



Trinity  
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The University of Dublin



# Lattice QCD and MUonE experiment

**Marina Krstić Marinković**  
Trinity College Dublin

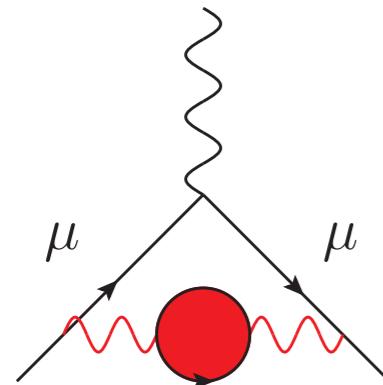
in collab. w. N. Cardoso (IST, Lisbon) and



Third Plenary Workshop of the Muon g-2 Theory Initiative  
Hadronic contributions to  $(g-2)_\mu$ , INT, Seattle, USA

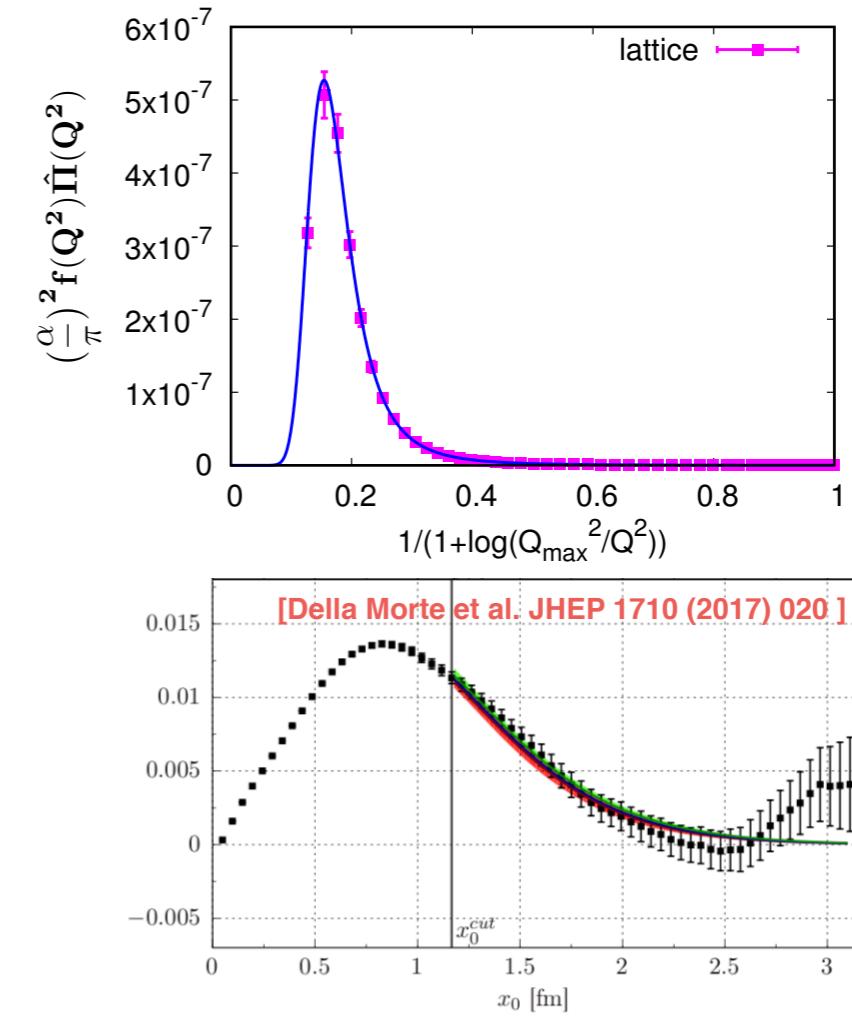
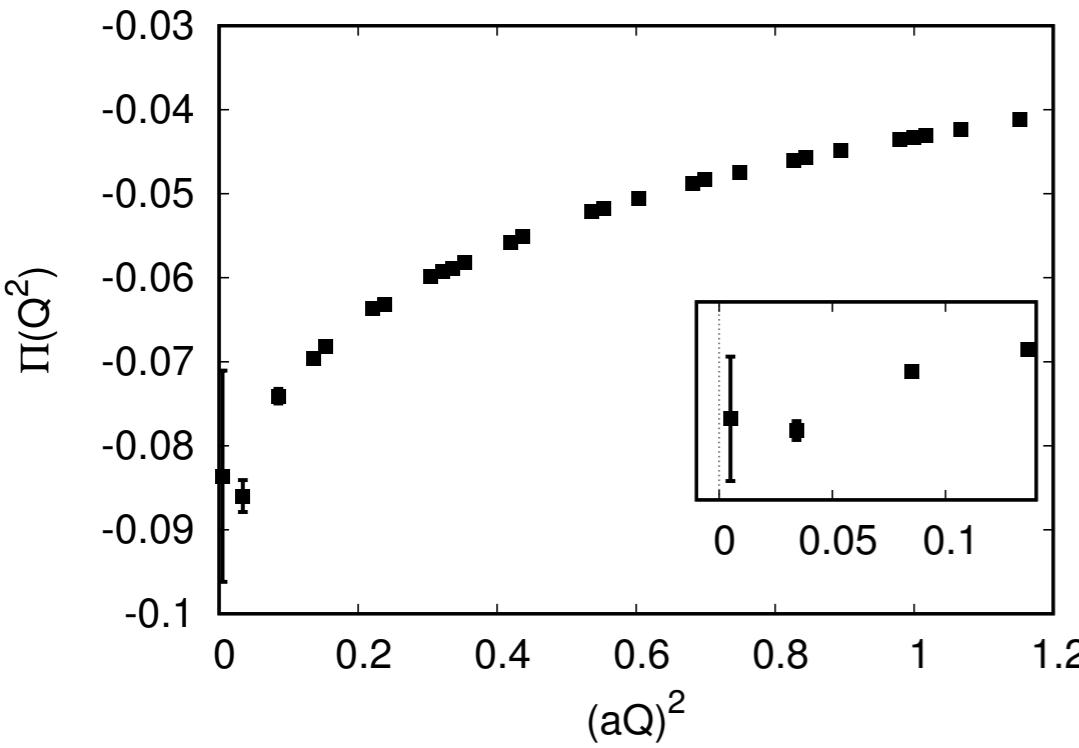
September 9 - 13, 2019

# The leading hadronic contribution - HVP



$$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 f(Q^2) \times \hat{\Pi}(Q^2)$$

$$f(Q^2) = m_\mu^2 Q^2 Z^3(Q^2) \frac{1 - Q^2 Z(Q^2)}{1 + m_\mu^2 Q^2 Z^2(Q^2)} \quad Z(Q^2) = \frac{\sqrt{Q^4 + 4m_\mu^2 Q^2} - Q^2}{2m_\mu^2 Q^2}$$



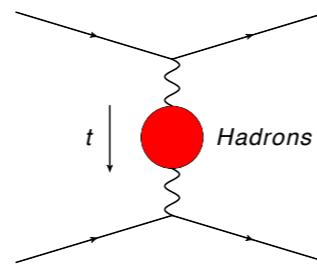
**Summary of ways to enhance StN ratio:**  
**[Talk by A. El-Khadra, Mon. 14.00]**

# MUonE project

- A high precision measurement of  $a_\mu^{\text{had, LO}}$   
with a 150 GeV  $\mu$  beam on  $e^-$  target at CERN



- In space-like momenta region
- Obtain  $a_\mu^{\text{had, LO}}$  by utilising the running of  $\alpha_{\text{QED}}$  in a space-like process

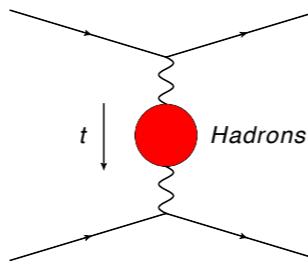


$$a_\mu^{\text{had, LO}} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta \alpha_{\text{had}}[Q^2(x)]$$

[Lautrup, de Rafael '69]

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$$a_\mu^{\text{had, LO}} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta \alpha_{\text{had}}[Q^2(x)]$$

[Lautrup, de Rafael '69]



- Proposal to measure precisely the  $Q^2$  - dependent fine-structure constant:

$$\alpha(Q^2) = \frac{\alpha(O)}{1 - \Delta \alpha(Q^2)}$$

[Phys.Lett. B746 (2015) 325-329 by Carloni, Passera, Trentadue, Venanzoni] @ $e^+e^-$  detector  
[Eur.Phys.J. C77 (2017) no.3, 139 by Abbiendi et al.] Physics Beyond Colliders@CERN

[See talk by C. Carloni Calame, Tue. 16.00]

# MUonE project

## Collaboration:

[LOI submitted to SPSC@CERN in June 2019]

<https://cds.cern.ch/record/2677471>

48 experiment + 20 theory and growing ...!

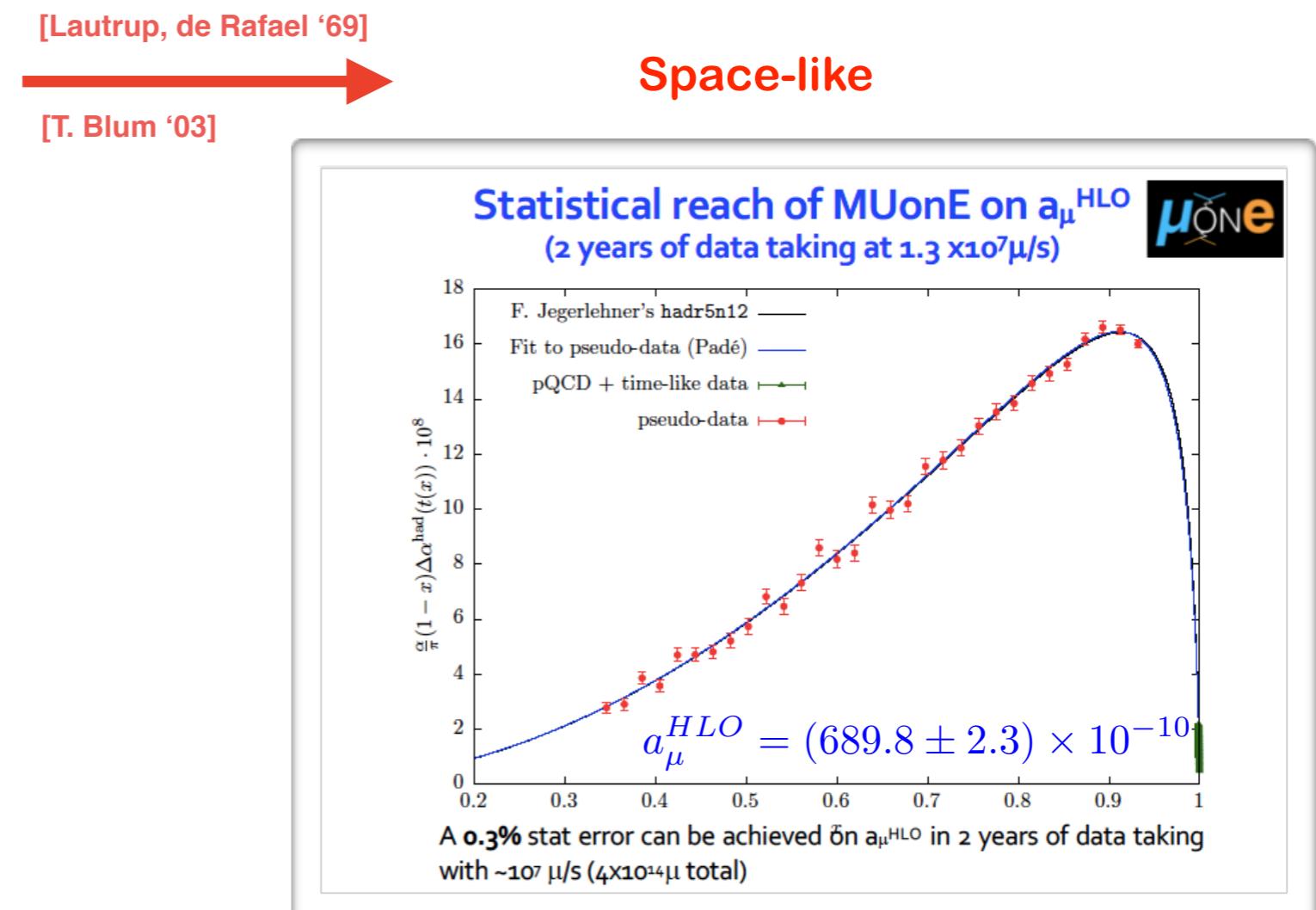
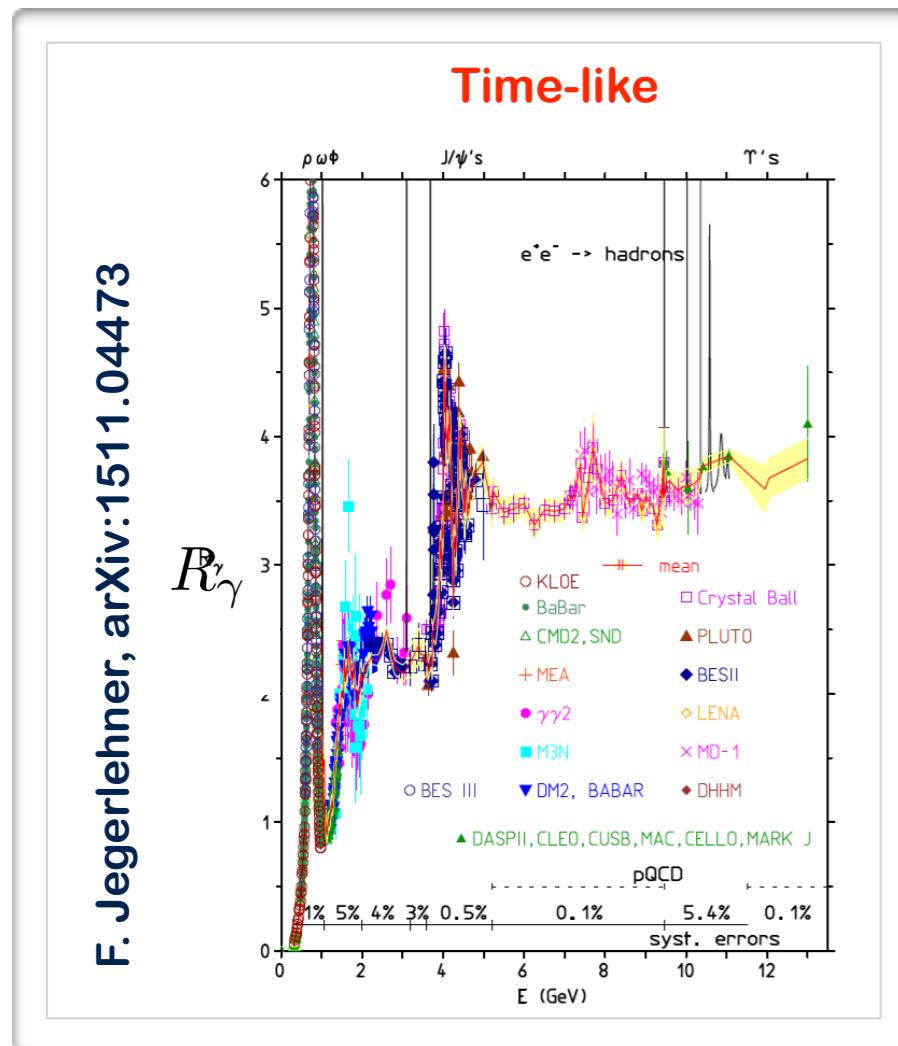
The screenshot shows the homepage of the MUonE Collaboration Meeting at CERN. At the top left is the MUonE logo. To its right, the title "1st MUonE Collaboration Meeting at CERN" is displayed. Below the title, the dates "25-26 March 2019" and location "CERN Europe/Zurich timezone" are listed. A search bar is located in the top right corner. On the left side, there is a sidebar with links: Overview (which is selected), Timetable, Contribution List, Registration, Participant List, and Videoconference Rooms. The main content area contains a detailed description of the meeting's purpose and schedule. It states: "This is the first Collaboration meeting of the MUonE experiment at CERN. MUonE plans to measure the running of alpha(t) space-like by means of the elastic scattering  $\mu e \rightarrow \mu e$ . The required high-intensity muon beam is available at the M2 beam line of the CERN SPS. Such a measurement will provide direct sensitivity to the leading hadronic contribution to the muon anomaly, allowing, together with the measurements of the muon g-2 at Fermilab and J-PARC, a stringent test of the Standard Model." It also specifies the start time as "Starts 25 Mar 2019, 09:00" and end time as "Ends 26 Mar 2019, 19:00", both in "Europe/Zurich" timezone.



$$\alpha(Q^2) = \frac{\alpha(O)}{1 - \Delta\alpha(Q^2)}$$

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# MUonE: space-like evaluation of $a_\mu^{\text{HVP, LO}}$



→ combination of many exp. data sets

→ smooth integrand

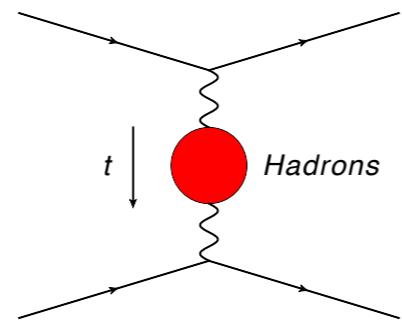
→ single experiment enough - but high accuracy needed (one-loop effect)

# MUonE: theoretical effort underway



→ Theory in low- $Q^2$  [dominating HVP integral]: To extract  $\Delta\alpha_{\text{had}}(t)$  from this measurement,  
the ratio of the SM cross sections in the signal and normalisation regions must be

known at  $\lesssim 10\text{ppm}!$



[NNLO amp.: Mastrolia, Passera, Primo, Schubert JHEP1711(2017),  
Di Vita, Laporta, Mastrolia, Primo, Schubert JHEP09(2018)016]

[NNLO had. contributions Fael JHEP1902(2019)027,  
Fael, Passera Phys. Rev. Lett. 122(2019) 19, 192001]

[MC@QED NLO+weak Alacevich, Carloni Calame, Chiesa,  
Montagna, Nicrosini, Piccinini, JHEP1902 (2019)155]

[MC@NNLO Carloni, Montagna, Nicrosini, Piccinini,Czyz]

[Fixed-order NNLO+ Resummation Broggio; Banerjee Engel,  
Gnendiger, Signer, Ulrich JHEP02(2019)118]

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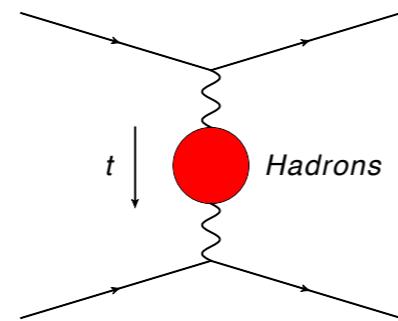


[0.001,0.14] GeV<sup>2</sup>

→ Theory in low-Q<sup>2</sup> [dominating HVP integral]: To extract  $\Delta\alpha_{\text{had}}(t)$  from this measurement,

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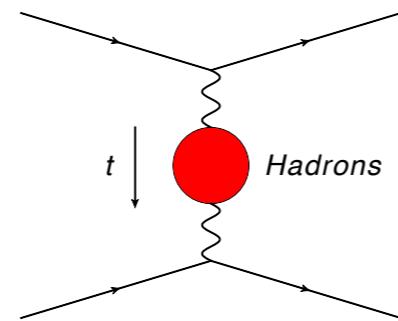


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[4,  $\infty$ ] GeV<sup>2</sup>

→ Theory in high-Q<sup>2</sup>: PT ✓

[Chetyrkin et al. '96]  
[Harlander&Steinhauser '02]

# MUonE: theoretical effort underway

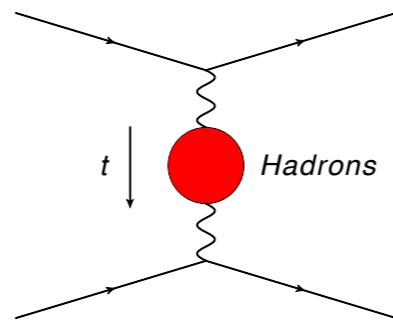


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[0.14,4] GeV<sup>2</sup>

→ Theory in intermediate-Q<sup>2</sup>: Lattice QCD or analytic continuation of the R-ratios

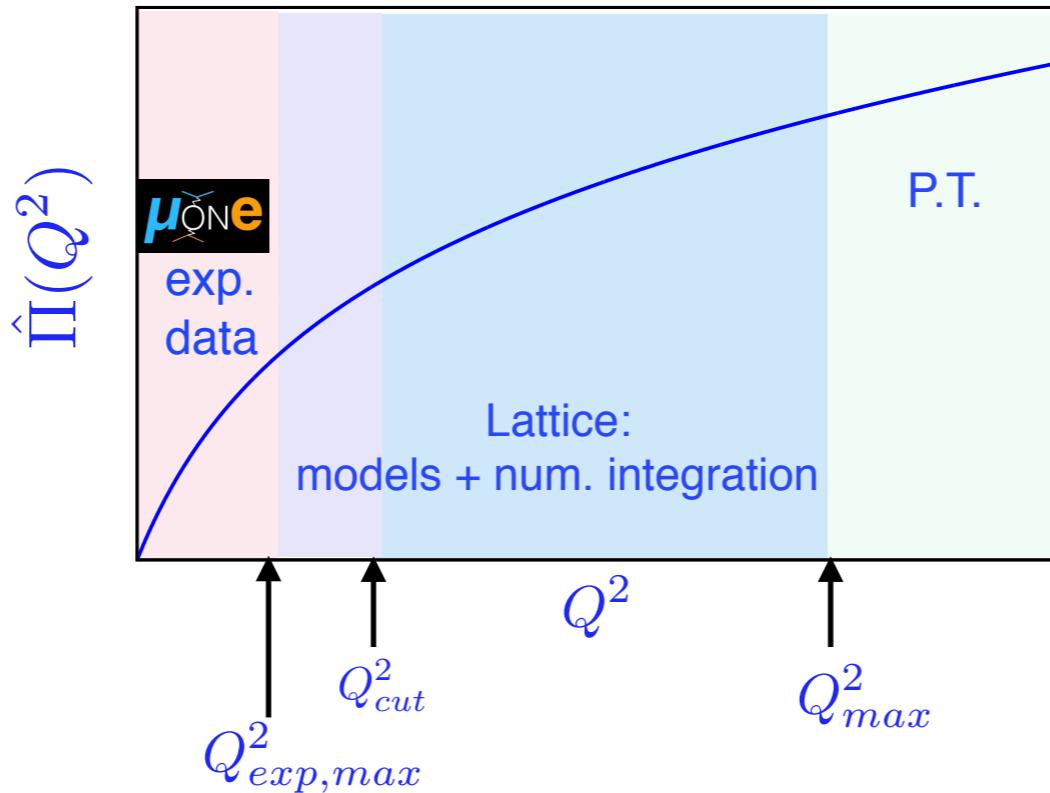
[4,  $\infty$ ] GeV<sup>2</sup>

→ Theory in high-Q<sup>2</sup>: PT ✓

[Chetyrkin et al. '96]

[Harlander&Steinhauser '02]

# Hybrid method: MUonE experiment +lattice



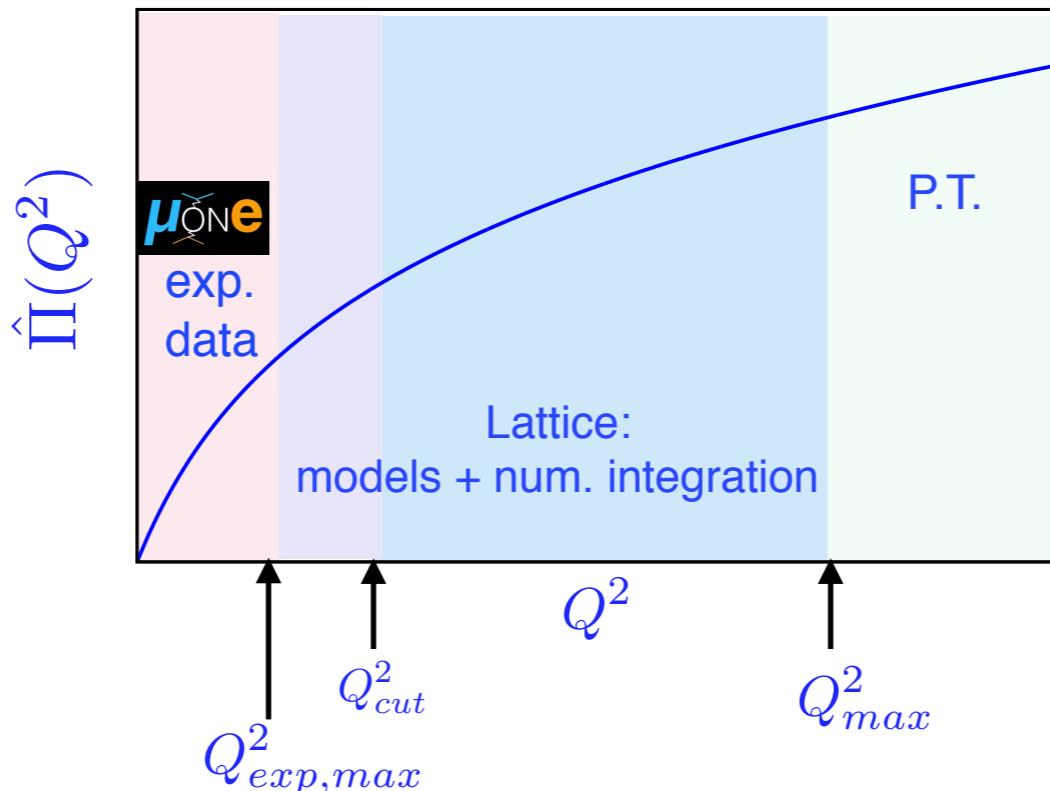
$$a_\mu^{had,LO} = \underbrace{\frac{\alpha}{\pi} \int_0^{0.93...} dx (1-x) \Delta \alpha_{had}[Q^2(x)]}_{I_0} + \underbrace{\left(\frac{\alpha}{\pi}\right)^2 \int_{0.14}^{Q^2_{max}} dQ^2 f(Q^2) \times \hat{\Pi}(Q^2)}_{I_1} + \underbrace{\left(\frac{\alpha}{\pi}\right)^2 \int_{Q^2_{max}}^{\infty} dQ^2 f(Q^2) \times \hat{\Pi}_{pert.}(Q^2)}_{I_2}$$



- lattice QCD
- R-ratios

[Chetyrkin et al. '96]  
 [Harlander&Steinhauser '02]

# Hybrid method: MUonE experiment +lattice



$$a_\mu^{had,LO} = \underbrace{\frac{\alpha}{\pi} \int_0^{0.93...} dx (1-x) \Delta \alpha_{had}[Q^2(x)]}_{I_0} + \underbrace{\left(\frac{\alpha}{\pi}\right)^2 \int_{0.14}^{Q_{max}^2} dQ^2 f(Q^2) \times \hat{\Pi}(Q^2)}_{I_1} + \underbrace{\left(\frac{\alpha}{\pi}\right)^2 \int_{Q_{max}^2}^{\infty} dQ^2 f(Q^2) \times \hat{\Pi}_{pert.}(Q^2)}_{I_2}$$

$I_1$

- lattice QCD
- R-ratios

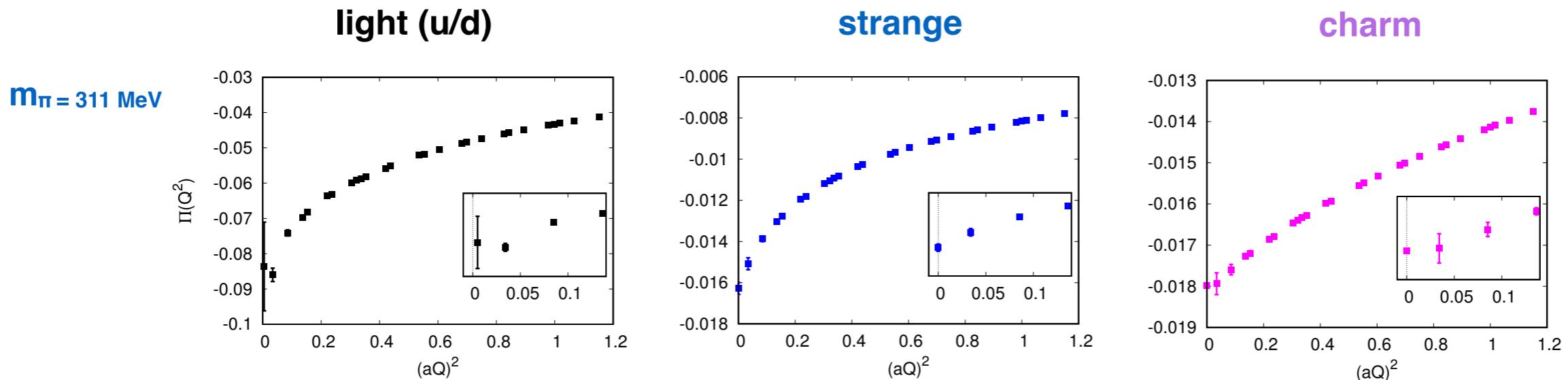
- $I_1$  contribution to the HVP from the lattice:

# HVP: Intermediate-Q<sup>2</sup> range integration [MKM & N. Cardoso '18]

$$a_\mu^{had,LO} = \underbrace{\frac{\alpha}{\pi} \int_0^{0.93...} dx (1-x) \Delta \alpha_{had}[Q^2(x)]}_{I_0} + \underbrace{\left(\frac{\alpha}{\pi}\right)^2 \int_{0.14}^{Q_{max}^2} dQ^2 f(Q^2) \times \hat{\Pi}(Q^2)}_{I_1} + \underbrace{\left(\frac{\alpha}{\pi}\right)^2 \int_{Q_{max}^2}^{\infty} dQ^2 f(Q^2) \times \hat{\Pi}_{pert.}(Q^2)}_{I_2}$$

- $I_1$  on CLS ensembles with  $N_f=2$  O(a) improved Wilson fermions (A5,E5,F6,G8,N6,O7)
- $m_\pi \approx 180-440 \text{ MeV}$ , continuum extrapolation (0.05-0.09 fm), chiral extrapolation to  $m_{\pi,\text{phys}}$
- Partially quenched: s, c ( $\kappa_s, \kappa_c$  taken from [CLS/Mainz, JHEP 1710 (2017)])
- Neglecting isospin breaking effects ( $m_u \neq m_d$  and  $\alpha_{em} \neq 0$ )
- $m_\pi L \geq 4$ , long-distance effects in  $I_1$  not yet explored explicitly

# HVP: Intermediate-Q<sup>2</sup> range integration [MKM & N. Cardoso '18]



→ Statistical precision in  $I_1$  (still) determined by  $\Pi(0)$

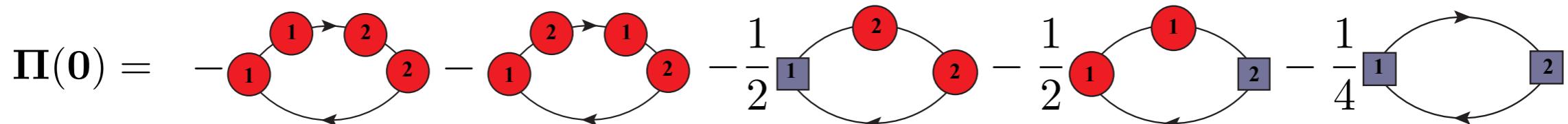
$$\hat{\Pi}(Q^2) = 4\pi^2(\Pi(Q^2) - \Pi(0))$$

→ Expansion of the quark propagator around zero spatial momentum

“derivative method”/“Rome method”

→ Strange & Charm contributions:  $\Pi(0) = -\frac{\partial \Pi_{\mu\nu}(Q)}{\partial Q_\mu \partial Q_\nu} \Big|_{Q^2=0}$

[de Divitiis, Petronzio, Tantalo; Phys.Lett. B718 (2012)]



→ Light contribution:  $Q^2 \rightarrow 0$  extrapolation

# HVP: Intermediate-Q<sup>2</sup> range integration [ETMC '18 and ETMC '19]

**light (u/d)**

[D. Giusti, F. Sanfilippo, S. Simula,  
Phys. Rev. D 98, 114504 (2018)]

Using the (dual +  $\pi\pi$ ) repr.

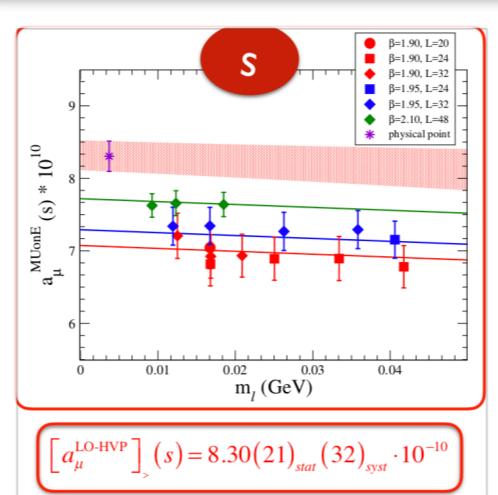
$$[a_\mu^{\text{LO-HVP}}]_> (ud) = 81.2(1.7) \cdot 10^{-10}$$

quark-connected  
terms only

[Talk by D. Giusti, Fri. 9.00]

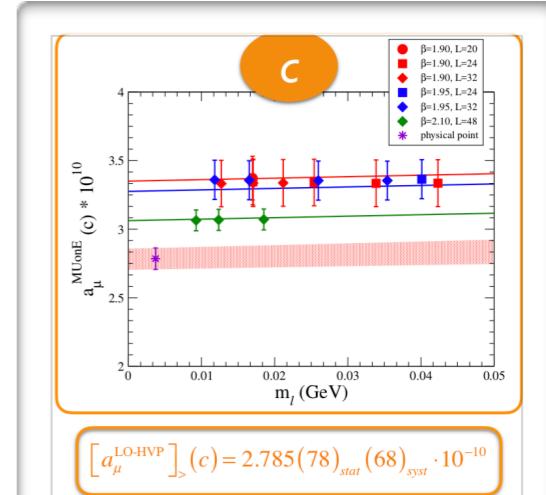
**strange**

[D. Giusti @ Lattice 2019]



**charm**

[D. Giusti @ Lattice 2019]



$$[a_\mu^{\text{LO-HVP}}]_> (s) = 8.30(21)_{\text{stat}}(32)_{\text{syst}} \cdot 10^{-10}$$

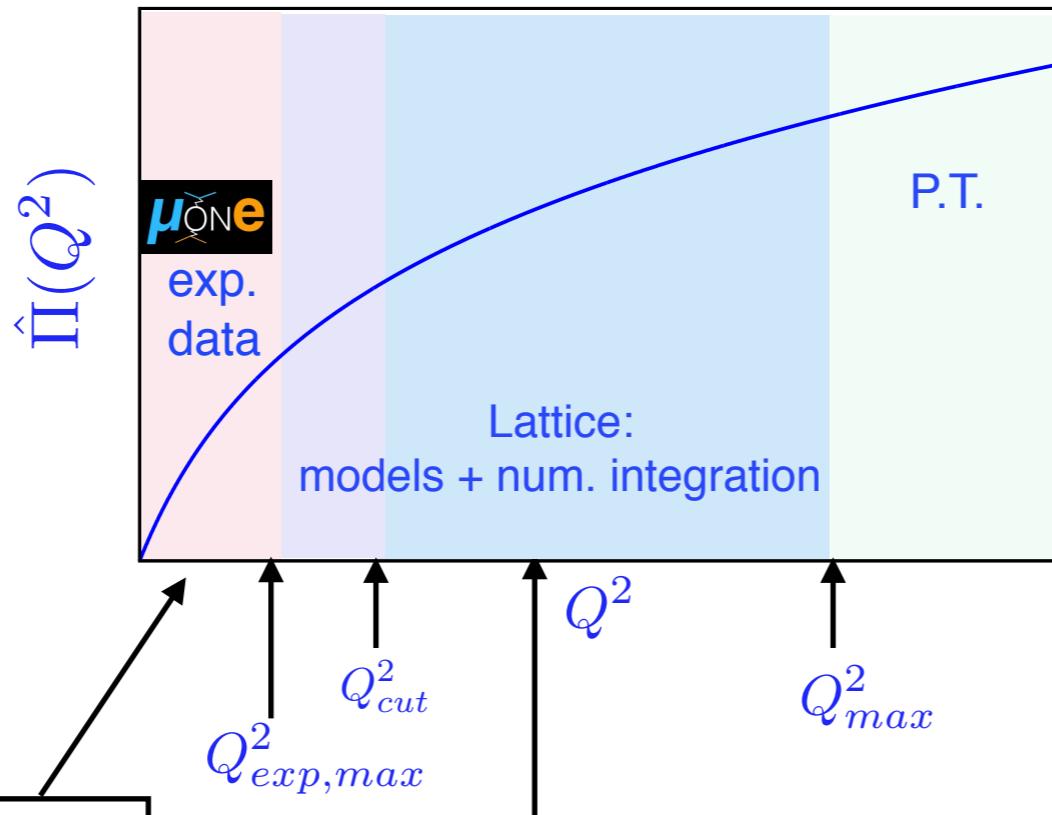
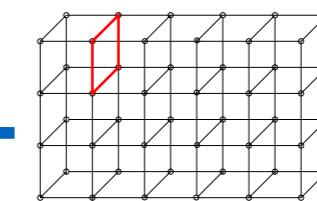
$$[a_\mu^{\text{LO-HVP}}]_> (c) = 2.785(78)_{\text{stat}}(68)_{\text{syst}} \cdot 10^{-10}$$

$f$	$[\delta a_\mu^{\text{HVP}}]_>(f) \cdot 10^{10}$
$ud$	0.9 (0.3)
$s$	-0.0005 (0.0004)
$c$	0.0034 (0.0007)
total	0.9 (0.3)

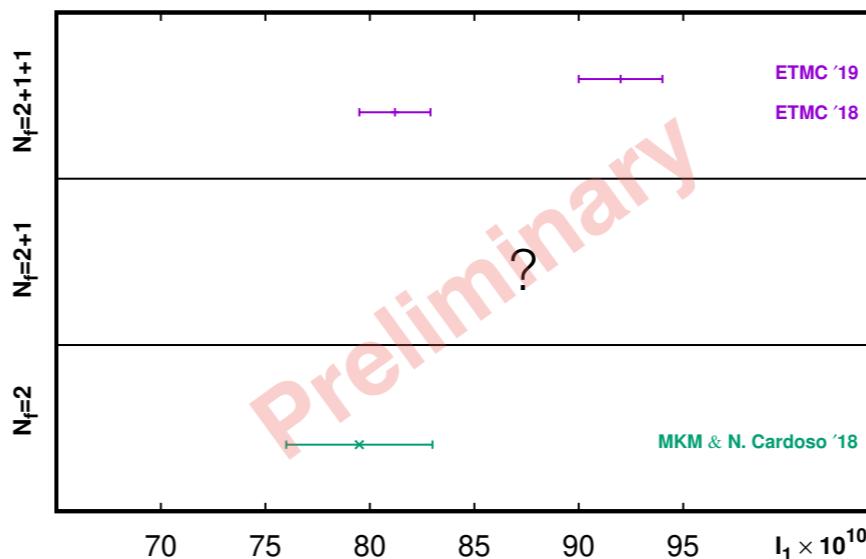
$$[a_\mu^{\text{HVP}}]_> = 92(2) \cdot 10^{-10}$$

preliminary

# Hybrid strategy for the HVP: $\mu_{\text{ONe}}$ + P.T.



- Low momentum region
  - Experiment (NLO, NNLO, radiative corrections ...)



[Talk by D. Giusti, Fri. 9.00]  
 [D. Giusti et al., Phys. Rev. D 98, 114504]

$$[\delta I_1 / a_\mu^{had, LO}] \simeq 0.3 - 0.5\%$$

MUonE after 2 yrs of data taking:

$$[\delta a_\mu^{had, LO} / a_\mu^{had, LO}] \simeq 0.3$$

# HVP: Intermediate- $Q^2$ range integration

$$a_{\mu}^{had,LO} = \underbrace{\frac{\alpha}{\pi} \int_0^{0.93...} dx(1-x)\Delta\alpha_{had}[Q^2(x)]}_{I_0} + \underbrace{\left(\frac{\alpha}{\pi}\right)^2 \int_{0.14}^{Q_{max}^2} dQ^2 f(Q^2) \times \hat{\Pi}(Q^2)}_{I_1} + \underbrace{\left(\frac{\alpha}{\pi}\right)^2 \int_{Q_{max}^2}^{\infty} dQ^2 f(Q^2) \times \hat{\Pi}_{pert.}(Q^2)}_{I_2}$$

- A recent proposal [ <https://cds.cern.ch/record/2677471> ] where  $I_1$  and  $I_2$  are evaluated directly from MUonE data [see talk by C. Carloni Calame, Tue. 16.00]

$$a_{\mu}^{had,LO} = \frac{\alpha}{\pi} \int_0^1 dx(1-x)\Delta\alpha_{had}[Q^2(x)]$$

- "Fermion-like" parametrization ansatz for  $\Delta\alpha_{had}[Q^2(x)]$

$$\Delta\alpha_{had}[Q^2(x)] = KM \left\{ -\frac{5}{9} - \frac{4M}{3Q^2} + \left( \frac{4M^2}{3Q^4} + \frac{M}{3Q^2} - \frac{1}{6} \right) \frac{2}{\sqrt{1 - \frac{4M}{Q^2}}} \log \left| \frac{1 - \sqrt{1 - \frac{4M}{Q^2}}}{1 + \sqrt{1 - \frac{4M}{Q^2}}} \right| \right\}$$

- Recursively used for the NNLO contribution [see talk by M. Fael, Tue. 16.50]
- Systematics estimate: conventional wisdom of lattice QCD calculations

# Summary & Outlook

- In the 2-3 years, 0.3% statistical uncertainty in HVP expected from 
- Measuring the running of  $\alpha_{\text{QED}}$  in the  $Q^2 \in [0.001, 0.14] \text{ GeV}^2$  which dominates the HVP
- Hybrid method (Exp.+Lat.+P.T.):  $I = I_0 + I_1 + I_2$ , first cross-checks with different lattice discretisations in progress
- Outlook: use  $m_{\pi, \text{phys}}$ , revisit strategy for  $a_\mu^{\text{light}}$  (TMR), isospin breaking corrections, estimate discr. effects

- MUonE: L.O.I. to SPSC@CERN submitted in June 2019!
- MUonE strategy to utilize low energy data for the entire  $Q^2$  range [Talk by C. Carloni Calame, Tue. 16.00]
- Lattice invaluable for estimating the systematics of this approach [Sections IVA and IVC of the Muon g-2 Theory Initiative white paper: Lattice QCD calculations of the HVP]

- Open postdoctoral position at the University of Munich, Germany available from 1.1.2020: MUonE-related topics and lattice QCD(+QED) conceptual developments. Please share with potential candidates:

<https://labs.inspirehep.net/jobs/1751820>



**Thank you!**

# MUonE Timeline

[LOI submitted to SPSC@CERN in June 2019]  
<https://cds.cern.ch/record/2677471>



## Planned activities and MUonE timescale

We expect a first feedback from SPSC in the next months, and hopefully have a pilot run approved for 2021.

### ~~> 2019-2020:

- ~~> prepare for a pilot run in 2021 (2 working modules)
- ~~> theory development, systematics studies (beam momentum scale, detector resolution, MS), design optimization, data analysis development, establish an “official” collaboration.

### ~~> 2021: pilot run.

### ~~> 2022: add stations in sync with CMS production plan (50% by spring, 50% by end of the year).

### ~~> 2023-2024: data taking with full apparatus.

[Talk by C. Carloni Calame, Tue. 16.00]