SHORT-DISTANCE CONTRIBUTION TO HLbL

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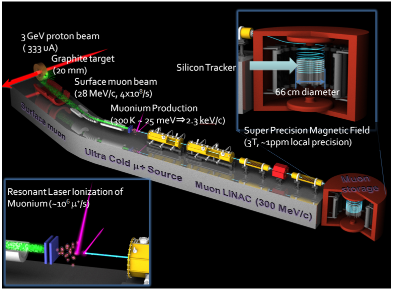
INT Workshop on Hadronic contributions to $(g-2)_\mu$ Seattle, USA 9-13 September 2019

Why do we do this?

The muon $a_\mu = \frac{g_\mu - 2}{2}$ will be measured more precisely



J-PARC

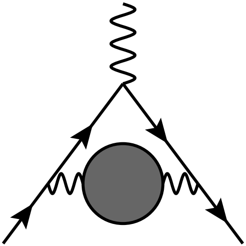
Fermilab

Why do we do this?

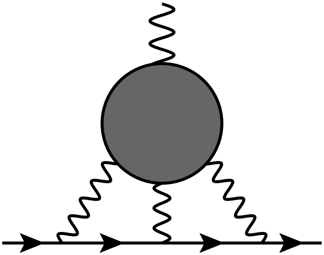
- Experiment dominated by BNL, FNAL error down by four
- Theory taken from PDG2018
- $a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{Had}}$
- $a_\mu^{\text{Had}} = a_\mu^{\text{LO-HVP}} + a_\mu^{\text{HO-HVP}} + a_\mu^{\text{HLbL}}$
- Impressive agreement with g_μ to 2×10^{-9}

Part	value	errors	units
a_μ^{EXP}:	116 592 091.x	(54)(33)	$\times 10^{-11}$
a_μ^{SM}:	116 591 823.x	(1)(34)(26)	$\times 10^{-11}$
Δa_μ:	268.x	(63)(43)	$\times 10^{-11}$
a_μ^{QED}:	116 584 719.0	(0.1)	$\times 10^{-11}$
a_μ^{EW} :	153.6	(1.0)	$\times 10^{-11}$
$a_\mu^{\text{LO-HVP}}$:	6 931.x	(33)(7)	$\times 10^{-11}$
$a_\mu^{\text{HO-HVP}}$:	-86.3	(0.9)	$\times 10^{-11}$
a_μ^{HLbL} :	105.x	(26)	$\times 10^{-11}$

Hadronic contributions



LO-HVP

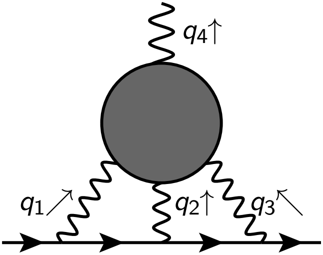


HLbL

- The blobs are hadronic contributions
- I will present some results for HLbL short-distance

  JB, N. Hermansson-Truedsson, A. Rodriguez-Sanchez, arxiv:1908.03331

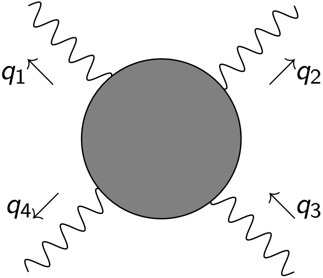
HLbL: the main object to calculate



- Muon line and photons: well known
- The blob: fill in with hadrons/QCD
- Trouble: low and high energy very mixed
- Double counting needs to be avoided: hadron exchanges versus quarks

General properties

$$\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3) \quad =$$



Actually we really need $\left.\frac{\delta \Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3)}{\delta q_{4\rho}}\right|_{q_4=0}$

General properties

$\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3)$:

- In general 138 Lorentz structures (136 in 4 dimensions)
- Using $q_{1\mu}\Pi^{\mu\nu\lambda\sigma} = q_{2\nu}\Pi^{\mu\nu\lambda\sigma} = q_{3\lambda}\Pi^{\mu\nu\lambda\sigma} = q_{4\sigma}\Pi^{\mu\nu\lambda\sigma} = 0$
  43 (41) gauge invariant structures
- 41 helicity amplitudes
- Bose symmetry relates some of them
- Compare HVP: one function, one variable
- General calculation from experiment via dispersion relations: recent progress
  Colangelo, Hoferichter, Kubis, Procura, Stoffer,...
- Well defined separation between different contributions
- Theory initiative: paper under preparation
- One remaining problem: intermediate- and short-distances

General

- Formalism of Colangelo et al., JHEP 1704 (2017) 161 [1702.07347]
- $\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3) = \sum_{i=1,54} \hat{T}^{\mu\nu\lambda\sigma} \hat{\Pi}_i$
- $Q_i^2 = -q_i^2$
- $Q_3^2 = Q_1^2 + Q_2^2 + 2\tau Q_1 Q_3$
- $a_\mu = \frac{2\alpha^3}{3\pi^2} \int_0^\infty dQ_1 dQ_2 Q_1^3 Q_2^3 \int_{-1}^{1} d\tau \sqrt{1-\tau^2} \sum_{i=1,12} \hat{T}_i \overline{\Pi}_i$
- The 12 $\overline{\Pi}_i$ are related to 6 $\hat{\Pi}_i$ with $q_4 \to 0$.

$$\overline{\Pi}_1 = \hat{\Pi}_1\,,\ \overline{\Pi}_2 = C_{23}\left[\hat{\Pi}_1\right]\,,\ \overline{\Pi}_3 = \hat{\Pi}_4\,,\ \overline{\Pi}_4 = C_{23}\left[\hat{\Pi}_4\right]\,,$$

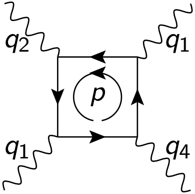
$$\overline{\Pi}_5 = \hat{\Pi}_7\,,\ \overline{\Pi}_6 = C_{12}\left[C_{13}\left[\hat{\Pi}_7\right]\right]\,,\ \overline{\Pi}_7 = C_{23}\left[\hat{\Pi}_7\right]\,,$$

$$\overline{\Pi}_8 = C_{13}\left[\hat{\Pi}_{17}\right]\,,\ \overline{\Pi}_9 = \hat{\Pi}_{17}\,,\ \overline{\Pi}_{10} = \hat{\Pi}_{39}\,,$$

$$\overline{\Pi}_{11} = -C_{23}\left[\hat{\Pi}_{54}\right]\,,\ \overline{\Pi}_{12} = \hat{\Pi}_{54}\,,$$

Quarkloop

- Use (constituent) quark loop
- Used for full estimates since the beginning (1970s)
- Used for short-distance estimates with mass as a cut-off
  JB, Pallante, Prades, 1996
- We recalculated:



- In agreement with quarkloop formulae from
  Hoferichter, Stoffer, private communication
- In agreement with known numerics

Charm loop

- Kühn et al., hep-ph/0301151, Phys.Rev. D68 (2003) 033018
- $a_\mu = \left(\frac{\alpha}{\pi}\right)^3 N_c e_q^4 \left[\frac{m_\mu^2}{M^2}\left(\frac{3}{3}\zeta_3 - \frac{19}{16}\right) + \cdots\right]$
- Up to m_μ^{10}/M^{10} in paper
- $m_c = 1.27$ GeV
- $a_\mu^{\mathrm{HLbLc}} = (3.165 - 0.0786 - 0.00033 + \cdots) \times 10^{-11}$
  $= 3.1(1) \times 10^{-11}$
- $m_b = 4.18$ GeV
- $a_\mu^{\mathrm{HLbLb}} = 1.8 \times 10^{-13}$

Quarkloop: u, d, s

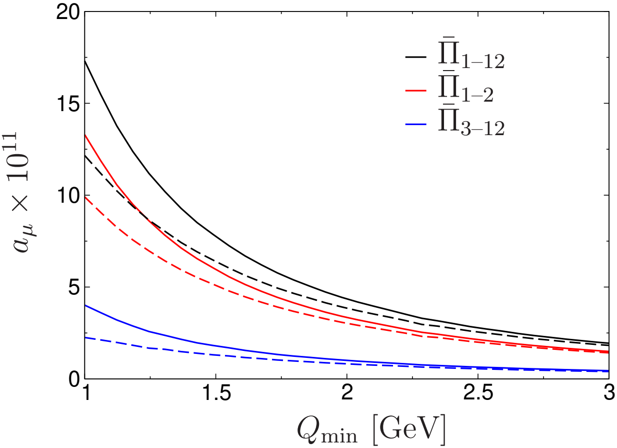


figure: Hoferichter

$Q_1, Q_2, Q_3 > Q_{\min}$

$M_Q = 0$: full

$M_Q = 0.3$ GeV dashed

$a_\mu^{\mathrm{HLbLQ}} = 54 \times 10^{-11}$

- M_Q provides an infrared cut-off, $M_Q \to 0$ divergent
- About 12×10^{-11} from above 1 GeV for $M_Q = 0.3$ GeV
- About 17×10^{-11} from above 1 GeV for $M_Q = 0$

Quarkloop

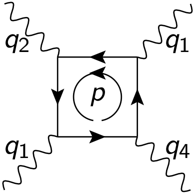
- Is it a first term in a systematic OPE?
- OPE has been used as constraints on specific contributions
    - $\pi^0\gamma^*\gamma^*$ asymptotic behaviour
    - Constraints on many other hadronic formfactors
    - $q_1^2 \approx q_2^2 \gg q_3^2$ Melnikhov, Vainshtein 2003
    - These are discussed in the next two talks

Short-distance: first attempt

$$\Pi^{\mu\nu\lambda\sigma} = -i\int d^4x d^4y d^4z e^{-i(q_1\cdot x+q_2\cdot y+q_3\cdot z)}\left\langle T\left(j^\mu(x)j^\nu(y)j^\lambda(z)j^\sigma(0)\right)\right\rangle$$

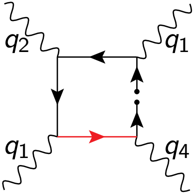
- Usual OPE: x, y, z all small
- First term in the expansion is the quark-loop
  no problem with $\partial/\partial q_4^\rho$ and $q_4 \to 0$

  p in loop $\Rightarrow$ no singular propagators:



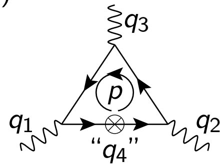
- Next term problems: no loop momentum;

  $q_4 \to 0$ propagator diverges:



Short-distance: correctly

- Similar problem in QCD sum rules for electromagnetic radii and magnetic moments
- Ioffe, Smilga, 1984
- For the q_4-leg use a constant background field and do the OPE in the presence of that constant background field
- Use radial gauge: $A_4^\lambda(w) = \frac{1}{2} w_\mu F^{\mu\lambda}$
  whole calculation is immediately with $q_4 = 0$.
- First term is exactly the usual quark loop
  (even including quark masses)



Quarkloop results: massless case

$$\hat{\Pi}_i^{\mathrm{ql}} = \sum_q \frac{N_c e_q^4}{16\pi^2} \int_0^1 dx \int_0^{1-x} dy \; I_i(x,y)$$

$$D = x(1-x-y)Q_1^2 + y(1-x-y)Q_2^2 + xyQ_3^2$$

$$I_1 = \frac{16}{D^2}\Big(y^2 - 3y^3 + 2y^4 - 4xy^2 + 4xy^3 - x^2y + 2x^2y^2 - \frac{Q_1^2}{Q_3^2}(y+x)(1-x-y)^2(-1+2x+2y)\Big)$$

$$I_7 = \frac{64}{D^3}xy(-1+2x)(x+y)(1-x-y)^2$$

$$I_{39} = \frac{64}{D^3}xy(1-x-y)(y-y^2+x-3xy+2xy^2-x^2+2x^2y)$$

$$I_{54} = \frac{16}{D^2}\left(\frac{1}{Q_1^2}y^2(1-y)(1-2y) - \frac{1}{Q_2^2}x^2(1-x)(1-2x)\right)$$

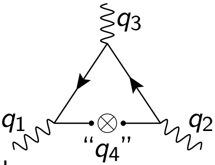
I_4 and I_{17} similar but slightly longer expressions

Short-distance: next term(s)

- Do the usual QCD sum rule expansion in terms of vacuum condensates
- There are new condensates, induced by the constant magnetic field: $\langle \bar{q}\sigma_{\alpha\beta}q\rangle \equiv e_q F_{\alpha\beta} X_q$
- Lattice QCD Bali et al., arXiv:1206.4205
  $X_u = 40.7 \pm 1.3$ MeV,
  $X_d = 39.4 \pm 1.4$ MeV,
  $X_s = 53.0 \pm 7.2$ MeV
- Could have started at order $1/Q$, only starts at $1/Q^2$ via $m_q X_q$ corrections to the leading quark-loop result
- X_q and m_q are very small, only a very small correction
- Next order: very many condensates contribute, work in progress

Short-distance

- Result derived from:



- $N_c = 3$ and one quark

$$\hat{\Pi}_1 = m_q X_q e_q^4 \frac{-4(Q_1^2 + Q_2^2 - Q_3^2)}{Q_1^2 Q_2^2 Q_3^4} \qquad \hat{\Pi}_7 = 0$$

$$\hat{\Pi}_4 = m_q X_q e_q^4 \frac{8}{Q_1^2 Q_2^2 Q_3^2} \qquad \hat{\Pi}_{17} = m_q X_q e_q^4 \frac{8}{Q_1^2 Q_2^2 Q_3^4}$$

$$\hat{\Pi}_{54} = m_q X_q e_q^4 \frac{-4(Q_1^2 - Q_2^2)}{Q_1^4 Q_2^4 Q_3^2} \qquad \hat{\Pi}_{39} = 0$$

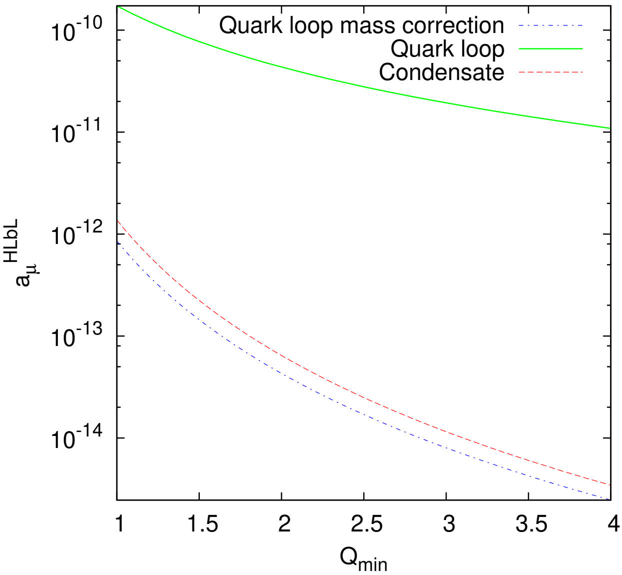
Short-distance: numerical results

- preliminary
- $Q_1, Q_2, Q_3 \geq Q_{\text{min}}$
- $m_u = m_d = m_s = 0$ for quark-loop
- $m_u = m_d = 5$ MeV and $m_s = 100$ MeV for $m_q X_q$

Q_{min}	quarkloop	$m_u X_u + m_d X_d$	$m_s X_s$
1 GeV	17.3×10^{-11}	5.40×10^{-13}	8.29×10^{-13}
2 GeV	4.35×10^{-11}	3.40×10^{-14}	5.22×10^{-14}

- Above 1 GeV still 15% of total value of HLbL
- Quarkloop goes roughly as $1/Q^2_{\text{min}}$
- $m_q X_q$ goes roughly as $1/Q^4_{\text{min}}$
- Naive suppression is $m_q X_q / Q^2_{\text{min}} \sim 2 \times 10^{-3}$
- Observed is roughly that

Numerical results



Short-distance: $1/Q^2_{\rm min}$

- Can we understand scaling with $Q_{\rm min}$?
- $a_\mu = \frac{2\alpha^3}{3\pi^2}\int_0^\infty dQ_1 dQ_2 Q_1^3 Q_2^3 \int_{-1}^1 d\tau\sqrt{1-\tau^2}\sum_{i=1,12}\hat{T}_i\overline{\Pi}_i$
- Do $Q_i \to \lambda Q_i$
- overall factor goes as λ^8
- Quark loop has no scale thus $\hat{\Pi}_i$ scale with their dimension
  $\hat{\Pi}_1, \hat{\Pi}_4 \sim \lambda^{-4}, \qquad \hat{\Pi}_7, \hat{\Pi}_{17}, \hat{\Pi}_{39}, \hat{\Pi}_{54} \sim \lambda^{-6}$
- $\Longrightarrow \overline{\Pi}_{1,\dots,4} \sim \lambda^{-4} \qquad \overline{\Pi}_{5,\dots,12} \sim \lambda^{-6}$
- Expand the T_i for $Q_i \gg m_\mu$: $T_1 \sim m_\mu^4$, $T_{i\neq 1} \sim m_\mu^2$
  $T_1 \sim \lambda^{-8}$, $T_{2,3,4} \sim \lambda^{-6}$, $T_{5,\dots,12} \sim \lambda^{-4}$
- Put all together: quark-loop scales as $a_\mu^{\rm SD\ ql} \sim \lambda^{-2}$
- $m_q X_q$ adds an overall factor $\Longrightarrow a_\mu^{{\rm SD}X_q} \sim \lambda^{-4}$

Short-distance: Conclusions

- We have shown that the quarkloop really is the first term of a proper OPE expansion for the HLbL
- We have calculated the next term which is suppressed by quark masses and a small X_q: negligible
- The next term contains both the usual vacuum and a large number of induced condensates but will not be suppressed by small quark masses
- Why do this: matching of the sum over hadronic contributions to the expected short distance domain
- Finding the onset of the asymptotic domain