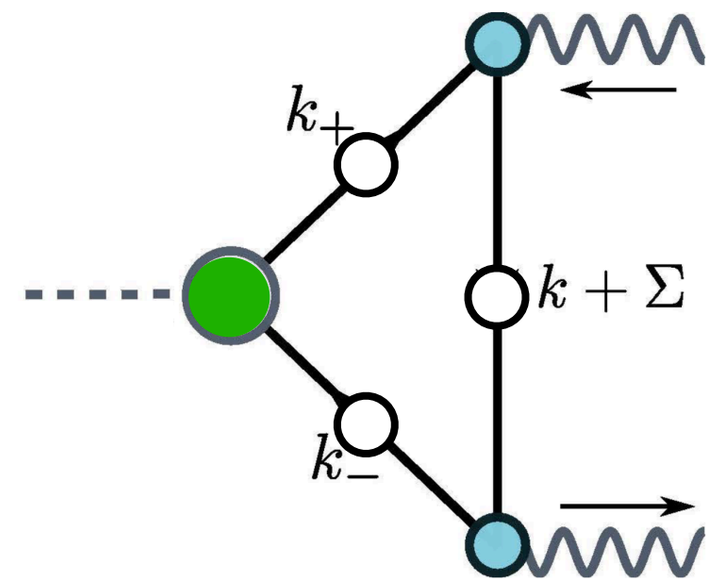
(Transition-) Form factors and decays

gauge fixed Euclidean QCD: quarks and gluons

Pion form factor:



Meson transition FF



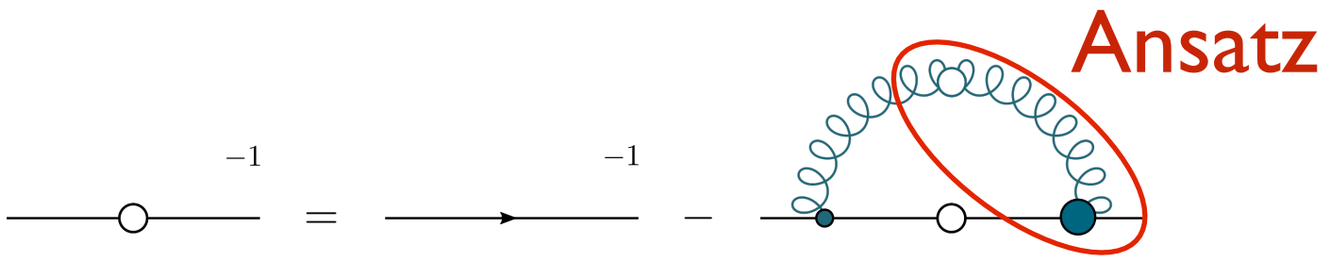
meson Bethe-Salpeter vertex

quark photon vertex

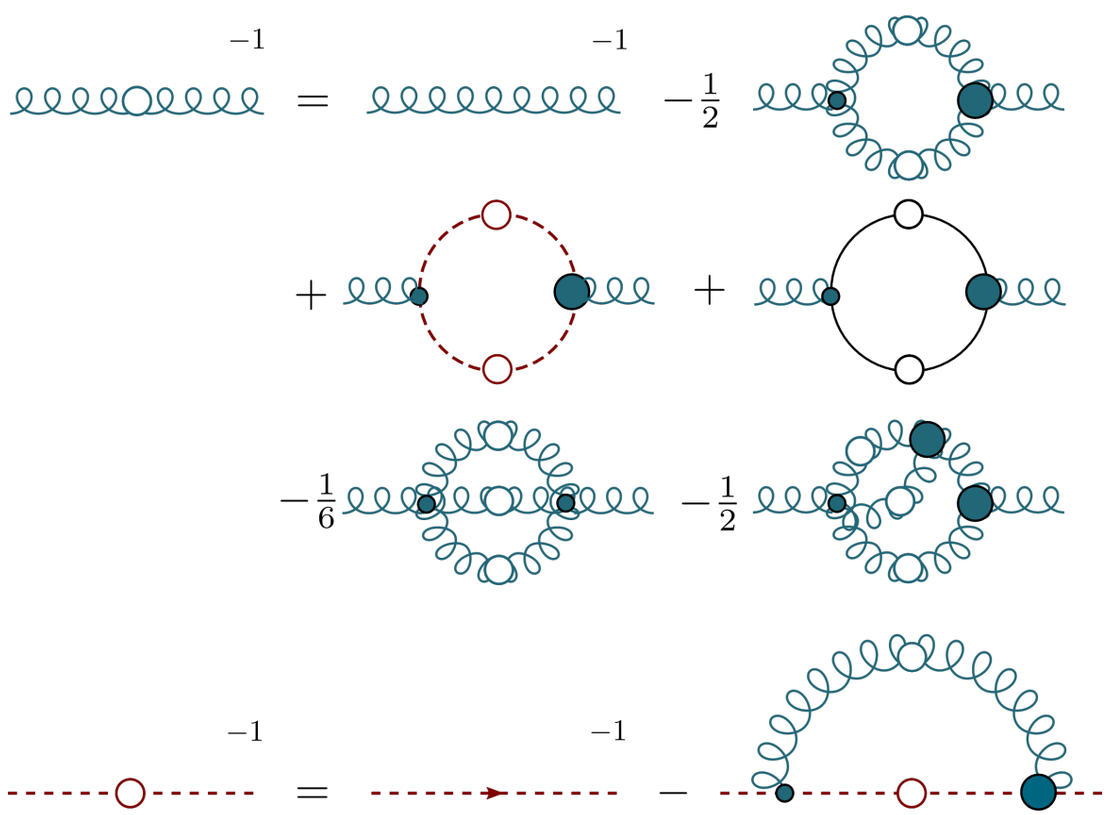
dressed quark propagator

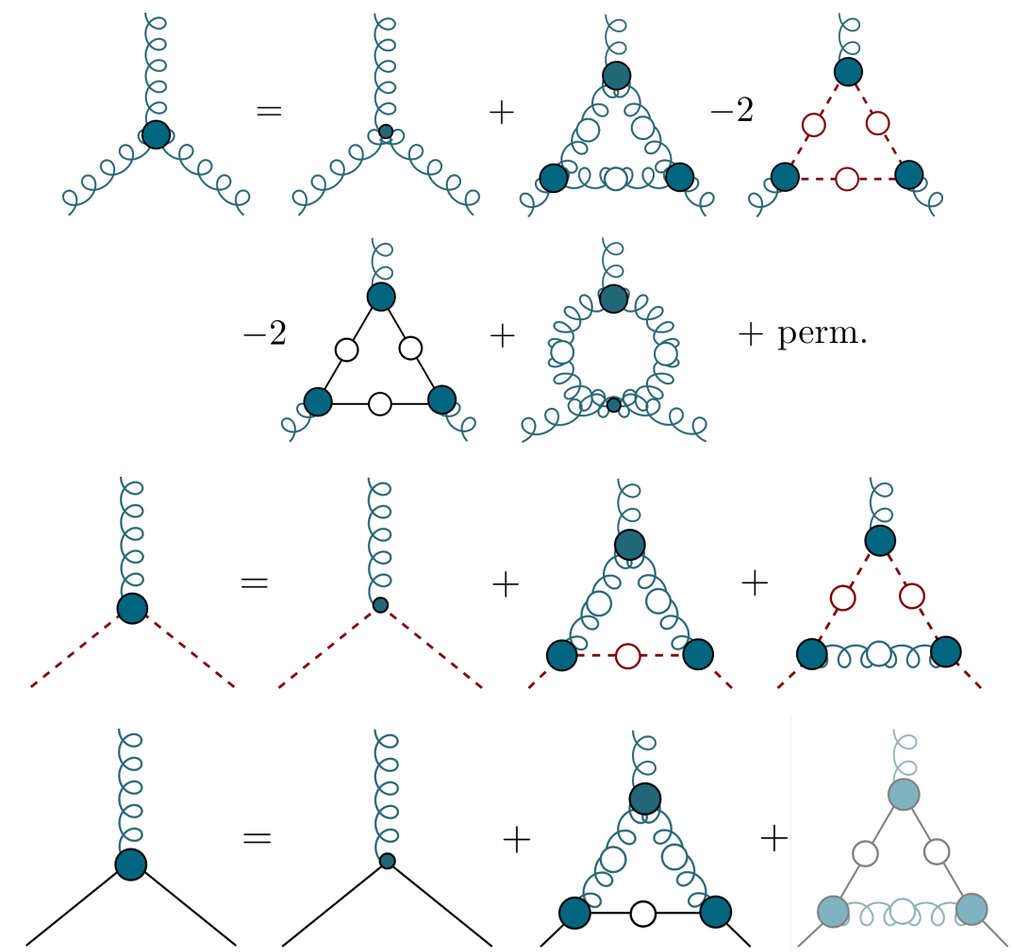
DSE-approach: truncations

Rainbow-Ladder (RL):



Beyond the rainbow (BRL):

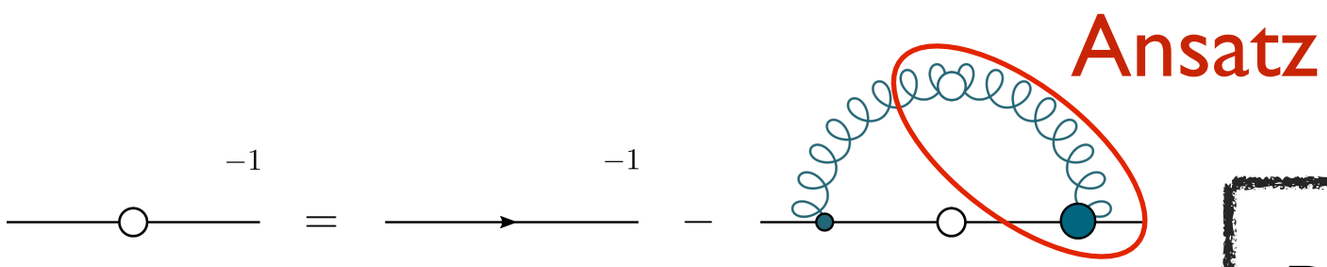




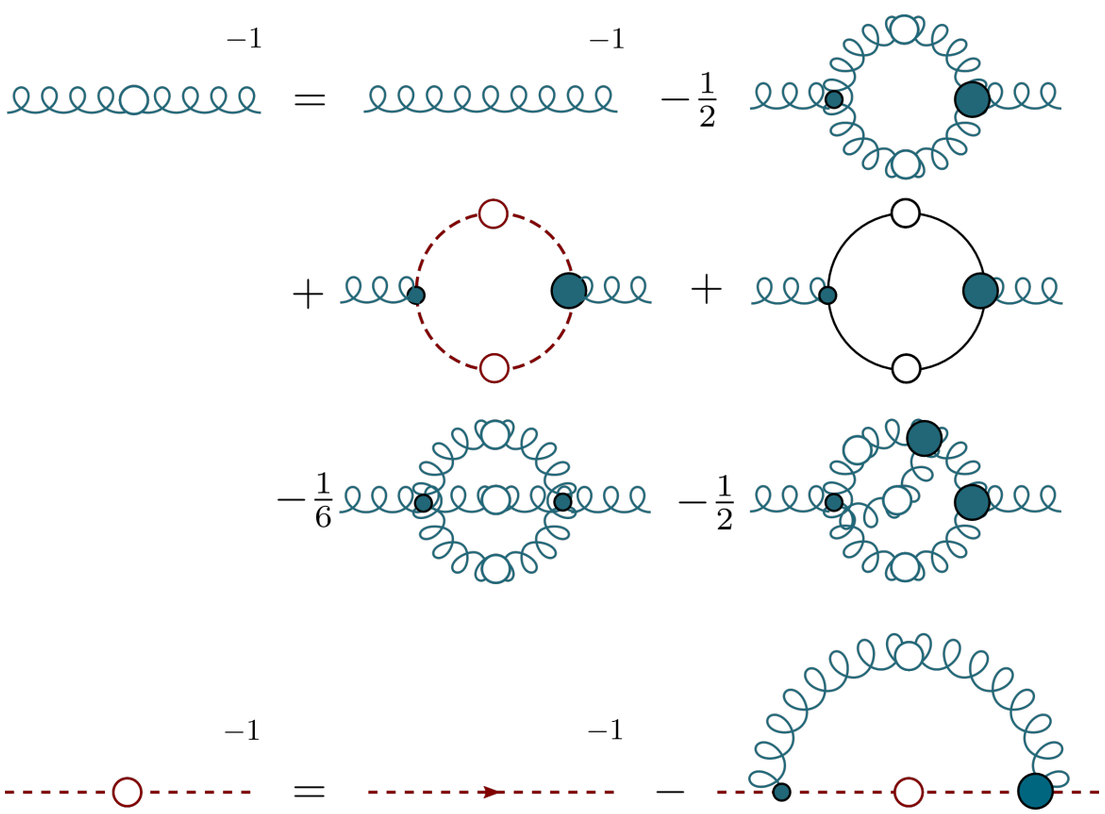
see e.g. Williams, CF, Heupel, PRD 93 (2016) 034026 and Refs. therein

DSE-approach: truncations

Rainbow-Ladder (RL):



Beyond the rainbow (BRL):

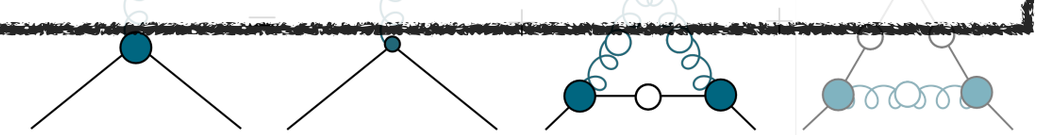


BRL-results for:

- light meson spectrum
- light baryon spectrum
- QCD phase diagram

Reviews: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]
CF, PPNP 105 (2019) 1 [1810.12938]
Huber, arXiv:1808.05227.

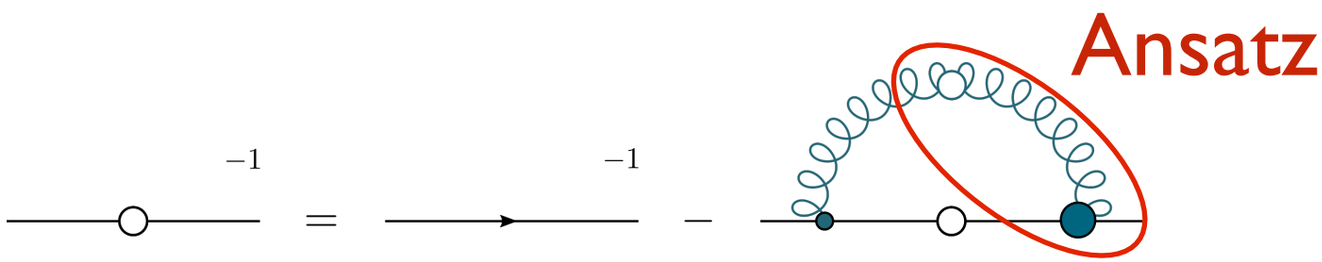
PS-mesons: RL is very good
V-mesons: RL is good
(ground states)

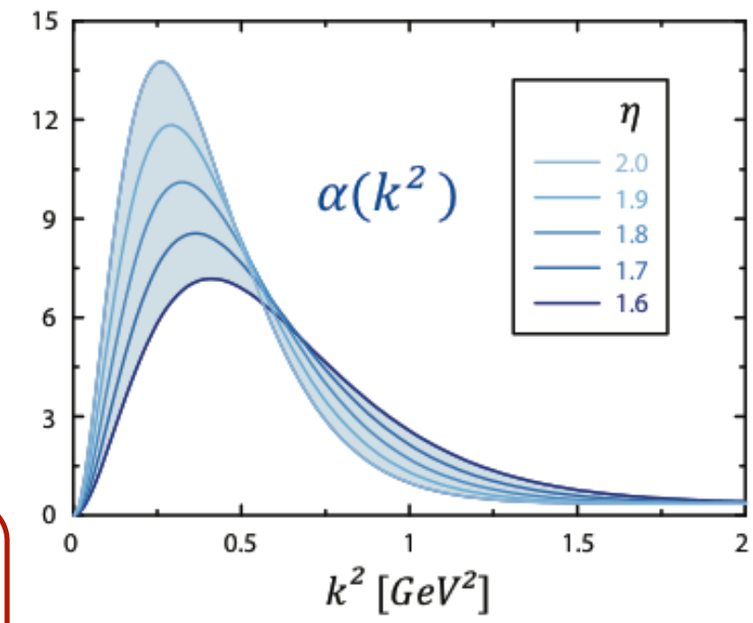


see e.g. Williams, CF, Heupel, PRD 93 (2016) 034026 and Refs. therein

DSE-approach: truncations

Rainbow-Ladder (RL):





Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

effective coupling

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

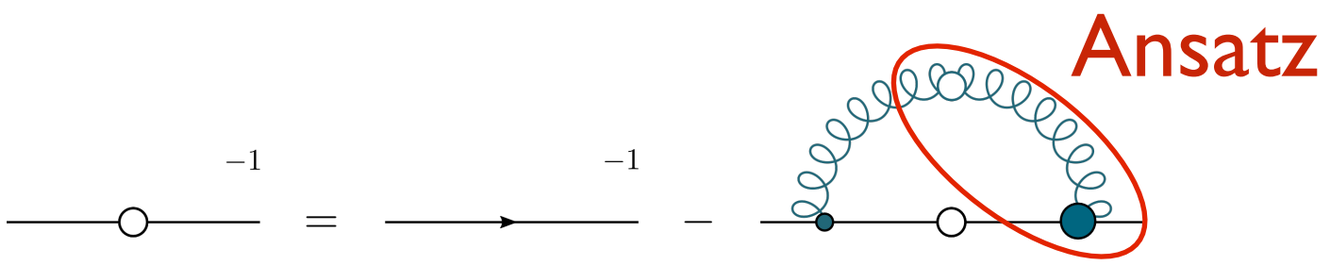
- α_{UV} from perturbation theory
- scale Λ from f_π
- parameter η : only small influence on spectra and FF
- qualitatively similar to explicit BRL calc.

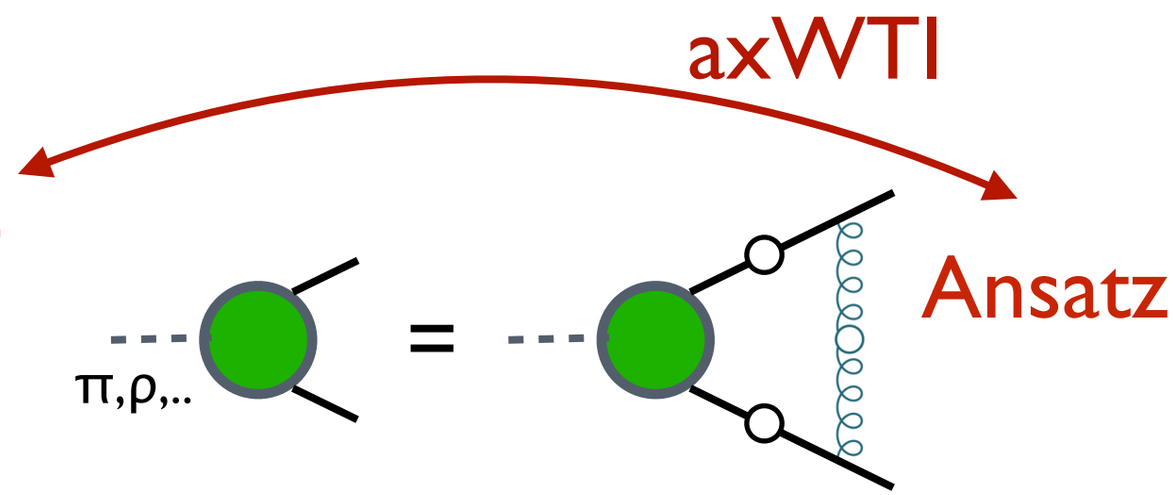
Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

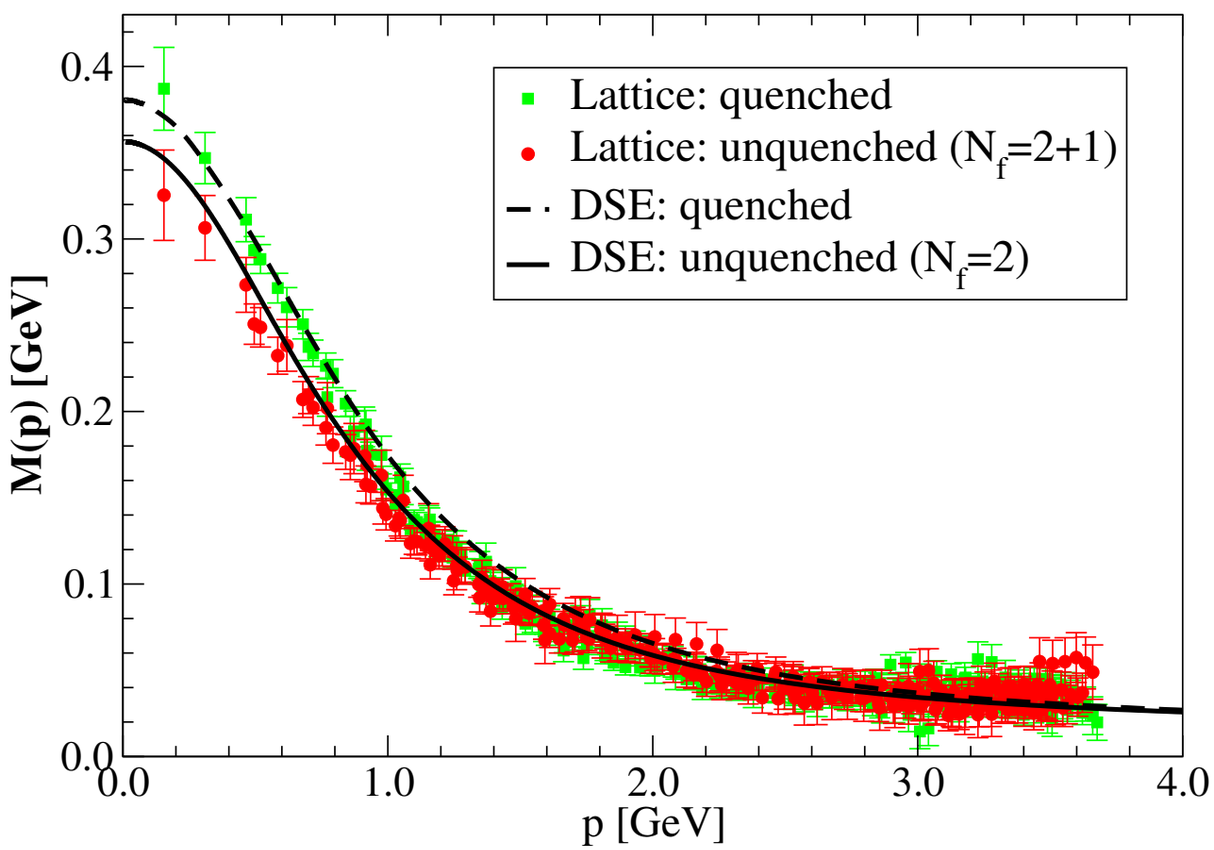
DSE-approach: truncations

$$[S(p)]^{-1} = [-i\not{p} + M(p^2)]/Z_f(p^2)$$

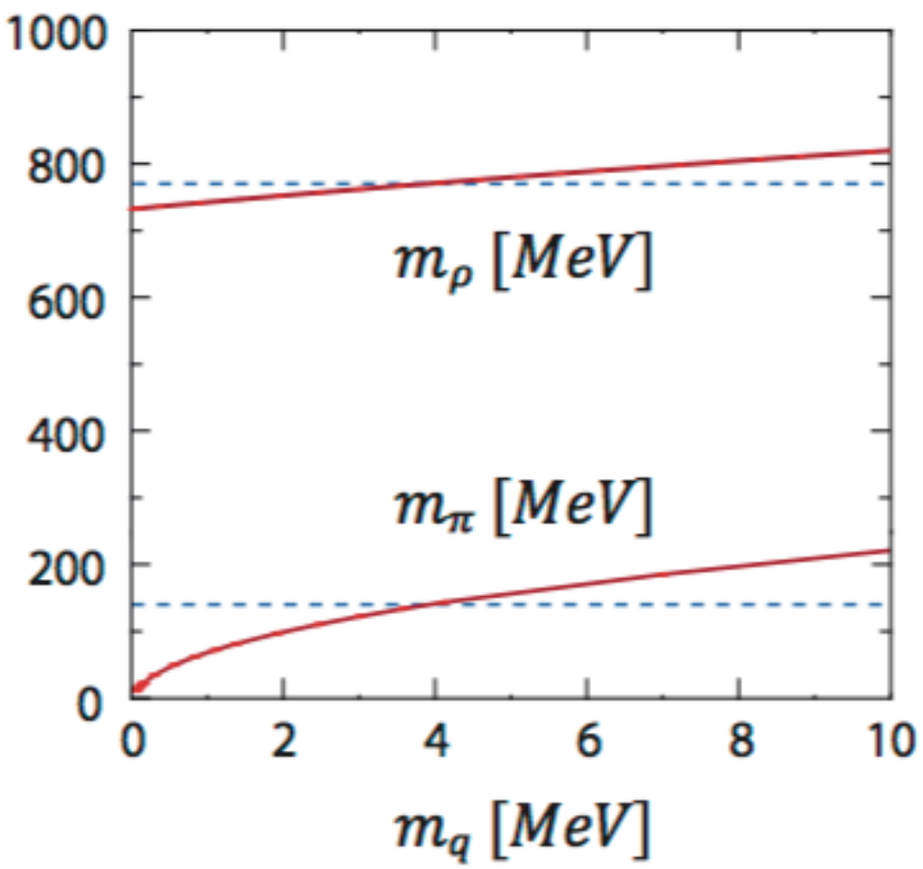
Rainbow-Ladder (RL):







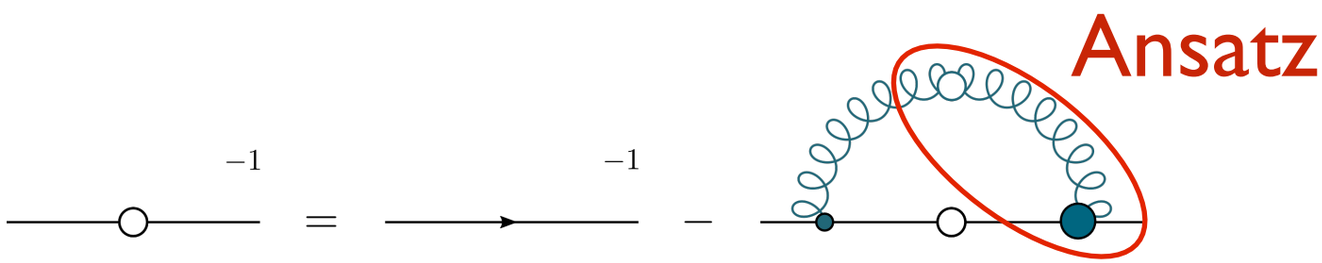
CF, Nickel, Williams, EPJ C 60 (2009) 47

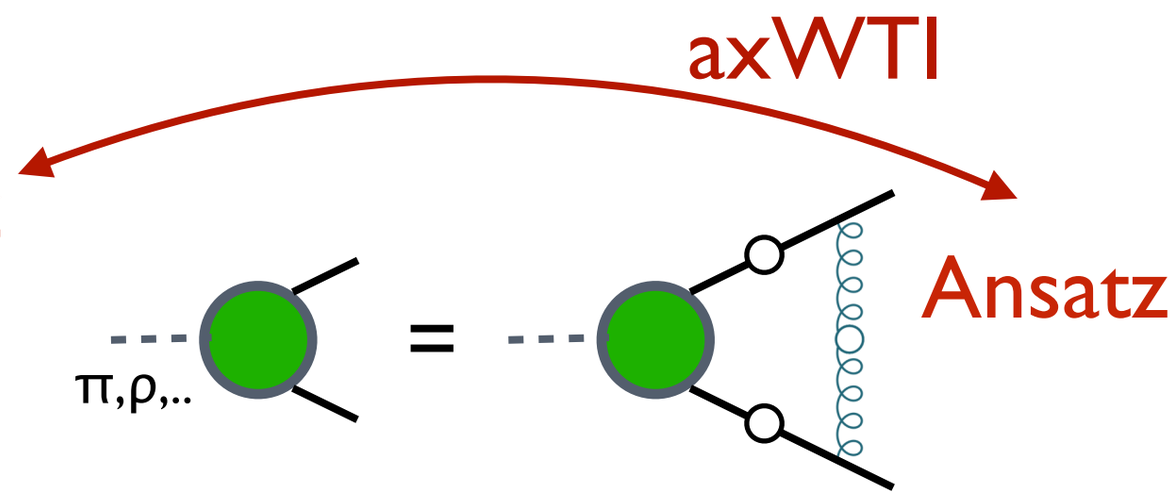


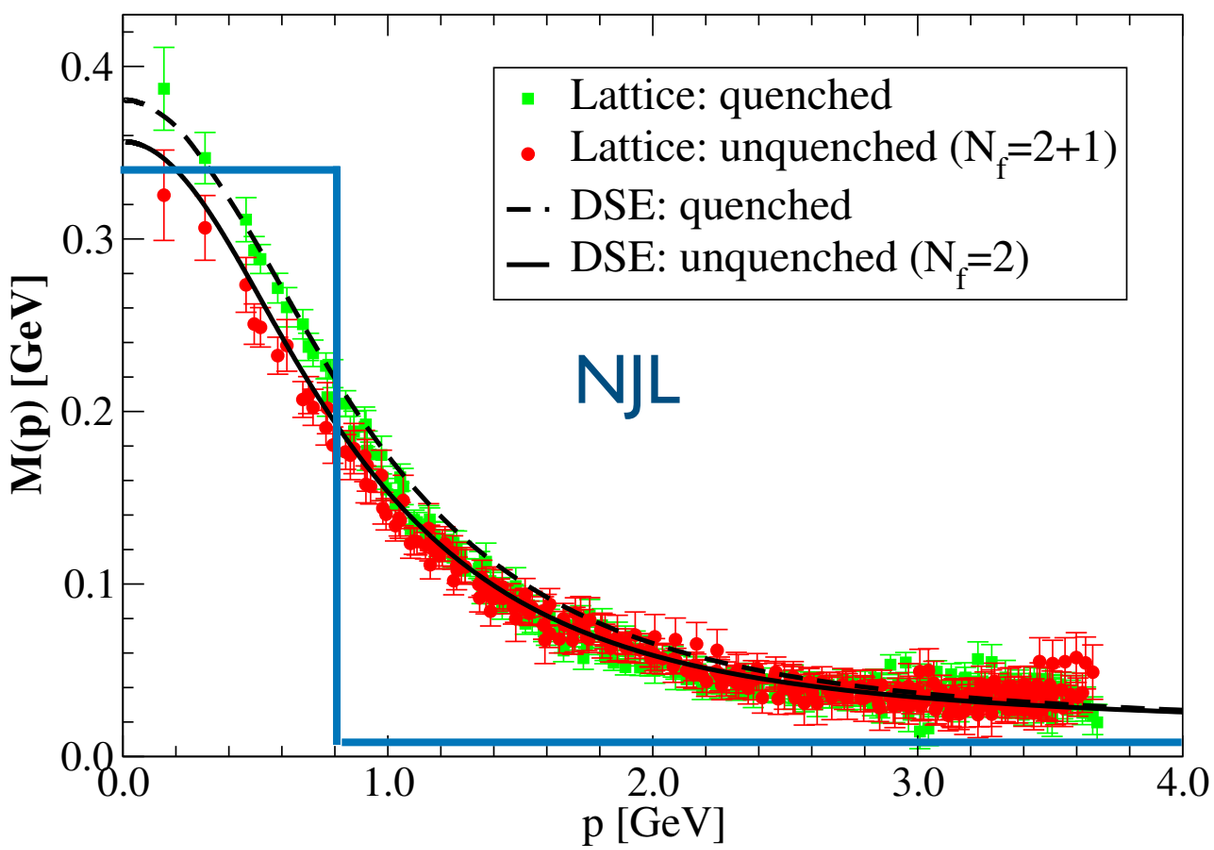
DSE-approach: truncations

$$[S(p)]^{-1} = [-i\not{p} + M(p^2)]/Z_f(p^2)$$

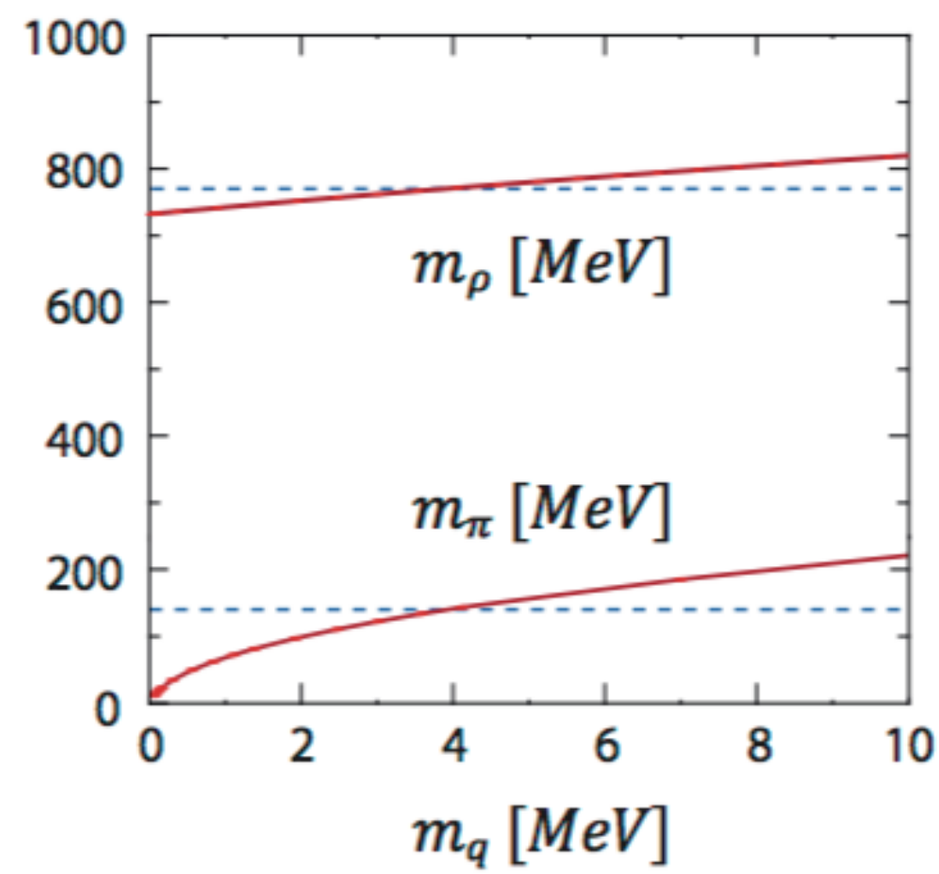
Rainbow-Ladder (RL):



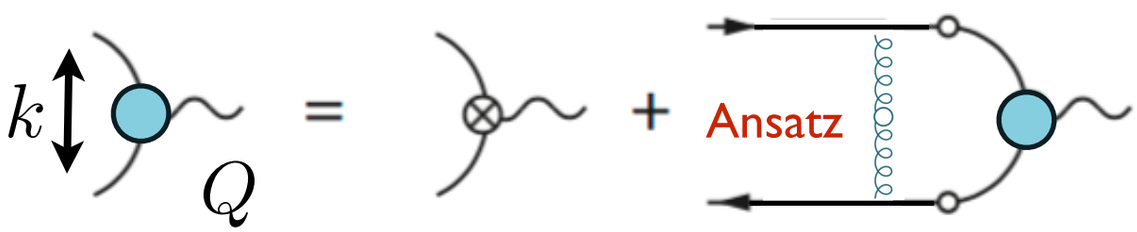




CF, Nickel, Williams, EPJ C 60 (2009) 47



Quark-photon vertex and dynamical vector mesons



Basis:

$$\{\gamma^\mu, Q^\mu, k^\mu\} \otimes \{\mathbb{1}, \not{Q}, \not{k}, \not{Q}\not{k}\}$$

$\longrightarrow$ 12 elements

$$\Gamma^\mu(k,Q) = \sum_{i=1,4} \lambda_i L_i^\mu(k,Q) + \sum_{i=i,8} \tau_i T_i^\mu(k,Q)$$

gauge part — transverse part

→ vector-mesons

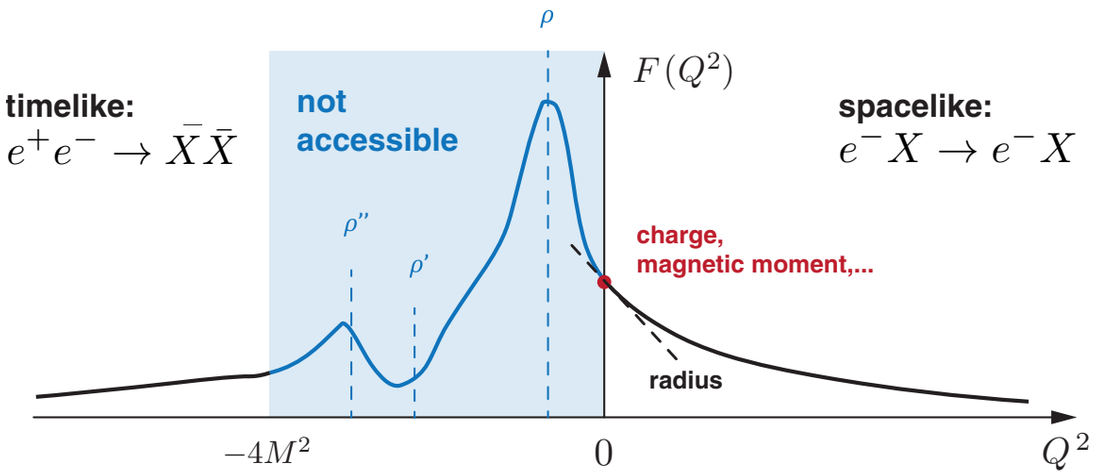
Ball and Chiu, PRD 22 (1980) 2542.

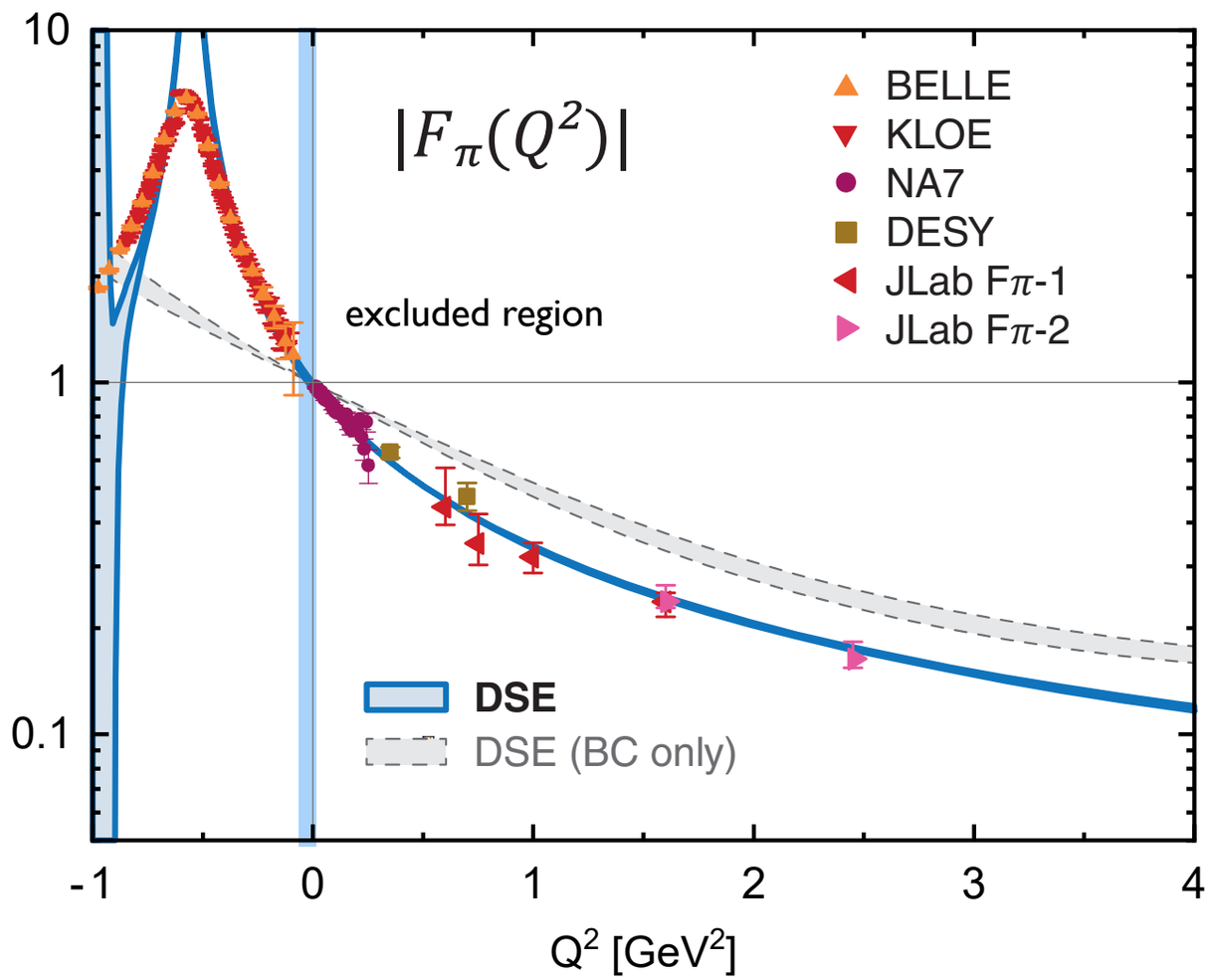
WTI: $Q^\mu \Gamma^\mu(k,Q) = S^{-1}(k+Q/2) - S^{-1}(k-Q/2)$

EM gauge invariance satisfied ✓

Vector mesons: dynamically generated ✓

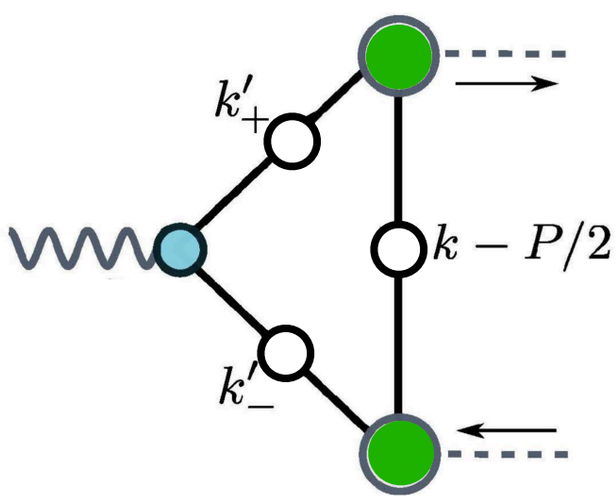
Pion form factor





Eichmann, CF, Weil and Williams, PLB 797 (2019), 134855 [arXiv:1903.10844]

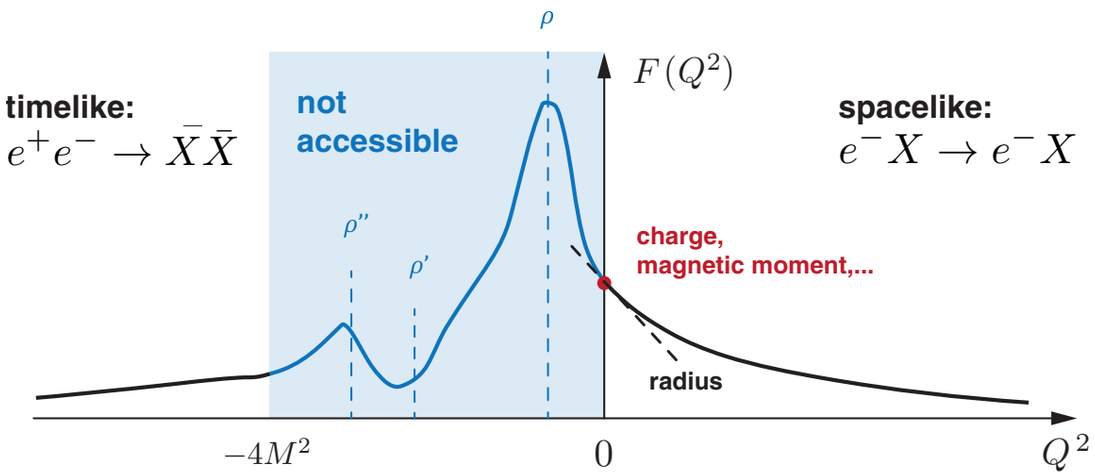
Pion form factor:

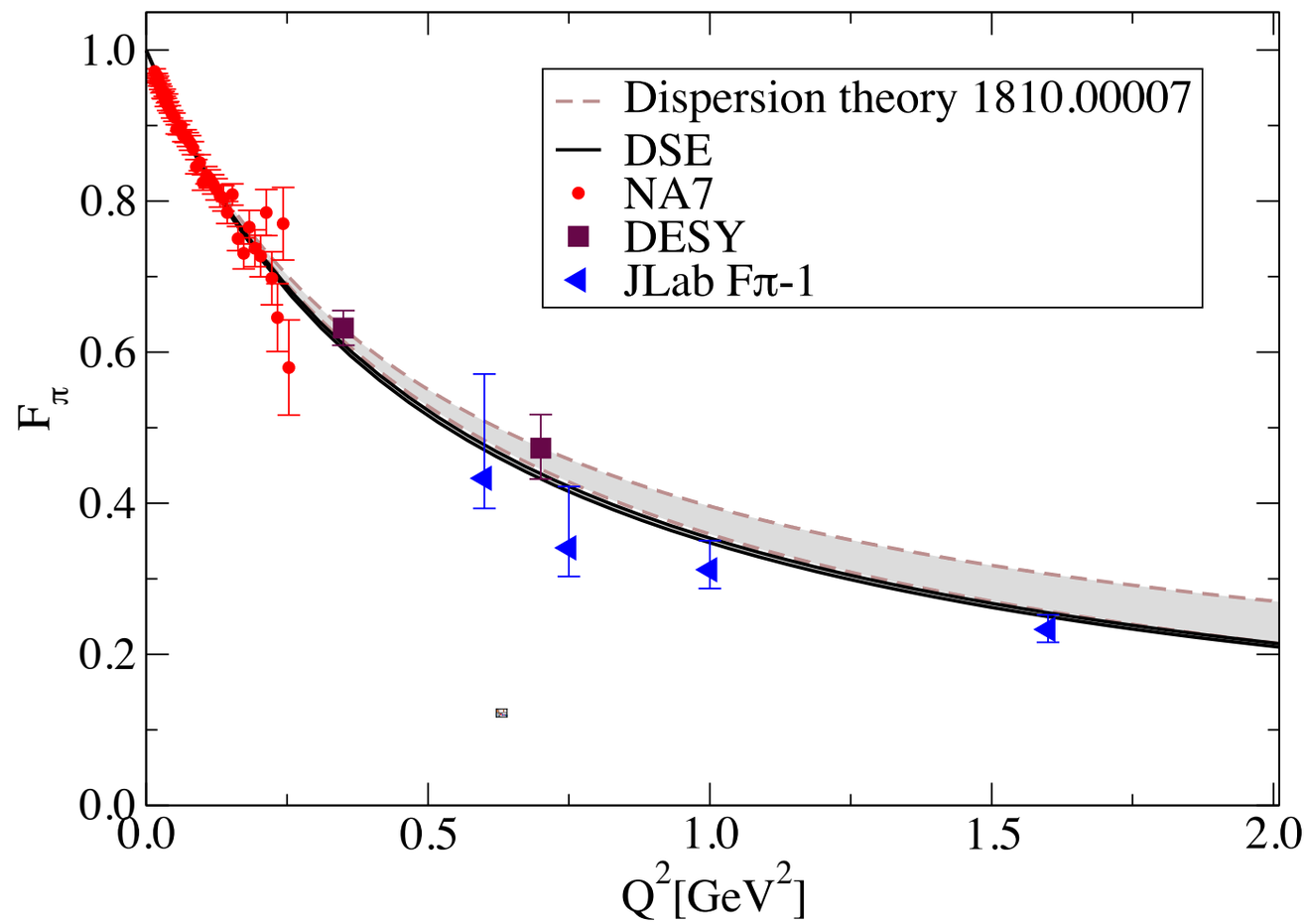


Including $\rho \to \pi\pi$ (BRL !) shifts ρ-pole onto 2nd Riemann sheet but hardly changes space-like quark-photon vertex

Williams, PLB in press, arXiv:1804.11161
Miramontes and Sanchis-Alepuz, EPJA in press, arXiv:1906.06227

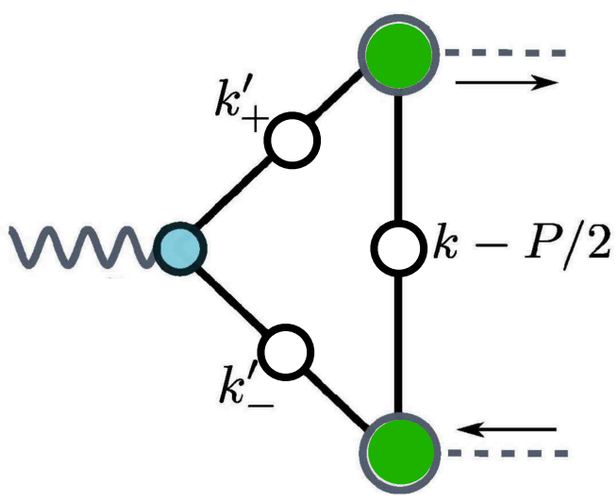
Pion form factor





Eichmann, CF, Weil and Williams, PLB 797 (2019), 134855 [arXiv:1903.10844]
Dispersive: Colangelo, Hoferichter and Stoffer, JHEP 1902 (2019) 006

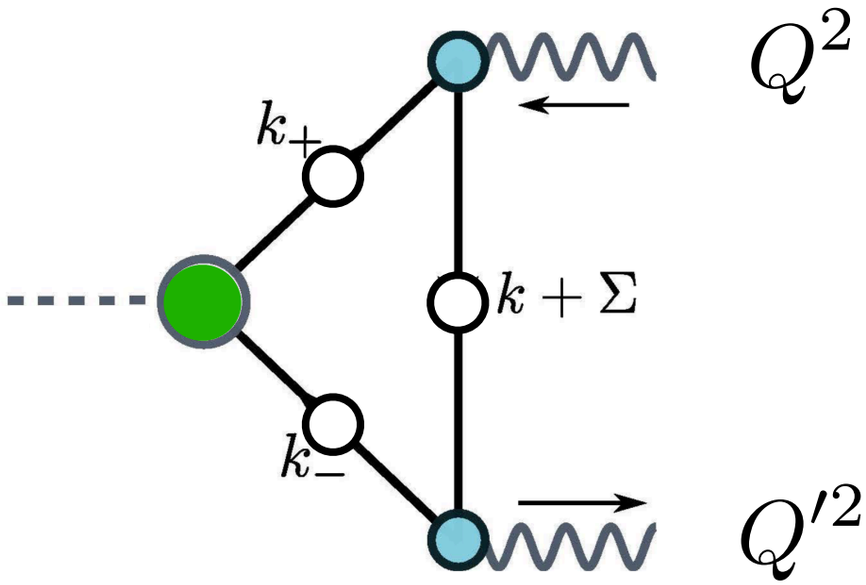
Pion form factor:



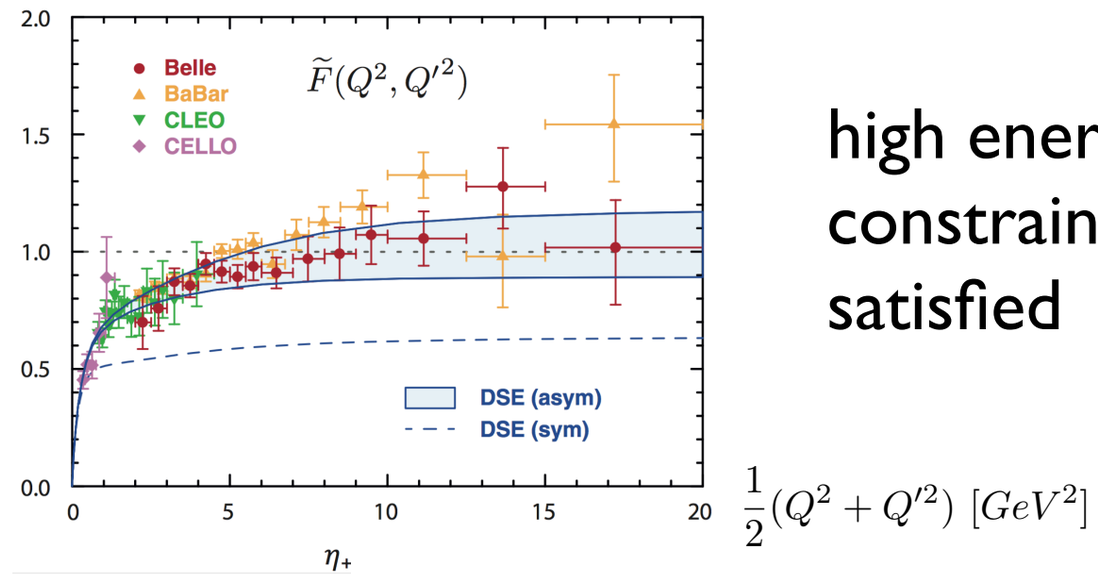
Including $\rho \to \pi\pi$ (BRL !) shifts ρ-pole onto 2nd Riemann sheet but hardly changes space-like quark-photon vertex

Williams, PLB in press, arXiv:1804.11161
Miramontes and Sanchis-Alepuz, EPJA in press, arXiv:1906.06227

Pion transition form factor

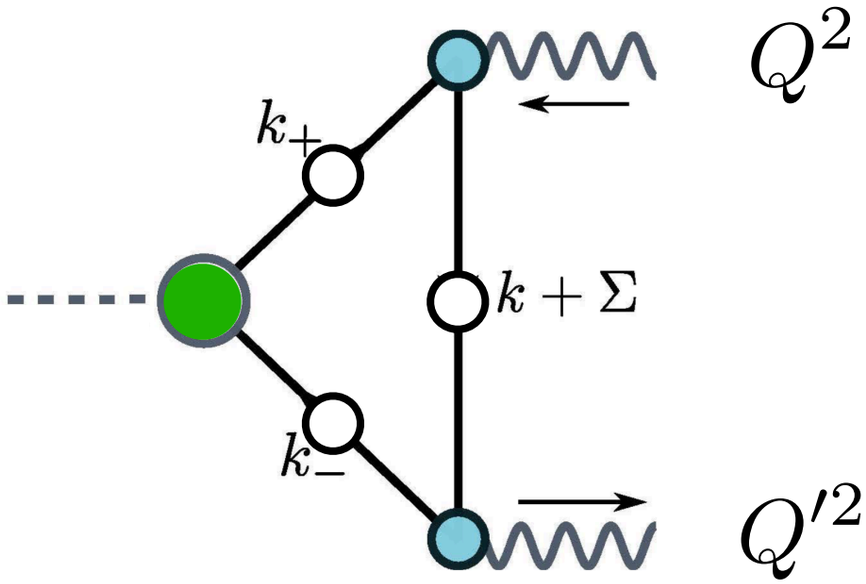


Eichmann, CF, Weil and Williams, PLB 774 (2017) 425

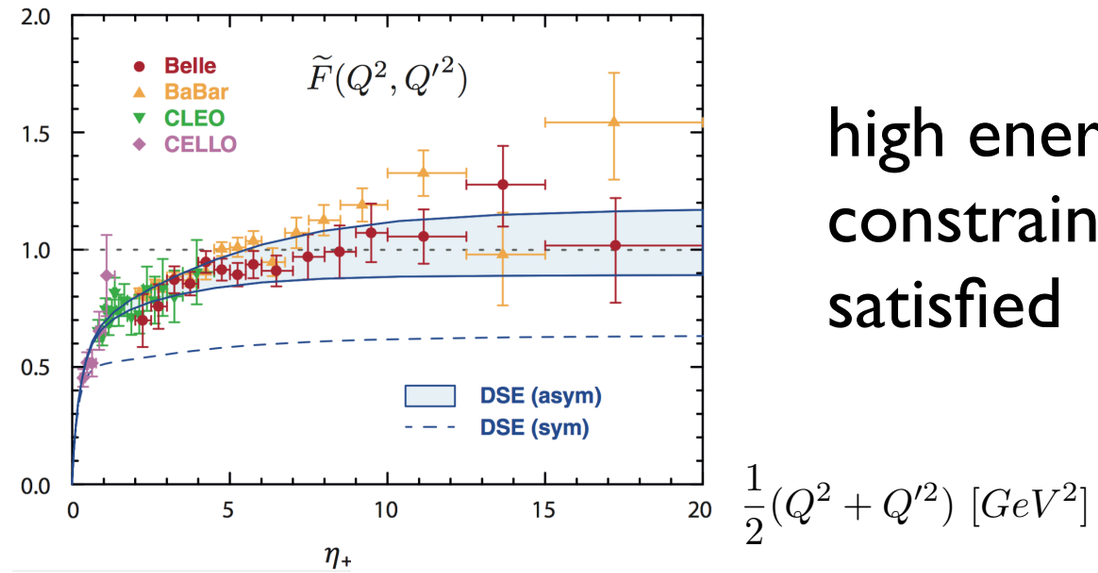


high energy constraints satisfied

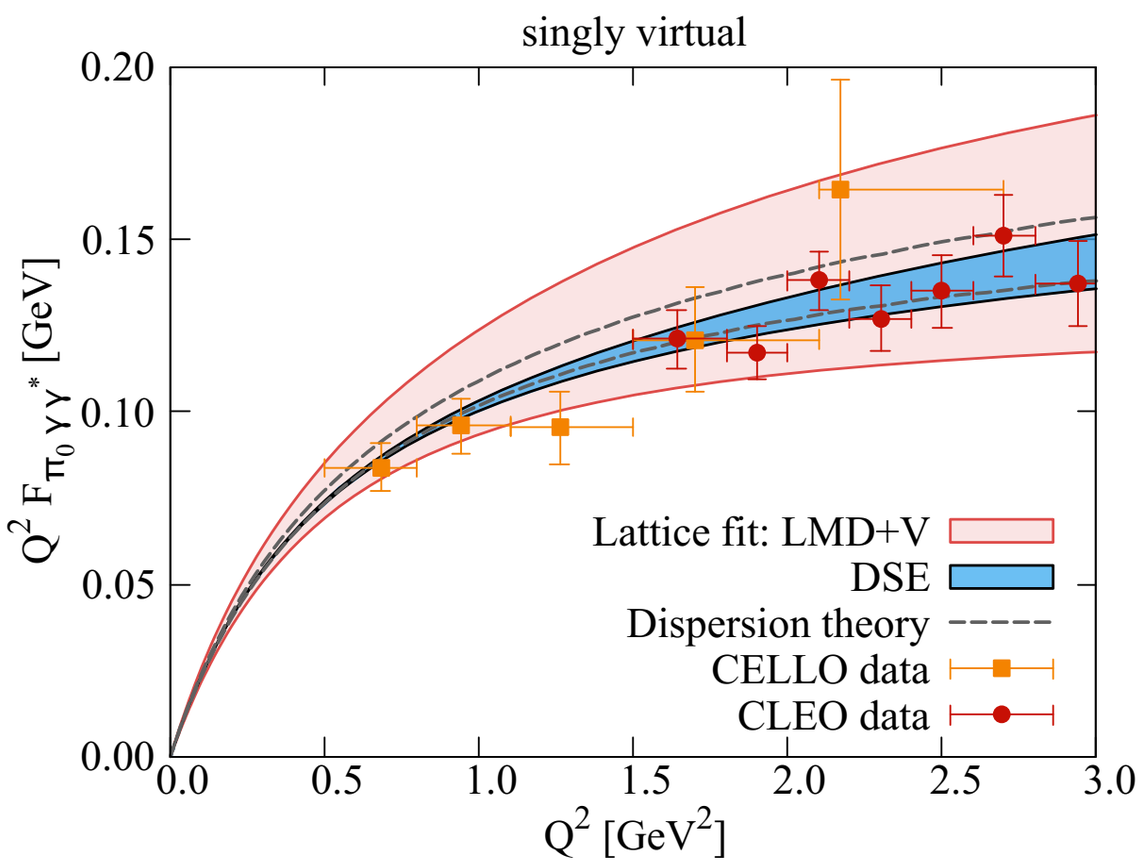
Pion transition form factor



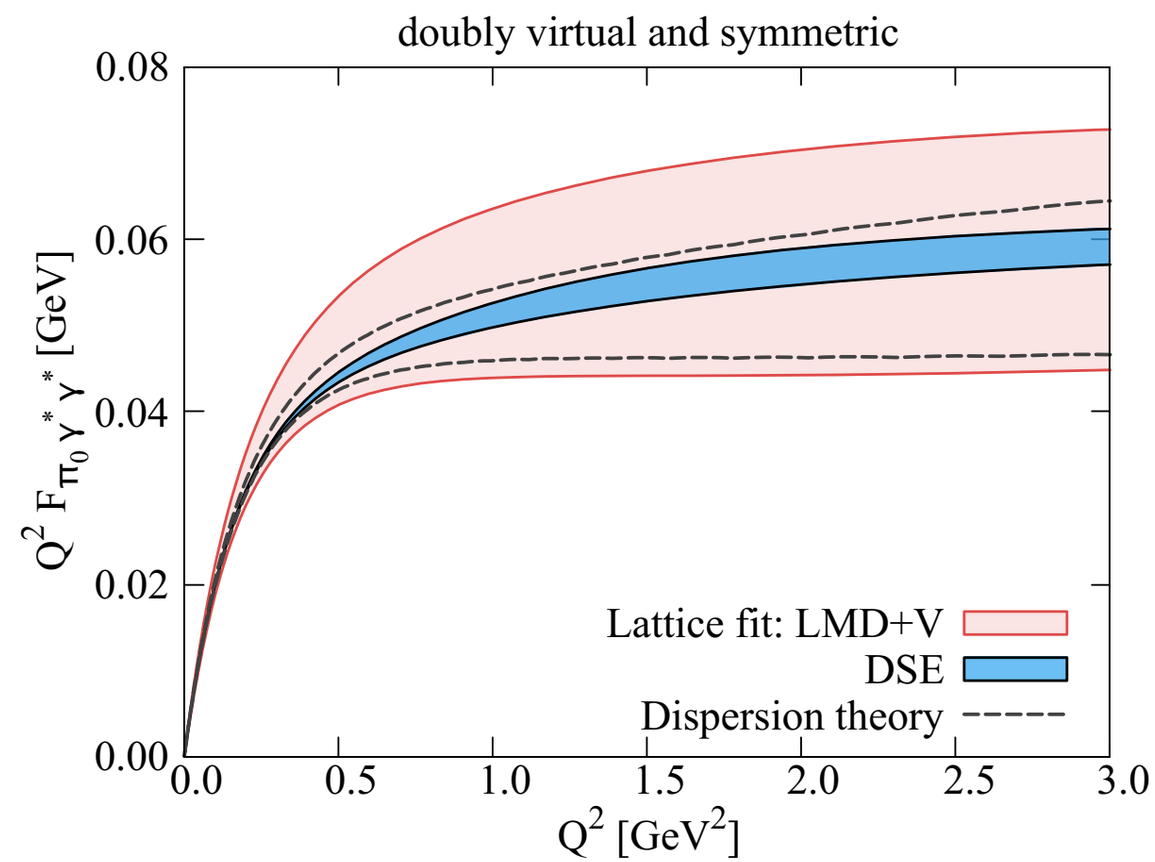
Eichmann, CF, Weil and Williams, PLB 774 (2017) 425



high energy constraints satisfied

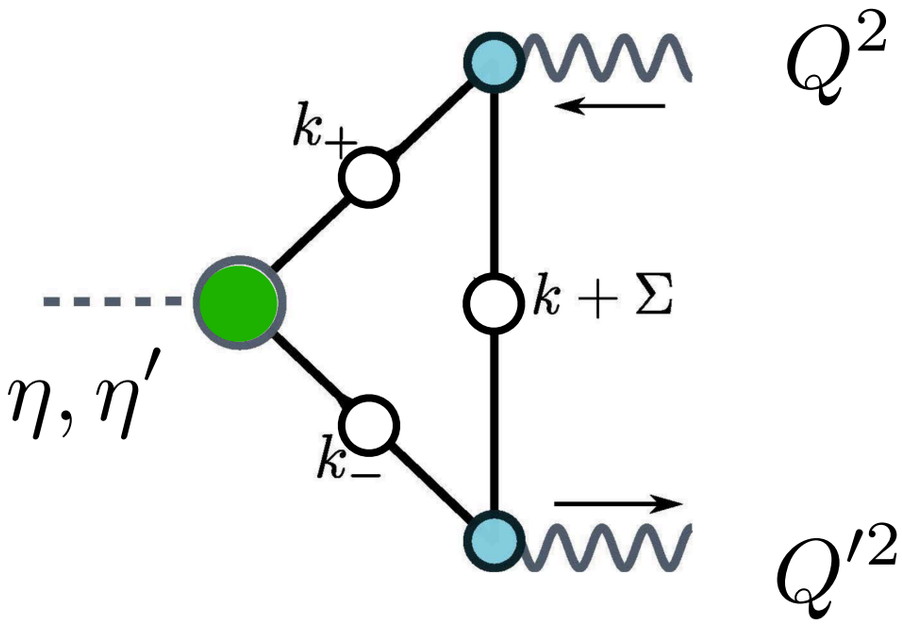


Eichmann, CF, Weil and Williams, PLB 797 (2019), 134855 [arXiv:1903.10844]



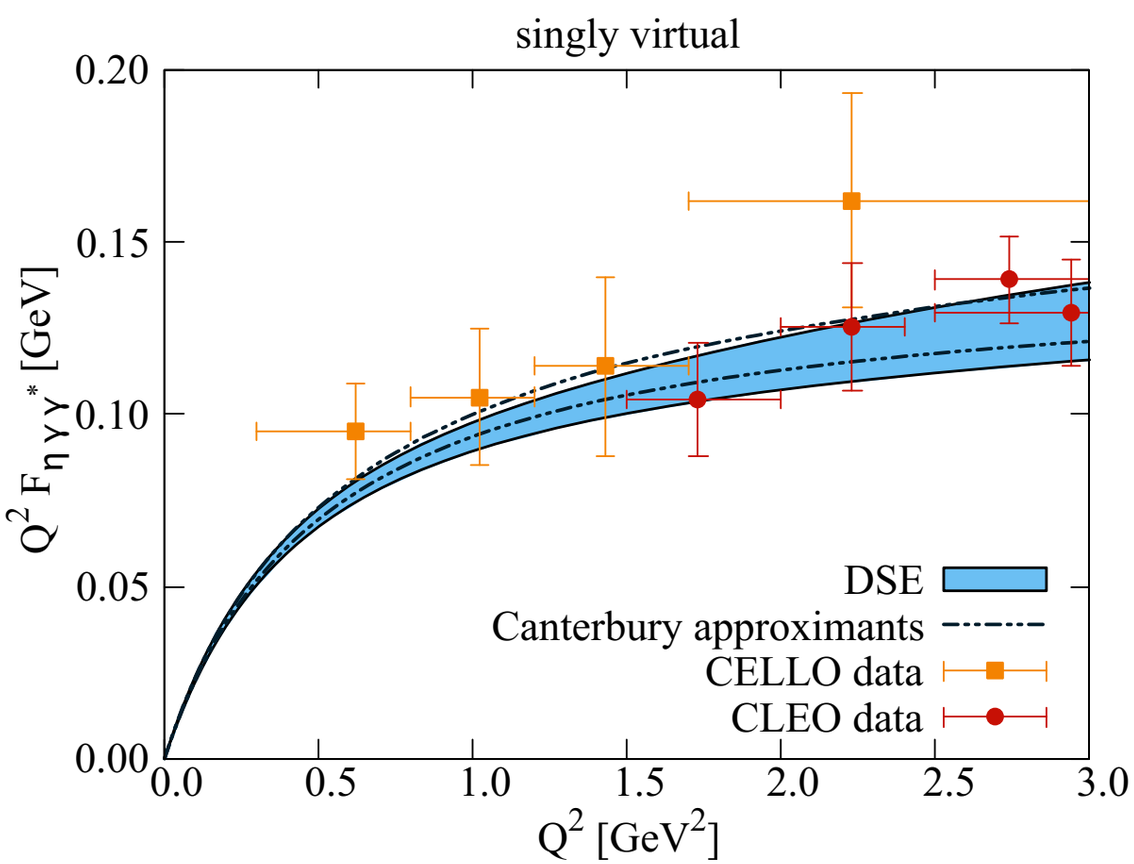
Lattice: Gerardin, Meyer, and Nyffeler, PRD94, 074507 (2016)
Dispersive: Hoferichter, Hoid, Kubis, Leupold, and Schneider, JHEP 10, 141 (2018)

eta/eta' transition form factor

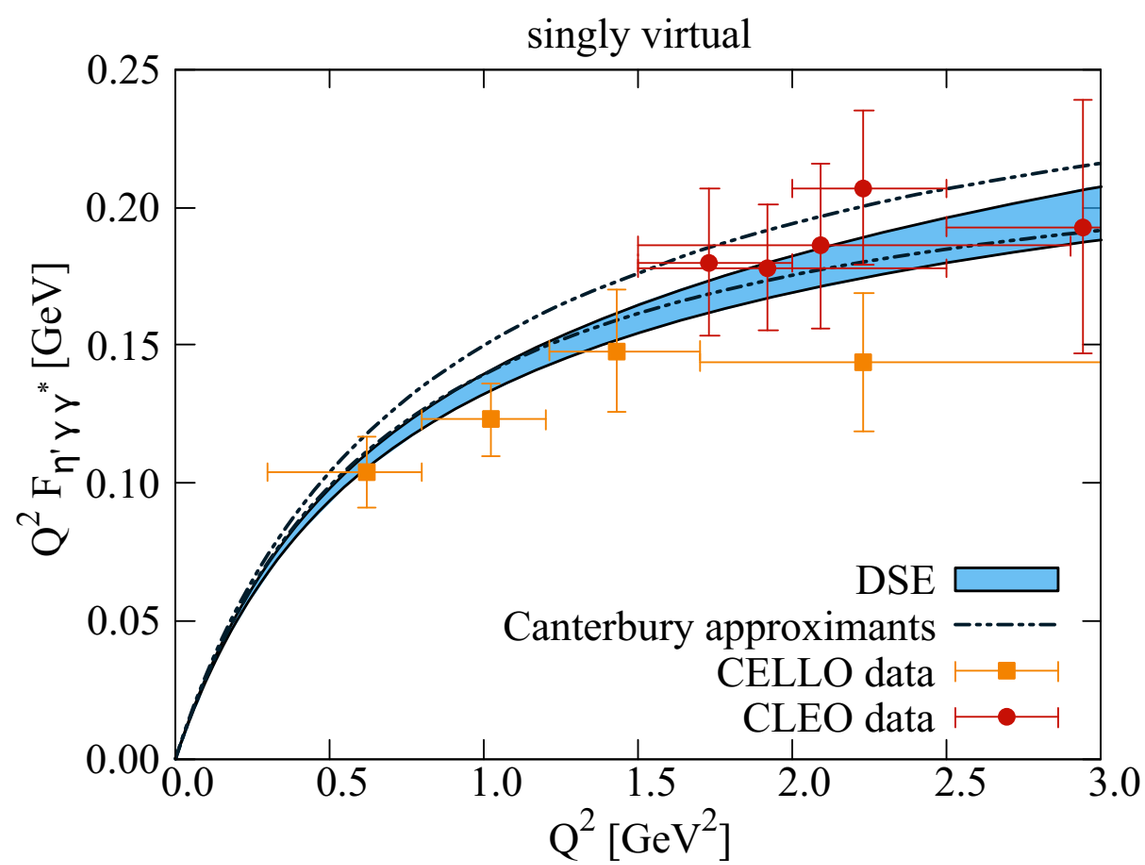


Two angle mixing scheme in quark flavour basis

Schechter, Subbaraman, and Weigel, PRD48, 339 (1993)
Feldmann, Kroll, and Stech, PLB 449,339 (1999)



Eichmann, CF, Weil and Williams, PLB 797 (2019), 134855 [arXiv:1903.10844]



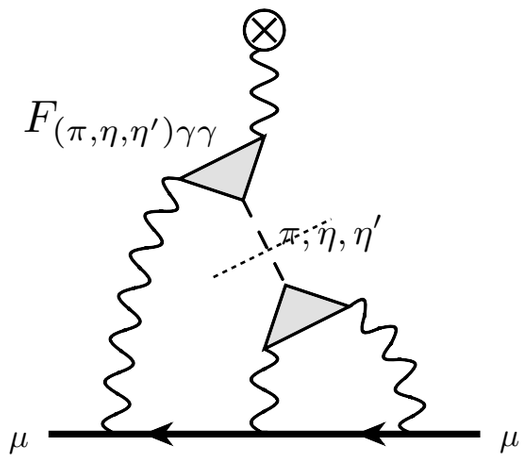
Canterbury: Masjuan and Sanchez-Puertas, PRD95, 054026 (2017)

Scalar and axial vector transition form factors

Scalar and axial vector transition form factors

in progress…

HLBL - results



off-shell; not unambiguous:

$$(a_\mu^\pi)_{DSE} = 57.5(0.6) \times 10^{-11}$$

$$(a_\mu^{\pi,\eta,\eta'})_{DSE} = 80.7(0.8) \times 10^{-11}$$

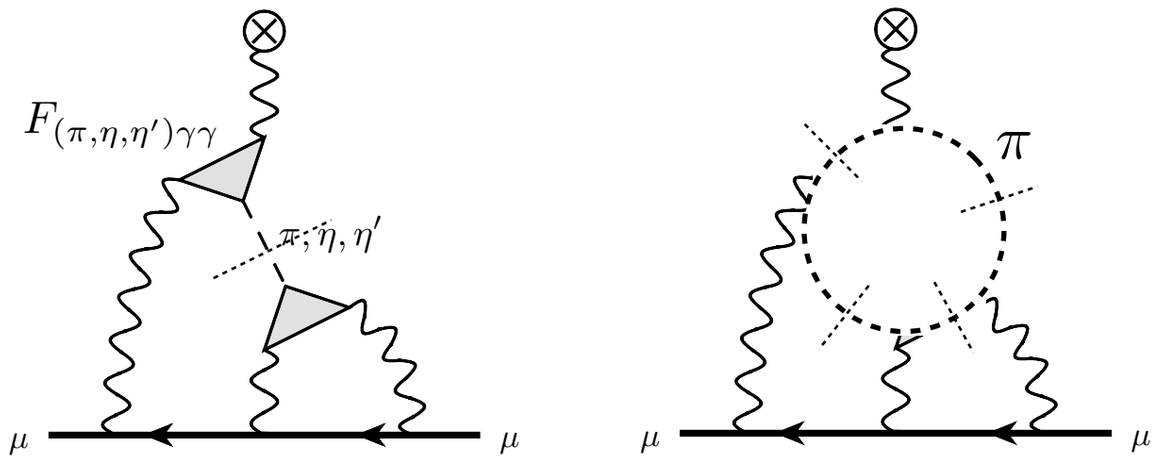
Goecke, CF, Williams, PRD 83 (2011) 094006

	DSE	Canterbury	Dispersion	Lattice +data
$a_\mu^{\pi-pole}$	$62.6\,(0.1)(1.3)$	$63.5\,(1.2)(2.3)$	$62.6^{+3.0}_{-2.5}$	$62.3\,(2.3)$
$a_\mu^{\eta-pole}$	$15.8\,(0.2)(0.3)(1.0)$	$16.2\,(0.9)(0.9)$		
$a_\mu^{\eta'-pole}$	$13.3\,(0.4)(0.3)(0.6)$	$14.5\,(0.7)(1.7)$		
	systematic numerical mixing angle			
$a_\mu^{PS-pole}$	$91.6(1.9)$	$94.3(5.3)$		

Masjuan and Sanchez-Puertas, PRD95, 054026 (2017) Gérardin, Meyer and Nyffeler, PRD 100 (2019) no.3, 034520

Eichmann, CF, Weil and Williams, PLB 797 (2019), 134855 Hoferichter, Hoid, Kubis, Leupold, and Schneider, JHEP 10, 141 (2018)

HLBL - results



off-shell; not unambiguous:

$$(a_\mu^\pi)_{DSE} = 57.5(0.6) \times 10^{-11}$$

$$(a_\mu^{\pi,\eta,\eta'})_{DSE} = 80.7(0.8) \times 10^{-11}$$

Goecke, CF, Williams, PRD 83 (2011) 094006

	DSE	Canterbury	Dispersion	Lattice +data
$a_\mu^{\pi-pole}$	$62.6\,(0.1)(1.3)$	$63.5\,(1.2)(2.3)$	$62.6^{+3.0}_{-2.5}$	$62.3\,(2.3)$
$a_\mu^{\eta-pole}$	$15.8\,(0.2)(0.3)(1.0)$	$16.2\,(0.9)(0.9)$		
$a_\mu^{\eta'-pole}$	$13.3\,(0.4)(0.3)(0.6)$	$14.5\,(0.7)(1.7)$		
	systematic numerical mixing angle			
$a_\mu^{PS-pole}$	$91.6(1.9)$	$94.3(5.3)$		
$a_\mu^{\pi-box}$	$-16.3(2)(4)$		$-15.9(2)$	

Masjuan and Sanchez-Puertas, PRD95, 054026 (2017)

Gérardin, Meyer and Nyffeler, PRD 100 (2019) no.3, 034520

Eichmann, CF, Weil and Williams, PLB 797 (2019), 134855

Hoferichter, Hoid, Kubis, Leupold, and Schneider, JHEP 10, 141 (2018)

Colangelo, Hoferichter, Procura, and Stoffer, JHEP 04, 161 (2017)

Summary

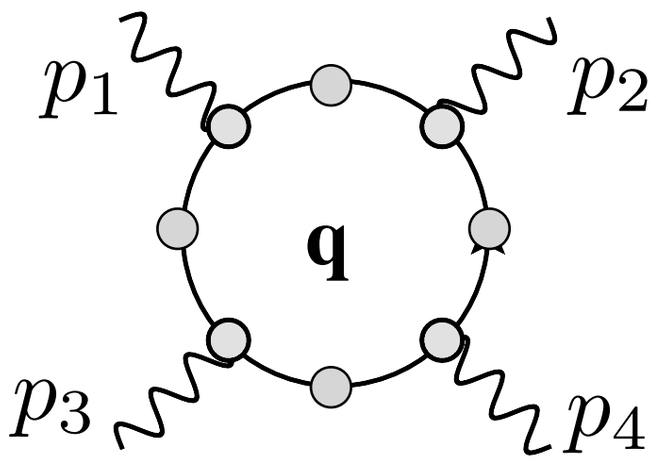
- (Euclidean) QCD based approach
  - renormalizable
  - full momentum dependence
  - multiscale problems feasible
- (Transition) form factors
  - calc. small and large Q,Q'
  - constraints satisfied

HVP	✓	(cross-check for us)
PS-pole	✓	
π-box	✓	
S-pole		(work in progress)
AV-pole		(work in progress)
K-box		interesting ?
quark-loop		on hold

676×10^{-10} no isospin breaking
Goecke, CF, Williams, PLB 704 (2011)

Backup

The quark loop



2013: $(10.7 \pm 0.3) \cdot 10^{-10}$

used only part of quark-photon vertex

$\longrightarrow$ calculation not complete !

$$\Gamma^\mu = \sum_{i=1,4} BC_i \, L_i^\mu + \sum_{i=1,8} F_i \, T_i^\mu$$

Goecke, CF, Williams, PRD 87 (2013) 034013

2018: Know how: Compton vertex

Eichmann, Ramalho, arXiv:1806.04579

Orthogonal basis: 136 elements - 41 transverse

Eichmann, CF, Heupel, PRD 92 (2015) no.5, 056006

$$\mathcal{M}^{\mu\nu\rho\sigma} = \sum_{i=1}^{41} a_i \, \tau_{\perp i}^{\mu\nu\rho\sigma} + \sum_{j=1}^{95} b_j \, \tau_j^{\mu\nu\rho\sigma}$$

$$\rightarrow \sum_{i=1}^{4} f_i \, X_i^{\mu\nu\rho\sigma} + \sum_{j=1}^{11} g_j \, K_j^{\mu\nu\rho\sigma}$$

with $f_i(S_0), g_i(S_0)$

$$S_0 = \frac{p_1^2 + p_2^2 + p_3^2 + p_4^2}{4}$$

when S4-Singlet S_0 only is considered

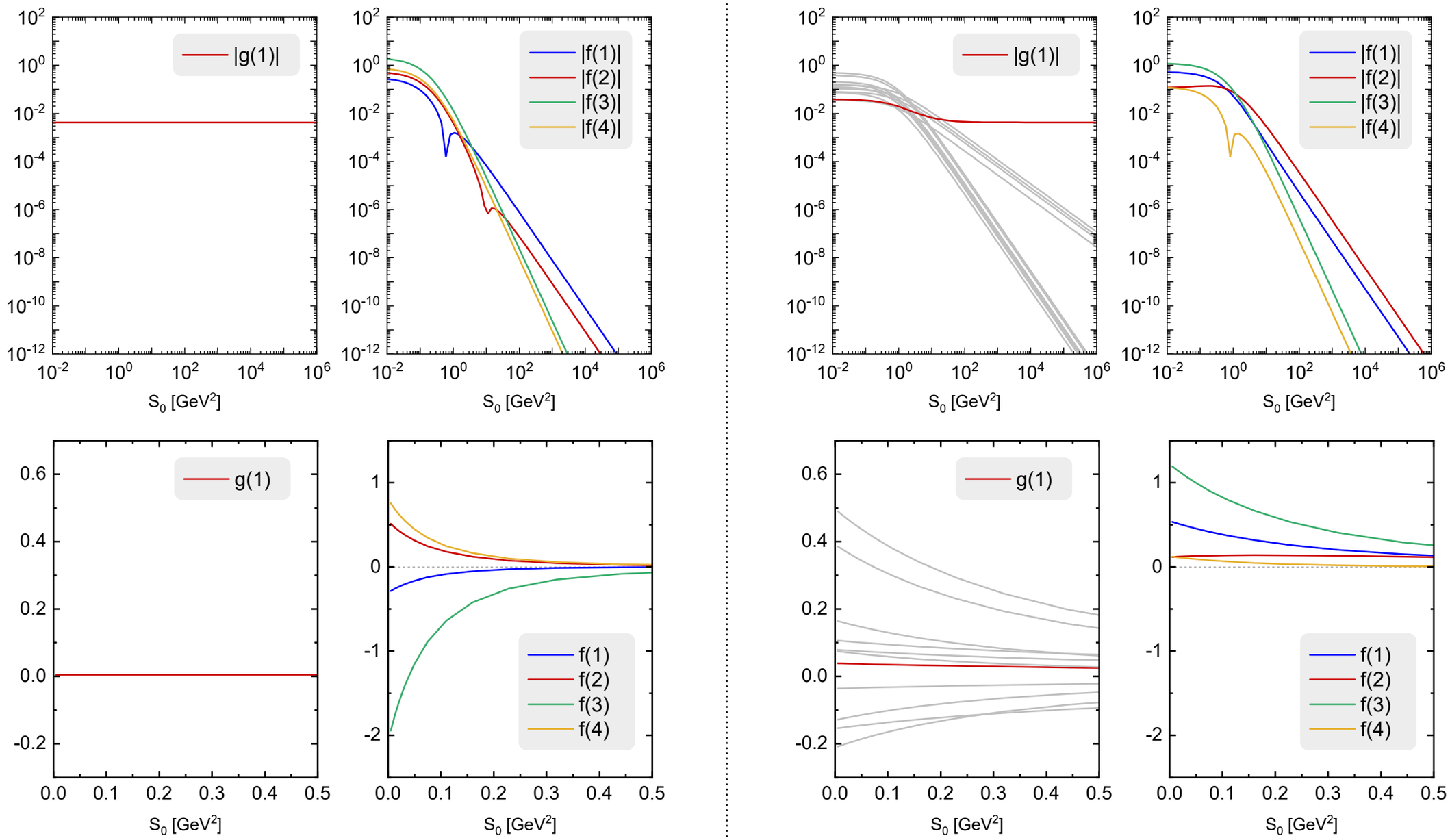
Results

NJL

DSE

log

lin

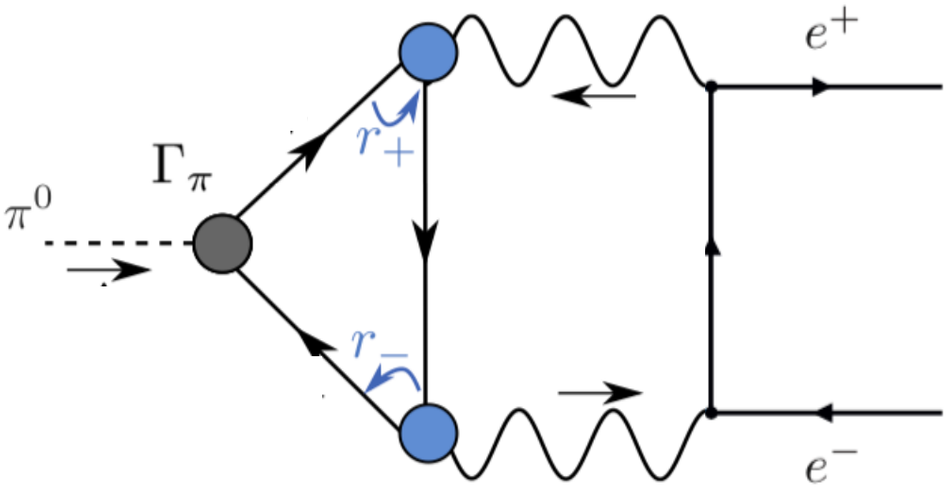


- agree with analytic result
- serve to check S_0-approx

- to do: use f(i) to calc. $(a_\mu^{quark-loop})_{DSE}$

Rare pion decay

Rare pion decay $\pi^0 \to e^+e^-$:



$$\frac{B(\pi_0 \to e^+e^-)}{B(\pi_0 \to \gamma\gamma)} = 2\left(\frac{m\alpha_{em}}{\pi m_\pi}\right)^2 \sqrt{1-4\frac{m^2}{m_\pi^2}}\ |A(-m_\pi^2/4)|^2$$

Usual: dispersive approach
DSE: direct calculation

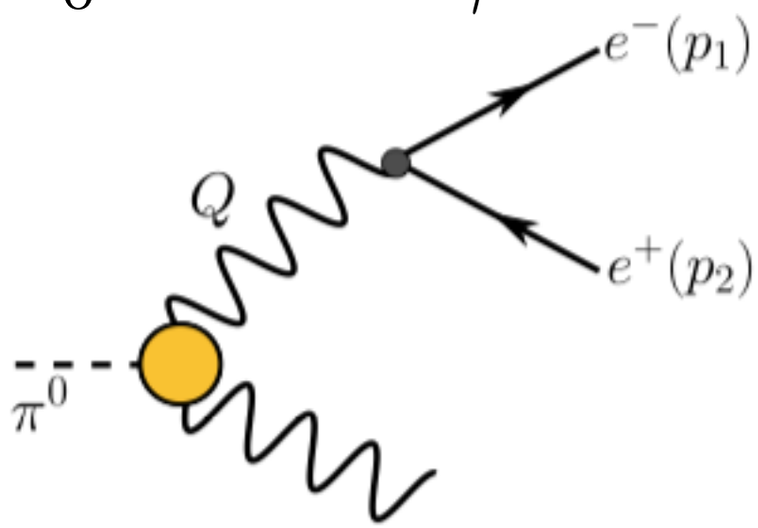
Collaboration	$B(\pi^0 \to e^+e^-)\,[10^{-8}]$
Experiment [1, 27, 28]	6.87(36)
Dorokhov et al. [2, 26]	6.23(9)
Husek et al. [35](THS)	6.14(8)
Masjuan et al. [29]	6.23(5)
Our result (DR)	6.21(3)
Our result (direct)	6.22(3)

Weil, Eichmann, CF and Williams, PRD96 (2017) no.1, 014021

- same result as everybody else
- discrepancy with exp. remains

More rare decays

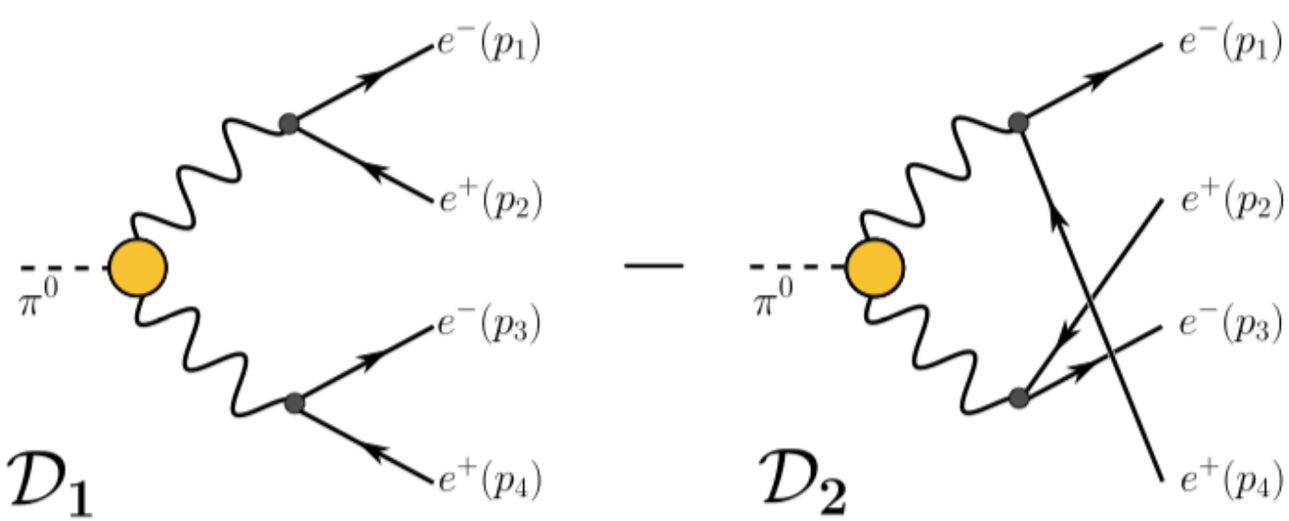
$$\pi_0 \to e^+e^-\gamma$$



Collaboration	$\Gamma_{\pi^0\to e^+e^-\gamma}$ [10^{-11} GeV]
PDG [21]	9.06(18)
Terschlüsen et al. [22]	9.26
Hoferichter et al. [23]	9.065
Our result	9.11(4)

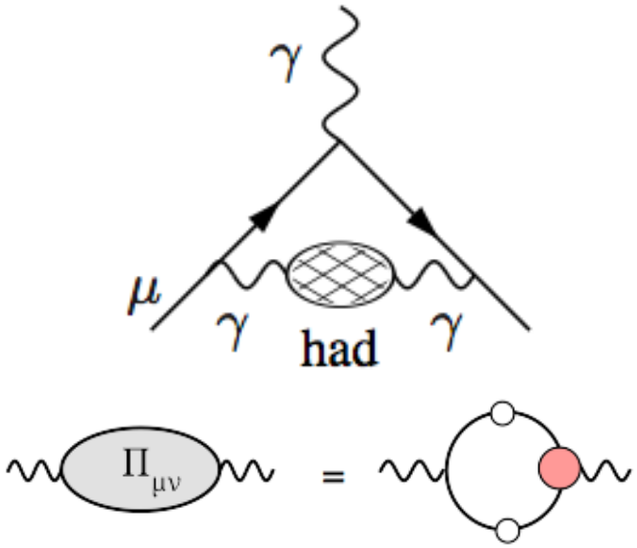
$$\pi_0 \to e^+e^-e^+e^-$$

Collaboration	$\Gamma_{\pi^0\to 2e^+2e^-}$ [10^{-13}GeV]
PDG [21]	2.58(12)
Terschlüsen et al. [22]	2.68
Escribano et al. [24]	2.62
Our result	2.63(1)

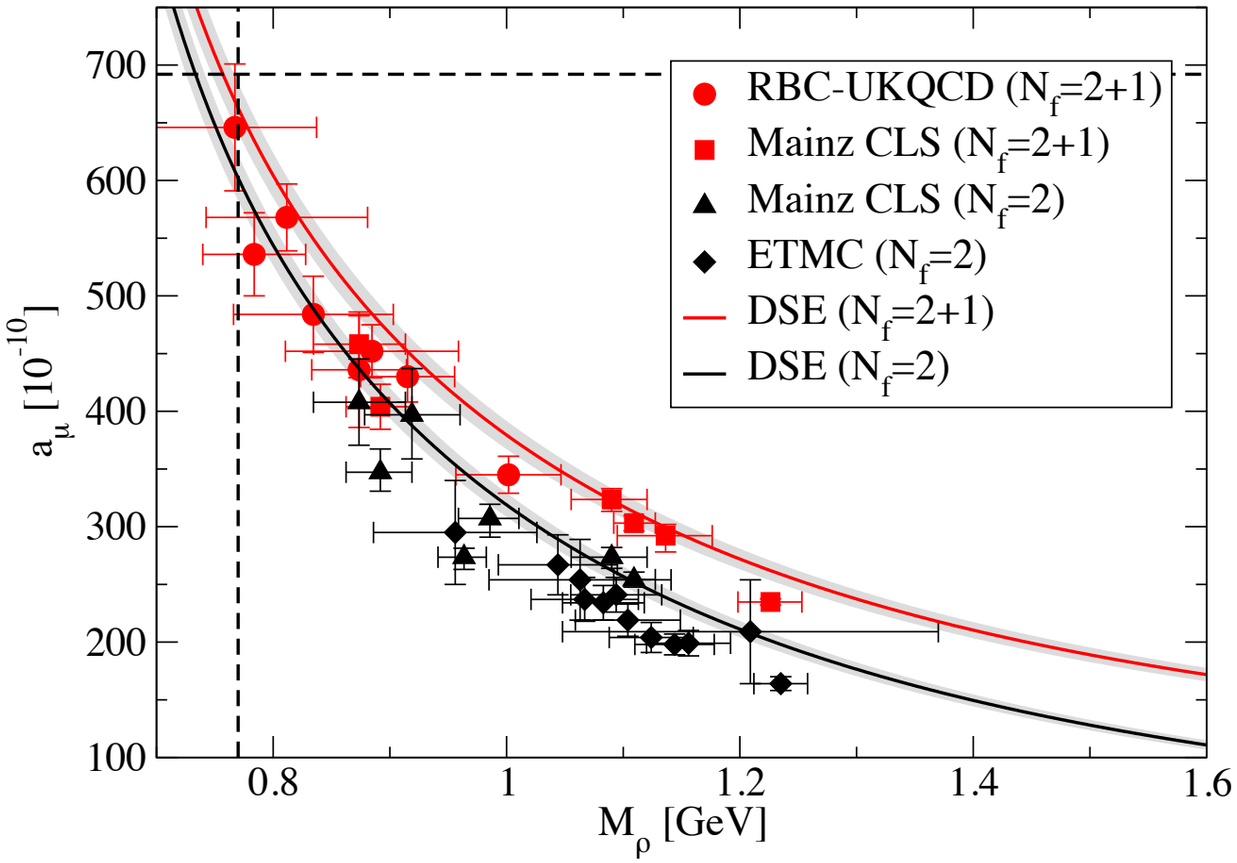


Weil, Eichmann, CF and Williams, Phys.Rev. D96 (2017) no.1, 014021

Results: Hadronic vacuum polarisation



10^{-10}	u/d	s	c	b
a_μ	600	60	15	1



DSE:

Goecke, CF, Williams, PLB 704 (2011)

$$a_\mu^{had.(1)} = (744.0 \pm 2) \cdot 10^{-10} \quad (m_\pi)$$

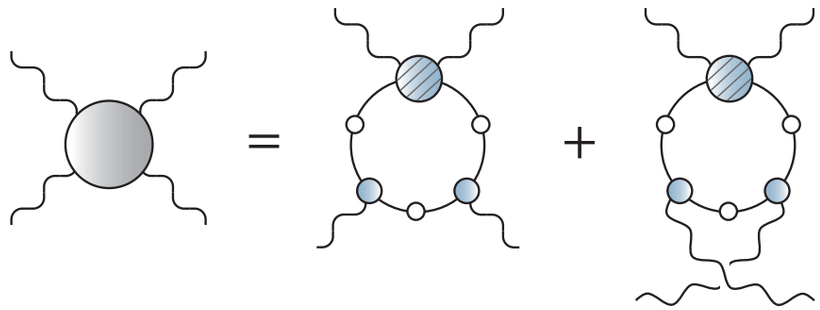
$$a_\mu^{had.(1)} = (676.0 \pm 2) \cdot 10^{-10} \quad (m_\rho)$$

Experiment:

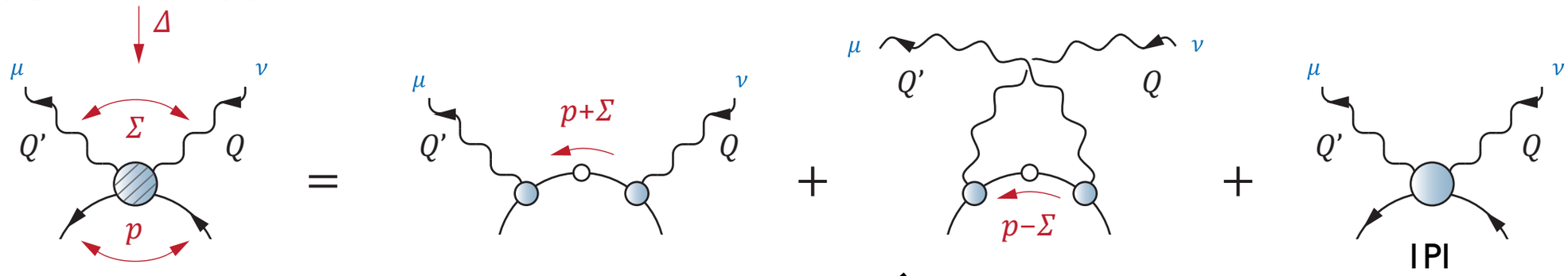
$$a_\mu^{had.(1)} = (692.1 \pm 5.6) \cdot 10^{-10}$$

Very reasonable agreement !

Quark-Compton-vertex



Compton-vertex:



Quark-Loop incl. "meson-exchange"

- basis constructed (128 elements)
- meson exchange contributions isolated and $F_{\pi\gamma\gamma}$ recovered

Eichmann and CF, PRD 87 (2013) 3, 036006

- applied to determine resonance contributions to nucleon Compton scattering

Eichmann, Ramalho, arXiv:1806.04579