Pseudoscalar-pole contributions to HLbL and longitudinal short-distance constraints

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Based on work done in collaboration with
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FOR FUNDAMENTAL PHYSICS

Motivation

- Mixed- and high-energy regions need to be estimated for a full evaluation of HLbL
- Issue: PS-pole contributions do not have the asymptotic behaviour dictated by QCD

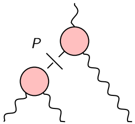
  Melnikov and Vainshtein, Phys. Rev. D70, 113006 (2004)
- Effective solution proposed by M&V: incompatible with low-energy properties of the HLbL tensor
- Here: summation over infinite tower of PS poles to restore asymptotic behaviour

Outline

- Review of the short-distance contraints (SDCs)
  - SDCs on pseudoscalars transition form factors
  - SDCs on HLbL
- Regge Model for the pseudoscalars transition form factors
  - pion
  - eta/eta'
- Outlook and conclusion

Dominant pseudoscalar-pole contributions

PS poles:



$$P = \pi^0,\ \eta,\ \eta'$$

Hadronic light-by-light tensor:

$$\Pi^{\mu\nu\lambda\sigma} = \sum_{i=1}^{54} \hat{T}_i^{\mu\nu\lambda\sigma} \hat{\Pi}_i$$

Colangelo et al., JHEP 1704 (2017) 161

Relevant for g-2:

$$\hat{\Pi}_1^{\pi^0\text{-pole}}(q_1, q_2, q_3) = -\frac{\mathcal{F}_{P\gamma^*\gamma^*}(-Q_1^2, -Q_2^2)\mathcal{F}_{P\gamma\gamma^*}(-Q_3^2)}{Q_3^2 + M_P^2}$$

Hadronic content in PS transition form factor (TFF):

$$i \int \mathrm{d}x\, e^{iq_1 \cdot x} \langle 0 | T \{ j_\mu^{\text{em}}(x) j_\nu^{\text{em}}(0) \} | P(q_1 + q_2) \rangle =: -\epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \mathcal{F}_{P\gamma^*\gamma^*}(q_1^2, q_2^2)$$

Normalized to the 2γ decays:

$$\boxed{\mathcal{F}_{P\gamma\gamma}^2(0,0) = \frac{4}{\pi\alpha^2 M_P^3}\Gamma(P \to 2\gamma)}$$

Short-distance constraints on the TFFs

Asymptotic behaviours:

- Singly-virtual TFFs

$$\lim_{Q^2\to\infty} Q^2 \mathcal{F}_{P\gamma\gamma^*}(-Q^2) = 12 \sum_{a=0,3,8} C_a F_P^a$$

Brodsky and Lepage (1979-1981)

- Doubly-virtual TFFs

$$\lim_{Q^2\to\infty} Q^2 \mathcal{F}_{P\gamma^*\gamma^*}(-Q^2,-Q^2) = 4 \sum_{a=0,3,8} C_a F_P^a$$

Novikov et al., Nucl. Phys. B 237, 525 (1984)

with the coefficients

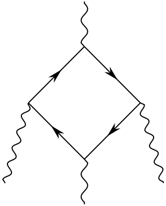
$$C_0 = \frac{2}{3\sqrt{6}},\ C_3 = \frac{1}{6},\ C_8 = \frac{1}{6\sqrt{3}}$$

and the decay constants (considering η-η' mixing):

$$\langle 0 | A^a_\mu(0) | P(p) \rangle =: i p_\mu F_P^a$$

Short-distance contraint on HLbL: high-energy region

Perturbative QCD quark loop:



- $Q_1^2 \approx Q_2^2 \approx Q_3^2 \gg \Lambda_{\text{QCD}}^2$: leading term in the OPE for HLbL corresponds to the quark loop

  Bijnens et al., 1908.03331 (2019)

- Analytical expression and decomposition into scalar functions of the quark loop is known

  Hoferichter, Stoffer

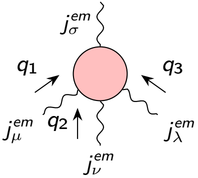
High-energy constraint on HLbL (isospin component $a = 0, 3, 8$):

$$\lim_{Q^2 \to \infty} Q^4\, \hat{\Pi}_1^{(a)}(-Q^2, -Q^2, -Q^2) = -\frac{4}{9\pi^2} \frac{C_a^2}{C_0^2 + C_3^2 + C_8^2}$$

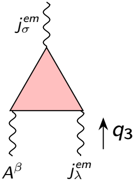
Short-distance constraint on HLbL: mixed region

$Q^2 \equiv Q_1^2 \approx Q_2^2 \gg Q_3^2$:

Vainshtein, Phys. Lett. B569, 187 (2003)
Czarnecki et al., Phys. Rev. D67, 073006 (2003)
Knecht et al., JHEP 03, 035 (2004)



$\xrightarrow{\text{OPE}}$ $\frac{2}{\hat{q}^2}\epsilon_{\mu\nu\alpha\beta}\,\hat{q}^\alpha$



with $\hat{q} := \frac{q_1 - q_2}{2}$

$$\Pi_{\mu\nu\sigma\lambda}(q_1, q_2, q_3) = \frac{8}{\hat{q}^2}\epsilon_{\mu\nu\alpha\beta}\hat{q}^\alpha W^\beta_{\sigma\lambda}(-q_3, q_4)\sum_a C_a^2$$

- $W_{\mu\nu\lambda}(q_1, q_2)$: 3 transversal and 1 longitudinal structure
- Axial anomaly: $\hat{\Pi}_1 \ni w_L(q_1^2, q_2^2, (q_1+q_2)^2) = \frac{2N_c}{(q_1+q_2)^2}$
- Mixed-region constraint on HLbL: Melnikov and Vainshtein, Phys. Rev. D70, 113006 (2004)

$$\lim_{Q_3^2\to\infty}\lim_{Q^2\to\infty} Q^2 Q_3^2\,\hat{\Pi}_1^{(a)}(-Q^2, -Q^2, -Q_3^2) = -\frac{2}{3\pi^2}\,\frac{C_a^2}{C_0^2 + C_3^2 + C_8^2}$$

Tension between the constraints

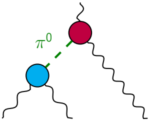
$Q^2 \equiv Q_1^2 \approx Q_2^2 \gg Q_3^2 \gg \Lambda^2_{\mathrm{QCD}}$, 3rd isospin component

- Constraint on the HLbL tensor:

$$\hat{\Pi}_1^{(3)} \longrightarrow -\frac{1}{6\pi^2 Q^2 Q_3^2}$$

- Constraints on the pion pole:

$$\hat{\Pi}_1^{\pi^0\text{-pole}} = -\frac{\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_1^2,-Q_2^2)\mathcal{F}_{\pi^0\gamma\gamma^*}(-Q_3^2)}{Q_3^2+M_\pi^2} \longrightarrow -\frac{2f_\pi}{3Q^2}\,\frac{1}{Q_3^2}\,\frac{2f_\pi}{Q_3^2}$$



Tension between the constraints

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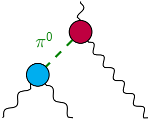
- Constraint on the HLbL tensor:

$$\hat{\Pi}_1^{(3)} \longrightarrow -\frac{1}{6\pi^2 Q^2 Q_3^2} = -\frac{2f_\pi}{3Q^2}\,\frac{1}{Q_3^2}\,\frac{1}{4\pi^2 f_\pi}$$

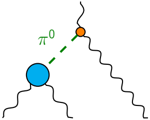
- Constraints on the pion pole:

$$\hat{\Pi}_1^{\pi^0\text{-pole}} = -\frac{\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_1^2,-Q_2^2)\mathcal{F}_{\pi^0\gamma\gamma^*}(-Q_3^2)}{Q_3^2+M_\pi^2} \longrightarrow -\frac{2f_\pi}{3Q^2}\,\frac{1}{Q_3^2}\,\frac{2f_\pi}{Q_3^2}$$

Melnikov & Vainshtein's effective solution:



$\xrightarrow{\text{M\&V}}$



$$a_\mu^{\pi^0-\text{pole}} = 62.6 \cdot 10^{-11} \qquad \longrightarrow \qquad 76.5 \cdot 10^{-11}$$

Incompatible with low-energy properties of HLbL

Solution consistent with the dispersive approach

Infinite tower of poles to restore asymptotic behaviour:

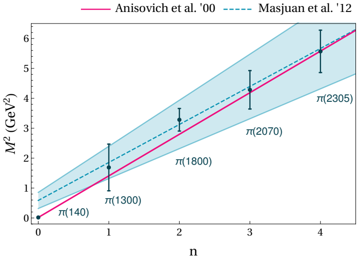
$$\lim_{Q^2\to\infty}\sum_{n=0}^{\infty}\hat{\Pi}_1^{\pi(n)\text{-pole}}(-Q^2,-Q^2,-Q^2)=\lim_{Q^2\to\infty}\sum_{n=0}^{\infty}\frac{\mathcal{F}_{\pi(n)\gamma^*\gamma^*}(-Q^2,-Q^2)\,\mathcal{F}_{\pi(n)\gamma\gamma^*}(-Q^2)}{-Q^2-M_\pi^2(n)}=\frac{-1}{9\pi^2Q^4}$$

$$\lim_{Q_3^2\to\infty}\lim_{Q^2\to\infty}\sum_{n=0}^{\infty}\hat{\Pi}_1^{\pi(n)\text{-pole}}(-Q^2,-Q^2,-Q_3^2)=-\frac{1}{6\pi^2Q^2Q_3^2}$$

Radial Regge trajectory:

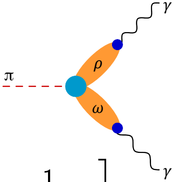
$$M^2_{\pi(n)}=\begin{cases}M_\pi^2 & n=0\\ M_{\text{fit}}^2+n\,\sigma_\pi^2 & n\geq 1\end{cases}$$

Masjuan et al., Phys. Rev. D85, 094006 (2012)



Aim: build minimal model for $\mathcal{F}_{\pi(n)\gamma^*\gamma^*}(-Q_1^2,-Q_2^2)$ that satisfies all SDCs

Regge model for the π^0 TFF



Standard large-N_c ansatz:

$$D_X^i := Q_i^2 + M_X^2$$

$$F_{\pi^0\gamma^*\gamma^*}(-Q_1^2, -Q_2^2) = \sum_{V_\rho, V_\omega} G_{\pi V_\rho V_\omega} F_{V_\rho} F_{V_\omega} \left[\frac{1}{D_{V_\rho}^1 D_{V_\omega}^2} + \frac{1}{D_{V_\omega}^1 D_{V_\rho}^2} \right]$$

- Assume Regge spectrum for vector mesons: $M_{V_{\rho/\omega}}^2 = M_{\rho(n)/\omega(n)}^2 = M_{\rho/\omega}^2 + n\,\sigma_{\rho/\omega}^2$

  Masjuan et al., Phys. Rev. D85, 094006 (2012)

- Compatible with B&L behaviour: $\mathcal{F}_{\pi^0\gamma\gamma^*}(\text{-}Q^2) \sim \frac{2F_\pi}{Q^2}$

- Possible to tune $G_{\pi V_\rho V_\omega}$, F_{V_ρ}, F_{V_ω} to have the required $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(\text{-}Q^2, \text{-}Q^2) \sim Q^{-2}$

  Broniowski and Ruiz Arriola, Phys. Rev. D74, 034008 (2006)

Regge model for the $\pi(n)$ TFFs

- n^{th} pion excitation only couples to n^{th} ρ and ω excitations
- Q^2-dependence in the numerator to account for eliminated resonances

$$\mathcal{F}_{\pi(n)\gamma^*\gamma^*}(-Q_1^2,-Q_2^2) = \frac{1}{8\pi^2 F_\pi}\Bigg\{\left(\frac{M_\rho^2 M_\omega^2}{D^1_{\rho(n)}D^2_{\omega(n)}} + \frac{M_\rho^2 M_\omega^2}{D^2_{\rho(n)}D^1_{\omega(n)}}\right) \times \left[c_{\text{anom}} + \left(c_A M^2_{+,\,n} + c_B M^2_{-,\,n}\right) + c_{\text{diag}}\frac{Q_1^2 Q_2^2}{(Q_+^2 + M^2_{\text{diag}})}\right] + \frac{Q_-^2}{Q_+^2}\left[c_{\text{BL}} + \left(c_A M^2_{-,\,n} + c_B M^2_{+,\,n}\right)\right] \times \left(\frac{M_\rho^2 M_\omega^2}{D^1_{\rho(n)}D^2_{\omega(n)}} - \frac{M_\rho^2 M_\omega^2}{D^2_{\rho(n)}D^1_{\omega(n)}}\right)\Bigg\}$$

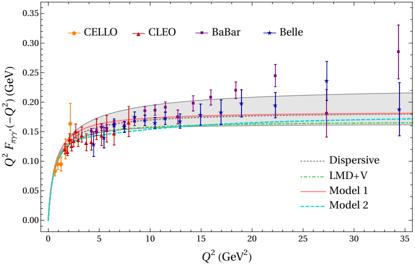
with $M^2_{\pm,\,n} = \frac{1}{2}\left(M^2_{\omega(n)} \pm M^2_{\rho(n)}\right)$, $Q_\pm^2 = Q_1^2 \pm Q_2^2$, $D_V^j = Q_j^2 + M_V^2$

- 5 parameters for 5 (short-distance) constraints
- 6^{th} parameter M_{diag} fitted to dispersive TFF

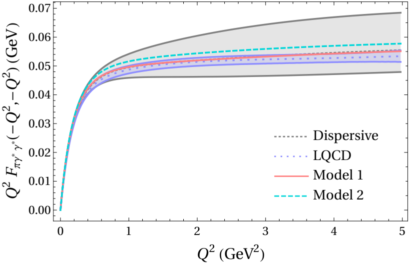
Hoferichter et al., JHEP 10, 141 (2018)

Pion TFF - comparison with data and literature

Singly-real TFF :



Doubly-virtual TFF:



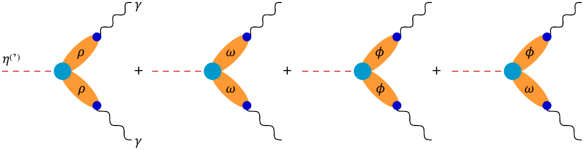
Dispersive: Hoferichter et al., JHEP 10, 141 (2018); LMD+V: Knecht and Nyffeler, Phys. Rev. D65, 073034 (2002)
Lattice QCD: Gérardin et al., 1903.09471 (2019)

Regge Model (Model 1) :

- satisfies all short-distance contraints
- is compatible with data and the dispersive pion TFF

Regge model for the eta/eta' TFFs

- Contributions also from same-mass vector mesons



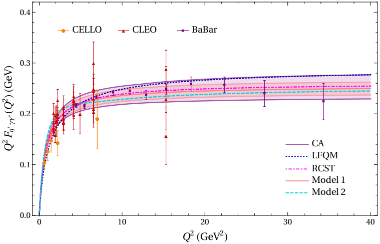
- Relative strength of the contributions determined from vector-meson dominance Lagrangians

Meißner, Phys. Rept. 161, 213 (1988)

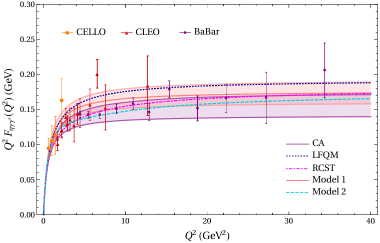
- η-η' and ω-ϕ mixings must be considered to fulfil all the constraints and be phenomenologically viable

Eta/eta' TFF - comparison with data and literature

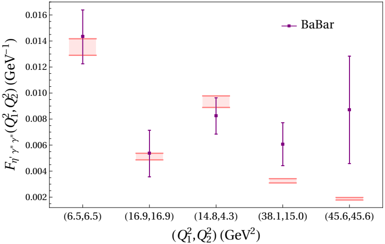
Singly-virtual η' TFF :



Singly-virtual η TFF:



Comparison to doubly-virtual η' data:



Canterbury Approximants:
Escribano et al. (2014-2016)

Light-front quark model:
Choi et al., 1903.01448v1 (2019)

Resonance chiral symmetric theory:
Czyż et al., Phys. Rev. D97, 016006 (2018)

(preliminary)

g-2 Results

The Regge models can now be used to compute the g-2 contributions

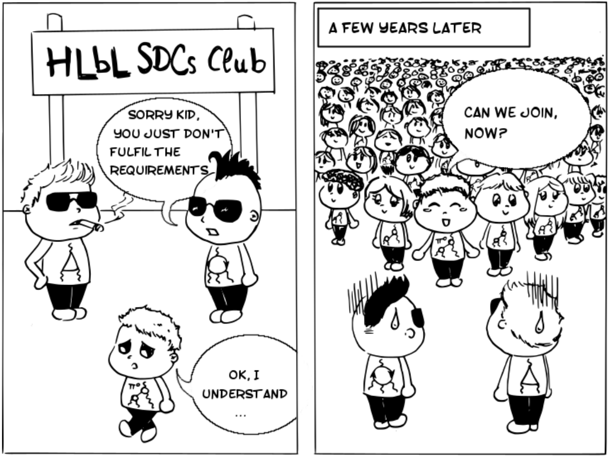
	$a_\mu^P(n=0)\cdot 10^{11}$	$\Delta a_\mu^P \cdot 10^{11} = \sum_{n=1}^{\infty} a_\mu^P(n)\cdot 10^{11}$	$\Delta a_\mu^P \cdot 10^{11}$ (M&V)
$P=\pi$	64.1	2.7	13.5
$P=\eta$	18.0	3.3	5.0
$P=\eta'$	14.8	6.6	5.0
Total	96.9	**12.6**	23.5

(preliminary!)

- The contribution Δa_μ^π is much smaller than the M&V prediction
- $\frac{\Delta a_\mu^\eta + \Delta a_\mu^{\eta'}}{\Delta a_\mu^{\pi^0}} \approx 3.7 \sim \frac{C_0^2 + C_8^2}{C_3^2} = 3$

Summary

- Short-distance constraints need to be implemented for a full evaluation of HLbL
- We implemented a Regge model for the pseudoscalar TFFs that satisfies all the requirements
- This leads to a shift $\Delta a_\mu^{\text{PS-poles}} \sim 12.6 \cdot 10^{-11}$ significantly smaller than the one predicted by M&V $\Delta a_\mu^{\text{PS-poles}} \sim 23.5 \cdot 10^{-11}$
- Uncertainty analysis to be finalized
- Matching to the pQCD quark loop $\rightarrow$ see talk by M. Hoferichter



Thank you for your attention!