$\eta^{(\prime)}$ Pole Contribution with Dispersive Methods

in collaboration with M. Hoferichter, B.-L. Hoid and B. Kubis

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Strategy - Components of TFF

Construct TFF from three pieces:

$$F_{\eta^{(\prime)}\gamma^\star\gamma^\star}(q_1^2, q_2^2) = F^{\text{disp}}_{\eta^{(\prime)}\gamma^\star\gamma^\star}(q_1^2, q_2^2) + F^{\text{eff}}_{\eta^{(\prime)}\gamma^\star\gamma^\star}(q_1^2, q_2^2) + F^{\text{asym}}_{\eta^{(\prime)}\gamma^\star\gamma^\star}(q_1^2, q_2^2)$$

Dispersive piece

- Low-energy description
- Reproduces all low-energy cuts and singularities

Effective Pole Term

- Parametrize higher intermediate states
- Full saturation of normalization sum rule
- Describe high-energy singly-virtual data

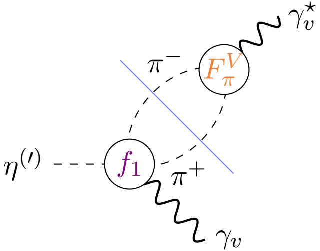
pQCD piece

- Induces leading-twist behavior of TFF ($\mathcal{O}(1/Q^2)$ asymptotics)
- Makes TFF respect asymptotic constraints

Transition form factor $\eta^{(\prime)} \to \gamma^\star\gamma$

Isospin decomposition: $F^{\text{disp}}_{\eta^{(\prime)}\gamma^\star\gamma^\star}(q_1^2, q_2^2) = F^{\eta^{(\prime)}}_{vv}(q_1^2, q_2^2) + F^{\eta^{(\prime)}}_{ss}(q_1^2, q_2^2)$

Reconstruction from the lowest-lying hadronic states:



Isovector part:

- largest contribution: $\pi^+\pi^-$ intermediate state
- Dispersively combine data on $\eta^{(\prime)} \to \pi^+\pi^-\gamma$ and $e^+e^- \to \pi^+\pi^-$

Isoscalar part:

- Dominated by narrow resonances:
  ω & ϕ Hanhart et al. 2013
- Employ VMD and fix couplings by exp. det. decay widths for
  - $\omega \to \eta\gamma$
  - $\eta' \to \omega\gamma$
  - $\phi \to \eta^{(\prime)}\gamma$
  - $\omega,\ \phi \to e^+e^-$

η: strong cancellation between ω and ϕ

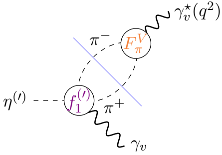
η': isoscalar contribution more significant than for η (e.g. in norm $\sim 20\,\%$)

From $\eta^{(\prime)} \to \pi^+\pi^-\gamma$ to $\eta^{(\prime)} \to \gamma^\star\gamma$

- Dominated at low energies by $\pi^+\pi^-$ in P-wave
- Differential decay width: Stollenwerk et al. 2012

$$\frac{\mathrm{d}\Gamma(t)}{\mathrm{d}t} = |P^{(\prime)}(t)\Omega(t)|^2\Gamma_0(t) \equiv |f_1^{(\prime)}(t)|^2\Gamma_0(t)$$

- where $t = M_{\pi\pi}^2$, Γ_0: phase space, and $P^{(\prime)}(t) = 1 + \alpha^{(\prime)}t$
  - $+\beta^{(\prime)}t^2$ (due to left-hand cut contribution) Kubis, Plenter 2015; Xiao et al. 2015
  - $+\,\rho-\omega$-mixing term (for η') Hanhart, SH, Kubis, Kupść, Wirzba, Xiao 2017



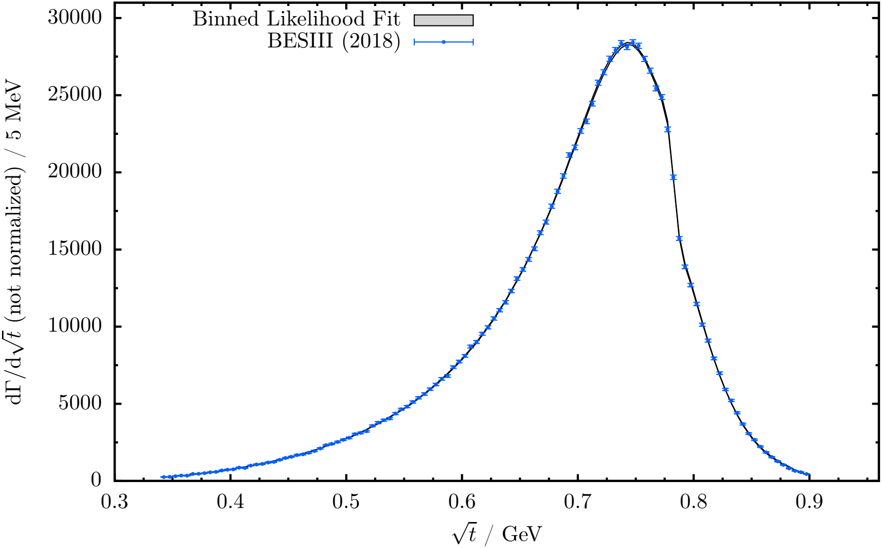
- input for isovector TFF $F^{(I=1)}_{\eta^{(\prime)}\gamma\gamma^\star}$
- $\operatorname{disc} F^{(I=1)}_{\eta^{(\prime)}\gamma\gamma^\star} \propto f_1^{(\prime)} \times (F_\pi^V)^*$ Hanhart et al. 2013

$\Rightarrow$ unsubtracted dispersion relation:

$$F^{(I=1)}_{\eta^{(\prime)}\gamma\gamma^\star}(q^2) = \frac{1}{96\pi^2}\int_{4M_\pi^2}^{\infty} \mathrm{d}t\, \frac{t\sigma_\pi^3(t)}{t-q^2} f_1^{(\prime)}(t)(F_\pi^V(t))^*$$

Fixing the parameters in $\eta' \to \pi^+\pi^-\gamma$

4 parameter binned maximum-likelihood fit to high-precision BESIII data:



Effective Pole Term

- Normalization of isovector + isoscalar part should be compared to exp. det. decay width $\Gamma(\eta^{(\prime)} \to \gamma\gamma)$ ($\Leftrightarrow F^{\text{exp}}_{\eta^{(\prime)}}(0,0)$)
- isovector + isoscalar contribution fulfill norm by around 90 % (for both η and η')

$\Rightarrow$ Introduce an effective pole term

$$F^{\text{eff}}_{\eta^{(\prime)}\gamma^\star\gamma^\star}(q_1^2, q_2^2) = g_{\text{eff}} F^{\text{exp}}_{\eta^{(\prime)}}(0,0) \frac{M_{\text{eff}}^4}{(M_{\text{eff}}^2 - q_1^2)(M_{\text{eff}}^2 - q_2^2)}$$

- g_{eff} fixed by fulfilling norm
  - $g_{\text{eff}} \sim 10\,\%$ $\Rightarrow$ small ($\pi^0: \ g_{\text{eff}} \sim 10\,\%$)
- M_{eff} fixed by fit to singly-virtual space-like data above 5 GeV^2
  BaBar; CELLO; CLEO; L3
  - $M_{\text{eff}} \sim 1$ GeV ($\pi^0: \ M_{\text{eff}} \sim 1.5 - 2$ GeV)

Doubly-virtual asymptotics

- Make use of analogy to pion transition form factor

  Hoferichter, Hoid, Kubis, Leupold, Schneider 2018

- From pQCD considerations:

$$F^{\text{asym}}_{\eta^{(\prime)}\gamma^\star\gamma^\star}(q_1^2, q_2^2) = F^{(\prime)}_{\text{asym}} \int_{s_m}^{\infty} \mathrm{d}x \, \frac{q_1^2 q_2^2}{(x - q_1^2)^2 (x - q_2^2)^2}$$

- Does not contribute to singly-virtual kinematics
- Complications due to η/η'-mixing — Escribano et al. 2014-2016
- However $F^{(\prime)}_{\text{asym}}$ fixed by Brodsky-Lepage-like limit:

  Brodsky, Lepage 1979-1981

$$\lim_{Q^2\to\infty} Q^2 F_{\eta^{(\prime)}\gamma^\star\gamma}(-Q^2, 0) = F^{(\prime)}_{\text{asym}} \qquad (= 2F_\pi \text{ for } \pi^0)$$

- By adding pQCD piece TFF fulfills operator product expansion constraint: Nesterenko, Radyushkin 1983; Novikov et al. 1984; Manohar 1990

$$\lim_{Q^2\to\infty} Q^2 F_{\eta^{(\prime)}\gamma^\star\gamma^\star}(-Q^2, -Q^2) = \frac{1}{3} F^{(\prime)}_{\text{asym}} \qquad \left(= \frac{2F_\pi}{3} \text{ for } \pi^0\right)$$

Uncertainty estimates

Normalization:

- Uncertainty in $F^{\text{exp}}_{\eta^{(\prime)}}(0,0)$ at 1.7 % for η, at 1.6 % for η' PDG 2018
  - Vary effective coupling g_{eff}

Dispersive input:

- Varying integral cutoffs between 1.5 and 2.5 GeV
- different phase shifts inputs:
  - Bern phase Colangelo, Caprini, Leutwyler
  - Madrid phase García-Martín et al. 2011
  - effective form factor phase (incl. ρ', ρ'') Schneider et al. 2012
    one of the most modern phase inputs → see P. Stoffer's talk
- Different representations of F_π^V and f_1 (via $R(s)$ and $P(s)$)

Singly-virtual asymptotics:

- Cover 2σ error band of M_{eff}

Doubly-virtual asymptotics:

- Vary threshold parameter $1 \leq s_m \leq 1.7\,\text{GeV}^2$

Khodjamirian 1999; Agaev et al. 2011; Mikhailov et al. 2016

Disclaimer

Preliminary assumptions for the following results:

- isovector & isoscalar subparts assumed to be factorizing
- $\rho - \omega$-mixing isospin-breaking effects excluded by hand
- Low values for M_{eff} under investigation
- Significant effect of ρ' in $e^+e^- \to \eta\pi\pi$

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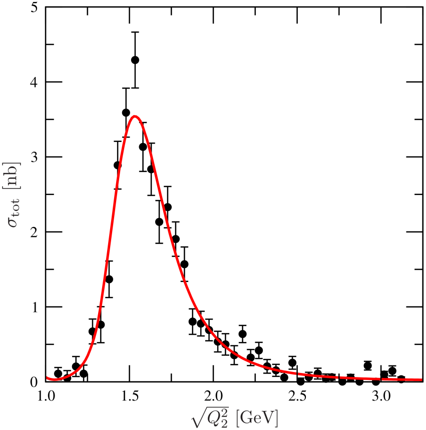
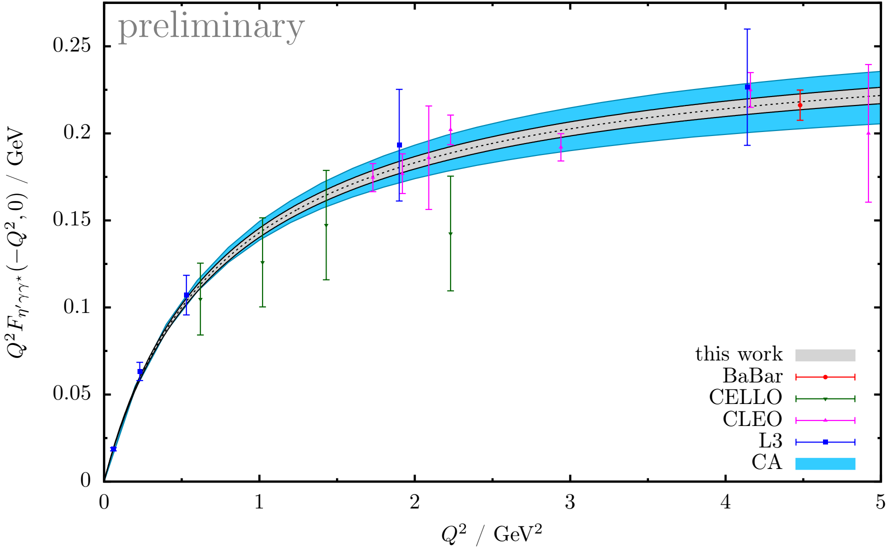


$e^+e^- \to \eta\pi\pi$ cross section.

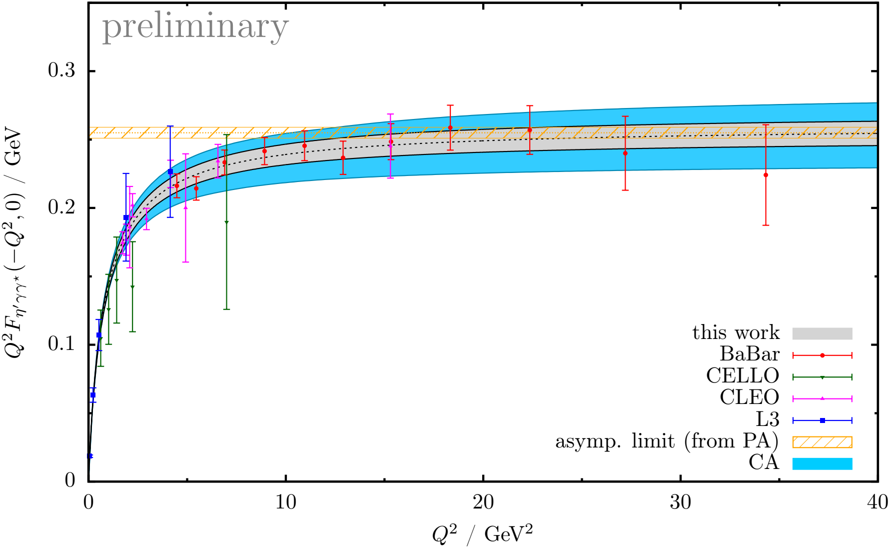
Xiao et al. 2015; data: BaBar 2008

Singly-virtual η' TFF



in comparison: Canterbury approximant from Masjuan, Sanchez-Puertas 2017

Singly-virtual η' TFF



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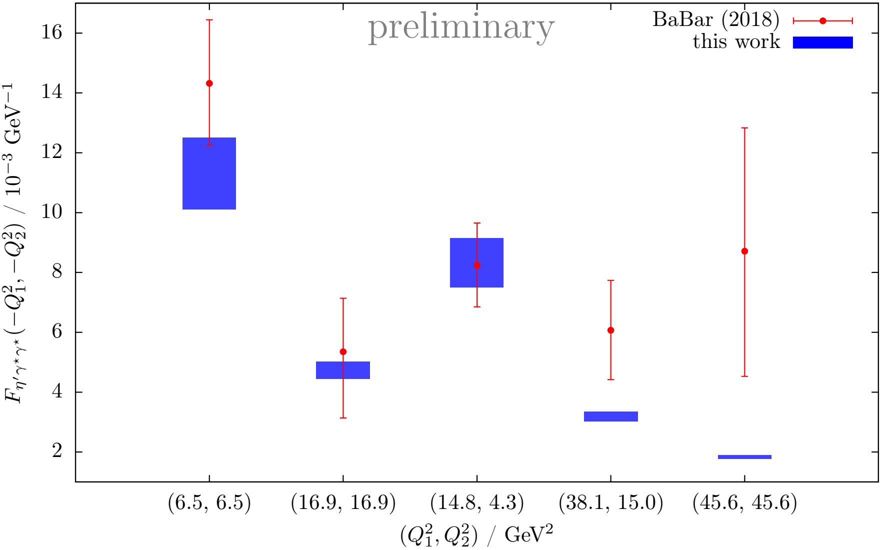
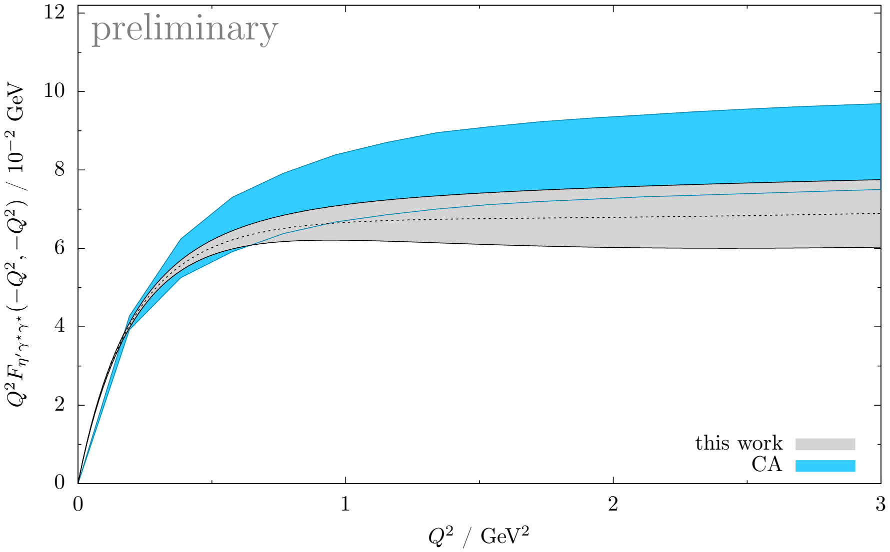
η' TFF comparison with doubly-virtual data



Doubly-virtual η' TFF



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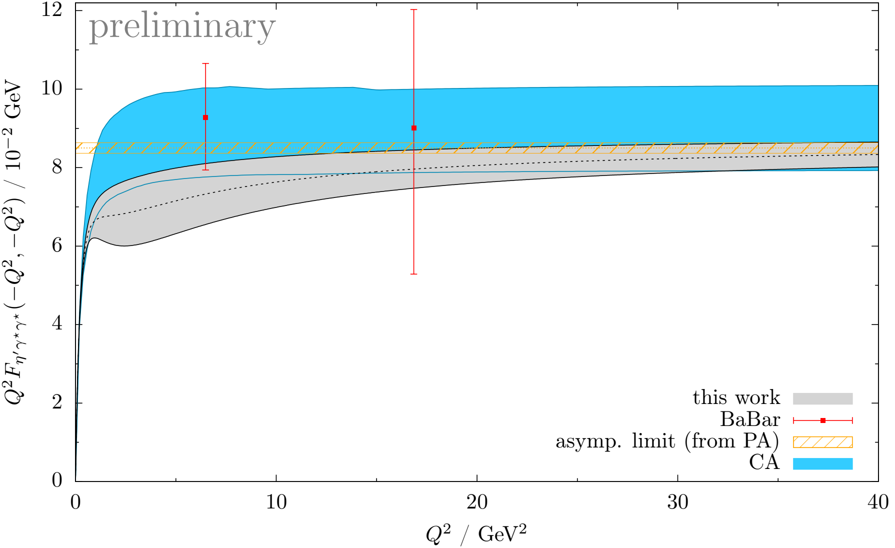

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(Preliminary) Numerical Results

η-pole contribution to a_μ

$$a_\mu^{\eta\text{-pole}} = 15.16\,(40)_{\text{Norm}}(27)_{\text{disp}}(58)_{\text{BL}}(36)_{\text{asym}} \times 10^{-11}$$

$a_\mu^{\eta\text{-pole}} \times 10^{11}$	this work	CA	DS
	15.2 ± 0.8	16.3 ± 1.4	15.8 ± 1.2

η'-pole contribution to a_μ

$$a_\mu^{\eta'\text{-pole}} = 13.57\,(23)_{\text{Norm}}(23)_{\text{disp}}(39)_{\text{BL}}(39)_{\text{asym}} \times 10^{-11}$$

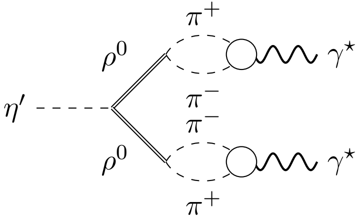
$a_\mu^{\eta'\text{-pole}} \times 10^{11}$	this work	CA	DS
	13.6 ± 0.6	14.5 ± 1.9	13.3 ± 0.8

CA: Canterbury approximant approach Masjuan, Sanchez-Puertas 2017

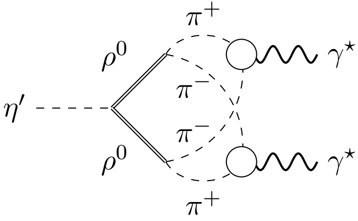
DS: Dyson-Schwinger approach Eichmann et al. 2019

Conclusions and outlook

- Preliminary dispersive results for $\eta^{(\prime)}$-pole contribution in good agreement with previous studies
  - Exact uncertainty of dispersive approach yet to be determined
- Factorization-breaking effects not considered yet. Try to account for them by starting at $\eta' \to 4\pi$ amplitude Guo, Kubis, Wirzba 2012
  - These effects have not been studied anywhere else either: source of systematic uncertainty in $\eta^{(\prime)}$ pole $\to$ work in progress J. Plenter, SH, MSc theses



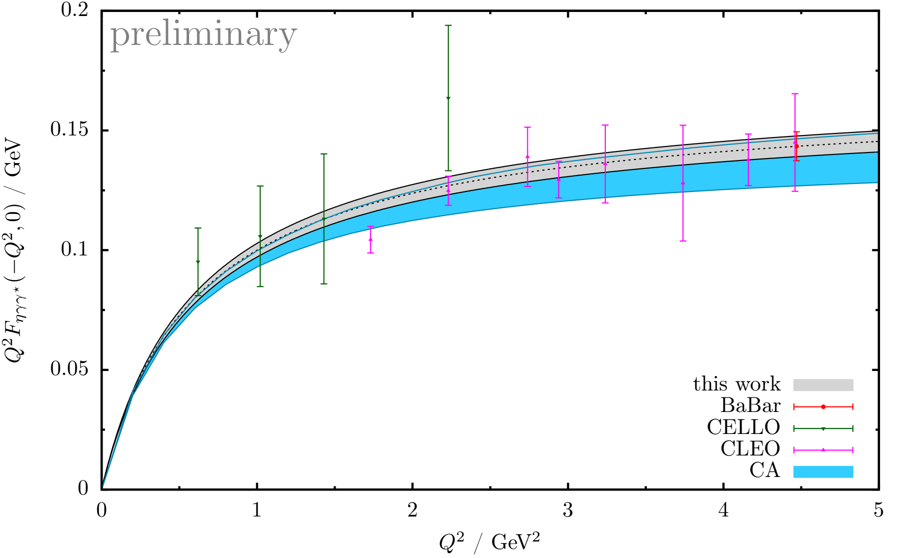
factorizing



non-factorizing

Spares

Singly-virtual η TFF



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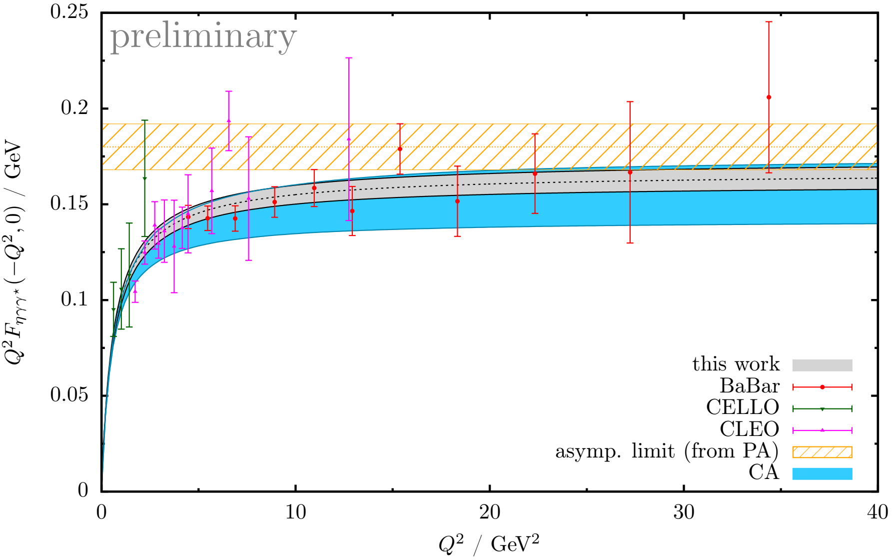
Singly-virtual η TFF



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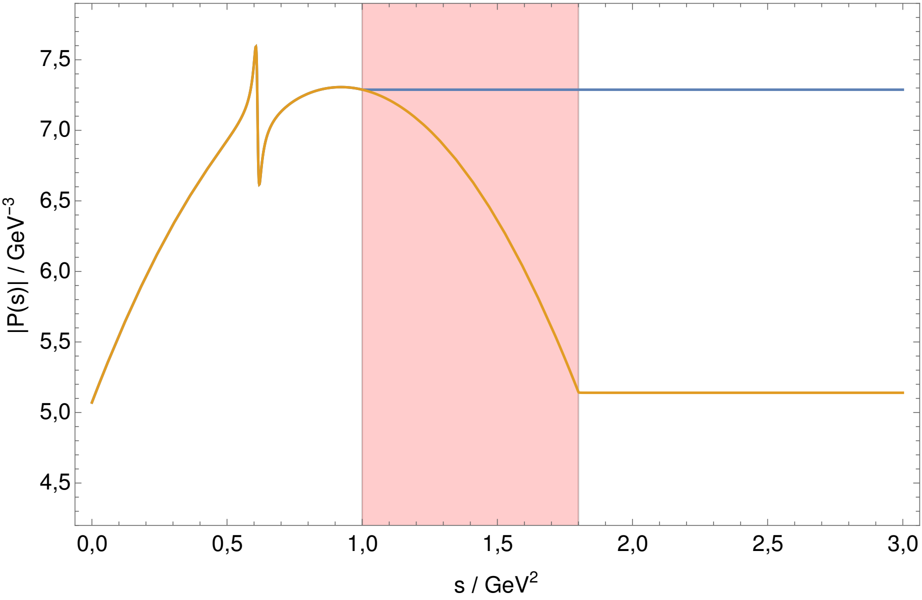
Example: Continuation of P' in f_1



Corresponding continuation of R in F_π^V