HVP status from KNT collaboration

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Muon g-2 Theory Initiative Workshop @ Seattle

September 11, 2019

Based on
A. Keshavarzi, DN and T. Teubner **(KNT)**
Phys. Rev. D97 (2018) 114025
[arXiv:1802.02995] and data updates (preliminary)
(no changes in the data combination method since KNT18)

Breakdown of SM prediction (as of 2018)

	2011		2018	
QED	11658471.81 (0.02)	$\longrightarrow$	11658471.90 (0.01)	[arXiv:1712.06060]
EW	15.40 (0.20)	$\longrightarrow$	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	$\longrightarrow$	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) 01016]
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]
	HLMNT11		**KNT18**	
LO HVP	694.91 (4.27)	$\longrightarrow$	693.27 **(2.46)**	this work
NLO HVP	-9.84 (0.07)	$\longrightarrow$	-9.82 (0.04)	this work
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144]
Theory total	11659182.80 (4.94)	$\longrightarrow$	11659182.05 (3.56)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	$\longrightarrow$	27.1 (7.3)	this work
Δa_μ	3.3σ	$\longrightarrow$	3.7σ	this work

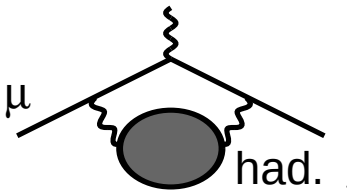
(HVP: Hadronic Vacuum Polarization)
(HLbL: Hadronic Light-by-Light)

(Numbers taken from KNT18, Phys. Rev. D97 (2018) 114025)

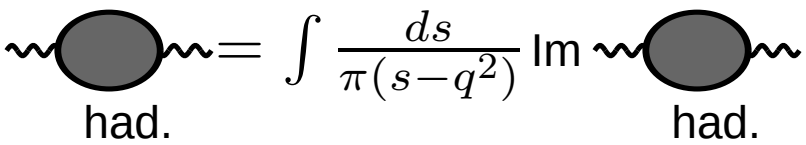
Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ Workshop' at Mainz, June 18-22, 2018

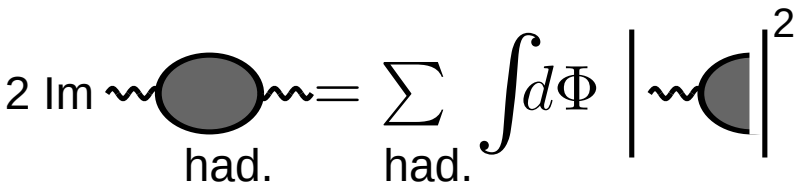
LO Hadronic Vacuum Polarization Contribution

The diagram to be evaluated:

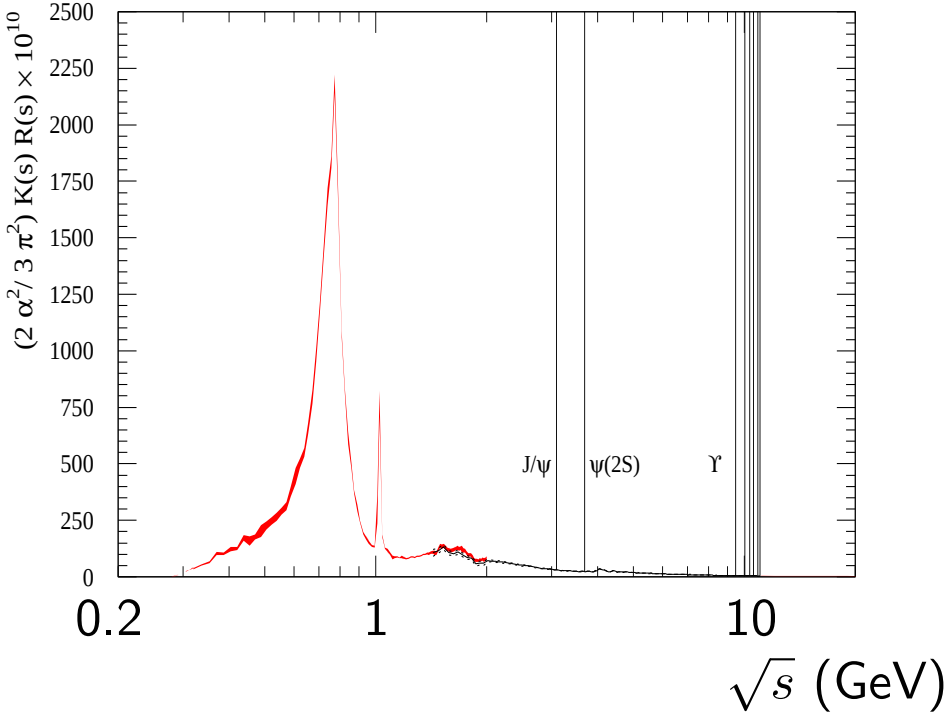


pQCD not useful. Use the dispersion relation and the optical theorem.





$$a_\mu^{\mathrm{had,LO}} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\mathrm{th}}}^{\infty} ds\ \frac{1}{s}\hat{K}(s)\sigma_{\mathrm{had}}(s)$$



- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\mathrm{had,LO}}$

Channel	Energy range [GeV]	$a_\mu^{\text{had,LO VP}} \times 10^{10}$	$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) \times 10^4$	New data
	Chiral perturbation theory (ChPT) threshold contributions			
$\pi^0\gamma$	$m_\pi \le \sqrt{s} \le 0.600$	0.12 ± 0.01	0.00 ± 0.00	...
$\pi^+\pi^-$	$2m_\pi \le \sqrt{s} \le 0.305$	0.87 ± 0.02	0.01 ± 0.00	...
$\pi^+\pi^-\pi^0$	$3m_\pi \le \sqrt{s} \le 0.660$	0.01 ± 0.00	0.00 ± 0.00	...
$\eta\gamma$	$m_\eta \le \sqrt{s} \le 0.660$	0.00 ± 0.00	0.00 ± 0.00	...
	Data based channels ($\sqrt{s} \le 1.937$ GeV)			
$\pi^0\gamma$	$0.600 \le \sqrt{s} \le 1.350$	4.46 ± 0.10	0.36 ± 0.01	[65]
$\pi^+\pi^-$	$0.305 \le \sqrt{s} \le 1.937$	502.97 ± 1.97	34.26 ± 0.12	[34,35]
$\pi^+\pi^-\pi^0$	$0.660 \le \sqrt{s} \le 1.937$	47.79 ± 0.89	4.77 ± 0.08	[36]
$\pi^+\pi^-\pi^+\pi^-$	$0.613 \le \sqrt{s} \le 1.937$	14.87 ± 0.20	4.02 ± 0.05	[40,42]
$\pi^+\pi^-\pi^0\pi^0$	$0.850 \le \sqrt{s} \le 1.937$	19.39 ± 0.78	5.00 ± 0.20	[44]
$(2\pi^+2\pi^-\pi^0)_{\text{no}\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.99 ± 0.09	0.33 ± 0.03	...
$3\pi^+3\pi^-$	$1.313 \le \sqrt{s} \le 1.937$	0.23 ± 0.01	0.09 ± 0.01	[66]
$(2\pi^+2\pi^-2\pi^0)_{\text{no}\eta\text{no}\omega}$	$1.322 \le \sqrt{s} \le 1.937$	1.35 ± 0.17	0.51 ± 0.06	...
K^+K^-	$0.988 \le \sqrt{s} \le 1.937$	23.03 ± 0.22	3.37 ± 0.03	[45,46,49]
$K_S^0K_L^0$	$1.004 \le \sqrt{s} \le 1.937$	13.04 ± 0.19	1.77 ± 0.03	[50,51]
$KK\pi$	$1.260 \le \sqrt{s} \le 1.937$	2.71 ± 0.12	0.89 ± 0.04	[53,54]
$KK2\pi$	$1.350 \le \sqrt{s} \le 1.937$	1.93 ± 0.08	0.75 ± 0.03	[50,53,55]
$\eta\gamma$	$0.660 \le \sqrt{s} \le 1.760$	0.70 ± 0.02	0.09 ± 0.00	[67]
$\eta\pi^+\pi^-$	$1.091 \le \sqrt{s} \le 1.937$	1.29 ± 0.06	0.39 ± 0.02	[68,69]
$(\eta\pi^+\pi^-\pi^0)_{\text{no}\omega}$	$1.333 \le \sqrt{s} \le 1.937$	0.60 ± 0.15	0.21 ± 0.05	[70]
$\eta2\pi^+2\pi^-$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.01	0.03 ± 0.00	...
$\eta\omega$	$1.333 \le \sqrt{s} \le 1.937$	0.31 ± 0.03	0.10 ± 0.01	[70,71]
$\omega(\to \pi^0\gamma)\pi^0$	$0.920 \le \sqrt{s} \le 1.937$	0.88 ± 0.02	0.19 ± 0.00	[72,73]
$\eta\phi$	$1.569 \le \sqrt{s} \le 1.937$	0.42 ± 0.03	0.15 ± 0.01	...
$\phi \to$ unaccounted	$0.988 \le \sqrt{s} \le 1.029$	0.04 ± 0.04	0.01 ± 0.01	...
$\eta\omega\pi^0$	$1.550 \le \sqrt{s} \le 1.937$	0.35 ± 0.09	0.14 ± 0.04	[74]
$\eta(\to \text{npp})K\bar{K}_{\text{no}\phi\to K\bar{K}}$	$1.569 \le \sqrt{s} \le 1.937$	0.01 ± 0.02	0.00 ± 0.01	[53,75]
$p\bar{p}$	$1.890 \le \sqrt{s} \le 1.937$	0.03 ± 0.00	0.01 ± 0.00	[76]
$n\bar{n}$	$1.912 \le \sqrt{s} \le 1.937$	0.03 ± 0.01	0.01 ± 0.00	[77]
	Estimated contributions ($\sqrt{s} \le 1.937$ GeV)			
$(\pi^+\pi^-3\pi^0)_{\text{no}\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.50 ± 0.04	0.16 ± 0.01	...
$(\pi^+\pi^-4\pi^0)_{\text{no}\eta}$	$1.313 \le \sqrt{s} \le 1.937$	0.21 ± 0.21	0.08 ± 0.08	...
$KK3\pi$	$1.569 \le \sqrt{s} \le 1.937$	0.03 ± 0.02	0.02 ± 0.01	...
$\omega(\to \text{npp})2\pi$	$1.285 \le \sqrt{s} \le 1.937$	0.10 ± 0.02	0.03 ± 0.01	...
$\omega(\to \text{npp})3\pi$	$1.322 \le \sqrt{s} \le 1.937$	0.17 ± 0.03	0.06 ± 0.01	...
$\omega(\to \text{npp})KK$	$1.569 \le \sqrt{s} \le 1.937$	0.00 ± 0.00	0.00 ± 0.00	...
$\eta\pi^+\pi^-2\pi^0$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.04	0.03 ± 0.02	...
	Other contributions ($\sqrt{s} > 1.937$ GeV)			
Inclusive channel	$1.937 \le \sqrt{s} \le 11.199$	43.67 ± 0.67	82.82 ± 1.05	[56,62,63]
J/ψ	...	6.26 ± 0.19	7.07 ± 0.22	...
ψ'	...	1.58 ± 0.04	2.51 ± 0.06	...
$\Upsilon(1S-4S)$	...	0.09 ± 0.00	1.06 ± 0.02	...
pQCD	$11.199 \le \sqrt{s} \le \infty$	2.07 ± 0.00	124.79 ± 0.10	...
Total	$m_\pi \le \sqrt{s} \le \infty$	693.26 ± 2.46	276.11 ± 1.11	...

Table from KNT18, Phys. Rev. D97 (2018) 114025

Breakdown of contributions to a_μ(had, LO VP) from various hadronic final states

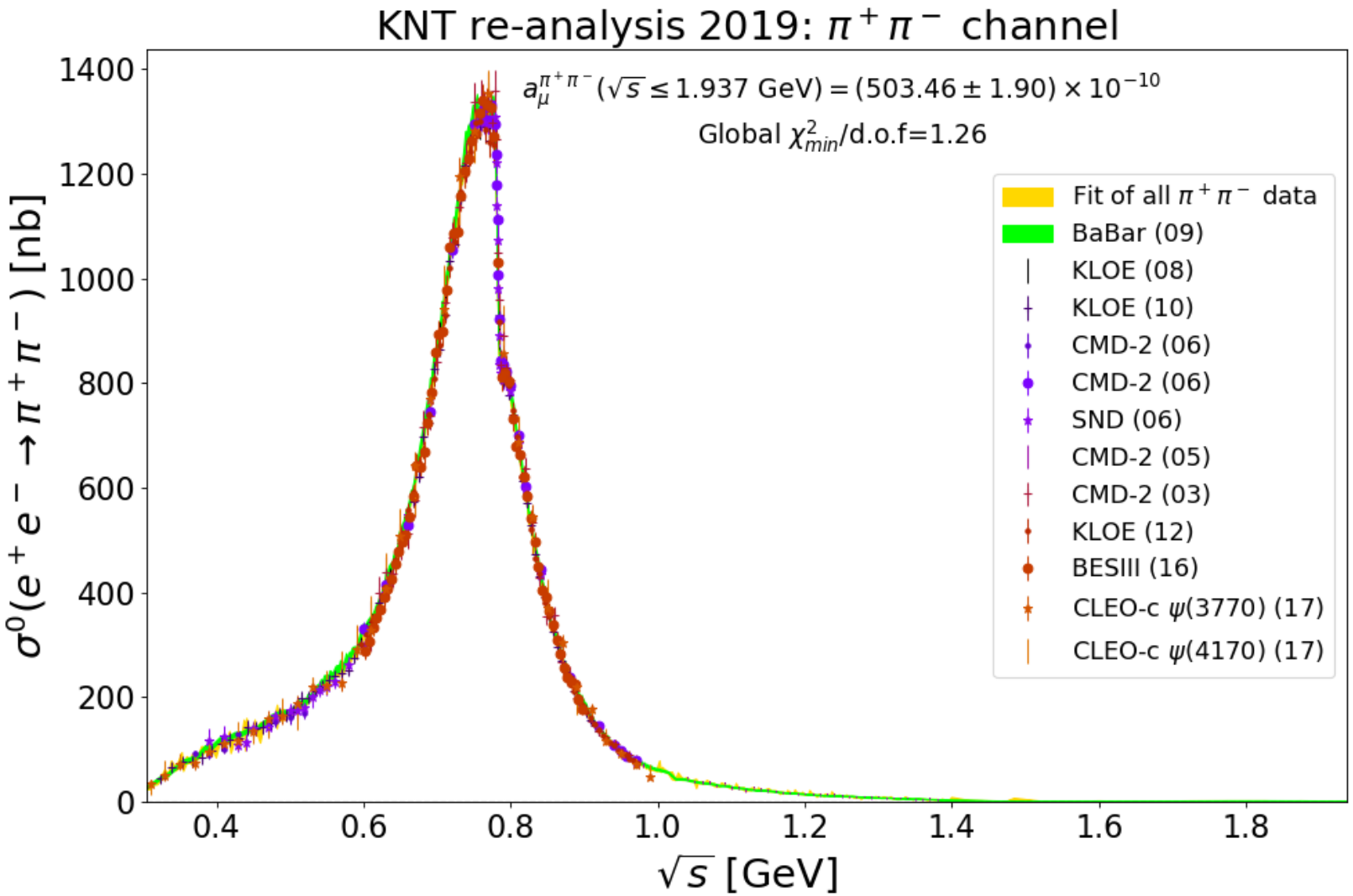
We have included new data sets from $\sim$ 30 papers (for KNT18) ($\sim$ 40 for KNT19 (preliminary)) in addition to those included in the HLMNT11 analysis

We have included $\sim$ 30 hadronic final states

At $2 \lesssim \sqrt{s} \lesssim 11$ GeV, we use inclusively measured data

At highest energies $\gtrsim 11$ GeV, we use pQCD

$\sigma^0(e^+e^- \to \pi^+\pi^-)$ data: overall picture

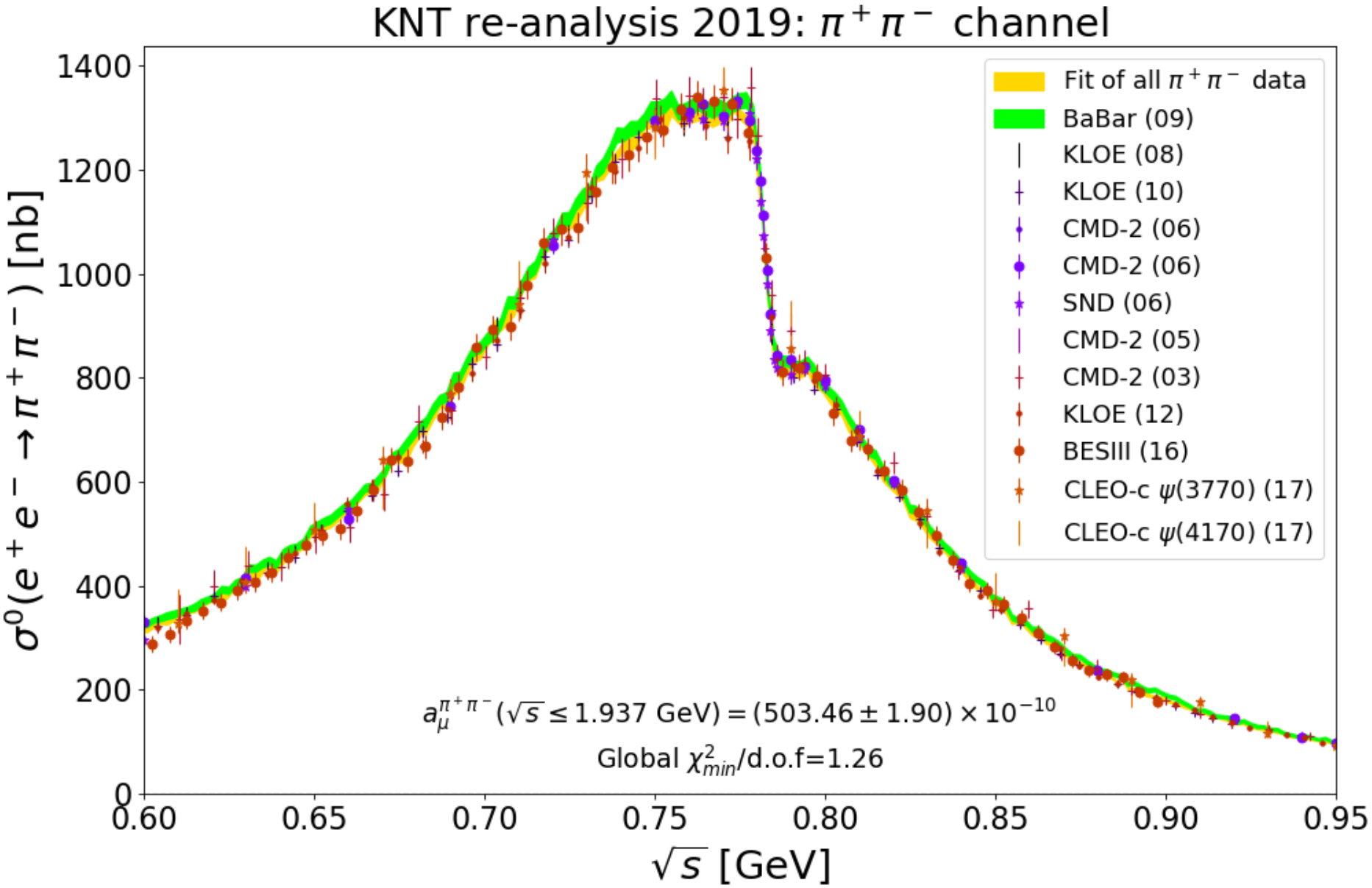


New data from CLEO-c [Phys. Rev. D97 (2018) 032012]

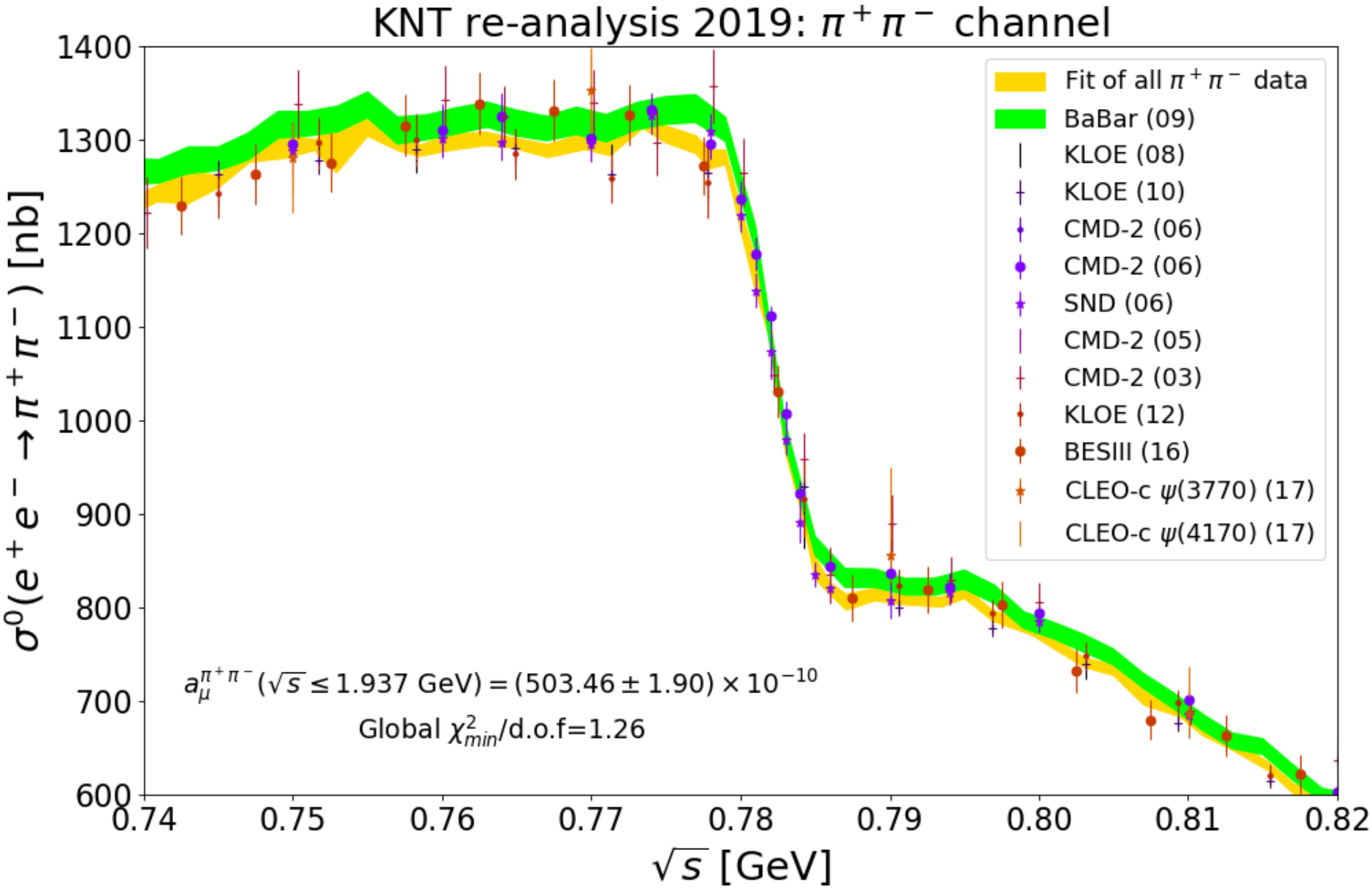
KNT18: $a_\mu(\pi^+\pi^-) = (503.11 \pm 1.97) \times 10^{-10}$

KNT19 (preliminary): $a_\mu(\pi^+\pi^-) = (503.46 \pm 1.90) \times 10^{-10}$

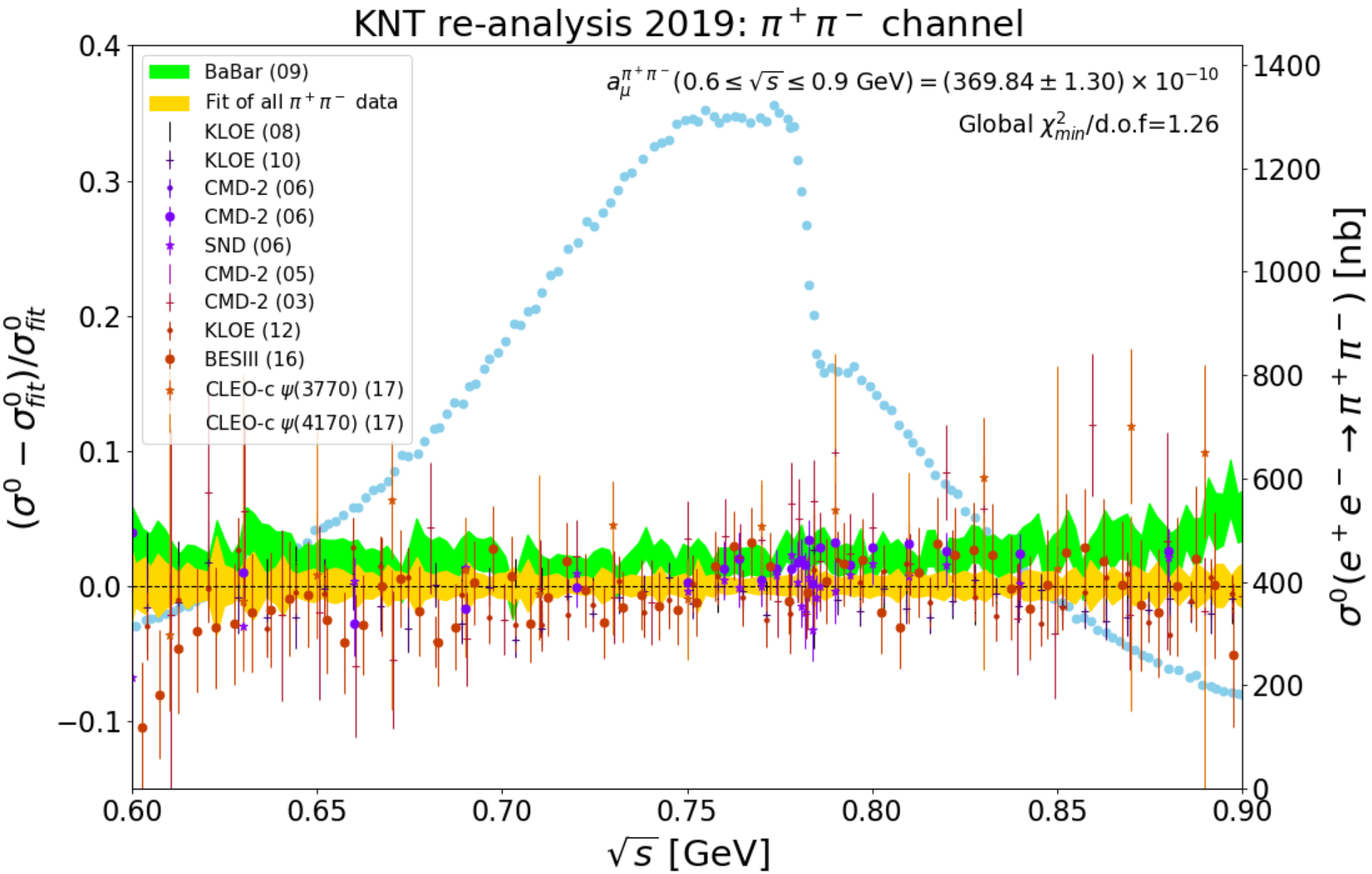
$\sigma^0(e^+e^- \to \pi^+\pi^-)$: ρ-meson region



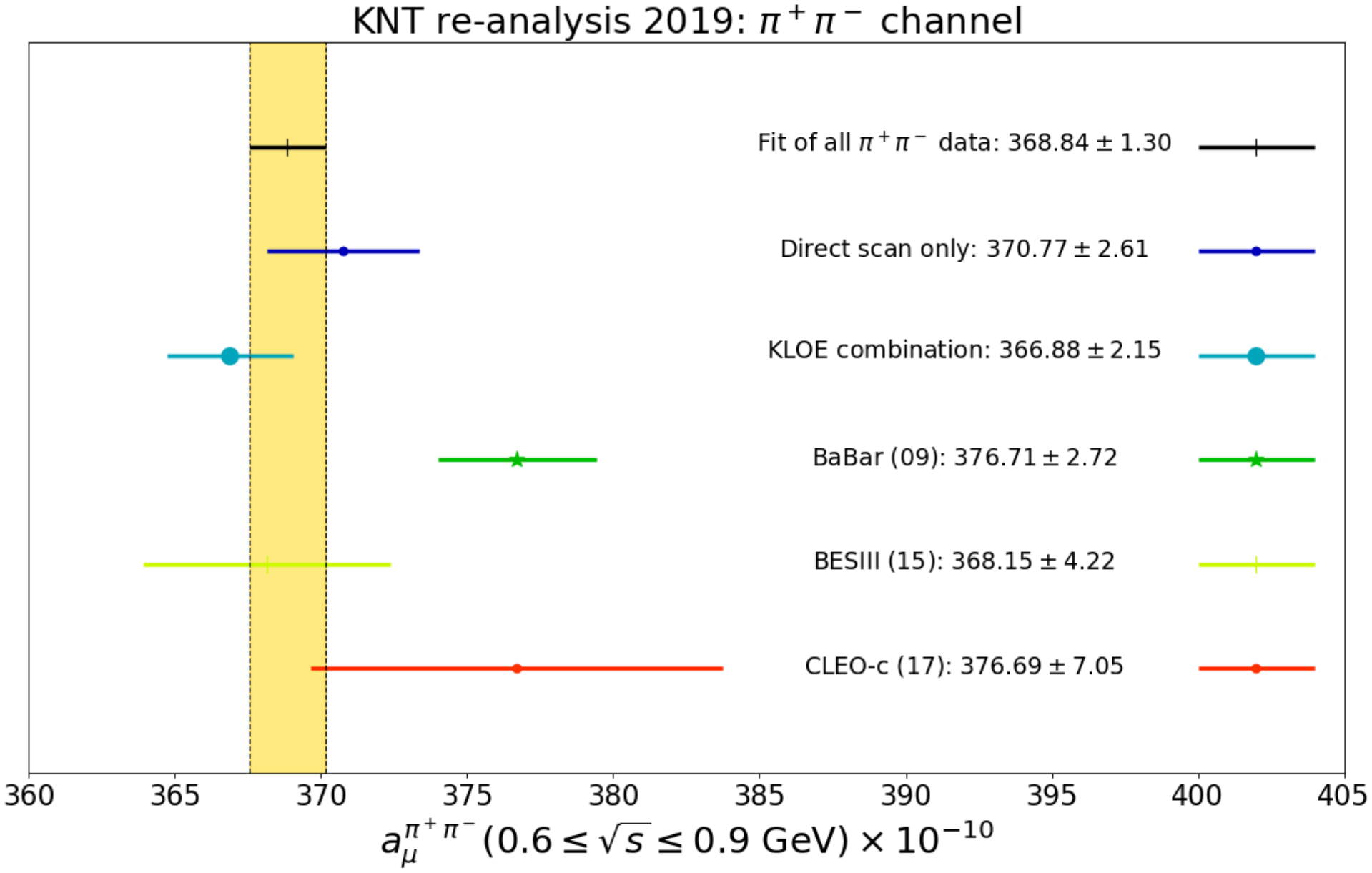
$\sigma^0(e^+e^- \to \pi^+\pi^-)$: ρ-ω interference region



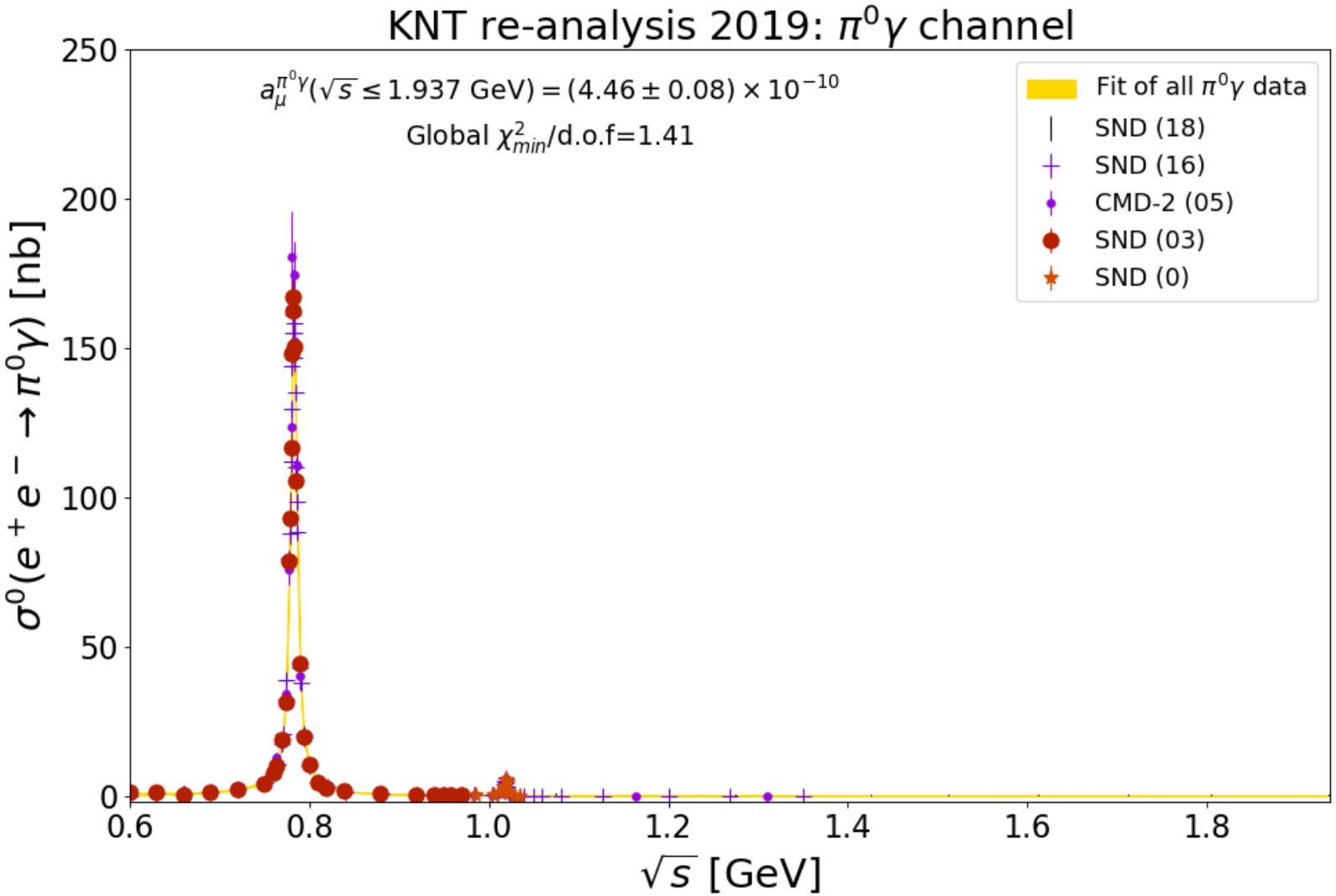
$\sigma^0(e^+e^- \to \pi^+\pi^-)$: relative differences



Contribution to $(g-2)_\mu$ from $\pi^+\pi^-$ channel



$\sigma^0(e^+e^- \to \pi^0\gamma)$ data

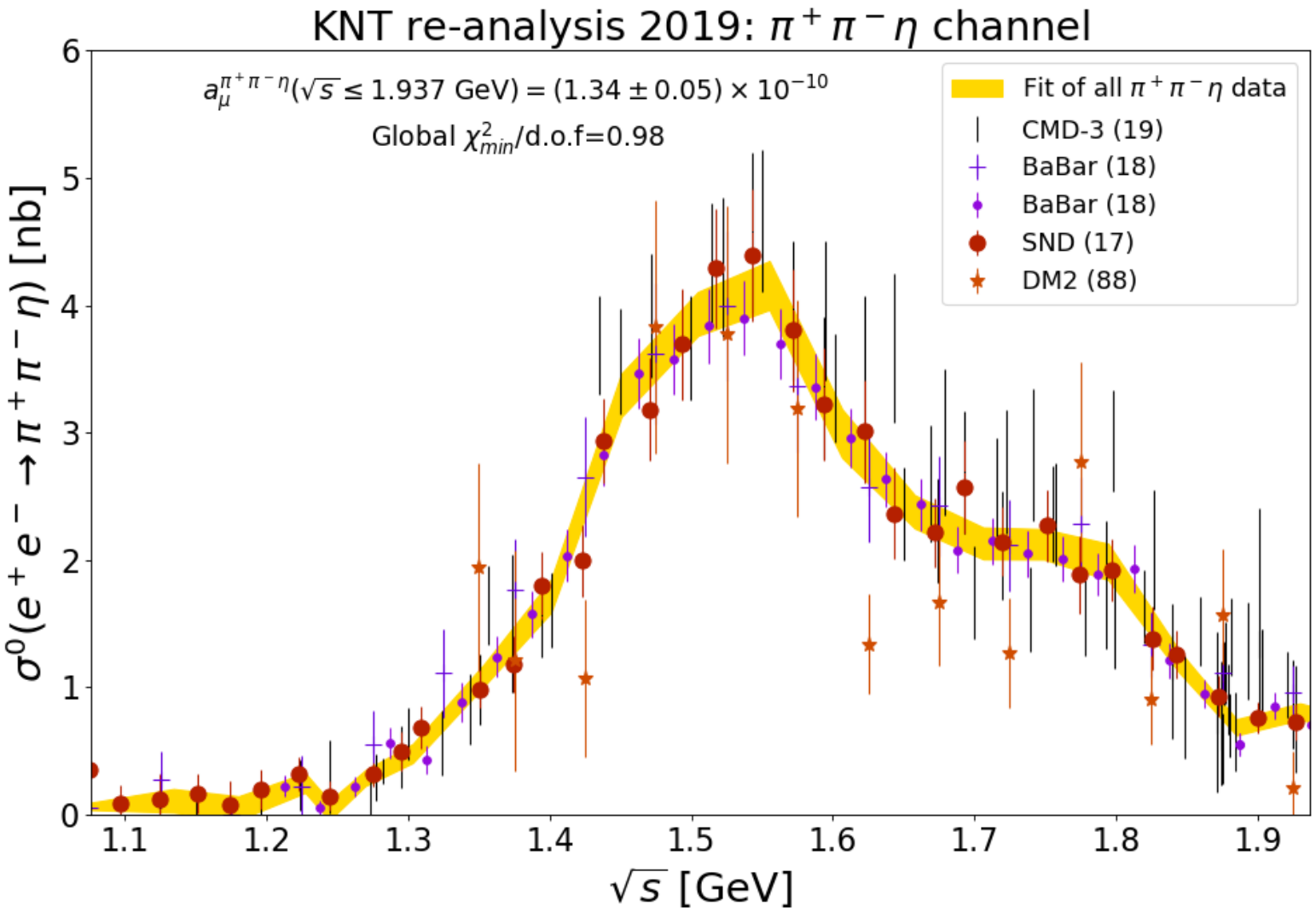


New data from SND [Phys. Rev. D98 (2018) 112001]

KNT18: $a_\mu(\pi^0\gamma) = (4.46 \pm 0.08) \times 10^{-10}$, $\chi^2_{\rm min}/{\rm dof} = 1.44$

KNT19 (preliminary): $a_\mu(\pi^0\gamma) = (4.46 \pm 0.08) \times 10^{-10}$, $\chi^2_{\rm min}/{\rm dof} = 1.41$

$\sigma^0(e^+e^- \to \pi^+\pi^-\eta)$ data

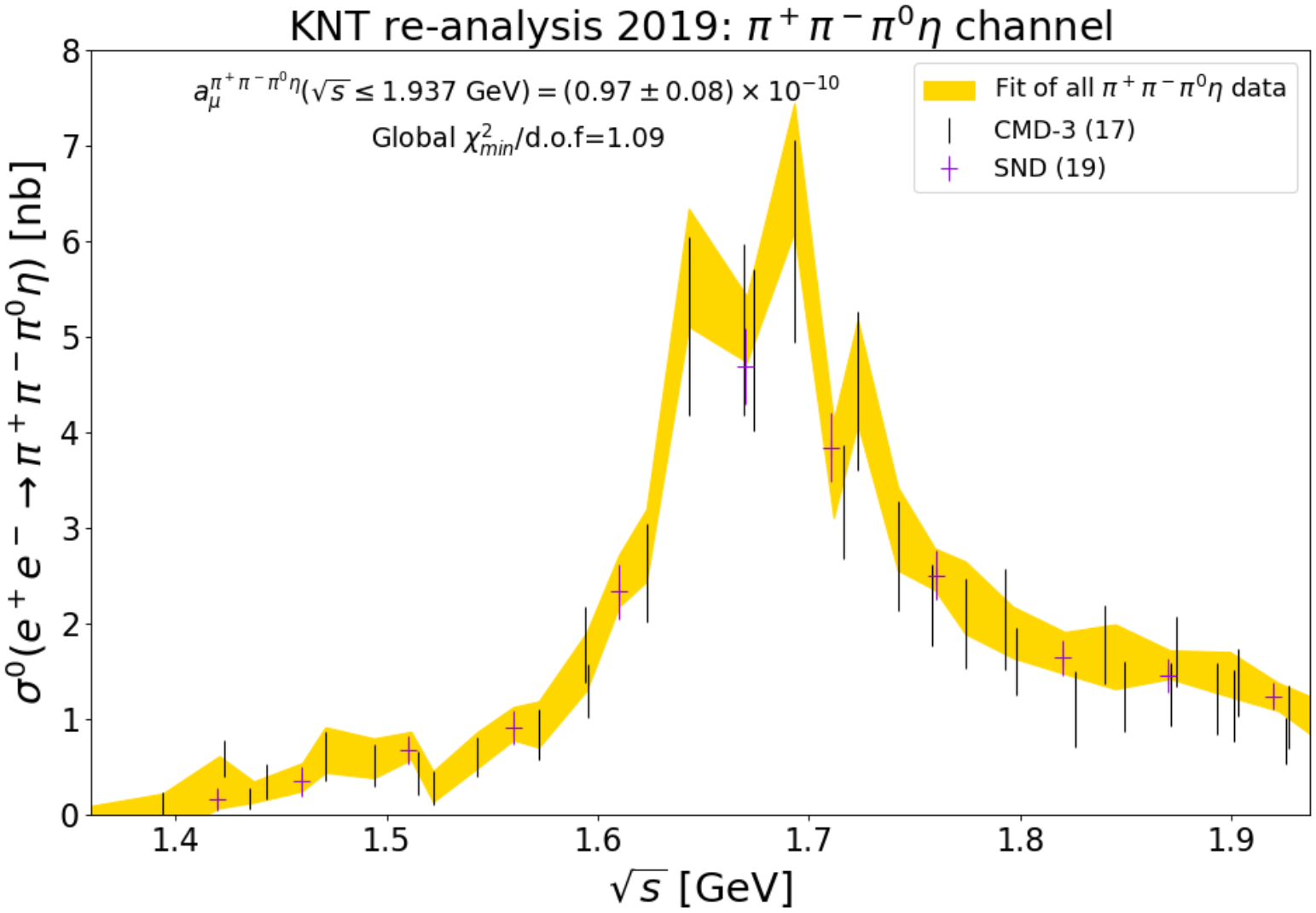


New data from BaBar [PRD98(2018)112015] & CMD-3 [arXiv:1907.08002]

KNT18: $a_\mu(\pi^+\pi^-\eta) = (1.29 \pm 0.06) \times 10^{-10}$, $\chi^2_{\rm min}/{\rm dof} = 1.06$

KNT19 (preliminary): $a_\mu(\pi^+\pi^-\eta) = (1.34 \pm 0.05) \times 10^{-10}$, $\chi^2_{\rm min}/{\rm dof} = 0.98$

$\sigma^0(e^+e^- \to \pi^+\pi^-\pi^0\eta)$ data

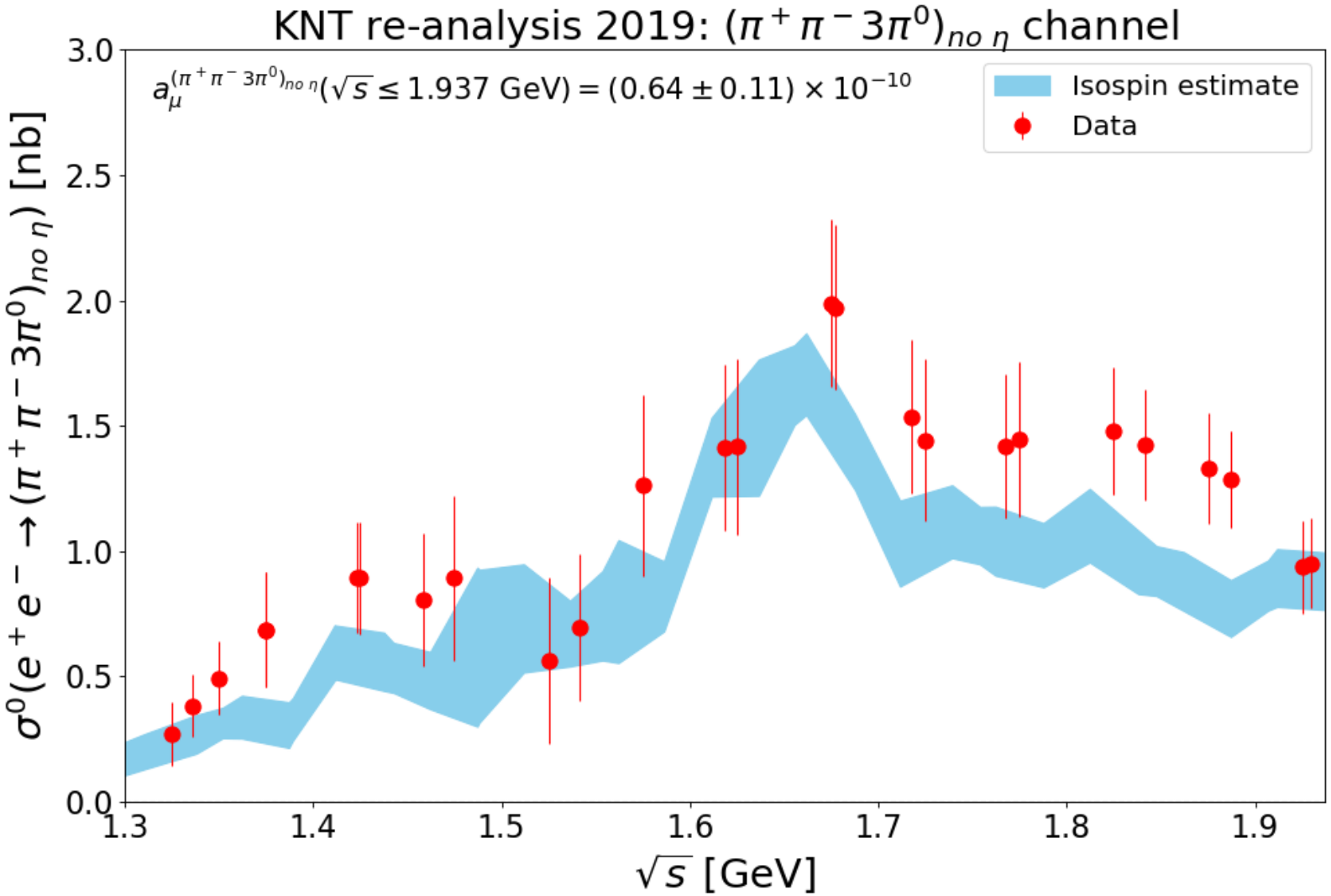


New data from SND [Phys. Rev. D99 (2019) 112004]

KNT18: $a_\mu((\pi^+\pi^-\pi^0\eta)_{\text{no }\omega}) = (0.60 \pm 0.15) \times 10^{-10}$

KNT19 (preliminary): $a_\mu((\pi^+\pi^-\pi^0\eta)_{\text{no }\omega}) = (0.71 \pm 0.08) \times 10^{-10}$, $\chi^2_{\text{min}}/\text{dof} = 1.09$

$\sigma^0(e^+e^- \to (\pi^+\pi^-3\pi^0)_{\text{no }\eta})$ data



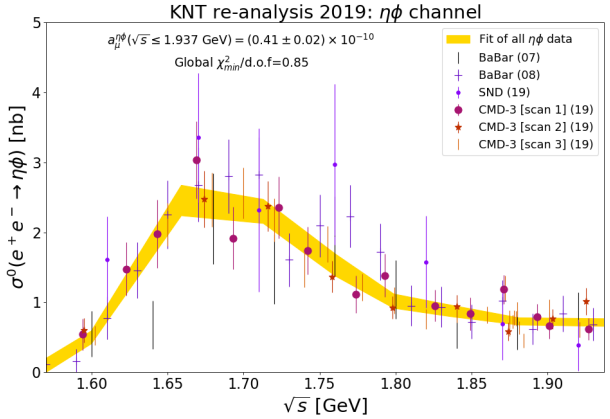
New data from BaBar [Phys. Rev. D98 (2018) 112015]

KNT18: $a_\mu((\pi^+\pi^-3\pi^0)_{\text{no }\eta}) = (0.50 \pm 0.04) \times 10^{-10}$ (isospin estimate)

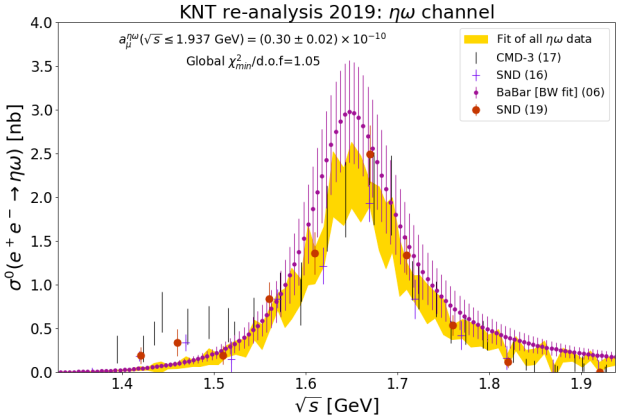
KNT19 (preliminary): $a_\mu((\pi^+\pi^-3\pi^0)_{\text{no }\eta}) = (0.64 \pm 0.11) \times 10^{-10}$

(Some of) other updated channels (KNT19 preliminary)

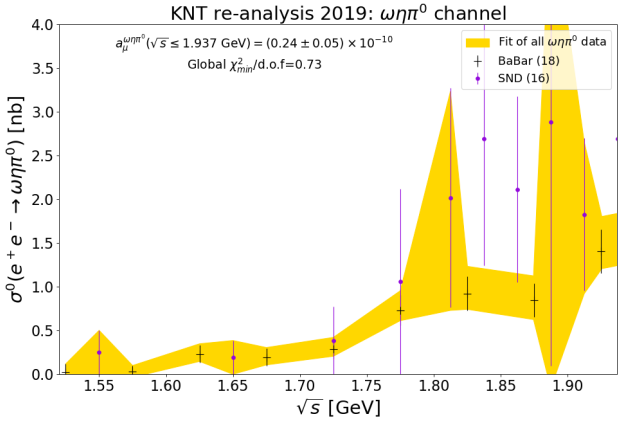
$\eta\phi$



$\eta\omega$

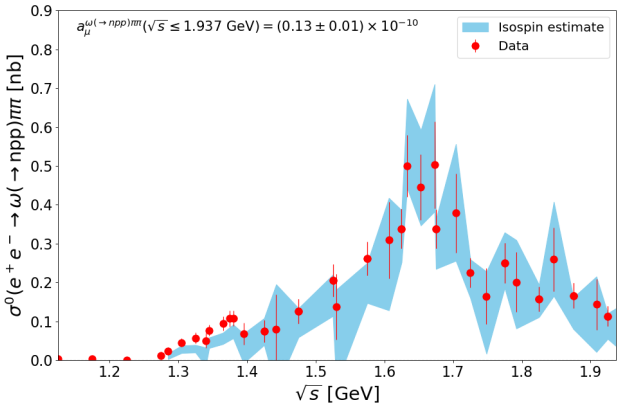


$\omega\eta\pi^0$

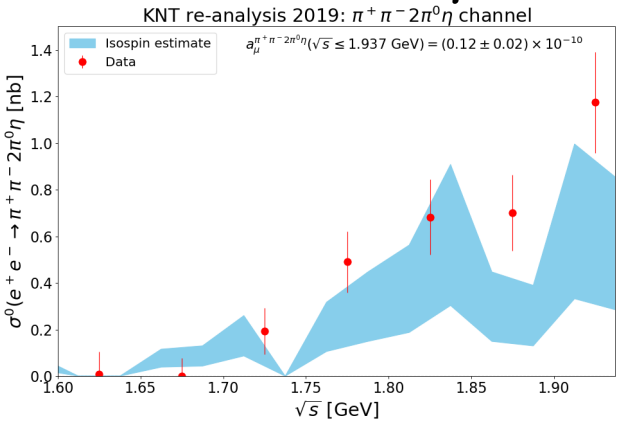


	$\eta\phi$	$\eta\omega$	$\omega\eta\pi^0$
KNT19 prelim.:	0.41(2)	0.30(2)	0.24(5)
KNT18:	0.42(3)	0.31(3)	0.35(9)

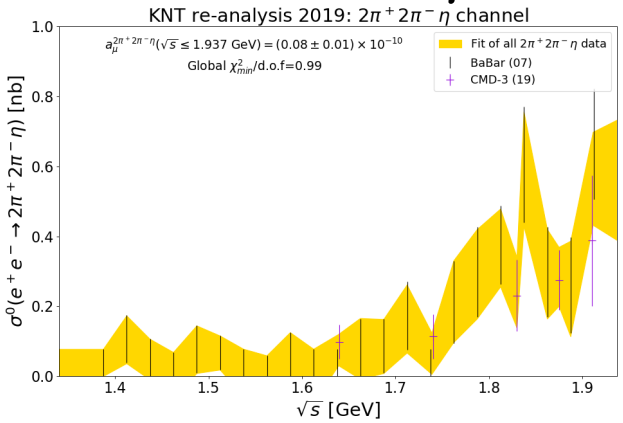
$\omega(\to$ non-purely-pionic$)\pi\pi$



$\pi^+\pi^-2\pi^0\eta$



$2\pi^+2\pi^-\eta$



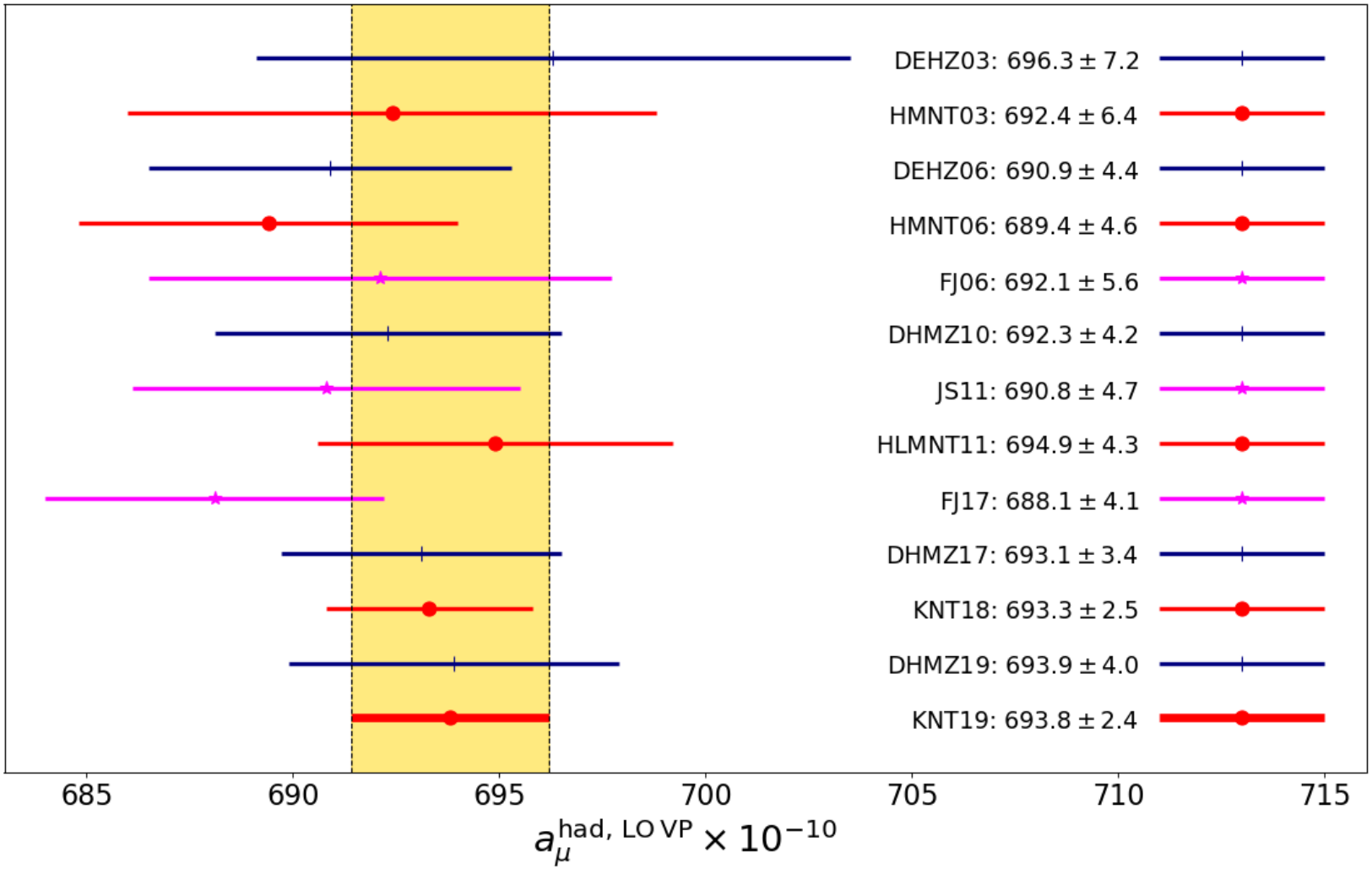
	$\omega(\to$ non-purely-pionic$)\pi\pi$	$\pi^+\pi^-2\pi^0\eta$	$2\pi^+2\pi^-\eta$
KNT19 prelim.:	0.13(1)	0.12(2)	0.08(1)
KNT18:	0.10(2) (isospin estimate)	0.08(4) (isospin estimate)	0.08(1)

Hadronic VP Contributions: comparison

Adding up all the channels, pQCD & narrow resonances contributions, we get

$a_\mu^{\text{had, LO VP}}(\text{KNT19 prelim.}) = (693.8 \pm 2.4) \times 10^{-10}$ (KNT18: $(693.3 \pm 2.5) \times 10^{-10}$)

$a_\mu^{\text{had, NLO VP}}(\text{KNT19 prelim.}) = (-9.83 \pm 0.04) \times 10^{-10}$ (KNT18: $(-9.82 \pm 0.04) \times 10^{-10}$)



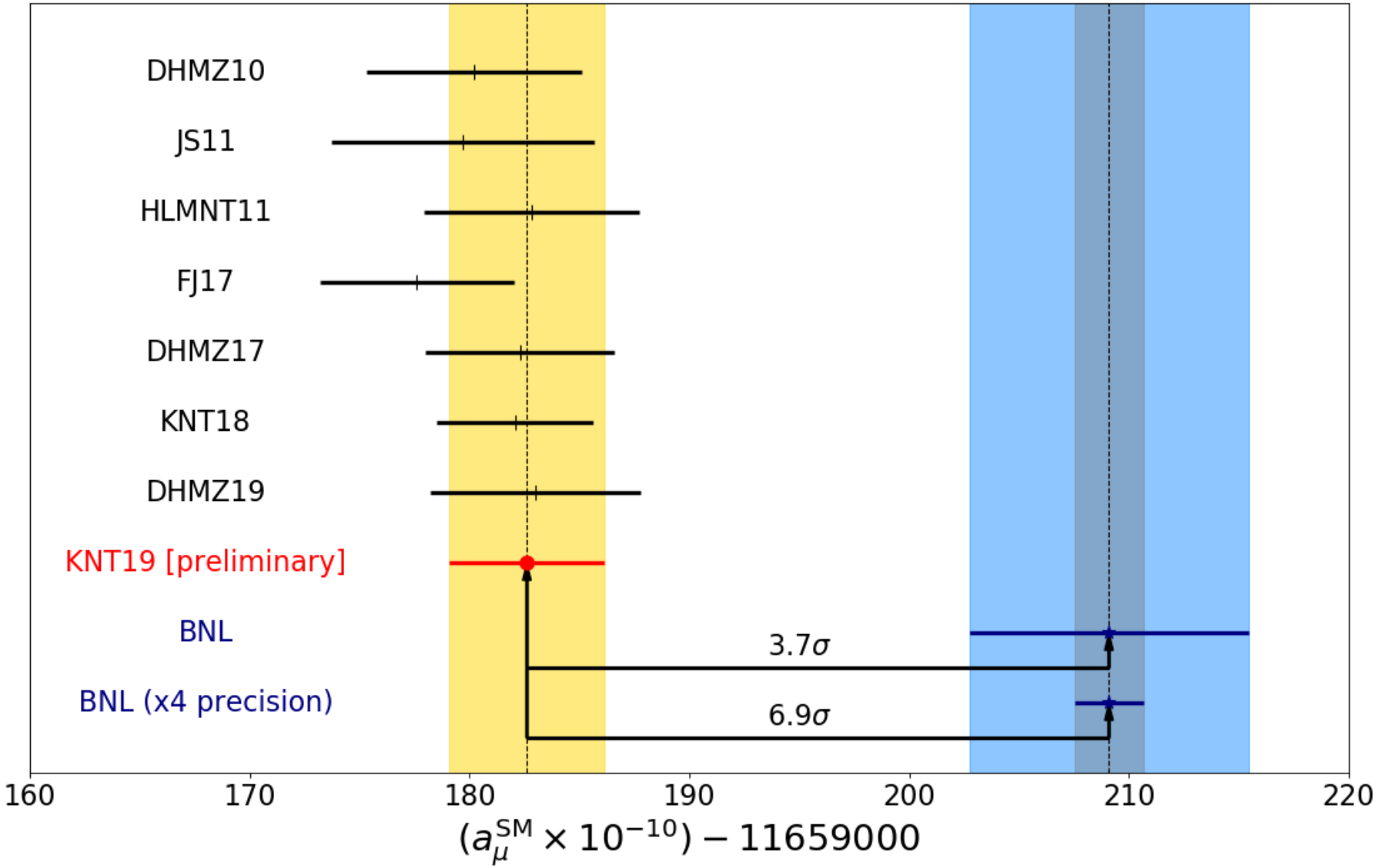
Breakdown of the SM prediction for $(g-2)_\mu$

Updating only the LO & NLO HVP wrt to KNT18…

	2018		2019	
QED			11658471.90 (0.01)	[Phys. Rev. D 97 (2018) 036001]
EW			15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL			9.80 (2.60)	[EPJ Web Conf. 118 (2016) 01016]
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]
	KNT18 [Phys.Rev. D97 (2018) 114025]		KNT19 [preliminary]	
LO HVP	693.27 (2.46)	→	693.84 (2.41)	This work
NLO HVP	-9.82 (0.04)	→	-9.83 (0.04)	This work
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144]
Theory total	11659182.04 (3.56)	→	11659182.61 (3.52)	This work
Experiment			11659209.10 (6.33)	World average
Exp - Theory	27.1 (7.3)	→	26.5 (7.2)	This work
Δa_μ	3.7σ	→	3.7σ	This work

Exp. vs SM prediction for $(g-2)_\mu$

Updating only the LO & NLO HVP wrt to KNT18…



Summary

- Standard Model prediction for $(g-2)_\mu$: $\gtrsim 3.5\sigma$ deviation from measured value $\Longrightarrow$ New Physics?
- KNT19 update (preliminary): 3.7σ.
  Only very slightly shifted (compared to KNT18) due to introduction of new $e^+e^- \to$ hadrons data
- To establish the $(g-2)_\mu$ anomaly more firmly, more data for $e^+e^- \to \pi^+\pi^-$ welcome (from CMD-3, SND, Belle II, ...)

Backup Slides

Data Combination in KNT18 (1)

Data combination: setup

⇒ Re-bin data into *clusters*

→ Scan cluster sizes for preferred solution (error, χ^2, check by sight...)

⇒ Correlated data beginning to dominate full data compilation...

→ Non-trivial, energy dependent influence on both mean value and error estimate

KNT18 prescription

- Construct full covariance matrices for each channel & entire compilation
  ⇒ Framework available for inclusion of any and all inter-experimental correlations
- If experiment does not provide matrices...
  → Statistics occupy diagonal elements only
  → Systematics are 100% correlated
- If experiment does provide matrices...
  → Use all information provided
- Use correlations to full capacity

Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ Workshop' at Mainz, June 18-22, 2018

Data Combination in KNT18 (2)

Data combination consideration

Question:

What are the main points of concern when combining experimental data to evaluate $a_\mu^{\text{had, VP}}$?

⇒ When combining data...

→ ...how to best combine large amounts of data from different experiments

→ ...the correct implementation of correlated uncertainties (statistical and systematic)

→ ...finding a solution that is free from bias

d'Agostini bias [Nucl.Instrum.Meth. A346 (1994) 306-311]

$$x_1 = 0.9 \pm \delta x_1 \quad x_2 = 1.1 \pm \delta x_2 \qquad C_{\text{sys}} = \begin{pmatrix} p^2 x_1^2 & p^2 x_1 x_2 \\ p^2 x_2 x_1 & p^2 x_2^2 \end{pmatrix} \qquad \Rightarrow \bar{x} \simeq 0.98 \text{ (systematic bias)}$$

(Normalisation uncertainties defined by data)

Effect worsened with full, iterative data combination

Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ Workshop' at Mainz, June 18-22, 2018

Data Combination in KNT18 (3)

Data combination consideration

Question:

What are the main points of concern when combining experimental data to evaluate $a_\mu^{\text{had, VP}}$?

⇒ When combining data...

→ ...how to best combine large amounts of data from different experiments

→ ...the correct implementation of correlated uncertainties (statistical and systematic)

→ ...finding a solution that is free from bias

Fixed matrix method [R. D. Ball et al. [NNPDF Collaboration], JHEP 1005 (2010) 075.]

$$x_1 = 0.9 \pm \delta x_1 \qquad C_{\text{sys}} = \begin{pmatrix} p^2\bar{x}^2 & p^2\bar{x}^2 \\ p^2\bar{x}^2 & p^2\bar{x}^2 \end{pmatrix} \qquad \Rightarrow \bar{x} = 1.00 \text{ (systematic bias)}$$

$$x_2 = 1.1 \pm \delta x_2$$

(Normalisation uncertainties defined by estimator)

Redefinition repeated at each stage of iterative data combination

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Data Combination in KNT18 (4)

Linear χ^2 minimisation [KNT18: arXiv:1802.02995, PRD (in press)]

⇒ Clusters are defined to have linear cross section

→ Fix covariance matrix with linear interpolants at each iteration (extrapolate at boundary)

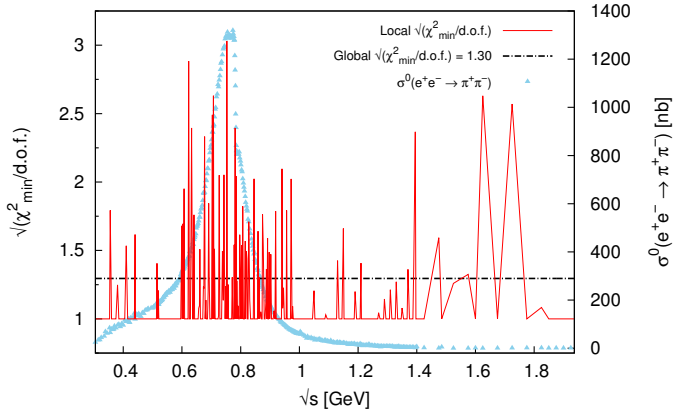
$$\chi^2 = \sum_{i=1}^{N_{\rm tot}} \sum_{j=1}^{N_{\rm tot}} \left(R_i^{(m)} - \mathcal{R}_m^i\right) \mathbf{C}^{-1}\left(i^{(m)}, j^{(n)}\right) \left(R_j^{(n)} - \mathcal{R}_n^j\right)$$

⇒ Through correlations and linearisation, result is the minimised solution of all available uncertainty information

→ ... through a method that has been shown to avoid d'Agostini bias

⇒ The flexibly of the fit to vary due to the energy dependent, correlated uncertainties benefits the combination

→ ... and any data tensions are reflected in a local and global $\chi^2_{\rm min}/{\rm d.o.f.}$ error inflation



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