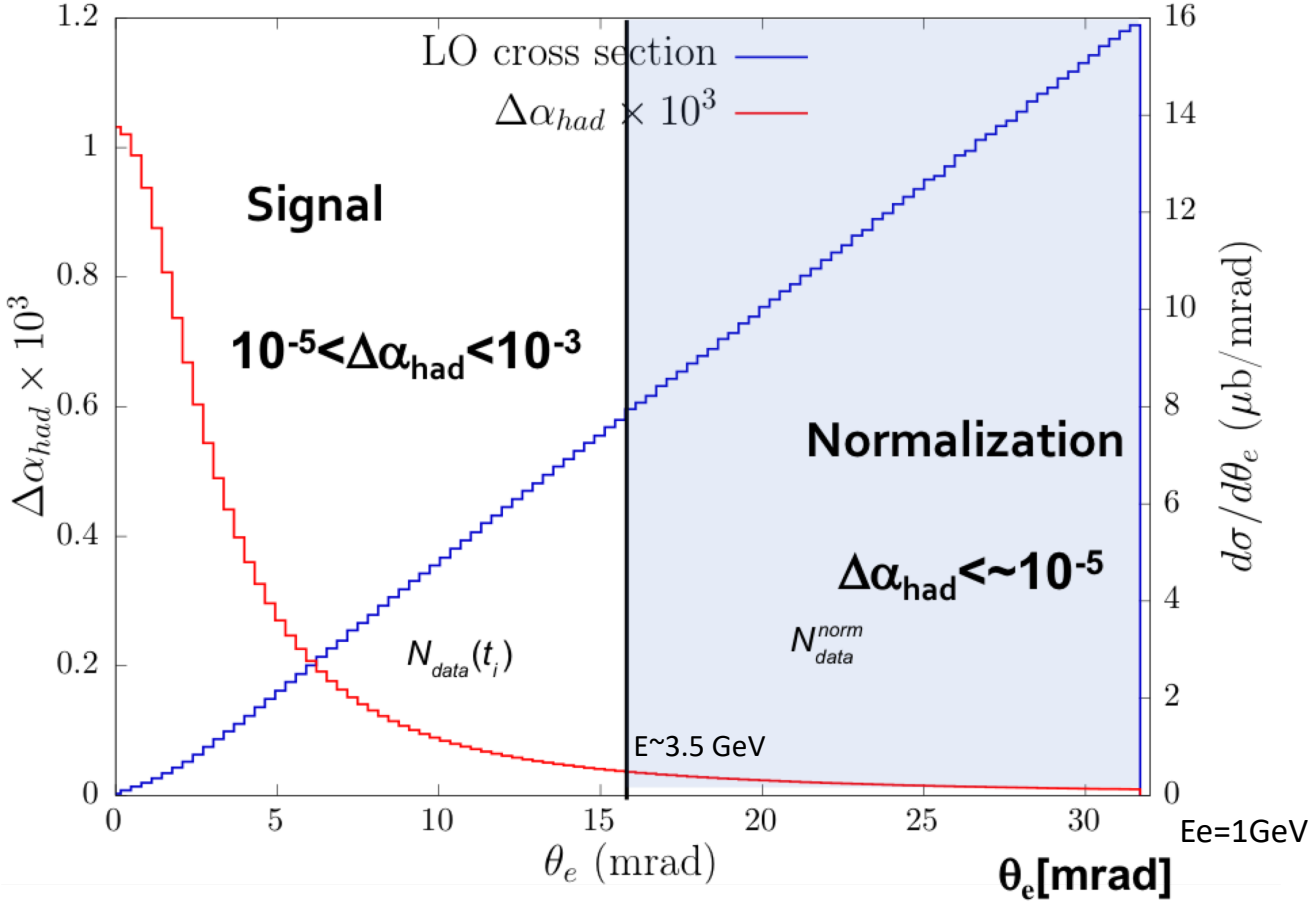
Radiative Corrections for MUonE

Matteo Fael

3rd Workshop of the Muon g-2 Theory Initiative

UNIVERSITÄT SIEGEN

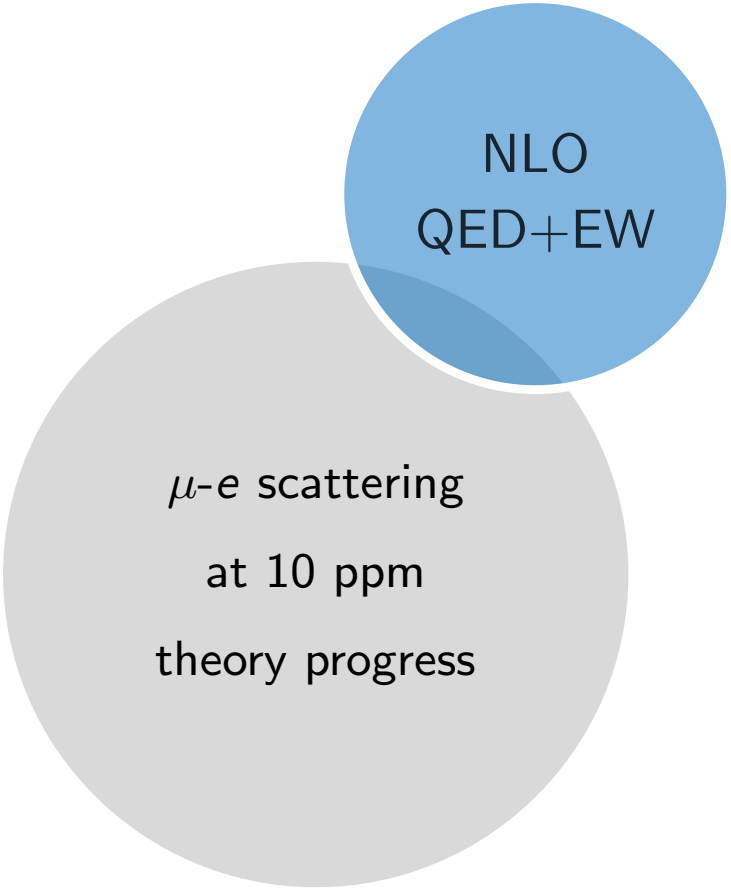


$$\left(1-\frac{\Delta\alpha^{\rm had}(t)}{1-\Delta\alpha^{\rm lep}(t)}\right)^{-2}=\frac{N_{\rm signal}(t_i)}{N_{\rm norm}}\times\frac{\sigma^{\rm MC}_{\rm norm}}{\sigma^{\rm MC}_0(t_i)}$$

μ-e scattering

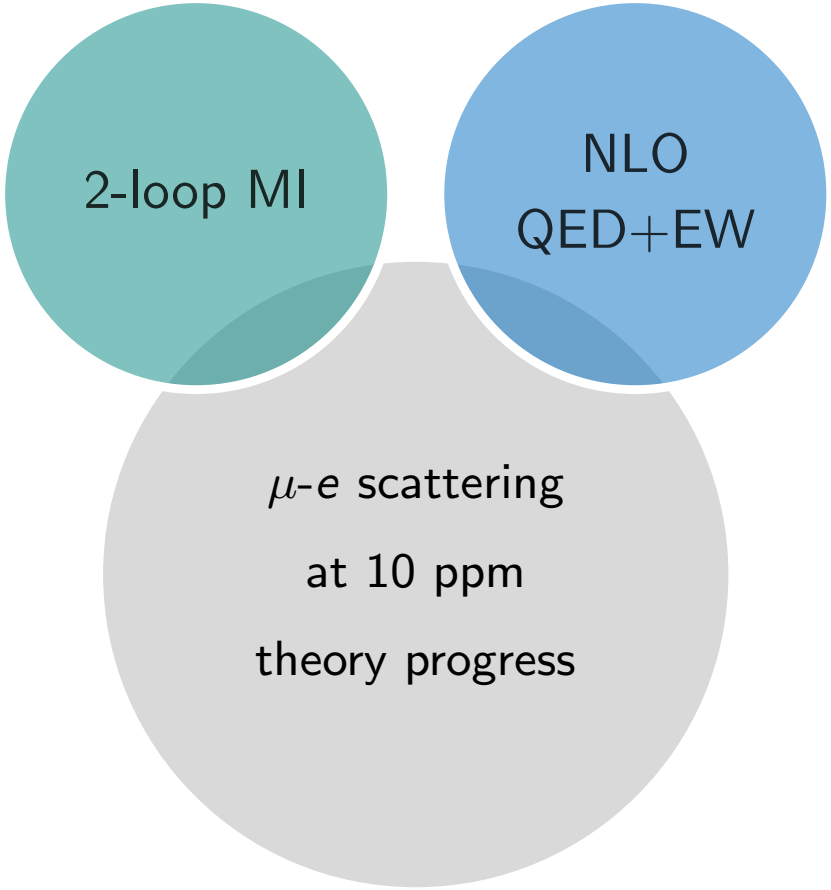
at 10 ppm

theory progress



M. Alacevich,
C. Carloni Calame,
M. Chiesa,
G. Montagna,
O. Nicrosini,
F. Piccinini.

JHEP 1902 (2019) 155

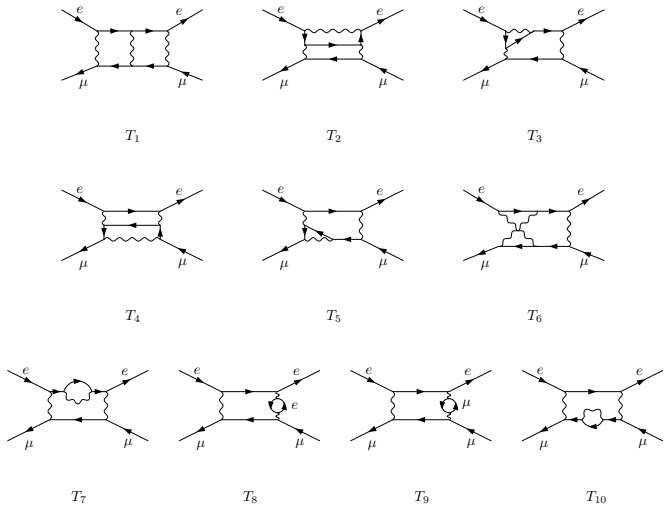


S. Di Vita,
S. Laporta,
P. Mastrolia,
M. Passera, A. Primo,
U. Schubert,
W.J. Torres
Bobadilla.

JHEP 1809 (2018) 016

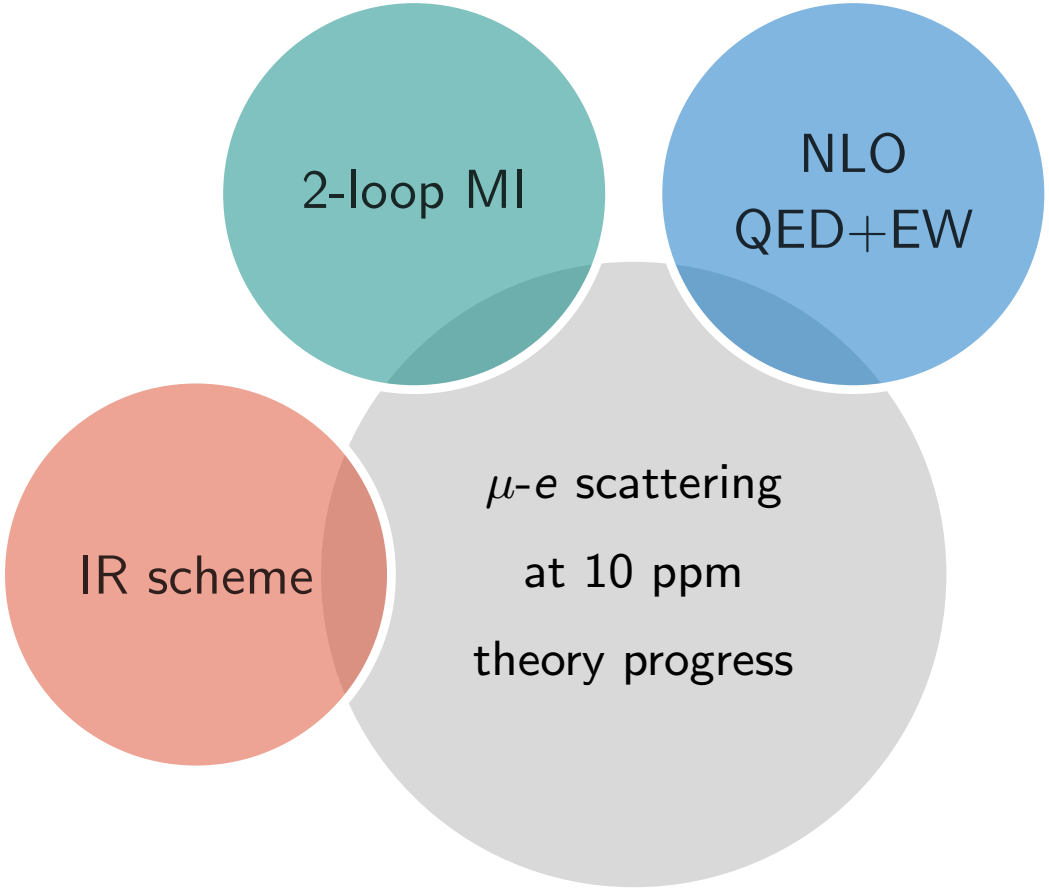
JHEP 1711 (2017) 198

QED two-loop master integrals



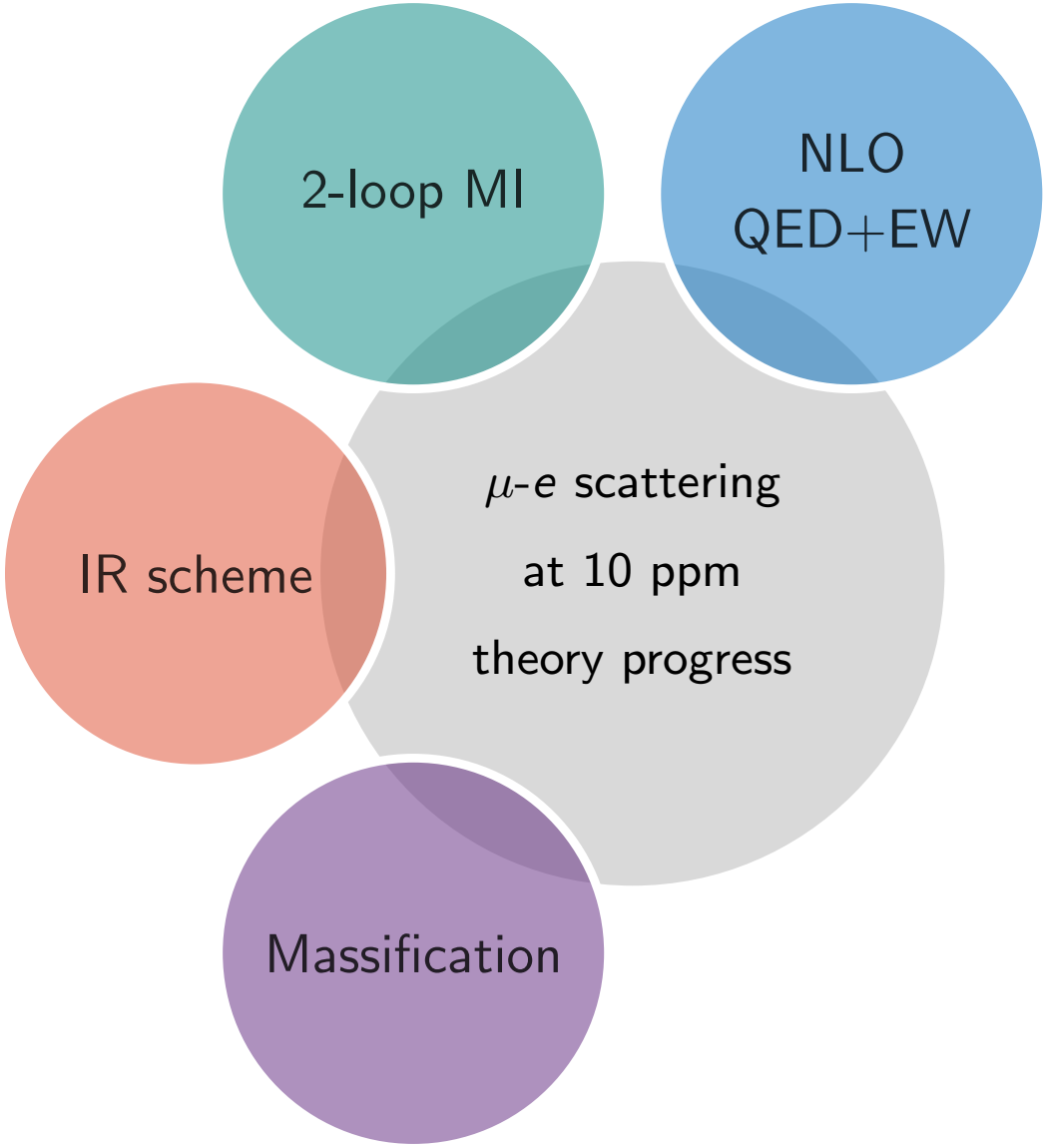
- Two-loop planar and non-planar master integrals for box diagrams computed.
- Full M_μ dependence, but electron taken massless.
- Method of differential equations and Magnus exponential series.
- Master integrals written in terms of generalized polylogarithms.
- Relevant also for $e^+e^- \to \ell^+\ell^-$ and $t\bar{t}$ production at hadron colliders.

non-planar: Di Vita, Laporta, Mastrolia, Primo, Schubert, JHEP 1809 (2018) 016;
planar: Mastrolia, Passera, Primo, Schubert, JHEP 1711 (2017) 198.



P. Banerjee T. Engel,
C. Gnendiger,
A. Signer, Y. Ulrich.

JHEP 1902 (2019) 118



P. Banerjee T. Engel,
C. Gnendiger,
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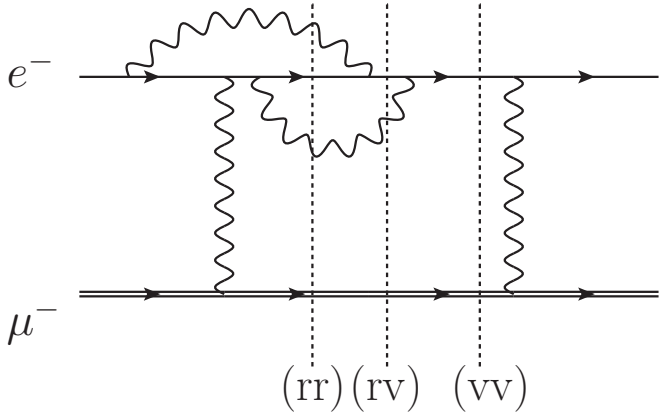
JHEP 1902 (2019) 118



$$d\sigma^{(2)} = \underbrace{d\sigma^{(\mathrm{vv})} + d\sigma^{(rv)} + d\sigma^{(rr)}}_{\text{separately IR divergent}}$$

- Collinear divergences are regulated by $\log(m_e)$
- double real emission ($d\sigma^{(rr)}$) and real-virtual ($d\sigma^{(rv)}$) easy to calculate with massive electron
- two-loop corrections ($d\sigma^{(vv)}$) close to impossible at present with $m_e \neq 0$
- FKS subtraction scheme extended to NNLO (FKS^2)

Banerjee, Engel, Gnendiger, Signer, Ulrich, in progress

massification /ˈmasɪˈfɪkeɪʃ(ə)n/

(noun)

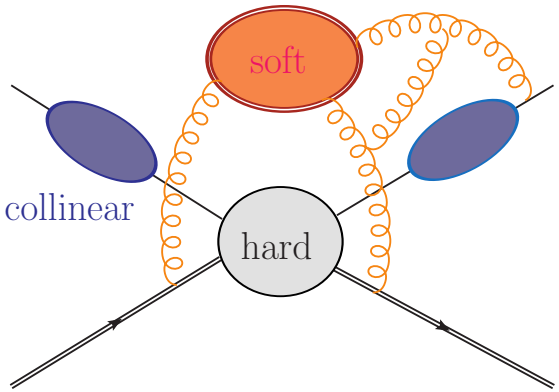
Methods to add leading mass effects to massless amplitudes in perturbation theory

Derivatives:

- **massify** verb
- **massified** adjective

Villigen English Dictionary (2019)

thanks to Y. Ulrich for the definition



- Previous works for Bhabha scattering:
  Penin, NPB 734 (2006) 185; Mitov, Moch, JHEP 05 (2007) 001;
  Becher, Melnikov, JHEP 06 (2007) 084.
- New features in μ-e scattering:
  $m_e \ll M_\mu \sim s$.

$$\mathcal{M}^{(n)}(s,t,M,m) = \prod_{i=1,2} \sqrt{Z_i(m)} \times \mathcal{S} \times \mathcal{M}^{(n)}(s,t,M,0)$$

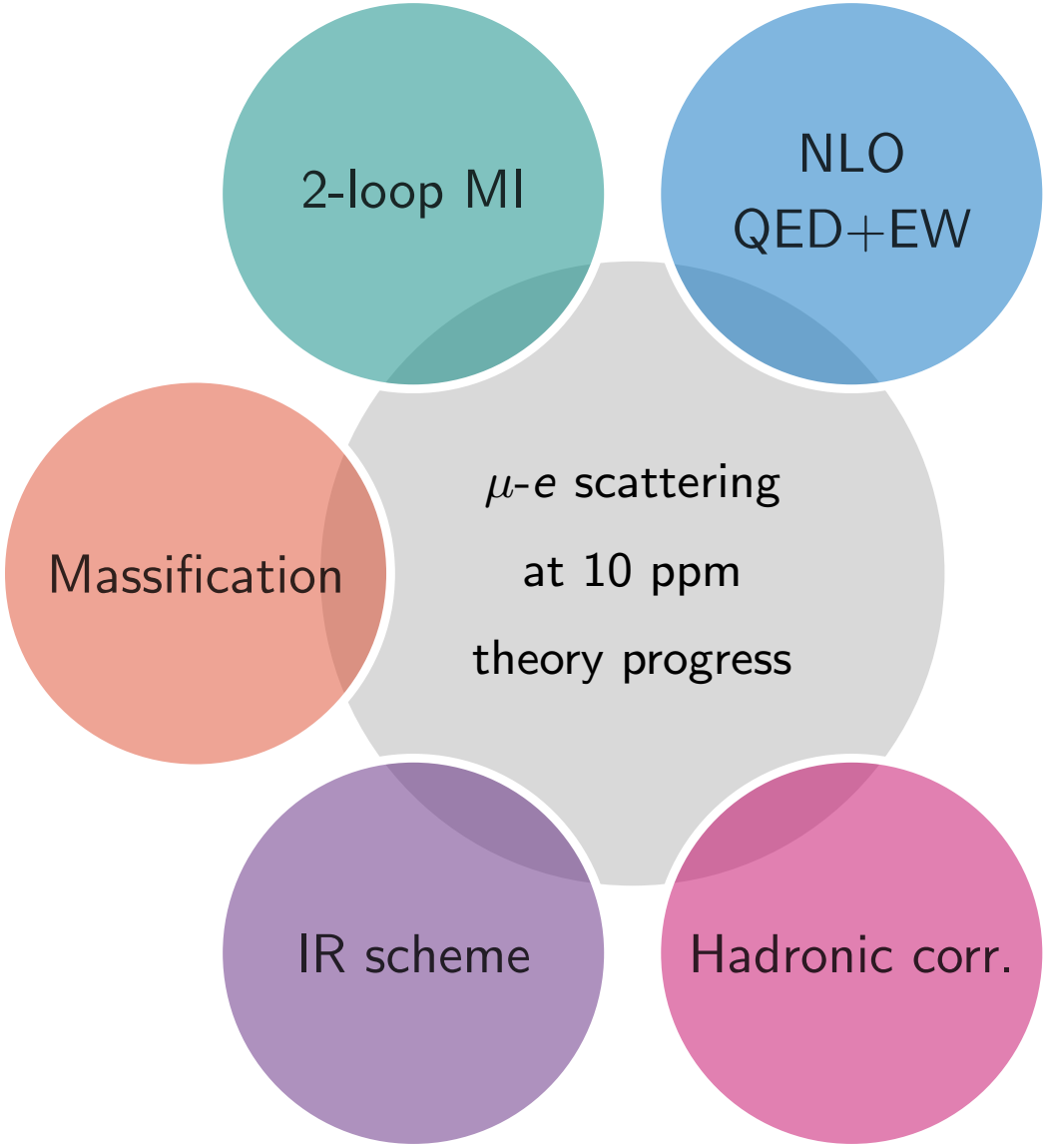
hard : $\mathcal{M}^{(n)}(s,t,M,0)$ massless amplitude.

soft : S, process dependent but 'easy'.

collinear : $\sqrt{Z_i(m)}$ process independent.

ultrasoft : cancel out!

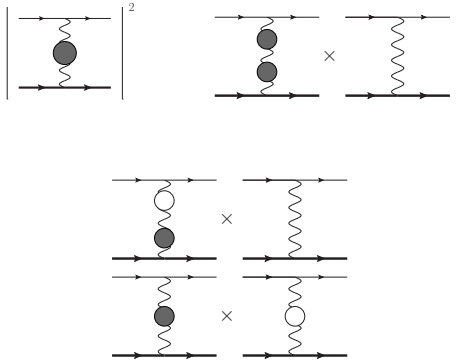
successfully applied to heavy-to-light form factors and muon decay
Engel, Gnendiger, Signer, Ulrich, JHEP 1902 (2019) 118



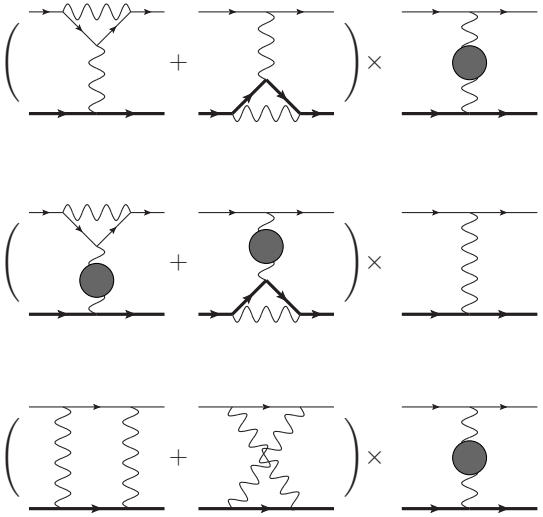
MF, M. Passera,

PRL 122 (2019) 192001
JHEP 1902 (2019) 027

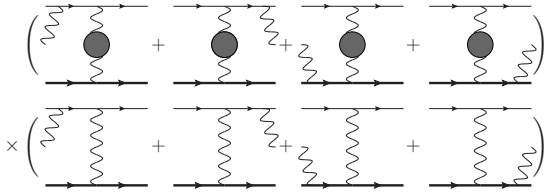
Class I



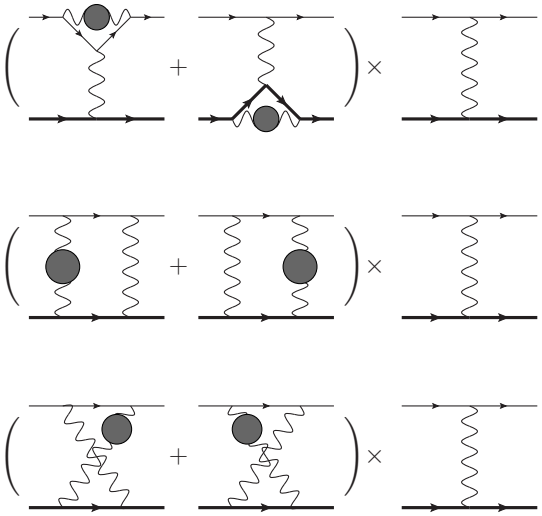
Class II



Class III



Class IV



← non-factorizable diagrams!

Inelastic scattering processes

μ-e scattering with hadrons in the final state:

- Phase space very small: $\sqrt{s} = 405$ MeV.
- $\mu e \to \mu e \pi\pi$, $\sqrt{s} - m_\mu - 2m_\pi = 22$ MeV
- $\mu e \to \mu e \pi^0$, $\sqrt{s} - m_\mu - m_\pi = 164$ MeV

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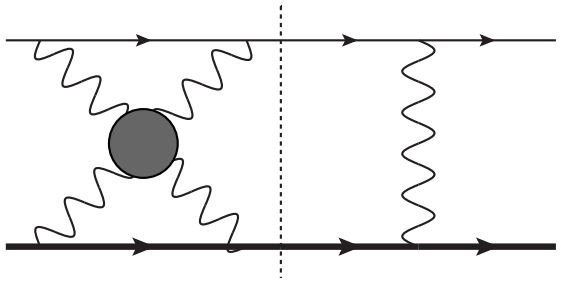
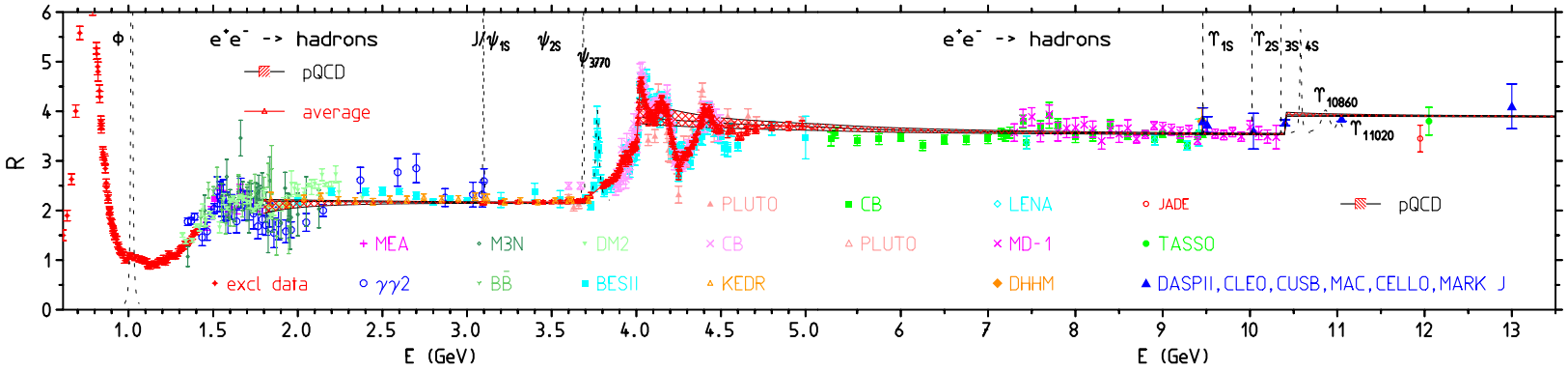


is of $O(\alpha^5)$

Two Roads:

- To $R(s)$:
  - Method: dispersion relation.
  - Based on $\mathrm{Im}\,\Pi^{\mathrm{had}}(s > 0)$.
  - input: $e^+e^- \to$ had data.



F. Jegerlehner, hep-ph/1804.07409

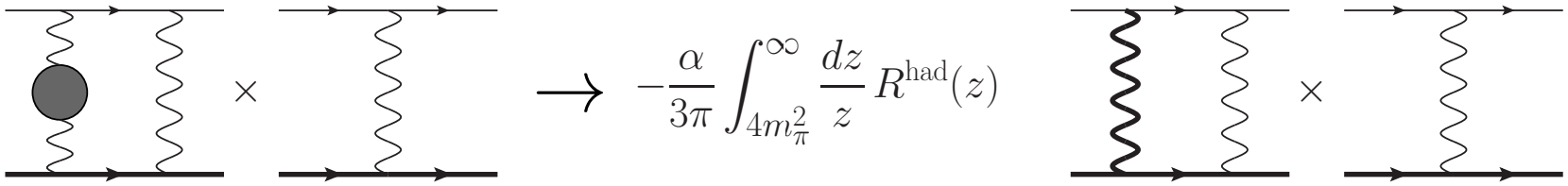
Two Roads:

- To $R(s)$:
  - Method: dispersion relation.
  - Based on $\mathrm{Im}\,\Pi^{\mathrm{had}}(s > 0)$.
  - input: $e^+e^- \to$ had data.

- Not to $R(s)$:
  - Method: hyperspherical integration.
  - Promote ansatz for $\Pi^{\mathrm{had}}(t < 0)$ also at NNLO.
  - input: MUonE's $e\mu \to e\mu$ data.

To $R(s)$

$$\frac{-ig^{\mu\nu}}{q^2}\Pi_{\rm had}(q^2) \to -\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{dz}{z}\,{\rm Im}\Pi_{\rm had}(z)\left[\frac{-ig^{\mu\nu}}{q^2-z}\right]$$



Hadronic NNLO for Bhabha scattering:

Actis, Czakon, Gluza, Riemann, PRL 100 (2008) 131602; Actis, Gluza, Riemann, Nucl. Phys. Proc. Suppl. 183 (2008) 174; Kühn, Uccirati, NPB 806 (2009) 300; Carloni Calame et al. JHEP 1107 (2011) 126.

HNNLO: Fortran Implementation

- `FeynArts` generates Feynman diagrams in QED+extra massive photon.
- `FormCalc` calculates tree-level and one-loop diagrams.
- `FormCalc` exports $|\mathcal{M}|^2$ as Fortran code.

T. Hahn, Comput. Phys. Commun. 140 (2001) 418;
T. Hahn, S. Passehr and C. Schappacher, J. Phys. Conf. Ser. 762 (2016) 012065

The Monte Carlo code:

- `Collier` evaluates one-loop functions.

  Denner, Dittmaier, Hofer, Comput. Phys. Commun. **212** (2017) 220

- Soft singularities with FKS subtraction.
  Frixione, Kunszt, Signer, NPB 467 (1996) 399
  Frederix, Frixione, Maltoni, Stelzer, JHEP 0910 (2009) 003

HNNLO: Mathematica Implementation

- One-loop functions `Package-X`

  Patel, Comput.Phys.Commun. 197 (2015) 276

- Checked with `LoopTools-quad`

  Hahn, Perez-Victoria, Comput.Phys.Commun. 118 (1999) 153

- Use of MATHEMATICAs arbitrary-precision numbers.
- Slicing method for IR.

Numerical Inputs:

- Jegerlehner's package: `alphaQEDc17`

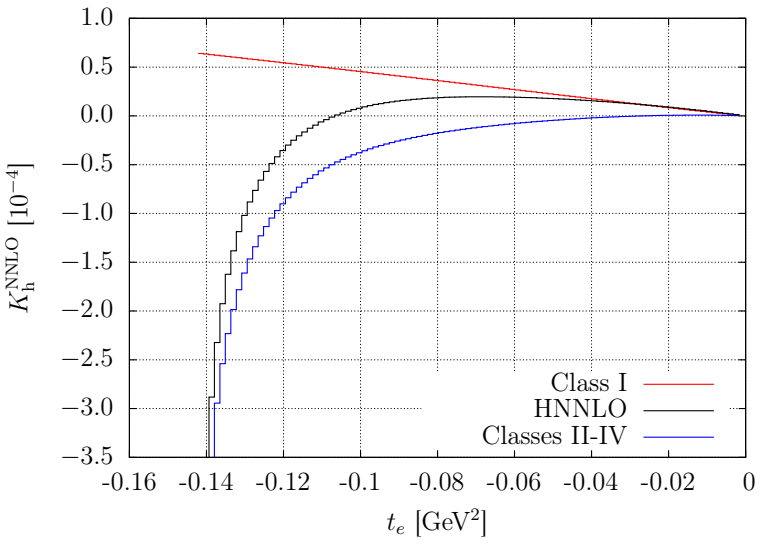
  hep-ph/0105283, `www-com.physik.hu-berlin.de/∼fjeger/alphaQEDc17.tar.gz`

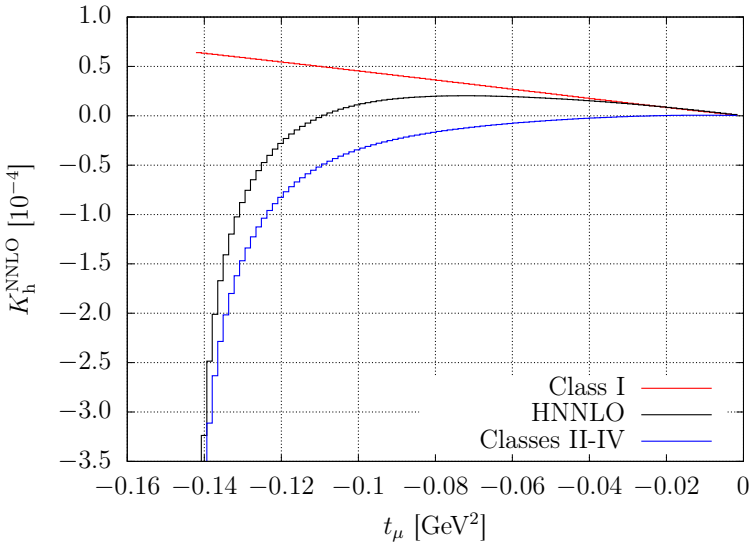
- Keshavarzi, Nomura, Teubner: `VP_KNT_v3_0`

  A Keshavarzi et al., PRD 97 (2018) 114025

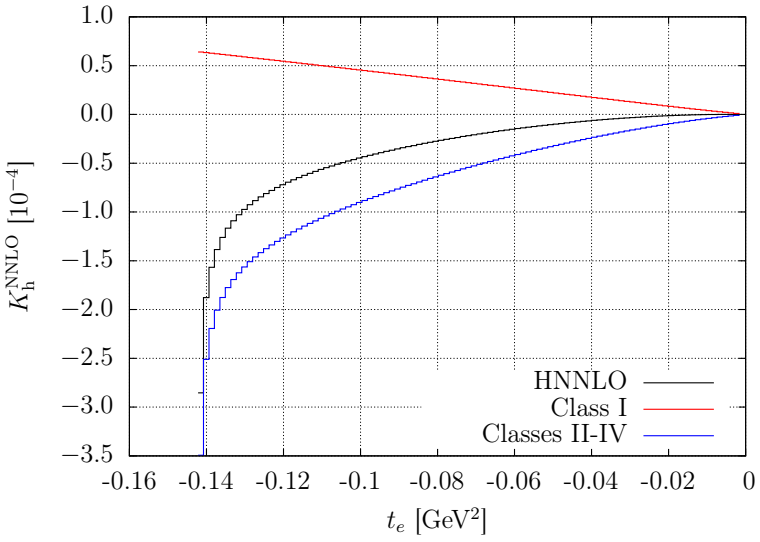
- Top quark analytic contribution included (but negligible!).

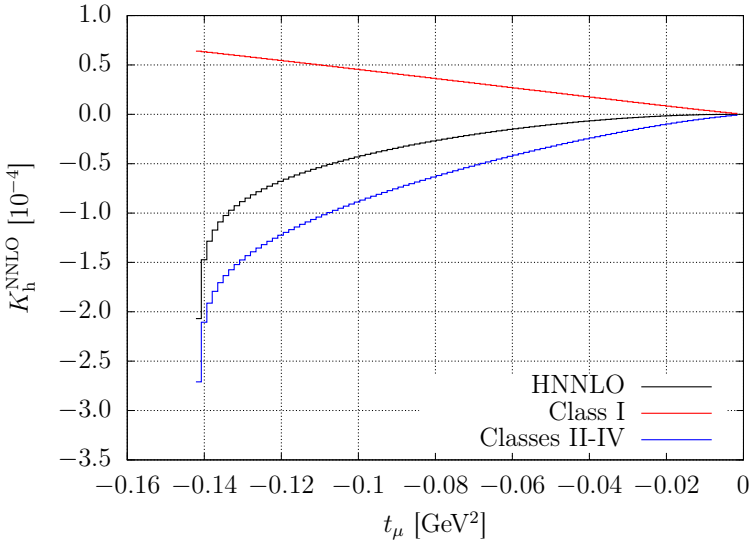
$e^-\mu^+ \to e^-\mu^+$





$e^-\mu^- \to e^-\mu^-$





Potential Pitfall:

- Good numerical stability with `Collier`.
- Disp. integration with `Quadpack`. All narrow resonances must be declared to the routine for precise numerical evaluation.
- IR poles cancel only if $\Pi^{\mathrm{had}}(q^2 < 0)$ and $\mathrm{Im}\,\Pi^{\mathrm{had}}(q^2)$ are *exactly* connected via disp. relation.
- Comparison FJ and KNT: difference in $d\sigma/dt$ is about 1% or less.

Not to $R(s)$

Can we use these ansatz at NNLO?

- Polynomial:

$$\Delta\alpha^{\rm had}(t) = c_1 t + c_2 t^2 + c_3 t^3$$

- Padé approximant:

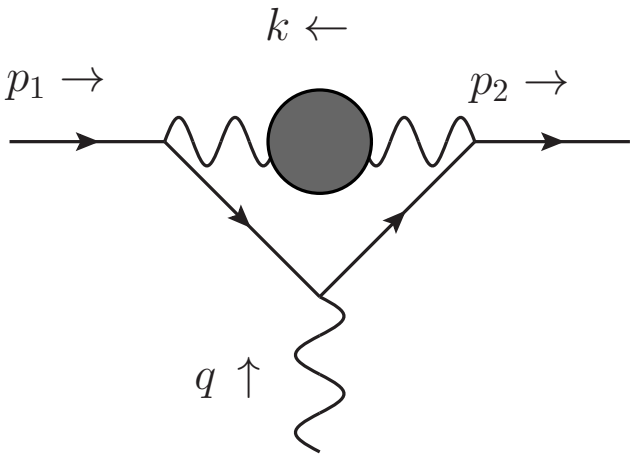
$$\Delta\alpha^{\rm had}(t) = at\frac{1+bt}{1+ct}$$

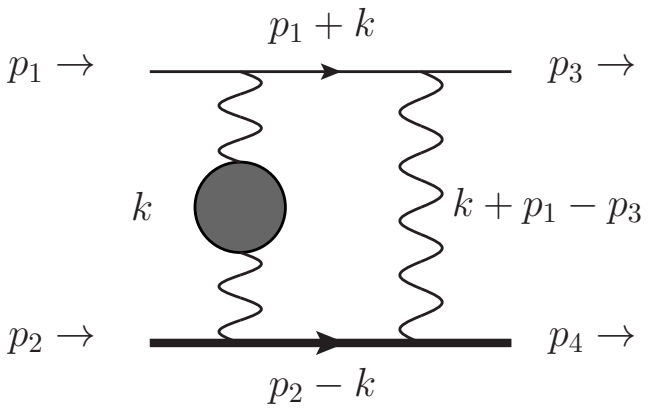
- Fermion-like function:

$$\Delta\alpha^{\rm had}(t) =$$

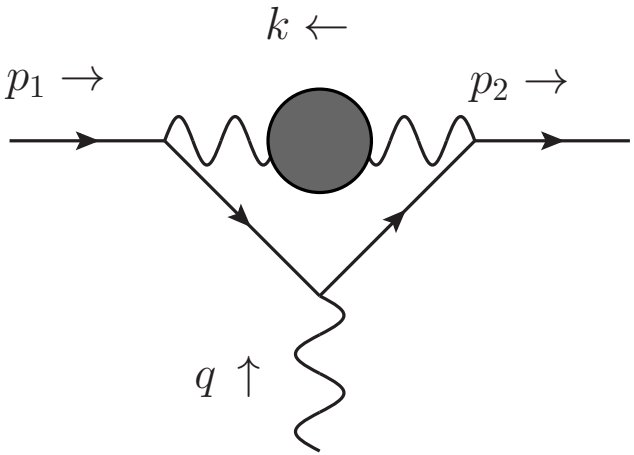
$$K\left[-\frac{5}{9} - \frac{4M}{t}\left(\frac{4M^2}{3t^2} + \frac{M}{3t} - \frac{1}{6}\right)\frac{2}{\sqrt{1-\frac{4M}{t}}}\log\left|\frac{1-\sqrt{1-\frac{4M}{t}}}{1+\sqrt{1-\frac{4M}{t}}}\right|\right]$$

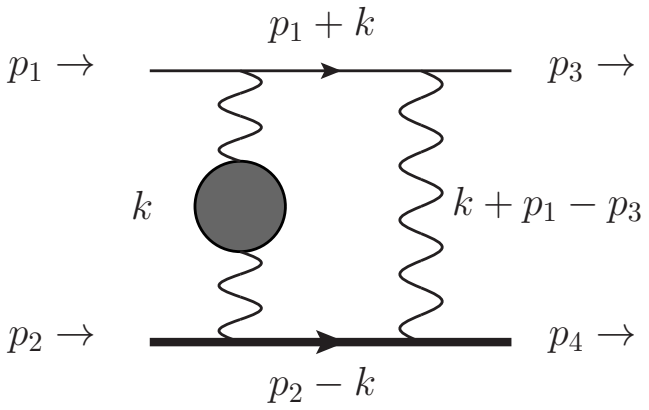
Hyperspherical integration method





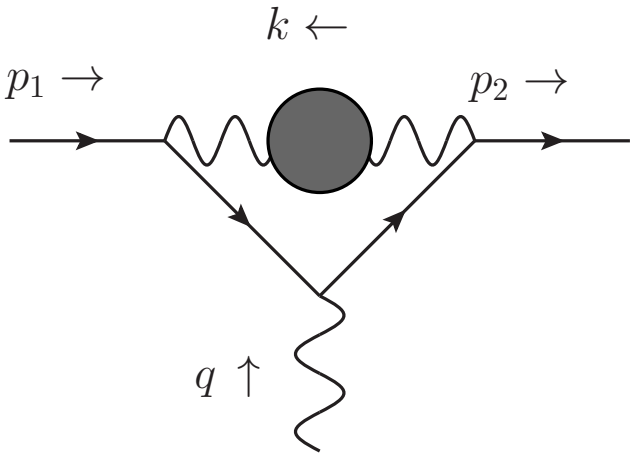
$$I(p_1, \ldots, p_n) = \int d^4q \frac{\Pi_{\mathrm{had}}(q^2)}{q^2 + i\varepsilon} \frac{\mathcal{N}(q, p_1, \ldots, p_n)}{\mathcal{D}_1 \cdots \mathcal{D}_n}$$

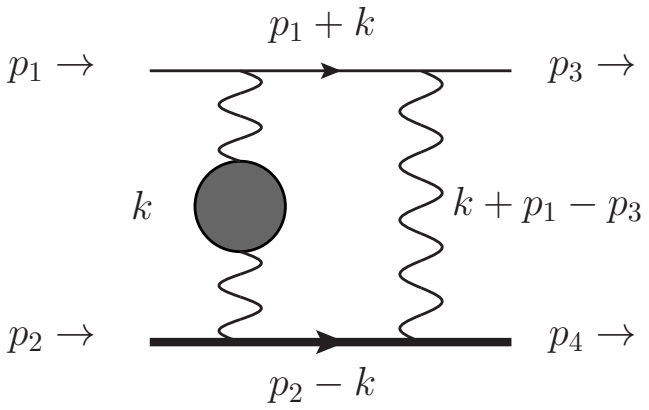




- Continuation to Euclidean region + Wick rotation.
- Introduce spherical coordinates:

$$I(P_1, \ldots, P_n) = \int \frac{dQ^2}{2} \Pi_{\mathrm{had}}(-Q^2) \int d\Omega_q \frac{\mathcal{N}(Q, P_1, \ldots, P_n)}{\mathcal{D}_1 \cdots \mathcal{D}_n}$$





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The Angular Integrals

- **Euclidean** denominators expanded in Gegenbauer polynomials:

$$\frac{1}{(Q-P)^2+m^2} = \frac{Z_{QP}}{|Q||P|}\sum_{n=0}^{\infty} Z_{QP}^n\, C_n^{(1)}(\hat{Q}\cdot\hat{P})$$

with

$$Z_{QP} = \frac{Q^2+P^2+m^2-\lambda^{1/2}(Q^2,P^2,-m^2)}{2|Q||P|}$$

The Angular Integrals

- Angular integrals:

$$\int \frac{d\Omega_Q}{2\pi^2}\, C_n^{(1)}(\hat{Q}\cdot\hat{P}_i)\, C_m^{(1)}(\hat{Q}\cdot\hat{P}_j) = \frac{\delta_{nm}}{n+1} C_n^{(1)}(\hat{P}_i\cdot\hat{P}_j)$$

- Continue the result back to the physical region:

$$p_i^2 = -P^2 \to m_i^2$$

- For the box diagrams we need also

$$\int \frac{d\Omega_Q}{[(Q-P_1)^2+m_1^2]\,[(Q-P_2)^2+m_2^2]\,[(Q-P_3)^2+m_3^2]}$$

Laporta, Nuovo Cim. A107 (1994) 1729

- 6th order contributions to the electron g-2:
  Levine and Roskies, Phys. Rev. Lett. **30** (1973) 772.
  Levine and Roskies, Phys. Rev. D **9** (1974) 421.
  Levine, Remiddi and Roskies, Phys. Rev. D **20** (1979) 2068.
  Roskies, Levine and Remiddi, Adv. Ser. Direct. High Energy Phys. **7** (1990) 162.
  Laporta and Remiddi, Phys. Lett. B **265** (1991) 182.
  Laporta and Remiddi, Phys. Lett. B **301** (1993) 440.

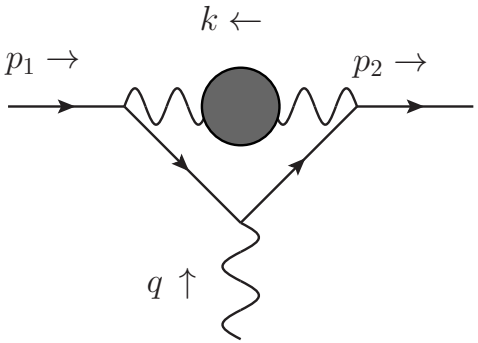
- Pion pole contribution to a_μ^{HLBL}:
  Knecht and Nyffeler, Phys. Rev. D **65** (2002) 073034.
  Nyffeler and Jegerlehner, Phys.Rept. 477 (2009) 1

- Dispersive approach to a_μ^{HLBL}:
  Colangelo, Hoferichter, Procura and Stoffer, JHEP **1409**, 091 (2014).
  Colangelo, Hoferichter, Kubis, Procura and Stoffer, Phys. Lett. B **738** (2014) 6.
  Colangelo, Hoferichter, Procura and Stoffer, JHEP **1509** (2015) 074.

- Integrand decomposition of scattering amplitudes:

  Mastrolia, Peraro, Primo, JHEP 1608 (2016) 164.

Vertex Corrections



- Two *radial* integrations (for both e and μ):

$$F_{1,2}^{\mathrm{had}}(t) = -\frac{\alpha}{\pi}\int_0^1 dx\, \Pi_{\mathrm{had}}\left(\frac{m_\ell^2 x^2}{x-1}\right)\; f_{1,2}\left(x, \frac{t}{m_\ell^2}\right)$$

with $Q^2 = \frac{m_\ell^2 x^2}{1-x}$.

Hadronic Contribution to the Form Factors

$$
\begin{aligned}
f_1(x,y) = & \frac{3x^3-4x^2+4}{4(1-x)x} + \frac{2-x}{1-x}\left\{\frac{6x^2}{(4-y)^2(x-1)} + \frac{x^2-6x+4}{2(4-y)(x-1)}\right. \\
& + \left[\frac{\left(x^2+8x-8\right)x}{(4-y)(1-x)^2} - \frac{12x^3}{(4-y)^2(1-x)^2} + \frac{4-y}{x} + \frac{2\left(x^2+x-1\right)}{(1-x)x}\right] \\
& \left.\times \frac{1}{\sqrt{y(y-4)}} \operatorname{arctanh}\left(\frac{(1-x)\sqrt{y\,(y-4)}}{2x+y-4-yx}\right)\right\}
\end{aligned}
$$

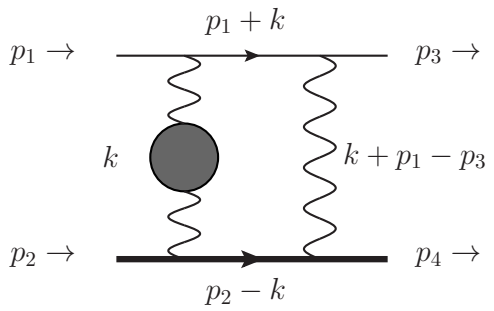
$$
\begin{aligned}
f_2(x,y) = & \frac{2-x}{1-x}\left\{\frac{6x^2}{(4-y)^2(1-x)} + \frac{2-x}{4-y} + \left[\frac{2x}{(4-y)(1-x)} + \frac{3x^3}{(4-y)^2(1-x)^2}\right]\right. \\
& \left.\times \frac{4}{\sqrt{y(y-4)}} \operatorname{arctanh}\left(\frac{(1-x)\sqrt{y\,(y-4)}}{2x+y-4-yx}\right)\right\}
\end{aligned}
$$

MF, JHEP 02 (2019) 027

- $\lim\limits_{q^2\to 0} f_2(x,q^2) = 1-x$ gives the spacelike formula for a_μ^{HLO}

  see also T. Blum, PRL 91 (2003) 052001

Box diagrams



- 14 *Radial* integrations (two box topologies):

$$I_\alpha = \frac{1}{i\pi^2} \int d^4q \, \Pi_{\rm had}(q^2) \cdots = \int_0^{+\infty} dQ^2 \, Q^2 \, \Pi_{\rm had}(-Q^2) \left\langle \ldots \right\rangle$$

- Angular integrals

$$\left\langle \ldots \right\rangle = \int \frac{d\Omega_Q}{2\pi^2} \cdots \Bigg|_{p_i^2 \to m_i^2}$$

An Example:

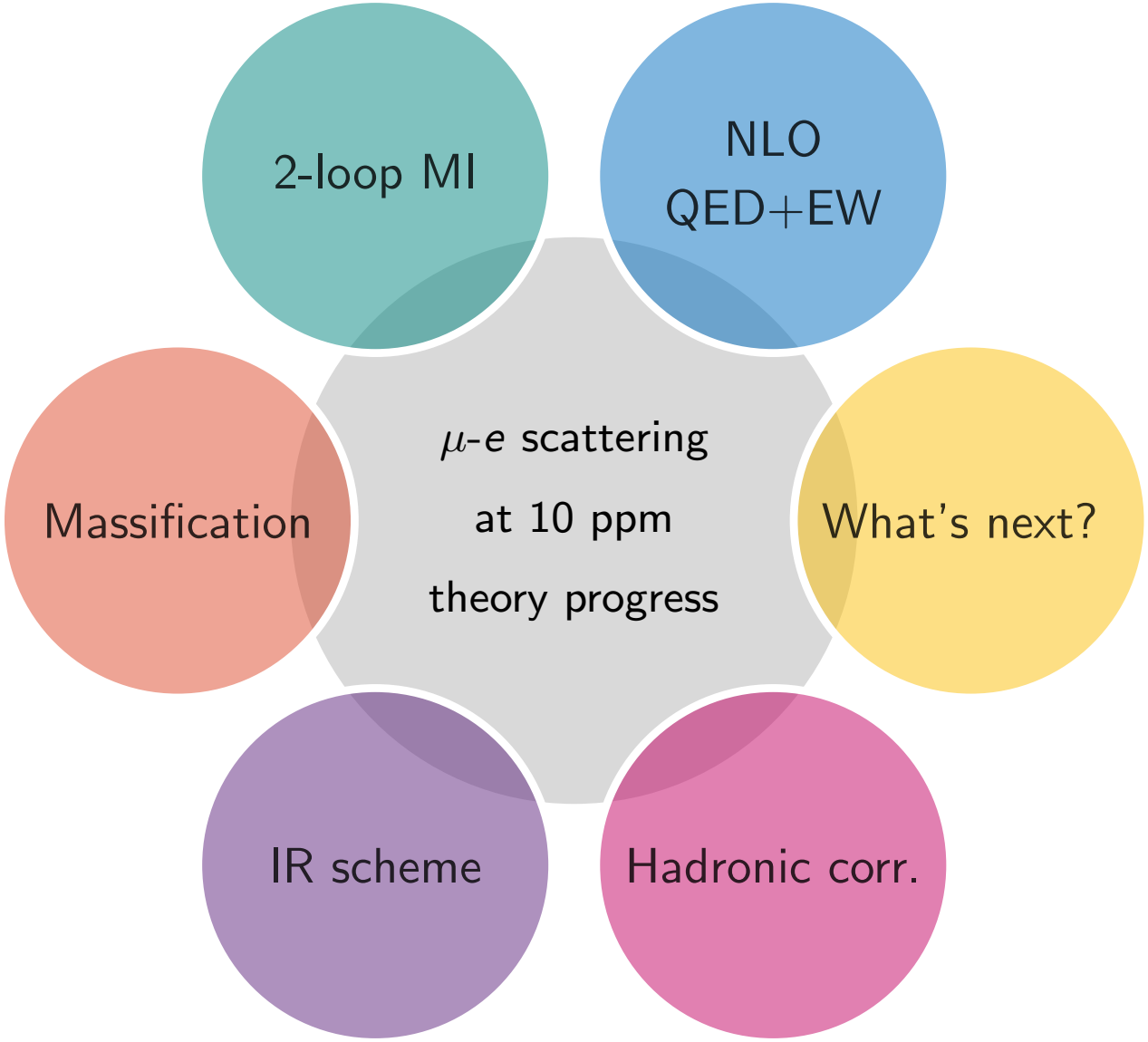
$$I_{013} = \frac{1}{i\pi^2}\int d^4q \frac{\Pi^{\rm had}(q^2)}{q^2[(q+p_1)^2-m^2][(q-p_2)^2-M^2]}$$

$$= \int dQ^2\, Q^2\, \Pi^{\rm had}(-Q^2)\left\langle \frac{1}{\mathcal{D}_0\mathcal{D}_1\mathcal{D}_3}\right\rangle$$

with

$$\left\langle \frac{1}{\mathcal{D}_0\mathcal{D}_1\mathcal{D}_3}\right\rangle = \frac{-2}{Q^2\lambda^{1/2}(s,M^2,m^2)}$$

$$\times \mathrm{Arctanh}\left(\frac{\lambda^{1/2}(s,M^2,m^2)}{s-M^2-m^2-8M^2m^2\Big/\left[Q^2\left(1-\sqrt{1+\frac{4m^2}{Q^2}}\right)\left(1-\sqrt{1+\frac{4M^2}{Q^2}}\right)\right]}\right)$$


2-loop MI
NLO
QED+EW
Massification
μ-e scattering
at 10 ppm
theory progress
What's next?
IR scheme
Hadronic corr.

To Do & Theory Open Issues

- QED double-virtual matrix elements.
- QED double real radiation & real-virtual corrections.
- Inelastic processes with leptonic and hadronic final states.
- $\Delta\alpha^{\rm had}$ ansatz and NNLO hadronic corrections with spacelike data. Study model dependence.
- Extend the MC to fixed order NNLO .
- Match the NNLO calculations with resummation of the log contributions.
- How to implement experimental elasticity cuts for resummation.
- Reliable estimate of th. uncertainty due to missing higher order corr.
- Bound electron effects.
- New physics sensitivity of the MUonE experiment.
- ...

Muon-electron scattering: Theory Kickoff Workshop

Padova, Sept. 4 - 5 2017

Organizers:
C. Carloni Calame, P. Mastrolia, O. Nicrosini P. Paradisi, M. Passera, F. Piccinini, L. Trentadue, G. Venanzoni

MITP T(r)opical Workshop

MITP Mainz, Feb. 19 - 23 2018

Organizers:

C. Carloni Calame, M. Passera, L. Trentadue, G. Venanzoni

2. Workstop/Thinkstart on Theory for μ-e scattering

Zurich, Feb. 4 - 7 2019

Organizers:
A. Signer and Y. Ulrich

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Mainz Institute for Theoretical Physics

The Evaluation of the Leading Hadronic Contribution to the Muon g-2: Toward the MUonE Experiment

30 March 2020 to 3 April 2020
Mainz Institute for Theoretical Physics, Johannes Gutenberg University
Europe/Berlin timezone

- Overview
- Scientific Program
- General Information
- Timetable
- Travel Information
- Application

Contact @ MITP : NN

mitp@uni-mainz.de

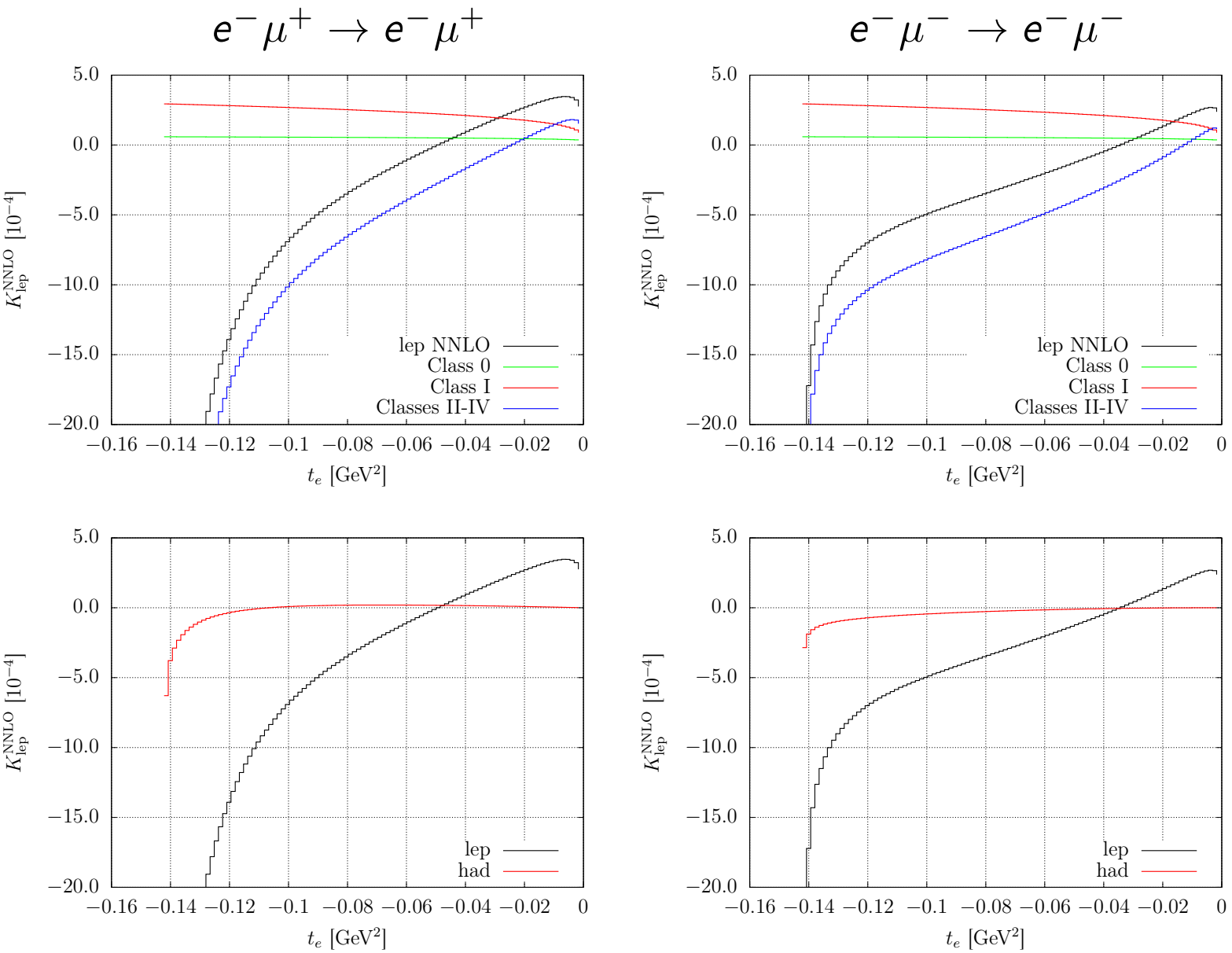
The aim of the topical workshop is to assess the state of the art of the theoretical determination of the muon anomalous magnetic moment, a_μ , in view of the anticipated new measurement of the E989 Muon g-2 experiment in Fermilab, which is expected to improve the present precision by a factor of four, and the new approach to measure a_μ under development by the E34 collaboration at J-PARC. The workshop will focus on a_μ^{HLO} , the leading hadronic contribution to the muon g-2, both analyzing the most recent experimental and theoretical results for its determination, as well as taking into consideration the most recent dedicated lattice calculations. Particular attention will be devoted to MUonE, a recently proposed experiment at CERN which aims at measuring the running of the electromagnetic coupling constant in the spacelike region by scattering high-energy muons on atomic electrons of a low-Z target through the process $\mu e \to \mu e$. The differential cross section of this process provides the direct measurement of a_μ^{HLO} and, therefore, an independent determination of the theoretical prediction for a_μ competitive with the timelike dispersive approach.

Starts 30 Mar 2020, 08:00
Ends 3 Apr 2020, 18:00
Europe/Berlin

Mainz Institute for Theoretical Physics, Johannes Gutenberg University
02.430
Staudingerweg 9 / 2nd floor, 55128 Mainz

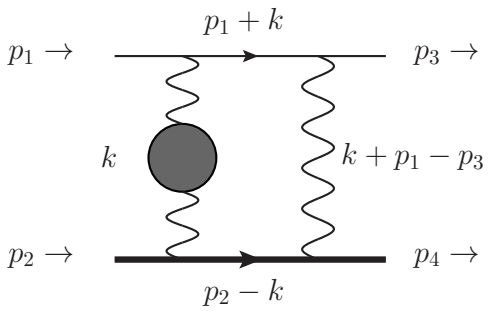
Backup

Leptonic Contribution



IR poles

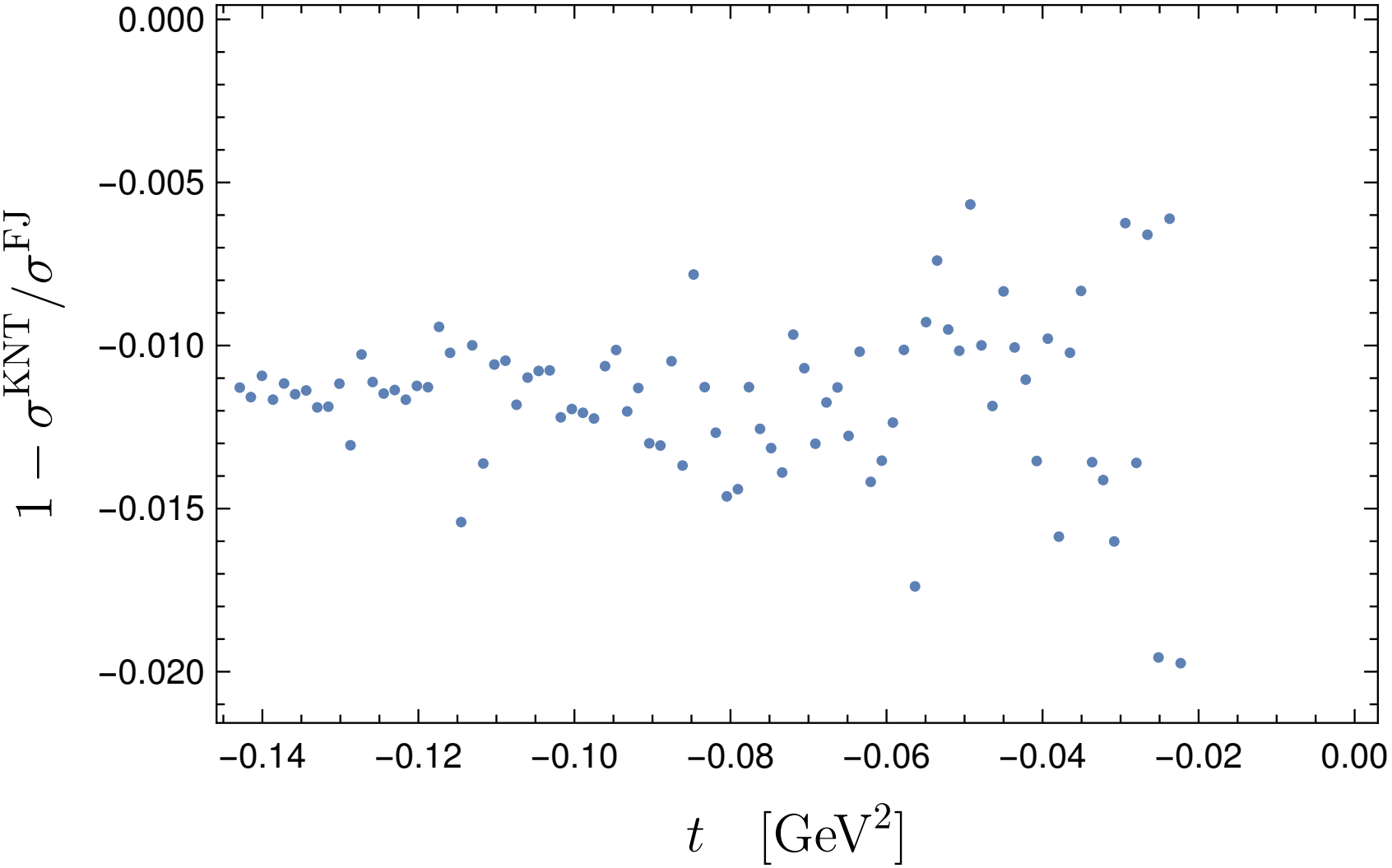
Subtraction Method:



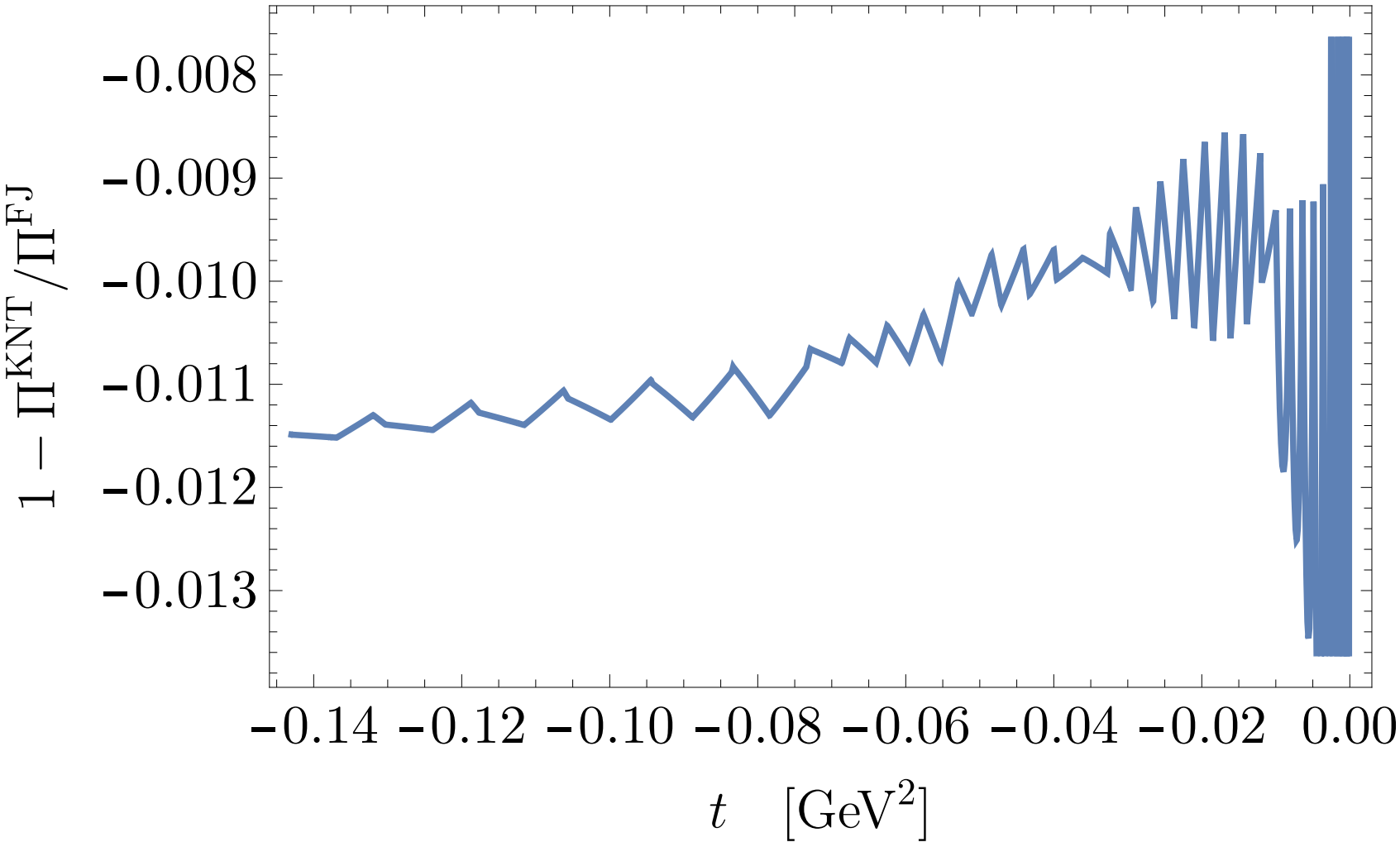
$$I_{ijk} = \int dQ^2\, Q^2 \left[\Pi^{\rm had}(-Q^2) - \Pi^{\rm had}(t)\right] \left\langle \frac{1}{\mathcal{D}_i \mathcal{D}_j \mathcal{D}_k} \right\rangle + \Pi^{\rm had}(t) \int \frac{d^D q}{i\pi^2} \frac{1}{\mathcal{D}_i \mathcal{D}_j \mathcal{D}_k}\,,$$

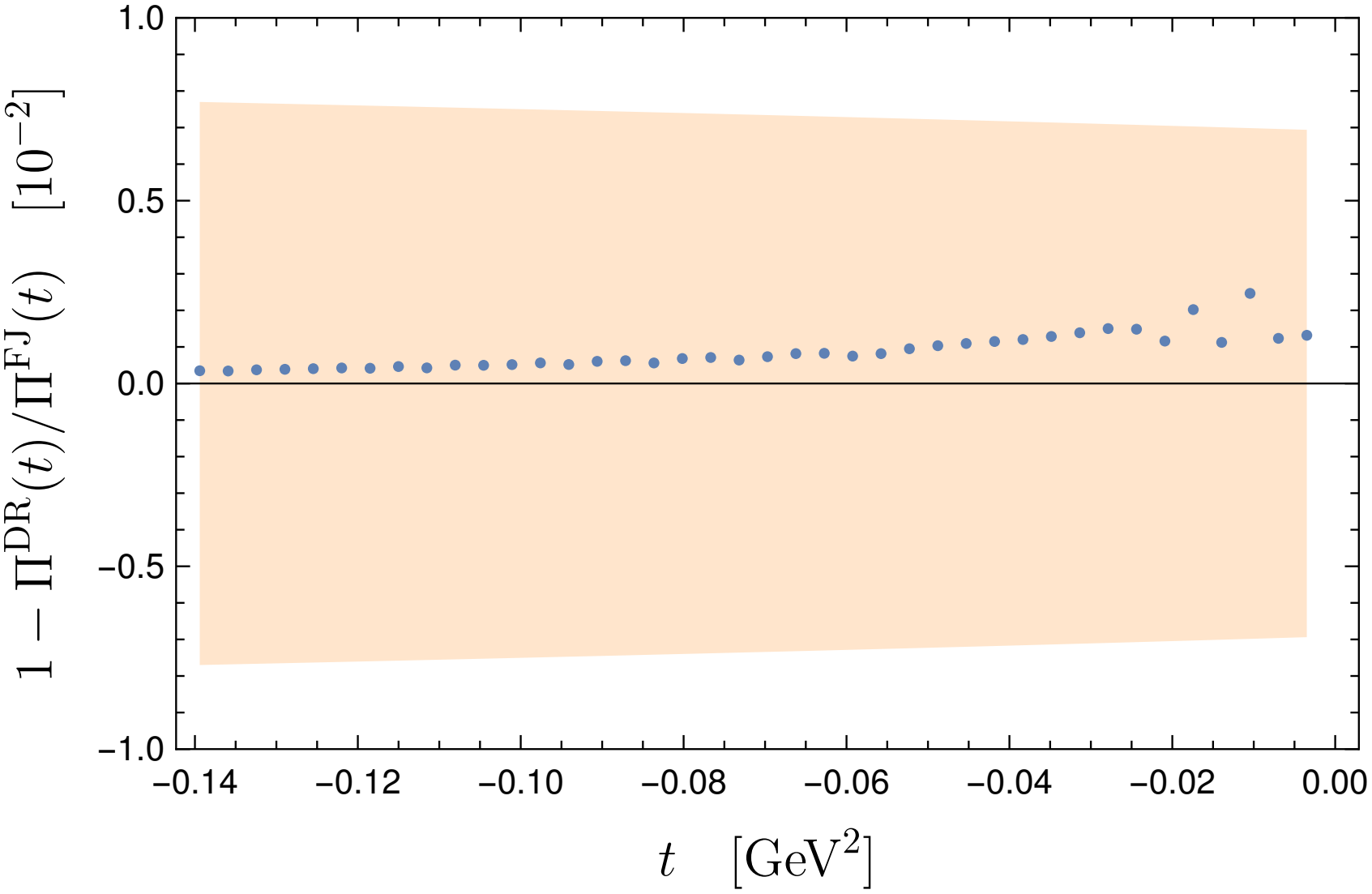
$$I_{0ijk} = \int dQ^2\, Q^2 \left[\Pi^{\rm had}(-Q^2) - \frac{2Q^2}{Q^2 + |t|} \Pi^{\rm had}(t)\right] \left\langle \frac{1}{\mathcal{D}_0 \mathcal{D}_i \mathcal{D}_j \mathcal{D}_k} \right\rangle + 2\,\Pi^{\rm had}(t) \int \frac{d^D q}{i\pi^2} \frac{1}{(q^2 - |t|)\mathcal{D}_i \mathcal{D}_j \mathcal{D}_k}\,,$$

Cross Section Uncertainty



Cross Section Uncertainty




1.0
0.5
0.0
−0.5
−1.0
−0.14
−0.12
−0.10
−0.08
−0.06
−0.04
−0.02
0.00
$1 - \Pi^{\mathrm{DR}}(t)/\Pi^{\mathrm{FJ}}(t) \quad [10^{-2}]$
$t \quad [\mathrm{GeV}^2]$