Present Status of Muonium HFS measurement in J-PARC （MuSEUM)

K. Shimomura (KEK)

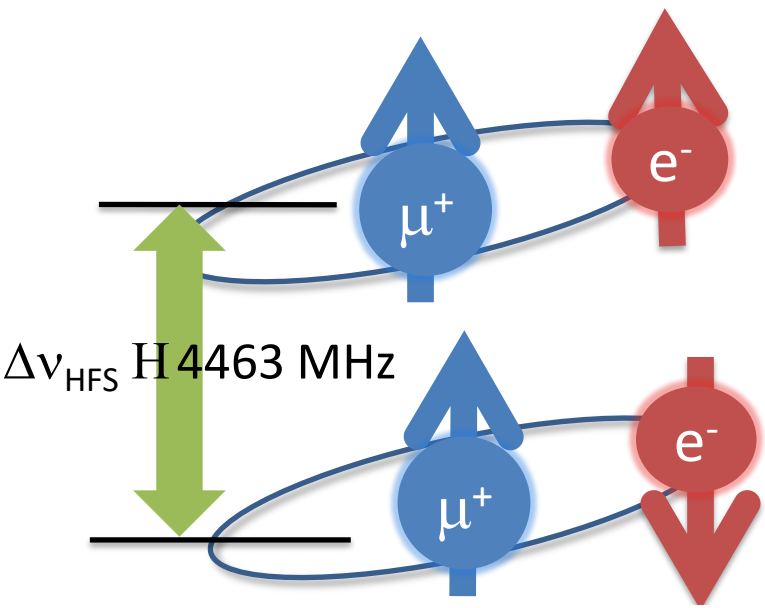
on behalf of the MuSEUM Collaboration

Muonium Hyperfine Structure

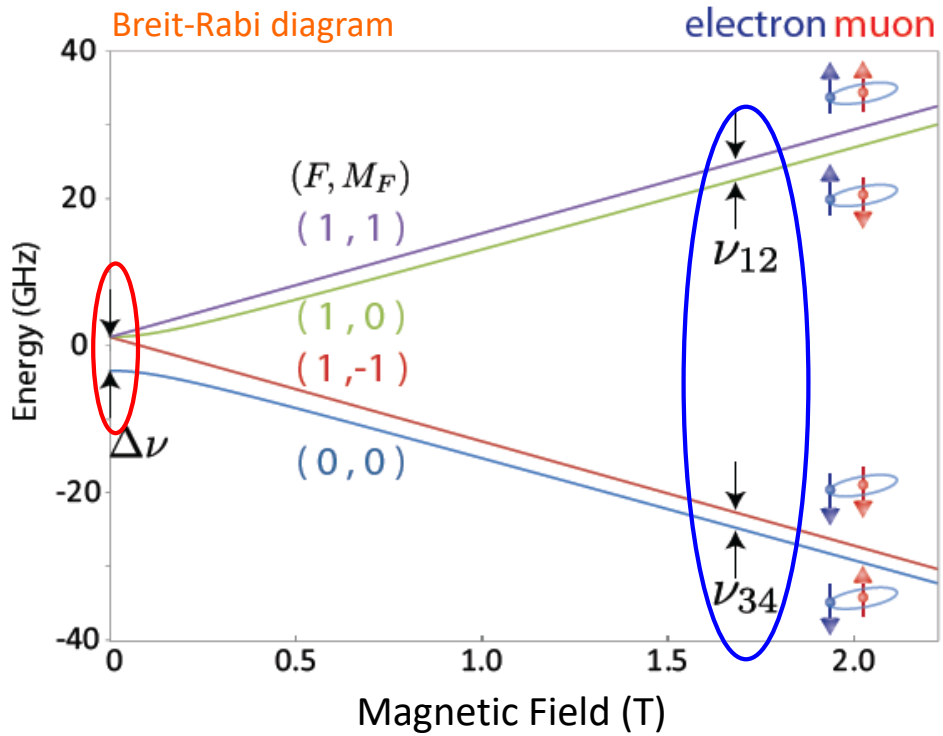
$$\mathcal{H} = h\Delta\nu \mathbf{I}_\mu \cdot \mathbf{J} - \mu_B^\mu g'_\mu \mathbf{I}_\mu \cdot \mathbf{H} + \mu_B^e g_J \mathbf{J} \cdot \mathbf{H}$$

$\Delta\nu_{HFS}$: Mu Hyperfine Structure

Zeeman Splitting



Pure lepton
= point particle



$\nu_{12}+\nu_{34} = \Delta\nu_{HFS}$ $\quad$ $\nu_{12}-\nu_{34} \propto \mu_\mu/\mu_p \propto m_\mu/m_p$

History of Mu HFS measurement

Time	Group	$\Delta\nu$	ppm	B field (T)	Ref
1961	Yale-Nevis	5500^{+2900}_{-1500} MHz		0.01-0.58	[33, 34]
1962	Yale-Nevis	4 461.3(2.0) MHz	450	1.1353	[35, 36]
1964	Yale-Nevis	4 463.24(12) MHz	27	0.5	[37, 38]
1966	Yale-Nevis	4 463.18(12) MHz	27	2.7×10^{-4}	[39]
1969	Yale-Nevis	4 463.26(4) MHz	9.0	3×10^{-4}	[40]
1969	Chicago	4 463.317(21) MHz	4.7	1.1353	[36, 41]
1970	Chicago	4 463.302 2(89) MHz	2.0	1.1353	[42]
1971	Yales-Nevis	4 463.308(11) MHz	2.5	3×10^{-4} and 1×10^{-6}	[43]
1973	Chicago-SREL	4 463 304.4(2.3) kHz	0.5	0	[44]
1975	LAMPF	4 463 302.2(1.4) kHz	0.3	very weak	[45]
1977	LAMPF	4 463 302.35(52) kHz	0.12	1.36	[46, 47]
1982	LAMPF	4 463 302.88(16) kHz	0.036	1.36	[48]
1999	LAMPF	4 463 302.765(53) kHz	0.012	1.7	[4]

Table 2.1: Comparison of Mu HFS measurements.

Magic Field

Spin Echo

Zero Field
World Record

Old Mu Method

High Field
World Record

K.S.Tanaka thesis

Most Precise Test of Bound State QED

Experiment:

LAMPF Experiment (1999)

ν_{HFS}(exp) 4463.302 765 (53) MHz [12 ppb]

μ_μ/μ_p = 3.18334524(37) [120ppb]

m_μ/m_e = 206.768277(24) [120ppb]

Theory:

ν_{HFS}(theory) 4463.302 868 (515) MHz [120 ppb]

ν_{HFS}(QED) 4463.302 720 (511) (70) (2) MHz

(m_μ/m_e) (QED) (α)

By Eides(2019) Two times lager than CODATA!

ν_{HFS}(weak) -65 Hz

ν_{HFS}(had. v.p.) 232 (1) Hz

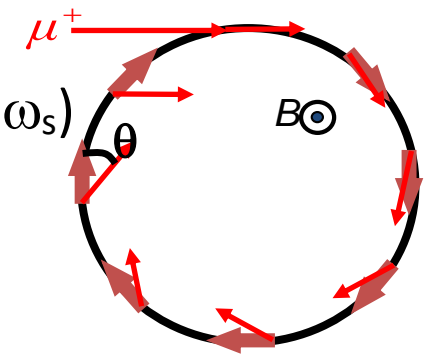
ν_{HFS}(had. h.o.) 5 (2) Hz

QED calculation: Effort for 10 Hz accuracy in progress (by Eides et al.)

Progress: PRA 86 (2012) 024501, PRL 112 (2014) 173004, PRD 89 (2014) 014034

Why Mu HFS measurement is so important?

g-2 E821(BNL) 0.5ppm 3.6σ deviation



- Measurement of the deviation of muon spin direction (ω_s) and muon momentum direction (ω_c) $\omega_a \propto (g-2)/2 = a_\mu$

$$\Rightarrow \vec{\omega}_a = \frac{e}{mc}\left[a_\mu \vec{B} - \cancel{\left(a_\mu - \frac{1}{\gamma^2-1}\right)\beta \times \vec{E}}\right]$$

a_μ *an independent precise muon mass measurement is required!*

- The ratio to the proton NMR frequency is important!

$$\Rightarrow \boxed{a_\mu = \frac{R}{\lambda - R}} \qquad \boxed{R \equiv \frac{\omega_a}{\omega_p}} \qquad \boxed{\lambda \equiv \frac{\mu_\mu}{\mu_p}}$$

From g-2 storage ring From Muonium HFS

$$\frac{\omega_a}{\omega_L(\mu)} = \frac{a_\mu\left(\frac{eB}{mc}\right)}{g_\mu\left(\frac{eB}{2mc}\right)} = \frac{a_\mu}{\left(\frac{g_\mu}{2}\right)} = \frac{a_\mu}{1+a_\mu}$$

$$= \frac{\omega_a}{\omega_L(p)}\frac{\omega_L(p)}{\omega_L(\mu)} = \frac{\omega_a}{\omega_p}\frac{\mu_p}{\mu_\mu} = R/\lambda$$

μ_μ/μ_p accuracy from direct measurement of 120 ppb.

W. Liu et al., Phys. Rev. Lett. 82 (1999) 711

Exiotic particle search

a pseudo vector boson

$$-\frac{\alpha}{r} \longrightarrow -\frac{\alpha + \alpha''(\mathbf{s}_1 \cdot \mathbf{s}_2)e^{-\lambda r}}{r},$$

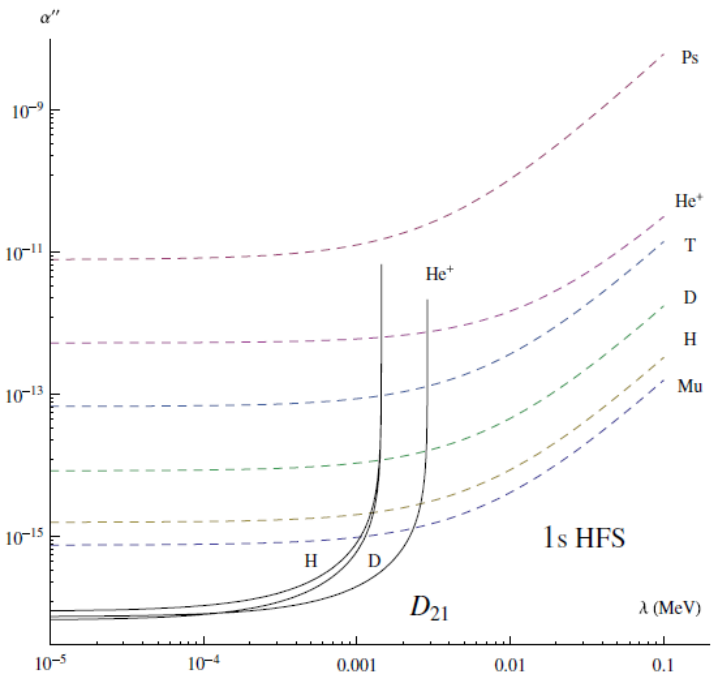


Fig. 2 on PRL 104, 220406 (2010)

a massive vector boson

$$\frac{\Delta E_{\rm hfs}}{E_{\rm hfs}} = \frac{8\alpha' m_e}{m_V} = \frac{8\alpha\kappa(\kappa + g_V/e)m_e}{m_V}.$$

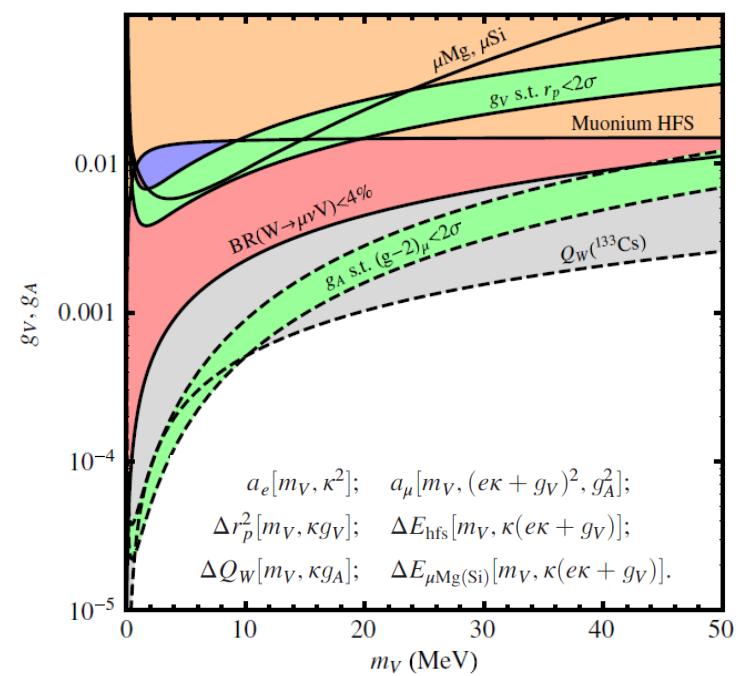


Fig. 6 on PRD90, 073004(2014).

S. G. Karshenboim *et al*., PRL 104, 220406 (2010), PRD82, 113013(2010).
PRA 84, 064502(2011) , PRD90, 073004(2014).

Test of CPT and Lorentz Invariance

CPT broken Theory ⇒ Lorentz symmetry is broken

O.W. Greenberg, PRL 89 (2002) 231602
R. Blihm, V.A. Kosteleky, C.D. Lane, PRL 84 (2000) 1098
V.W. Hughs et al., PRL 87(2002) 111804

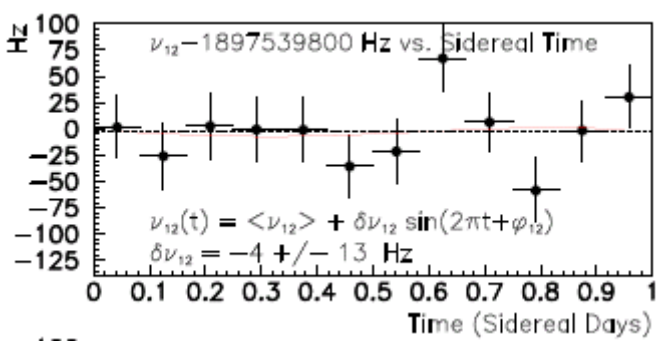
CPT violation search

Ex: Muon difference $g_{\mu+}/ g_{\mu-}$ 10^{-8}

g_μ-2/MuHFS precise measurement

Lorentz symmetry violating term in STE Lagrangian b
Corresponding MuHFS $\Delta\nu_{12/34}$
These value might change in sidereal time (23h56m)

$$\tilde{b}_3^{\mu}/\pi=-\delta\Delta\nu_{12}=\delta\Delta\nu_{34}$$



LAMPF Exp. Figure of Merit

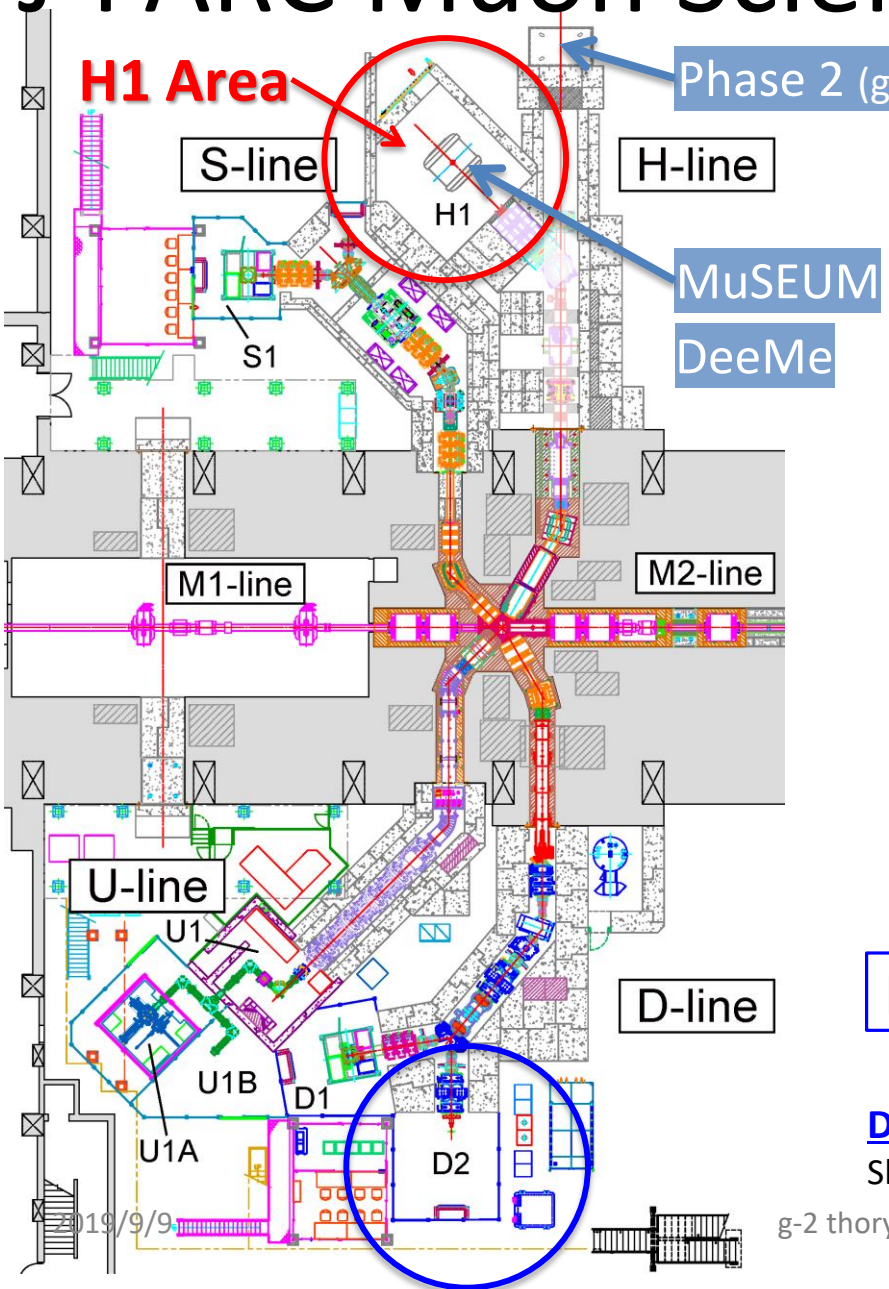
$$2\sqrt{(b^{\mu+}{}_X)^2+(b^{\mu+}{}_Y)^2}\,/\,m_\mu < 5\times10^{-22}$$

$m_\mu/M_P \sim 10^{-20}$

Plank scale sensitivity

V.A. Kostelecky, A.J. Vargas, PRD 92 (2015) 056002

J-PARC Muon Science Facility (MUSE)



Under construction

H-Line: for particle and atomic physics large scale experiments, "precison frontier".

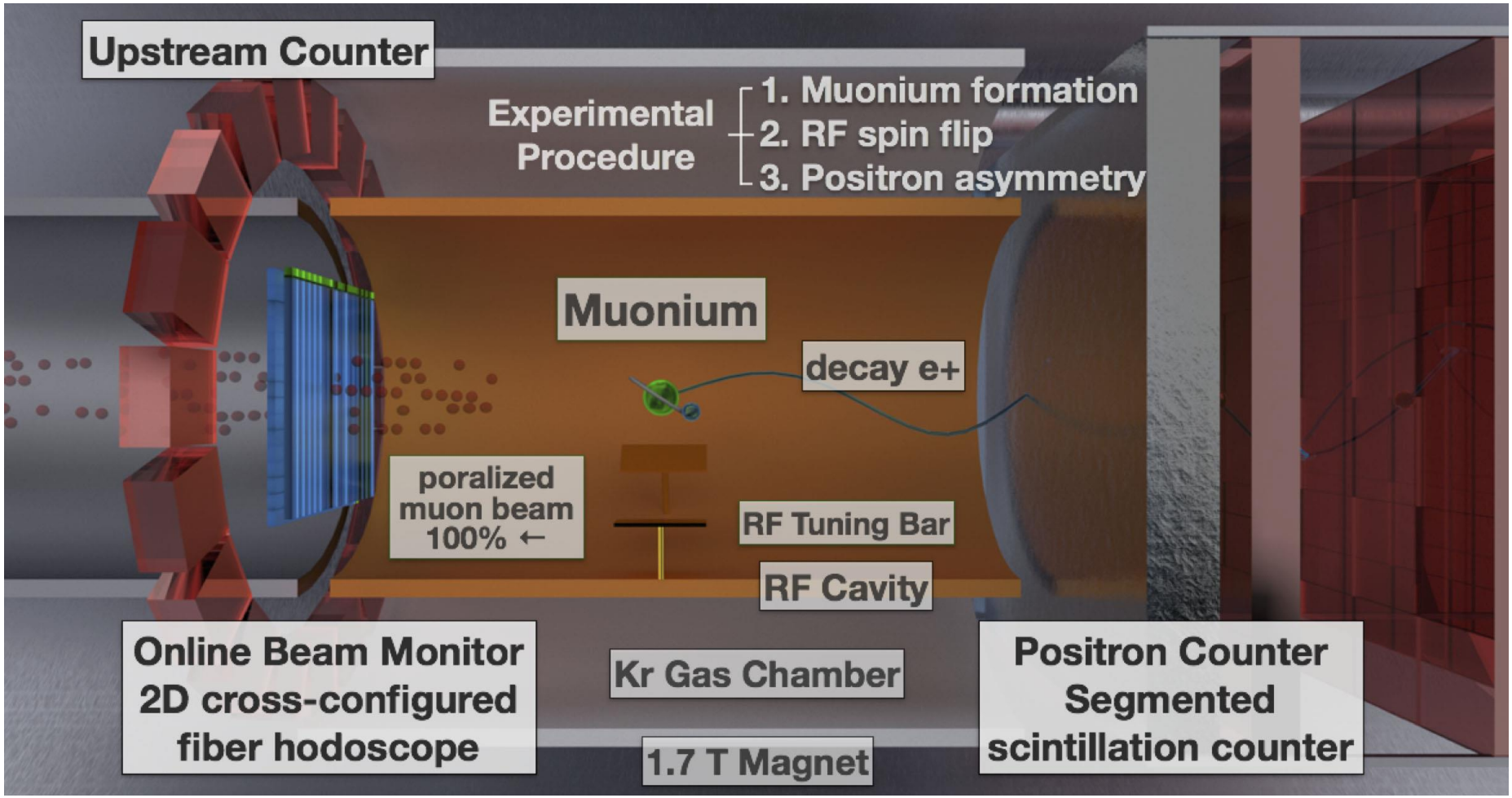
Higher intensity tunable (4 – 50 MeV) μ^+ & μ^- beam.
(Exp.: MuSEUM, Deeme, g-2, …)

MLF Experimental Hall No. 1 (2018)

Beamline in Operation

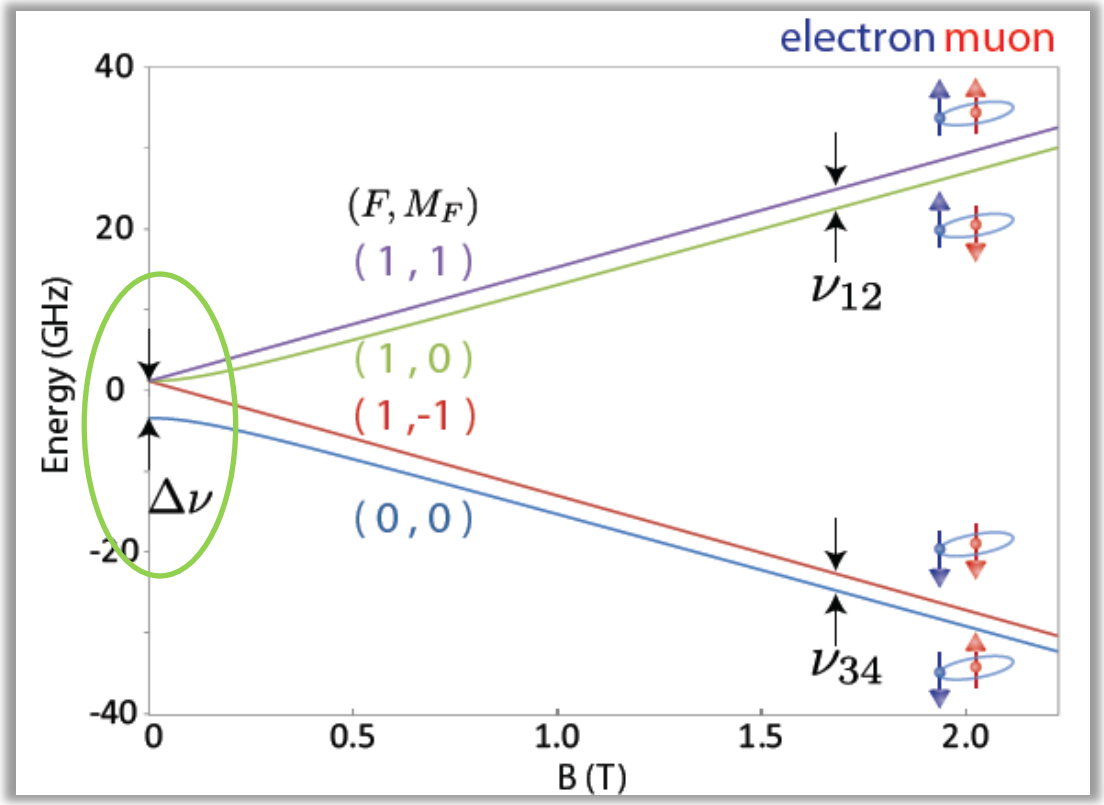
D-Line: Decay and Surface muon (μ^+ & μ^-)
Slow (50 keV) – fast (50 MeV) beam, general purpose.

MuSEUM Experimental Layout



Zero field measurement at D Line

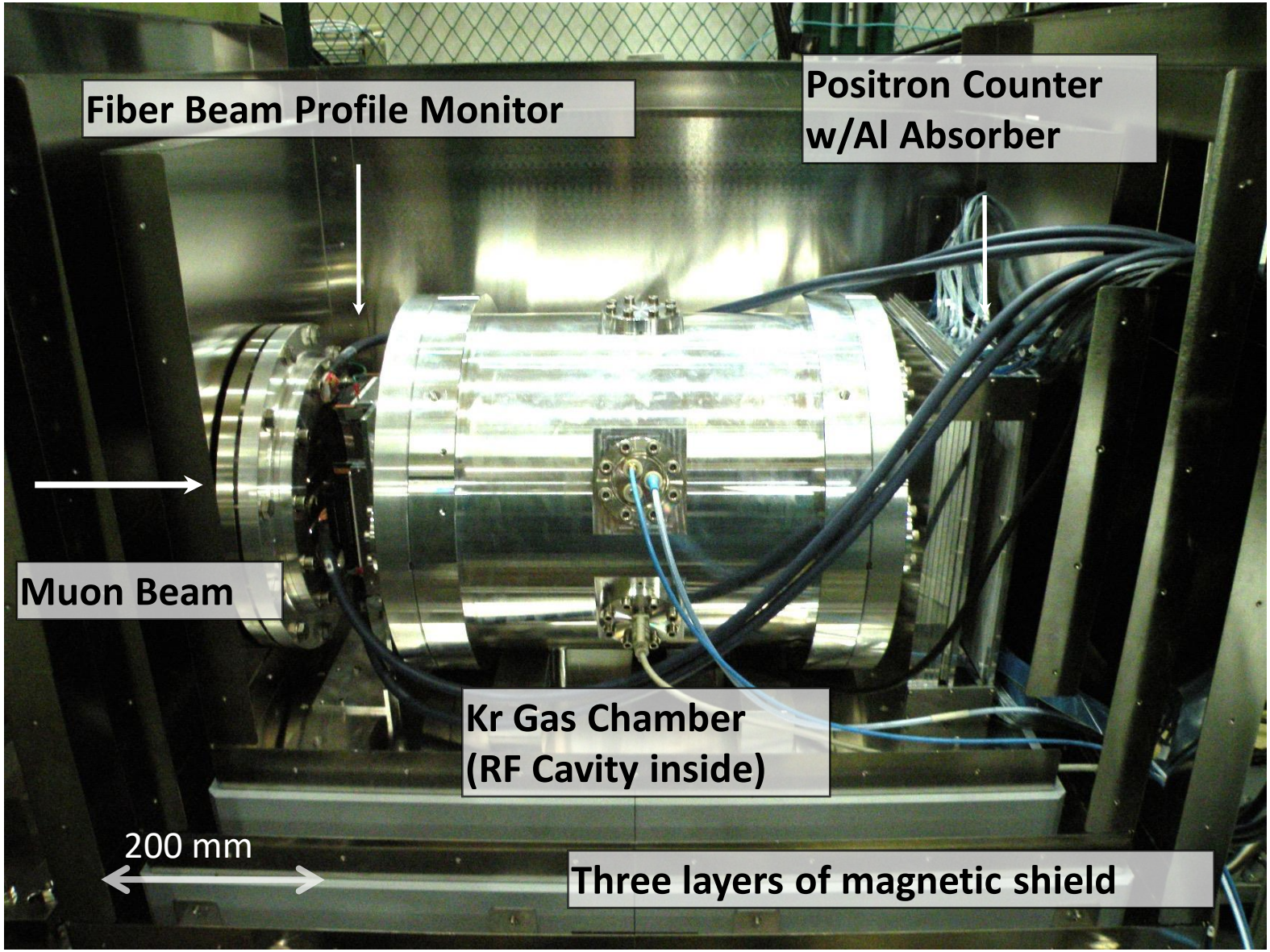
Important milestone for HF measurement-



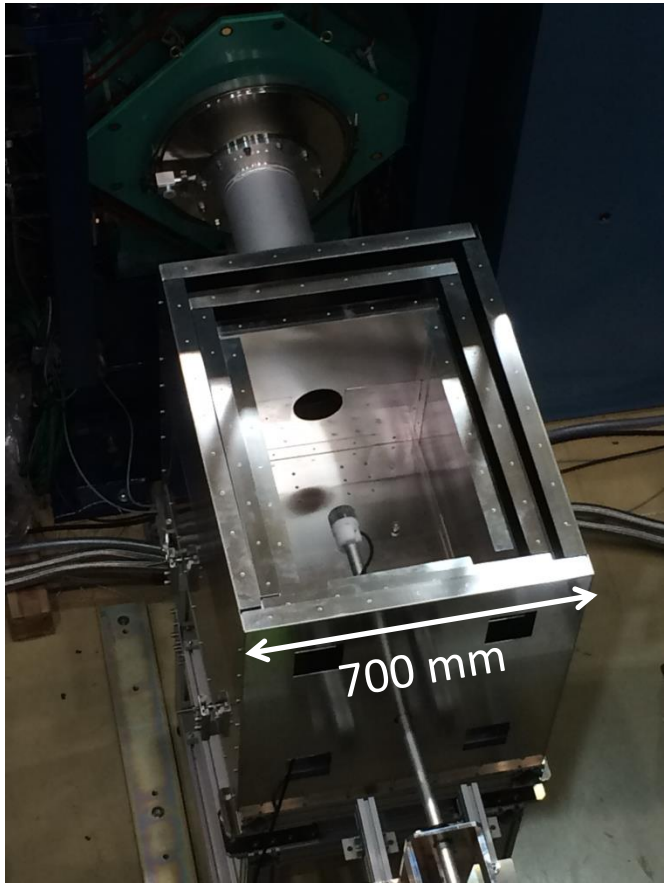
Two additional components are required.

- Permalloy magnetic shield
- RF cavity for ZF

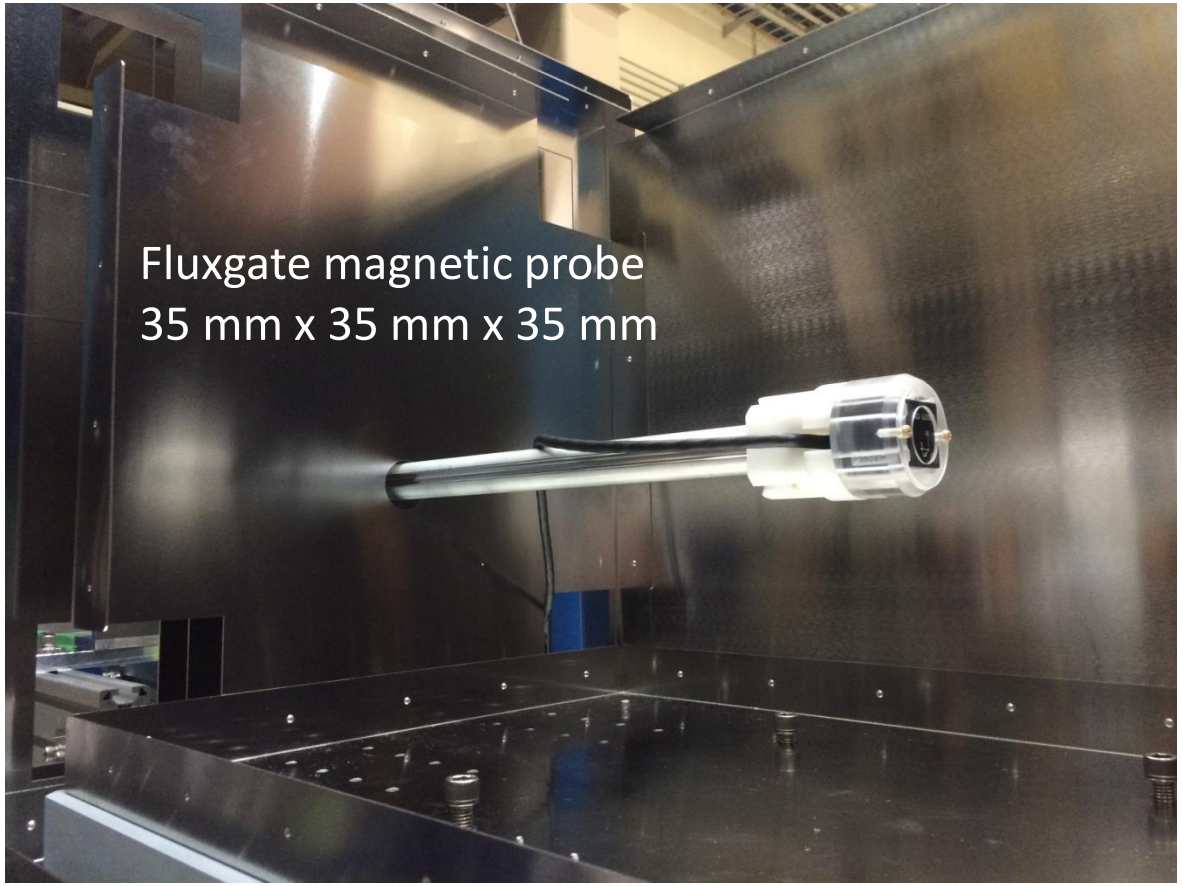
Resonance Measurement Setup



Magnetic Shield and Field Probe

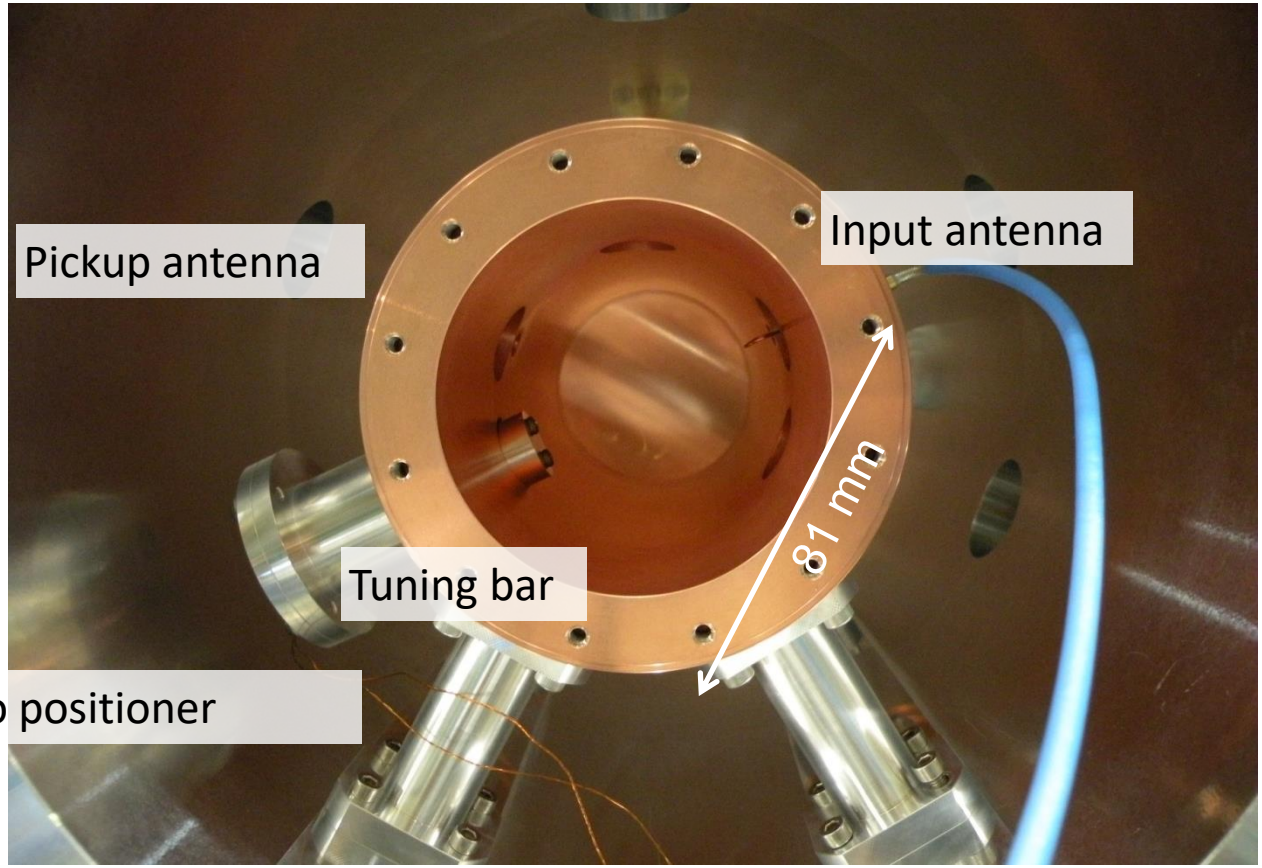


Assembled magnetic shield without top boards



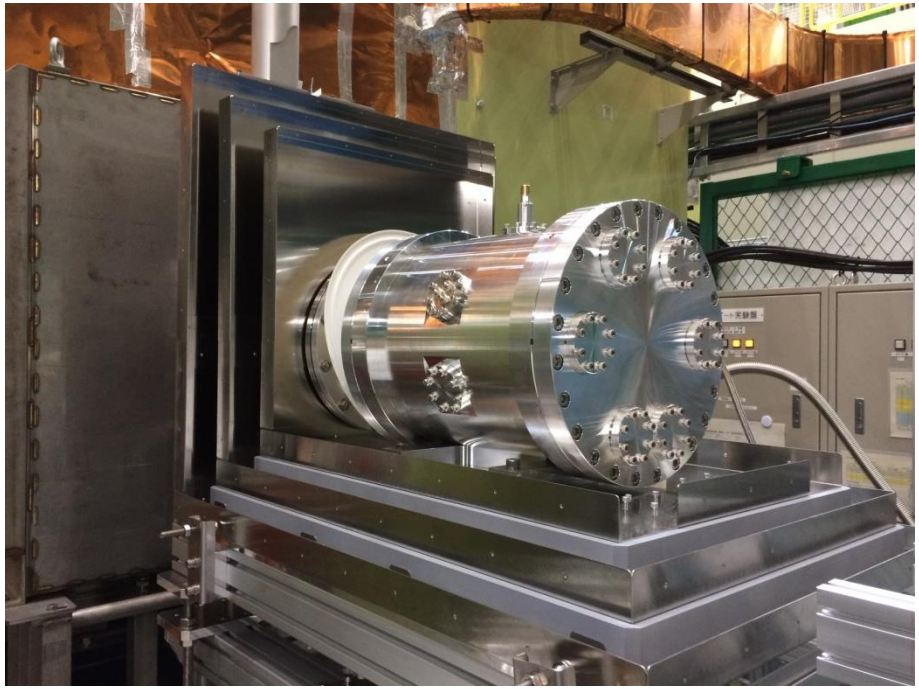
Magnetic field probe holder and its moving system

Inside of the Gas Chamber



- Expected Q-value is 10000, microwave power is up to 3 W
- TM110 mode at 4.463 GHz, +-1 MHz tuning by a piezo positioner

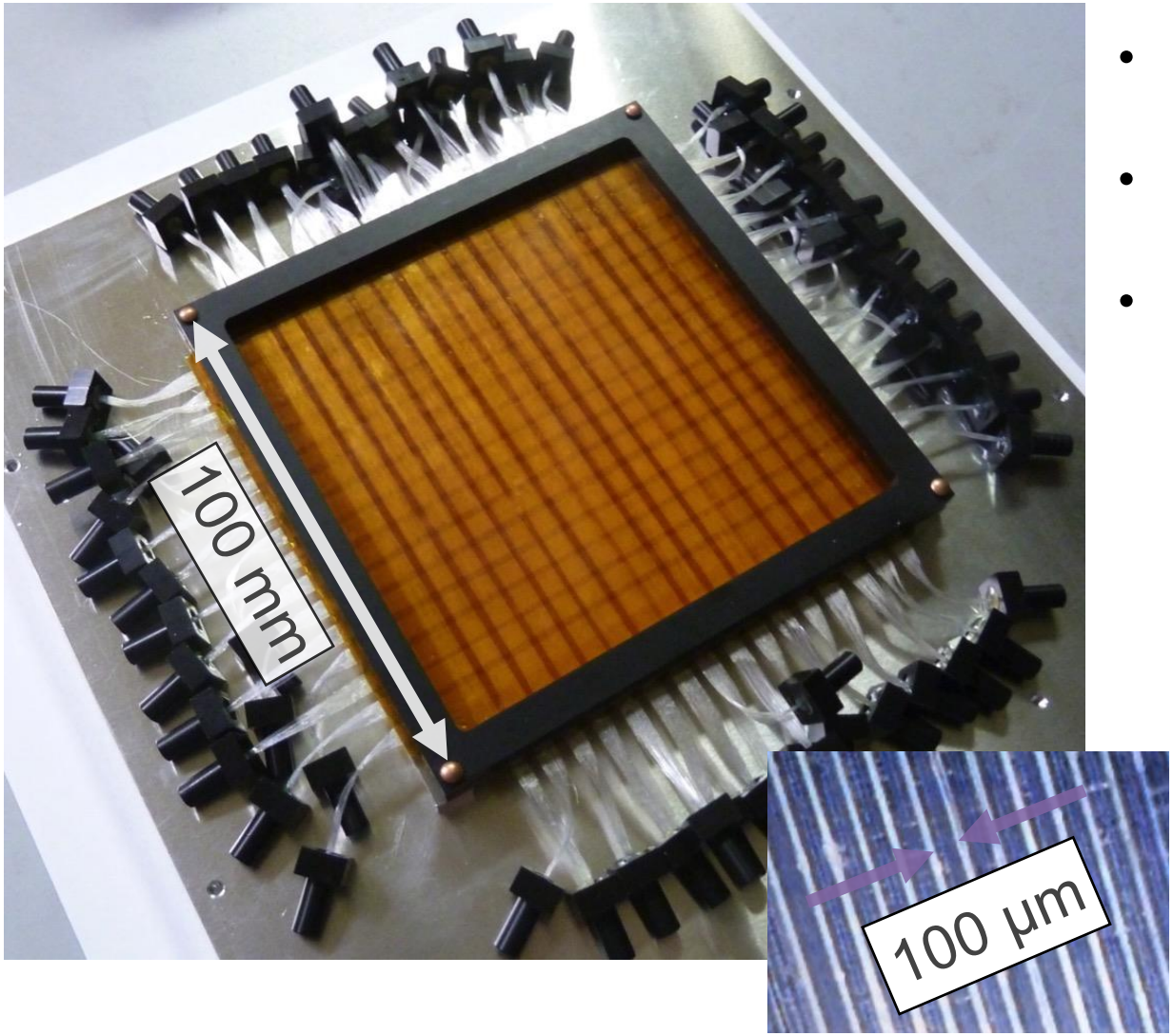
Gas Chamber and Gas Handling



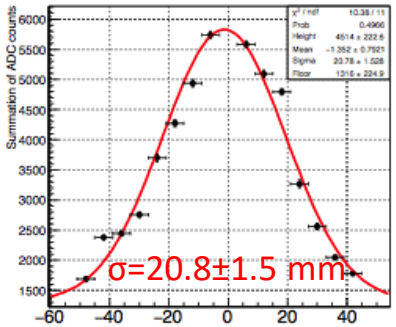
- 425 mm length, 280 mm diameter, 100 μm Al beam window
- Gas pressure is monitored by a capacitance gauge
- Gas purity is measured by Q-Mass spectrometer

Muon Beam Profile Monitor

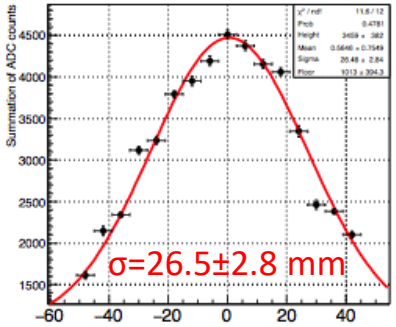
Kanda, Ueno, Toyoda



- Two layers of 100-μm fiber hodoscope (2x16ch).
- 3 x 3mm^2 active area MPPC with 15-μm pixel pitch.
- EASIROC readout



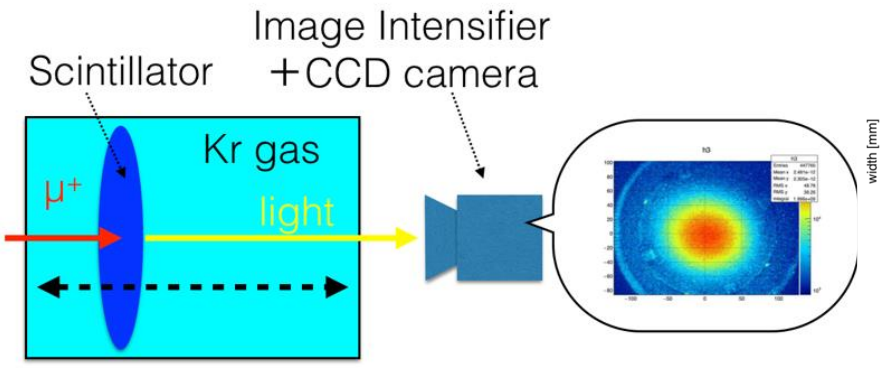
Horizontal position (mm)



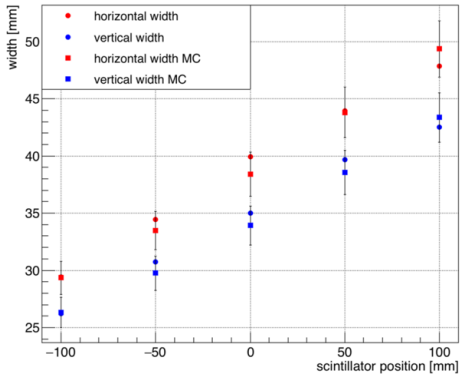
Vertical position (mm)

Offline 3D Beam Profile Monitor

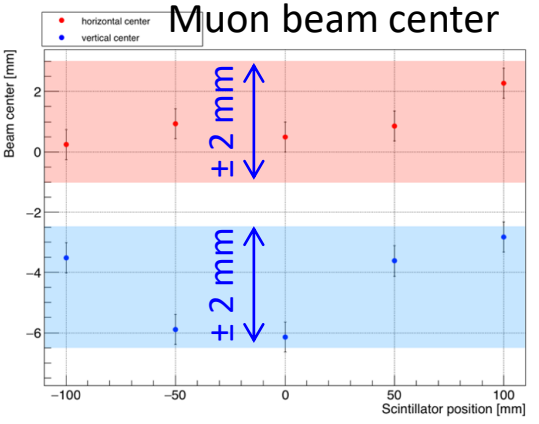
Ueno, Kanda, Toyoda, Ito

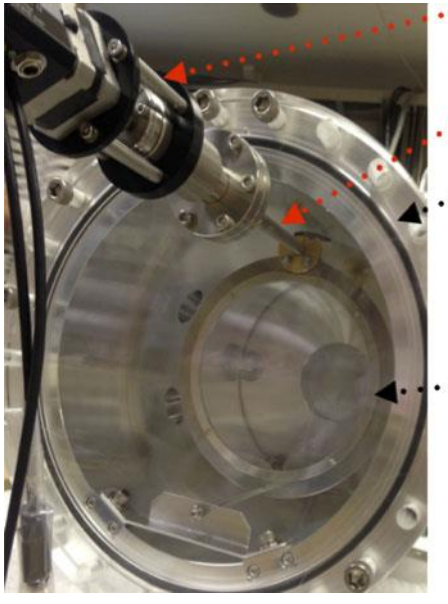


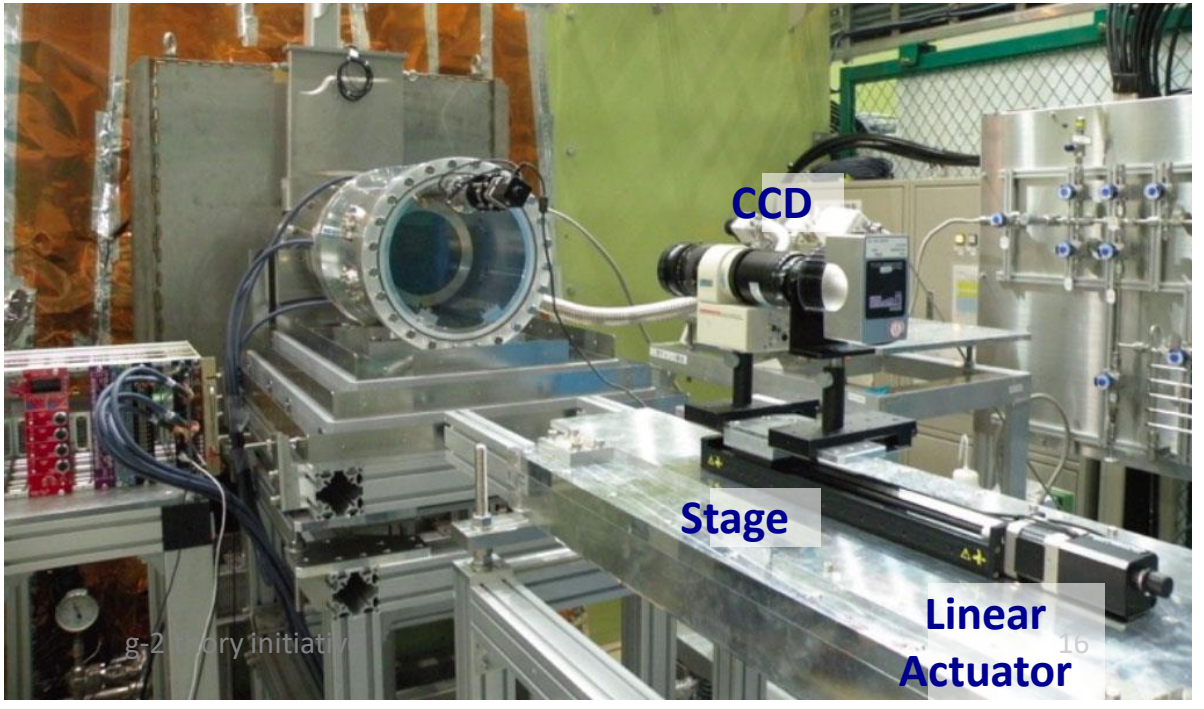
Muon beam width



Muon beam center

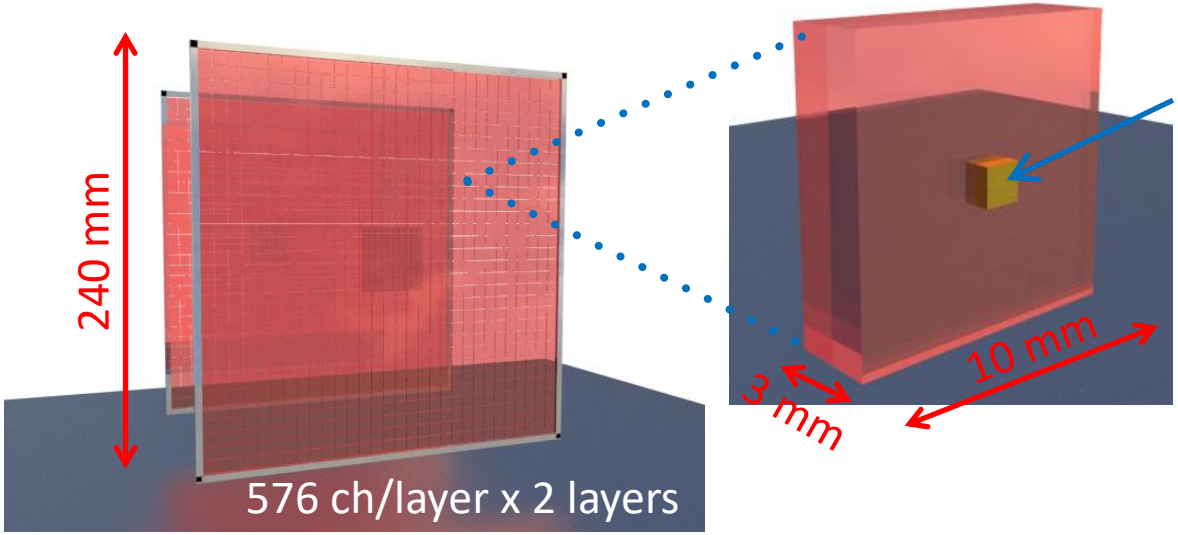




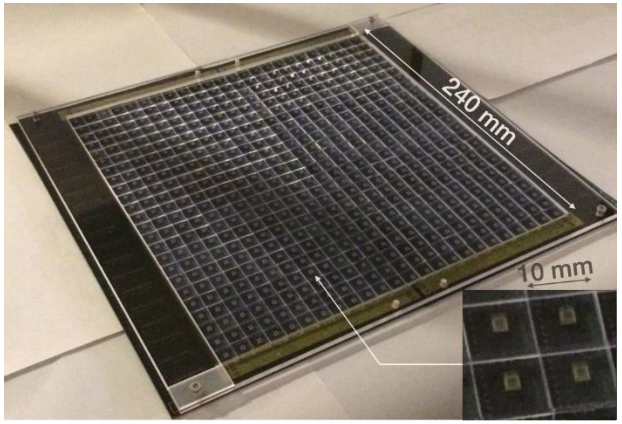


Positron Counter (1): Scintillation Position Detector

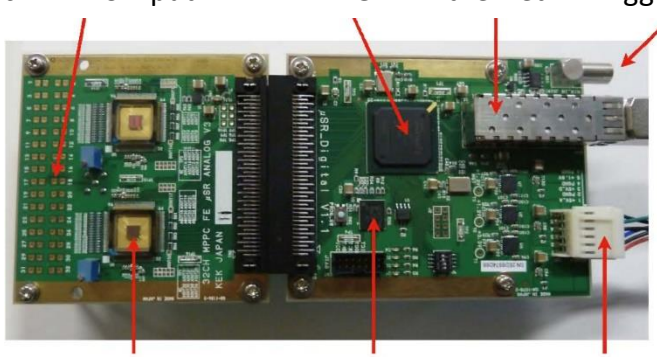
Kanda, Kojima



MPPC (Multi-Pixel Photon Counter)
1.3 mm x 1.3 mm active area
(Hamamatsu)

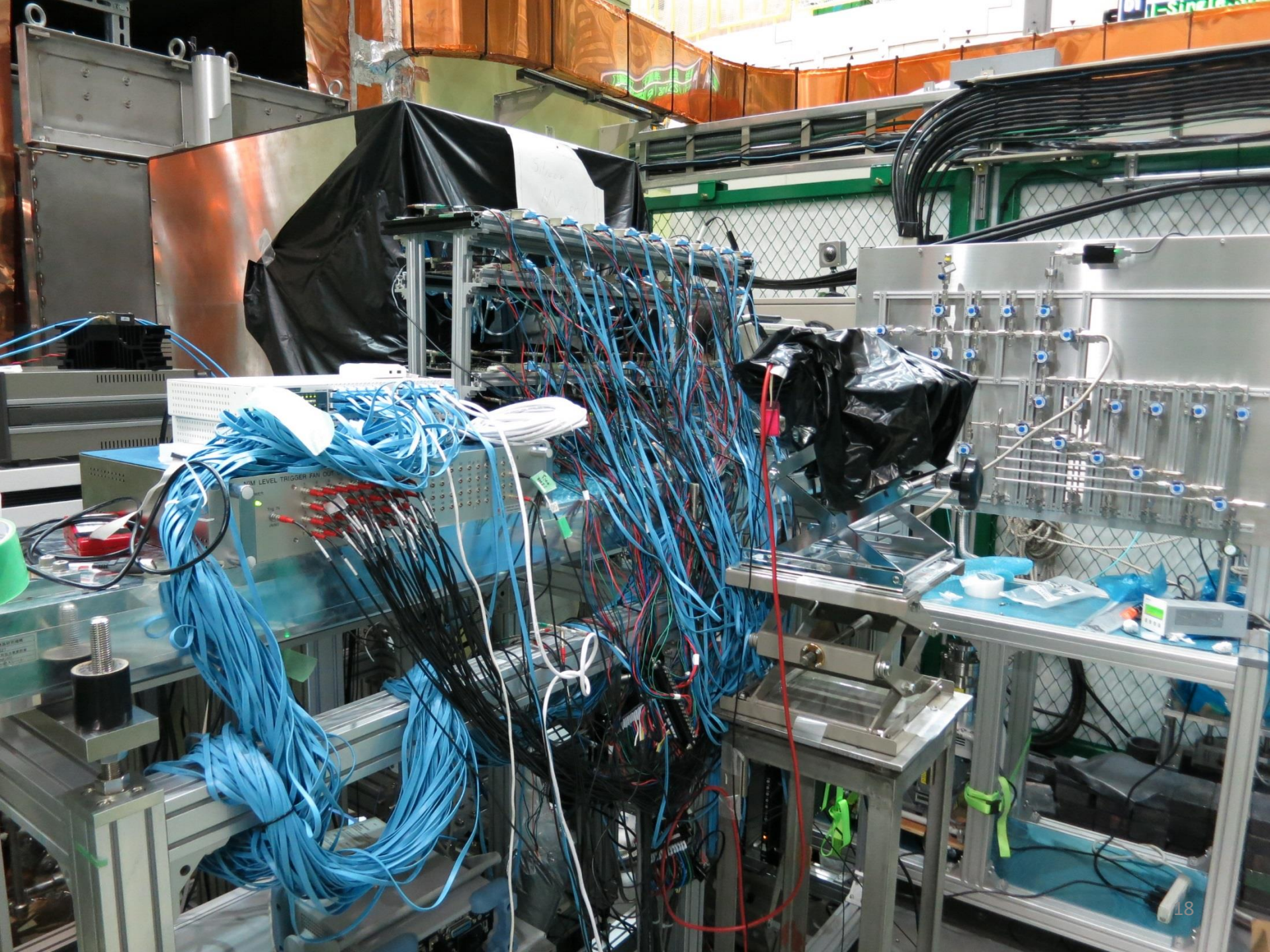


Plastic scintillator + MPPC + Kaliope readout circuit



Segmented scintillation detector

- Scintillation counter with SiPM readout
- Unit cell: 10 mm × 10 mm × 3 mm^t
- Area: 240 mm × 240 mm
- 24x24 segments x 2 layers = 1152 ch
- High-rate capability required
- Pileup loss at 3 MHz/ch ~ 2%


NIM LEVEL TRIGGER FAN OUT

First Resonance Search

2016. June. 3(24h)
Beam profile measurement

2016. June. 12-14(60h)
Muonium Resonance Search

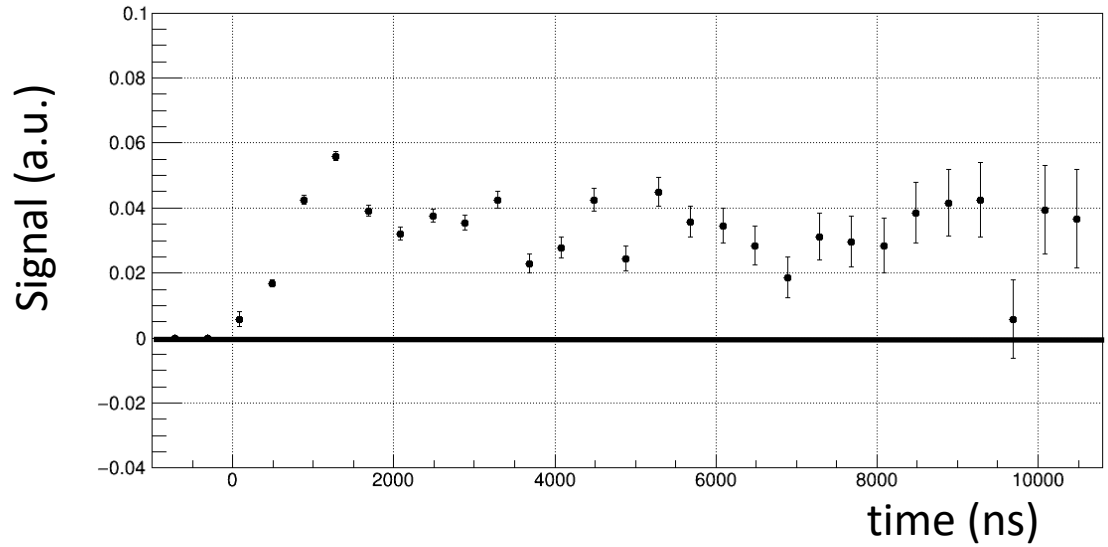
Results

1 Good zero field condition (~100nT) was obtained.
2 Beam profile was successfully measured.
3 Enough RF power and cavity Q value were obtained
4 All detectors were working properly.
5 After several trial, Muonium HFS resonances was obtained!

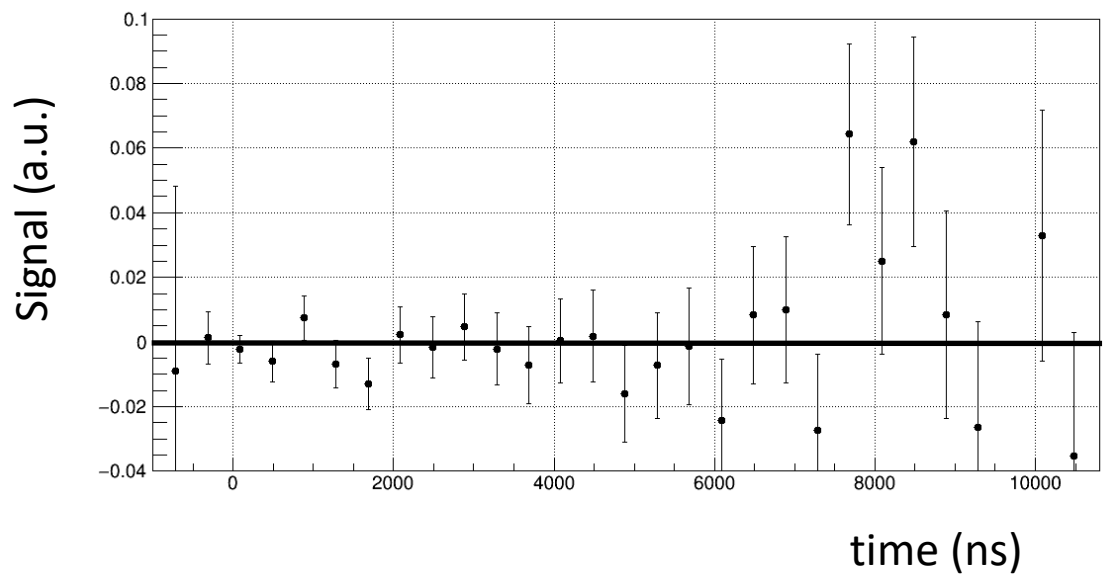
Time Dependent Spin Flip Signal

Kanda thesis

Near at Resonance
4463.1 MHz
RF frequency
1.0 Kr atm
27.4 MeV/c muon



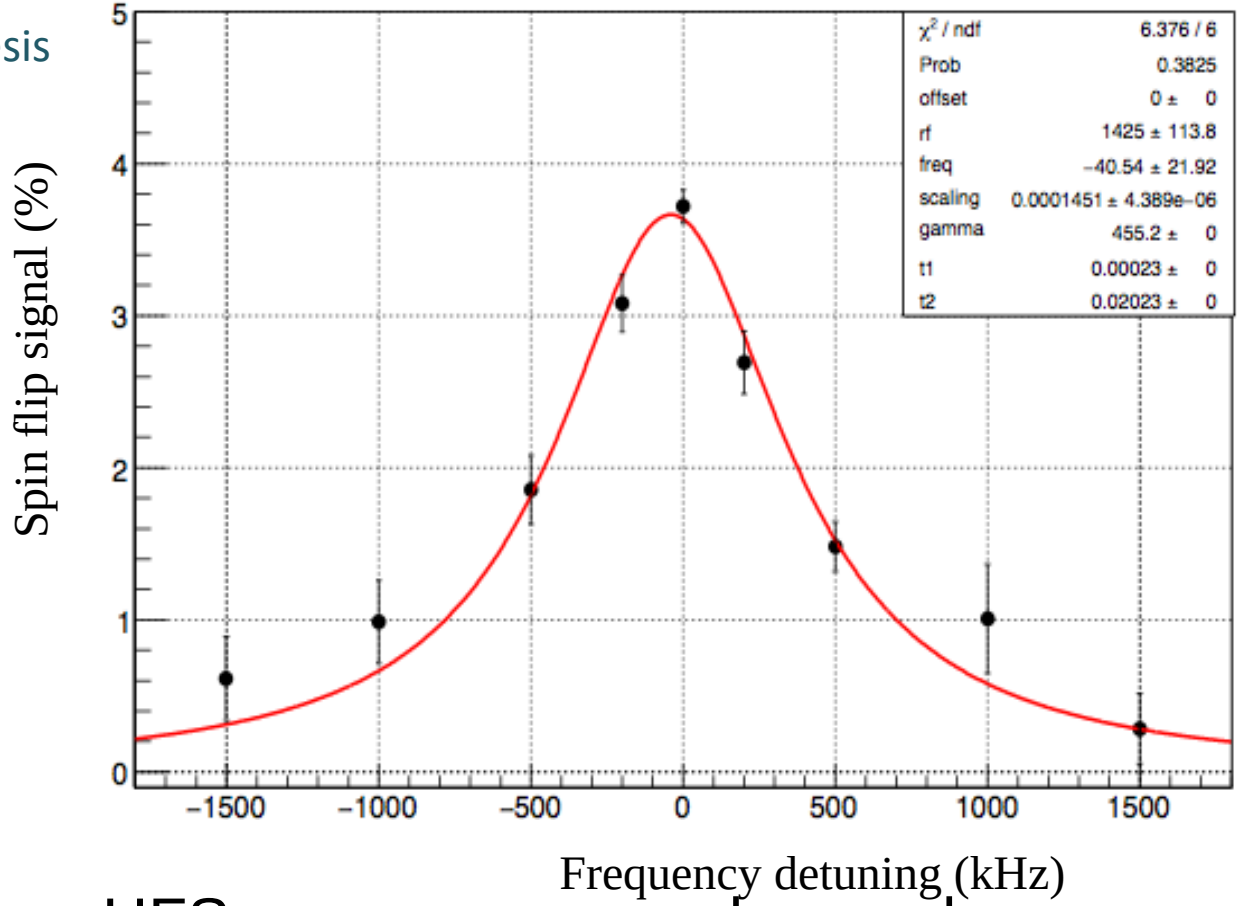
Off-Resonance
RF frequency was far detuned
1.0 Kr atm
27.4 MeV/c muon



Signal = ON/OFF-1

Resonance Lineshape

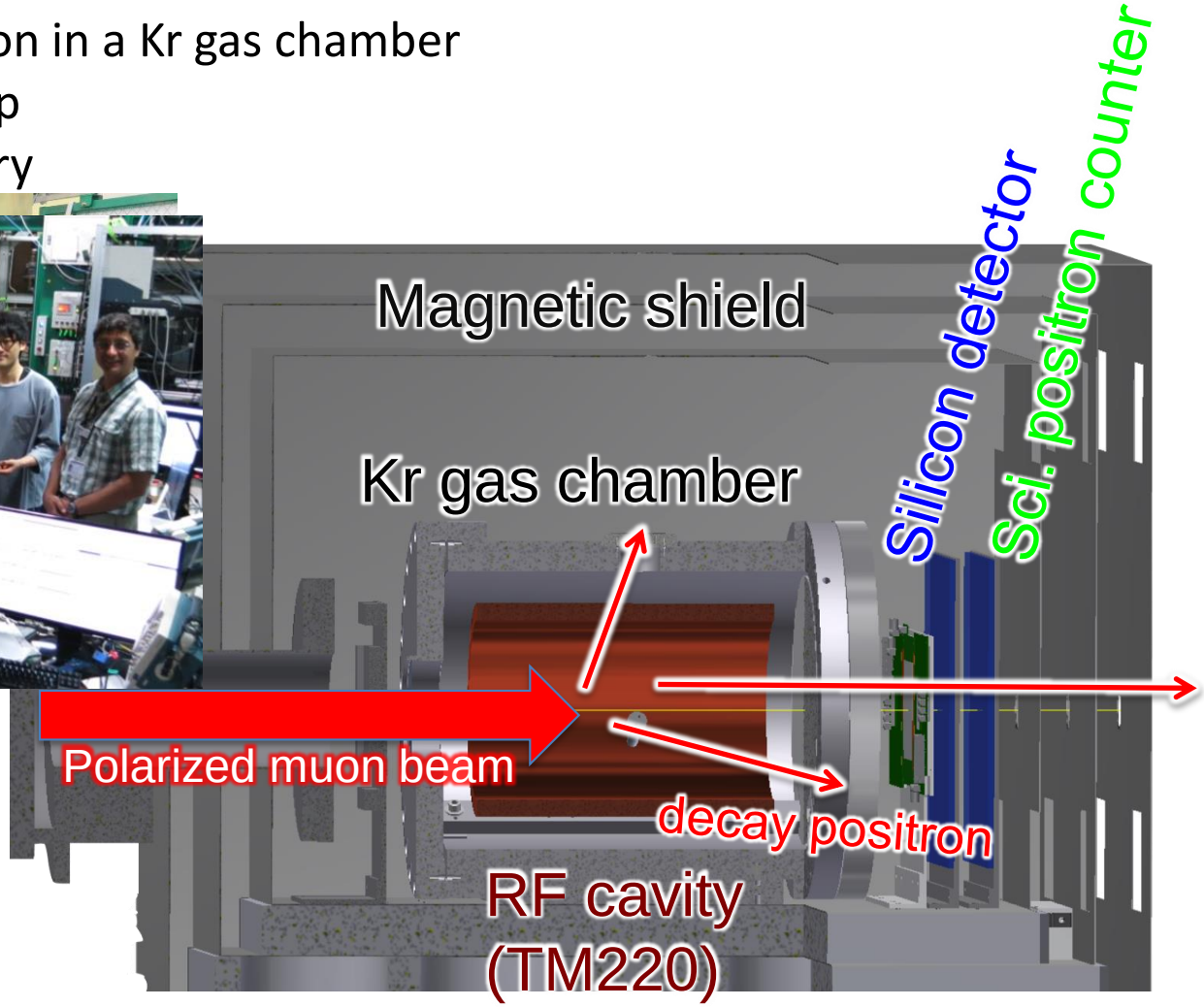
Kanda thesis



- Muonium HFS resonance was observed
- Fitted by Lorenzian and freq. center was 4463.1+-0.02 MHz
- Expectation from precursor experiments is 4463.1 MHz

MuSEUM Zero Field Experiment

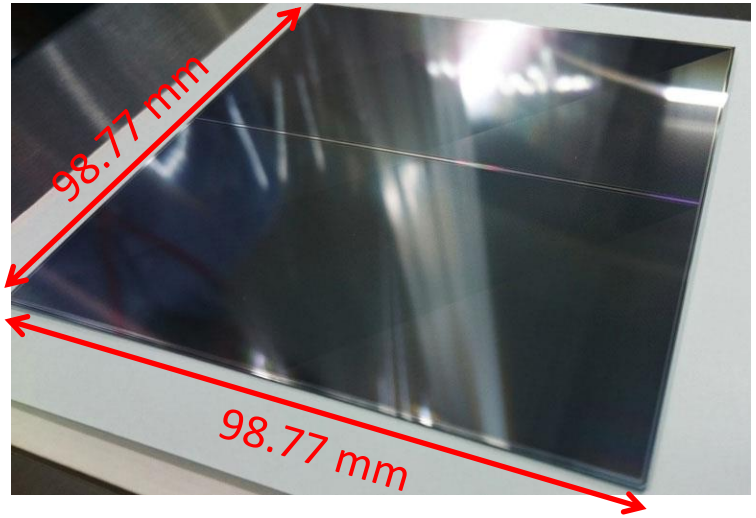
- Experimental Procedure
  - Muonium formation in a Kr gas chamber
  - Microwave spin flip
  - Positron asymmetry

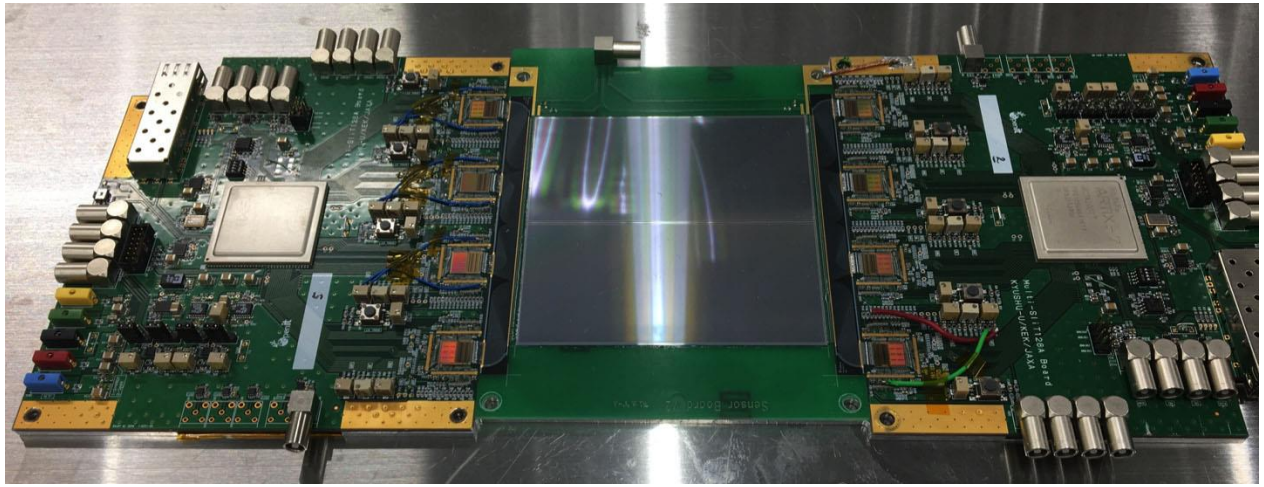


Positron Counter (2): Silicon Strip Detector

Nishimura

Item	Specification
Sensor type	single-sided, p+ on n
Size	98.77 mm × 98.77 mm
Active Area	97.28 mm × 97.28 mm
Strip pitch	0.19 mm
Strip length	48.575 mm
No. of strips	512 x 2 blocks
Thickness	0.32 mm



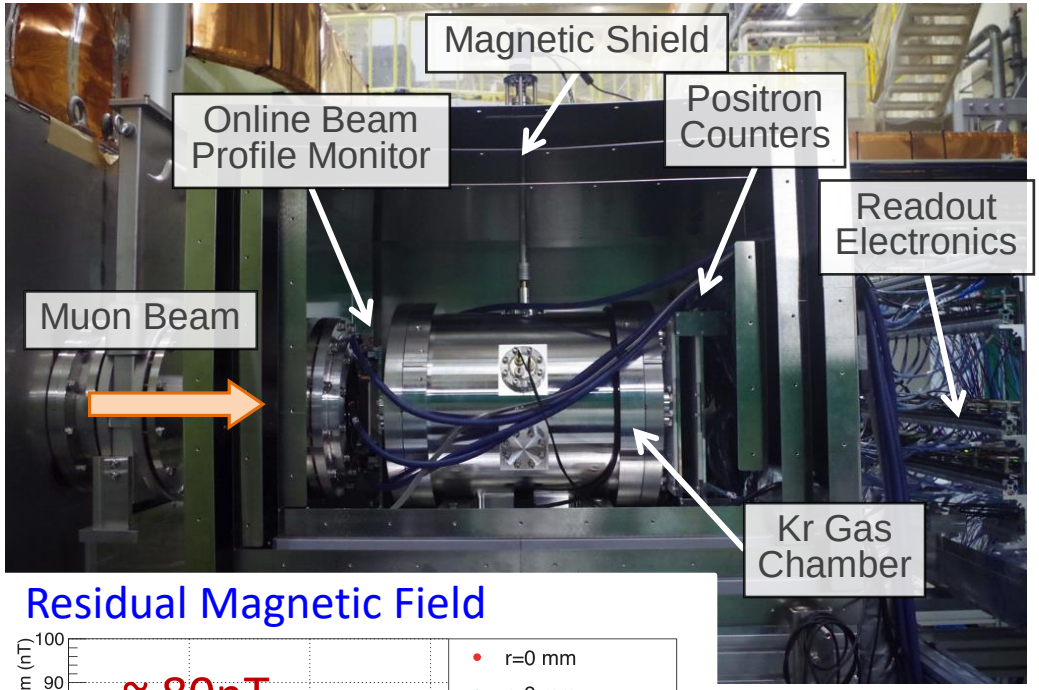


Silicon strip detector

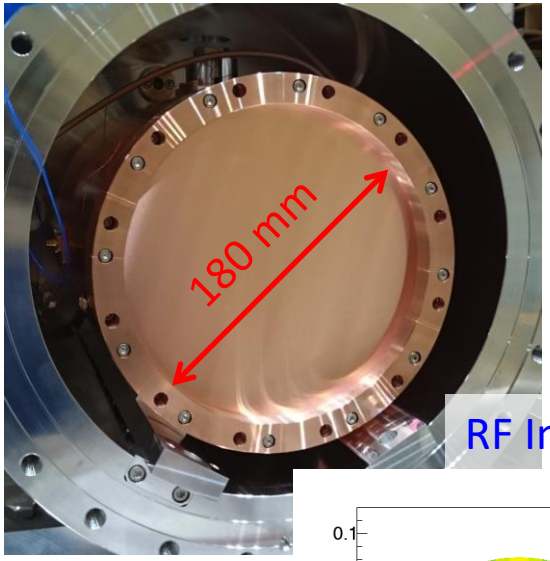
- Readout chips (SliT128A, 128 ch/chip)
- Developed for J-PARC g−2/EDM experiment
- Highly-segmented
- High-rate capability
- S/N ~ 21

Zero Field Measurements at D-Line

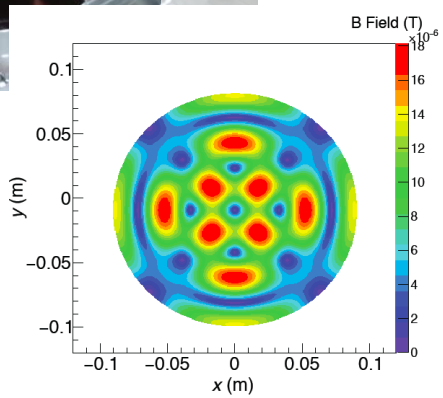
Experimental Setup



New RF Cavity for Zero Field



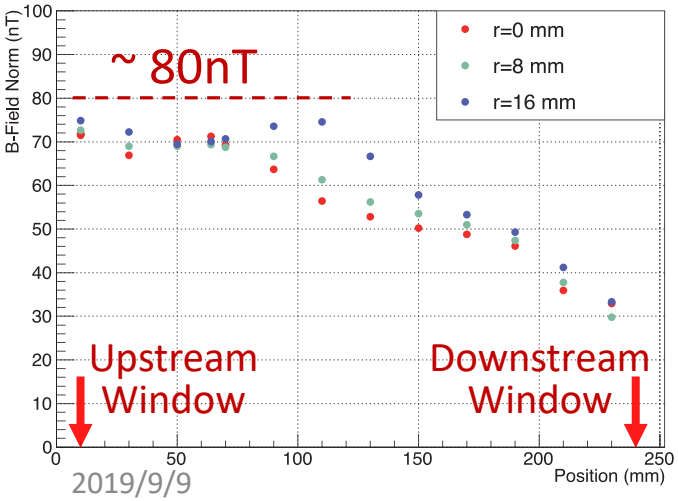
RF Intensity

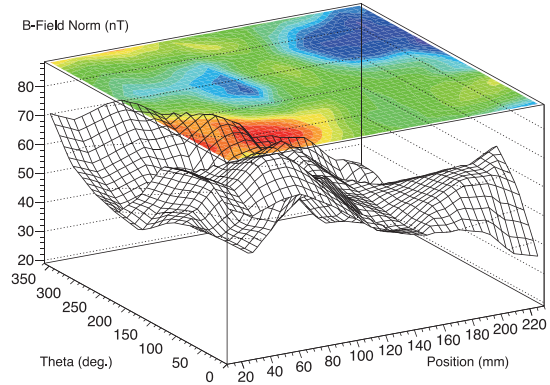


$\Delta\nu$ = 4.463 GHz

TM220 mode
Larger cavity
More muon stop
Q-Value: 20,000 (calc.)

Residual Magnetic Field

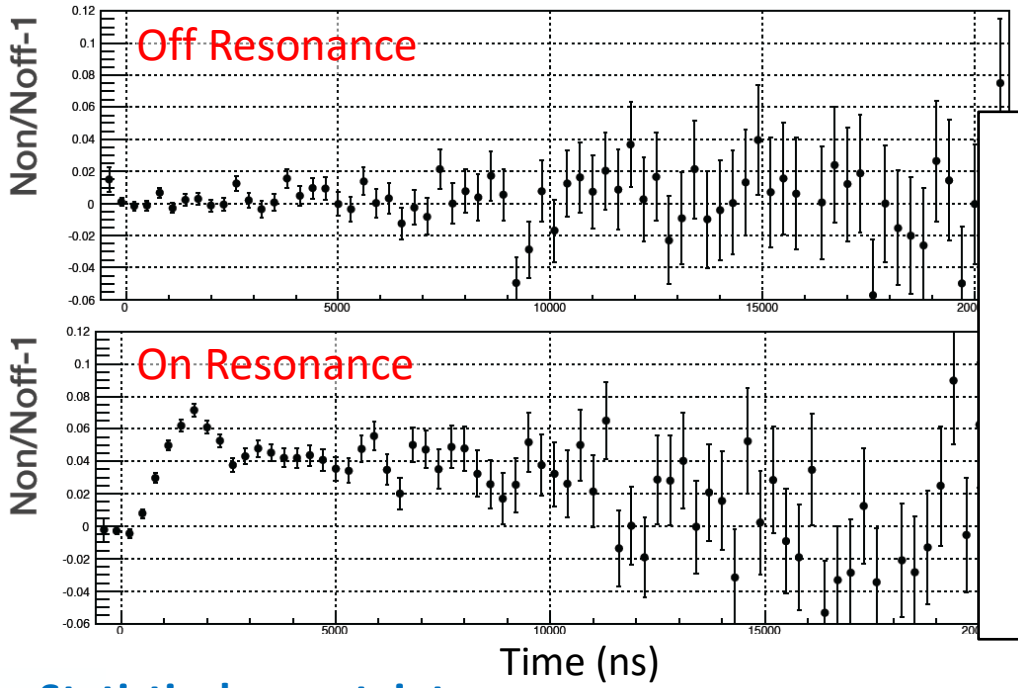




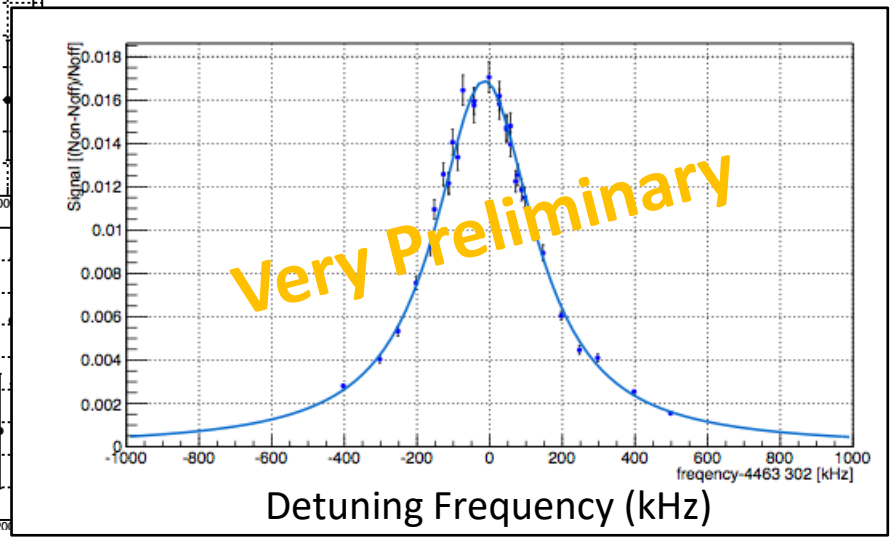
Results (1): Time Integral Method

Conventional Method

– Scintillation Position Detector Data –



Resonance Line Shape (2018 March)



Statistical uncertainty:

2016 Feb. ~ 20 kHz (5ppm)
2017 Feb. ~ 4 kHz (1ppm)
2017 June ~ 2 kHz (0.5ppm)
2018 March ~ 1kHz, measured at 0.4, 0.55, 0.7 atm.
2018 June ~ 1kHz, measured at 0.3 atm Kr gas pressure.

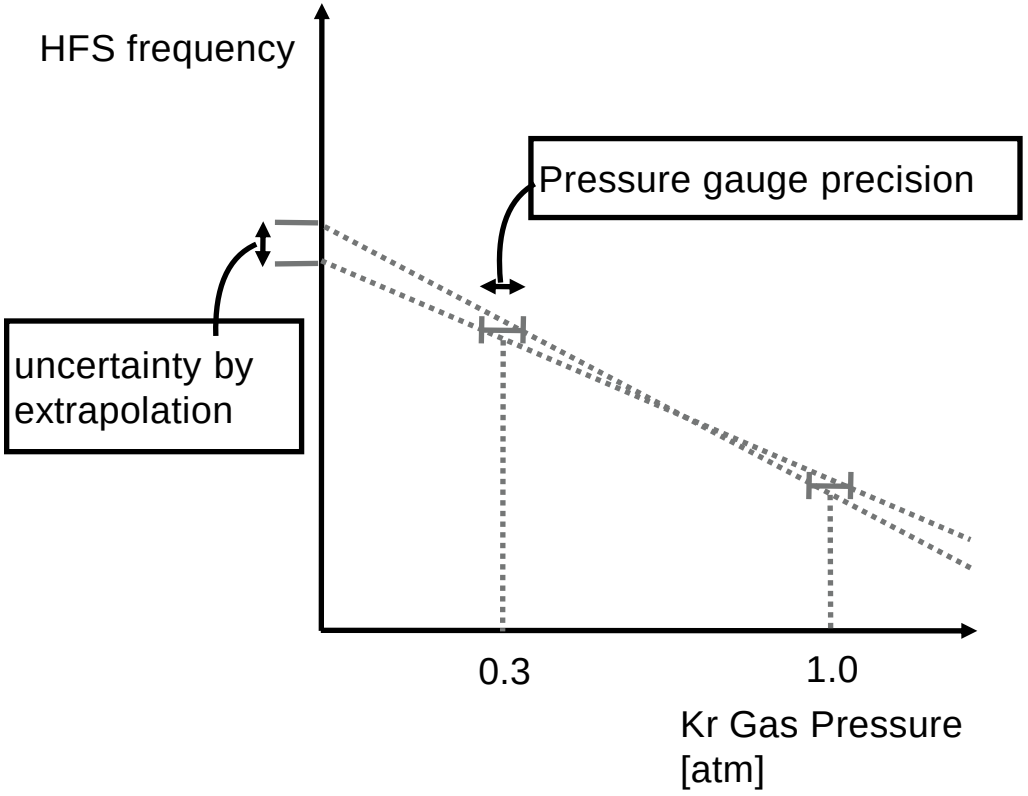
New world record at ZF ???
Data analysis on going

Previous ZF Experiment at LAMPF:
$\Delta\nu_{HFS} = 4\,463\,302.2 \pm 1.4$ kHz (0.3 ppm)

Systematic uncertainty: Estimation in progress

Latest Experiment

- 2018 Mar 9th - 16th @ J-PARC D2-Line
- Purpose : Measure the HFS value in vacuum by extrapolation
- Measured with 0.4, 0.55 and 0.7 atm Kr gas pressure

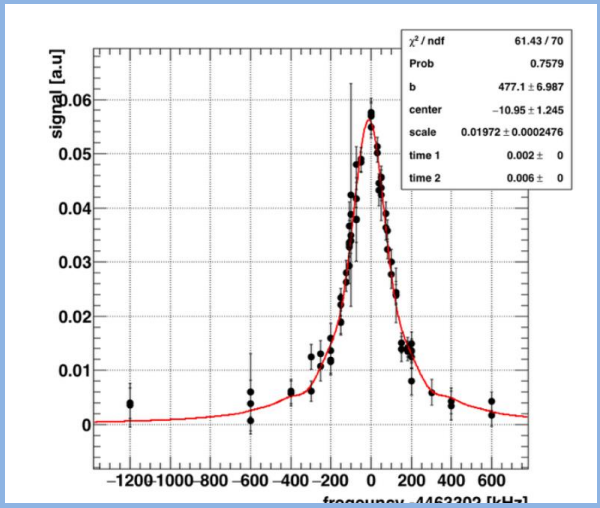


Mu HFS Measurement in 2018

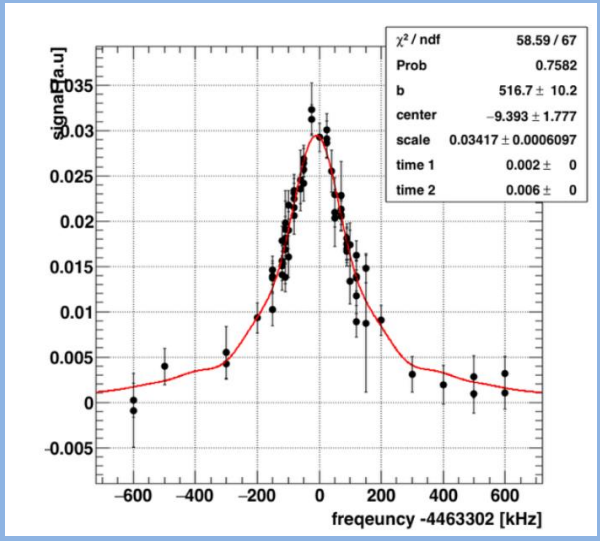
Ueno thesis

- Kr gas pressure shift
  - Resonance frequency is shifted due to collision of muonium & the Kr atom
  - Gas pressure in the experiment in 2018 0.3, 0.4, 1.0 atm
  - Spin flip resonance signal was obtained for each gas pressure

0.4 atm



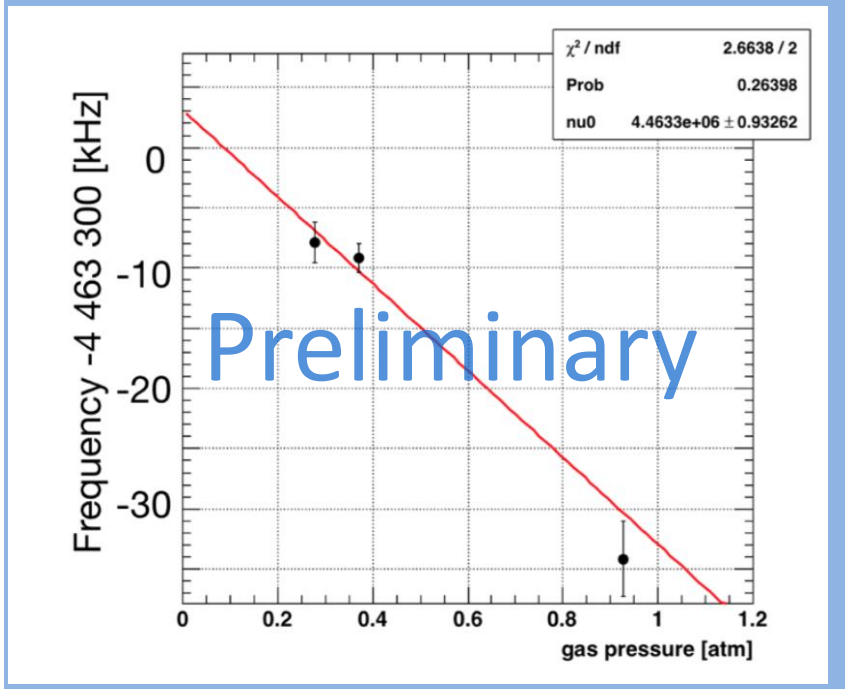
0.3 atm



Mu HFS Measurement in 2018

Ueno thesis

- Kr gas pressure shift
  - Resonance frequency is shifted due to collision of muonium & the Kr atom
  - Gas pressure in the experiment in 2018 0.3, 0.4,1.0 atm
  - Spin flip resonance signal was obtained for each gas pressure
- Recent analysis achieved 0.9kHz ! (Assume previous pressure)



Collitional Shift

- Systematic uncertainty from the collisional shift is dominant
- There are two major effect from the collision

- Pauli exclusion effect (Short-range)
  -> positive shift
- Van der Waals interaction effect (Long-range)
  -> negative shift
  dominant for large Z atom

J. Chem. Phys. 32, 972 (1960)

H-HFS pressure dependence in noble gas

B. K. Rao *et al.*, Phys. Rev. A 2(4) 1411(1970).

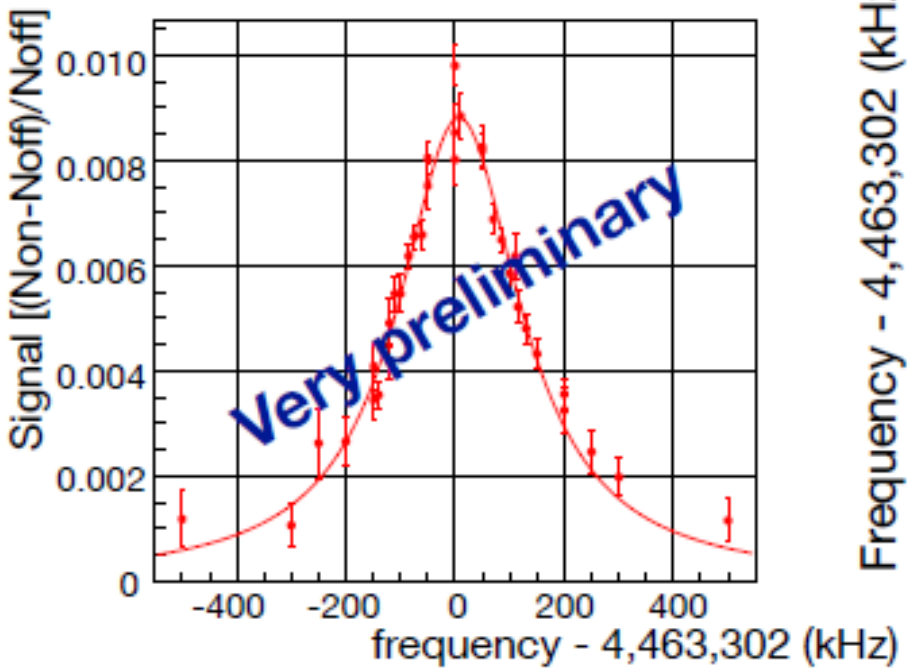
TABLE V. Values of the fractional pressure shift f_p for all five systems at 273 °K.

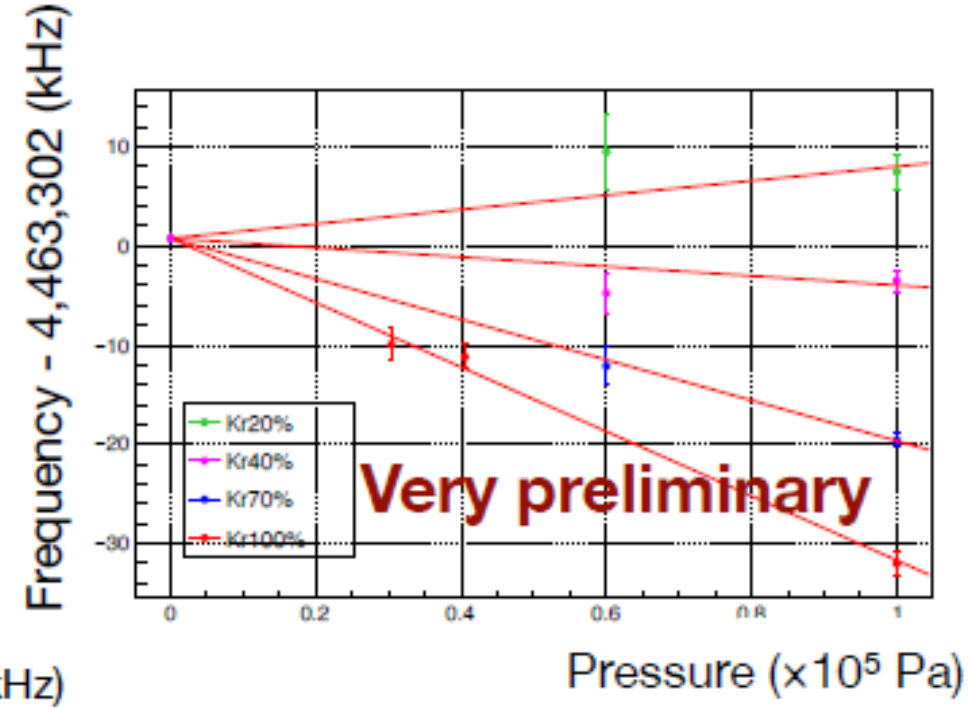
System	f_p (in 10^{-9}/ Torr) Short-range	Long-range	Total	Experiment
H-He[a]	2.130	− 0.952	1.178	4.80
H-He[b]	2.679	− 0.950	1.729	4.80
H-Ne	2.059	− 1.952	0.107	2.88
H-Ar	4.556	− 7.827	−3.081	−4.77
H-Kr	7.525	−11.842	−4.317	−10.40
H-Xe	10.922	−17.560	−6.638	−20.00

- **He-Kr mixture gas can reduce the collisional shift**
- From Hydrogen HFS, Kr:He=4.8:10.4=Kr32% is ideal

Kr-He mixture measurement

- Used mixture gas Kr-He to suppress the extrapolation uncertainty
- Confirmed Mu formation in Kr-He gas, and observed MuHFS resonance





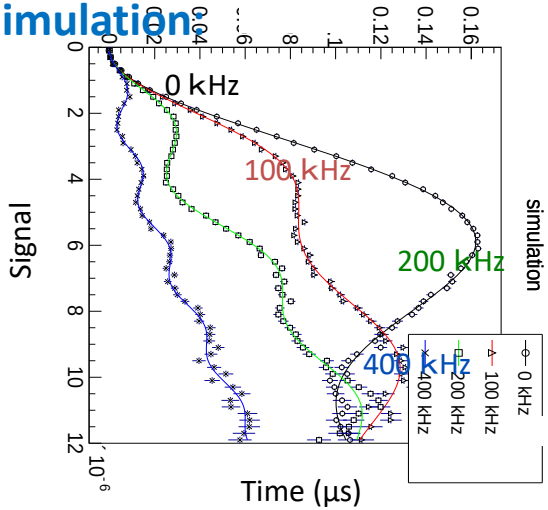
resonance with Kr 20% gas at 1.0 atm

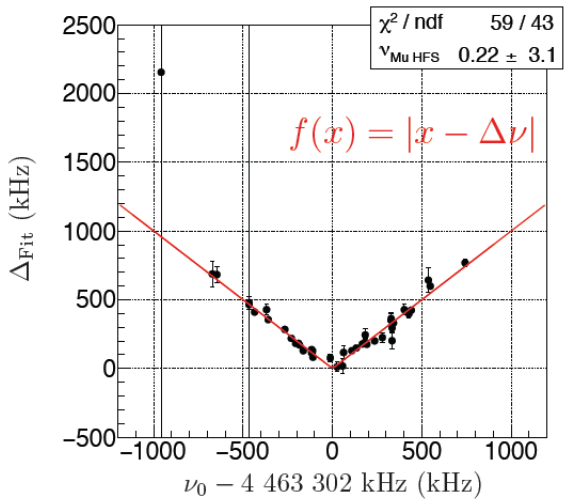
Results (2): Time Differential Method

– Silicon Strip Detector Data –

Nishimura thesis

Simulation:





Preliminary

$\Delta\nu_{HFS} = 4\,463\,302.2$ kHz $\pm$ 3.1 $\pm$ 0.2 kHz

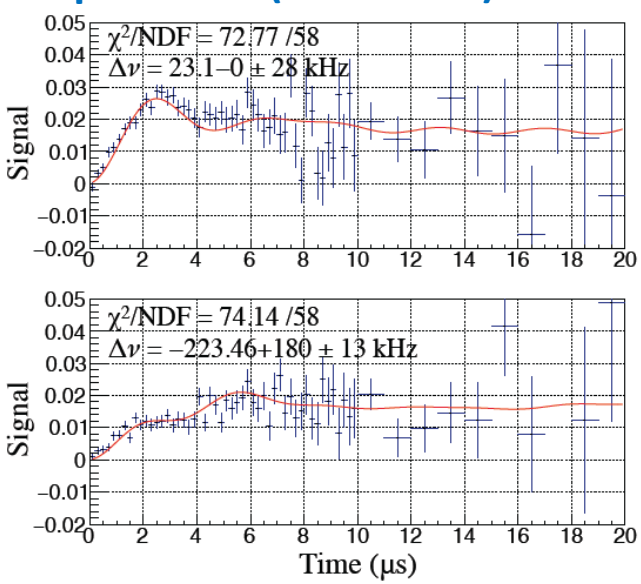
Statistics:

- less data (smaller detector area)

Systematics (main):

- RF power drift (200 Hz)
- gas pressure extrapolation (66 Hz) (only one pressure data !)

Experiment (2017 June):



Possible advantages of this method:

- Each detuning frequency data fitted individually.
- Can determine $\Delta\nu_{HFS}$ with only one frequency data.
- Most sensitive detuning frequency is ~60 kHz.
- Can improve statistical uncertainty by 3.2 times compared to the conventional method.
- Can reduce systematics of RF power variation (free fitting parameter).
- Need high-statistics data.

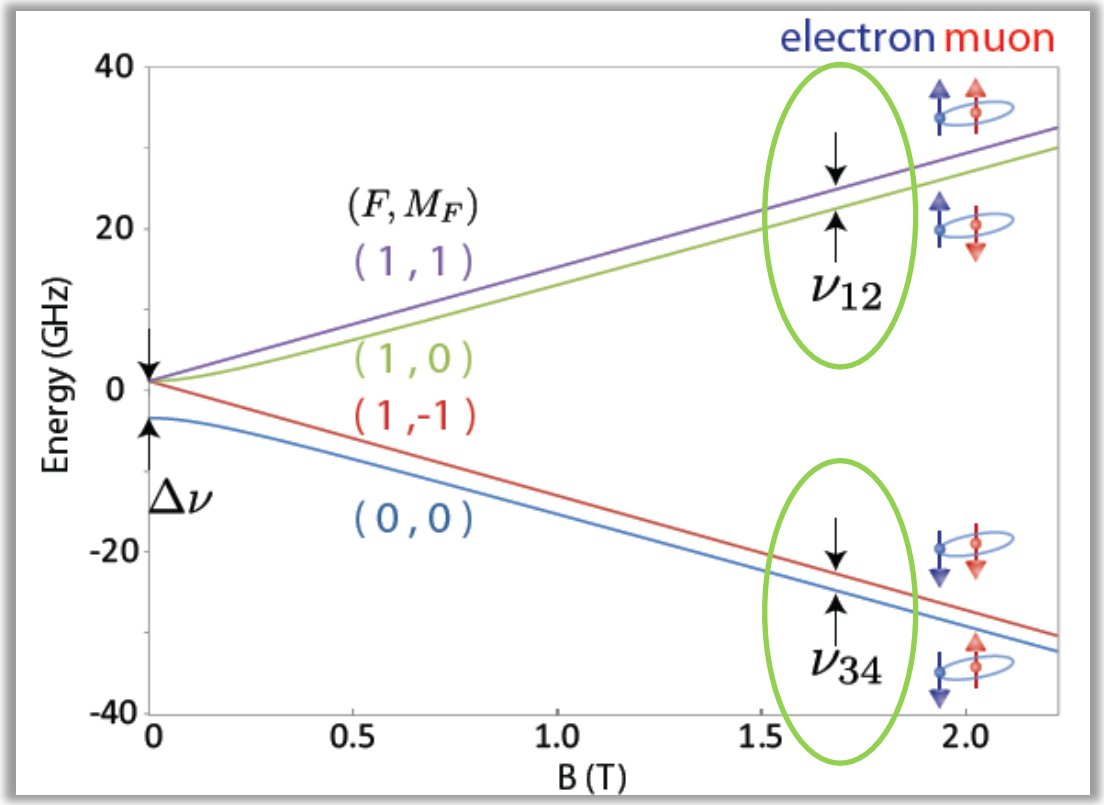
Systematic Uncertainty

Nishimura

Item	June 2017	Prospects
Gas pressure fluctuation	7 Hz	7 Hz
Gas pressure extrapolation	66 Hz	7 Hz
Gas impurity	0 Hz	0 Hz
Static magnetic field	0 Hz	0 Hz
Microwave power drift (including muon beam profile)	200 Hz	1 Hz
Pileup event loss	1 Hz	1 Hz
Time Calibration	1 Hz	1 Hz
Total	200 Hz	10 Hz

- Systematic uncertainty was much smaller than statistical uncertainty in June 2017
- Systematic can be as small as the previous experiment

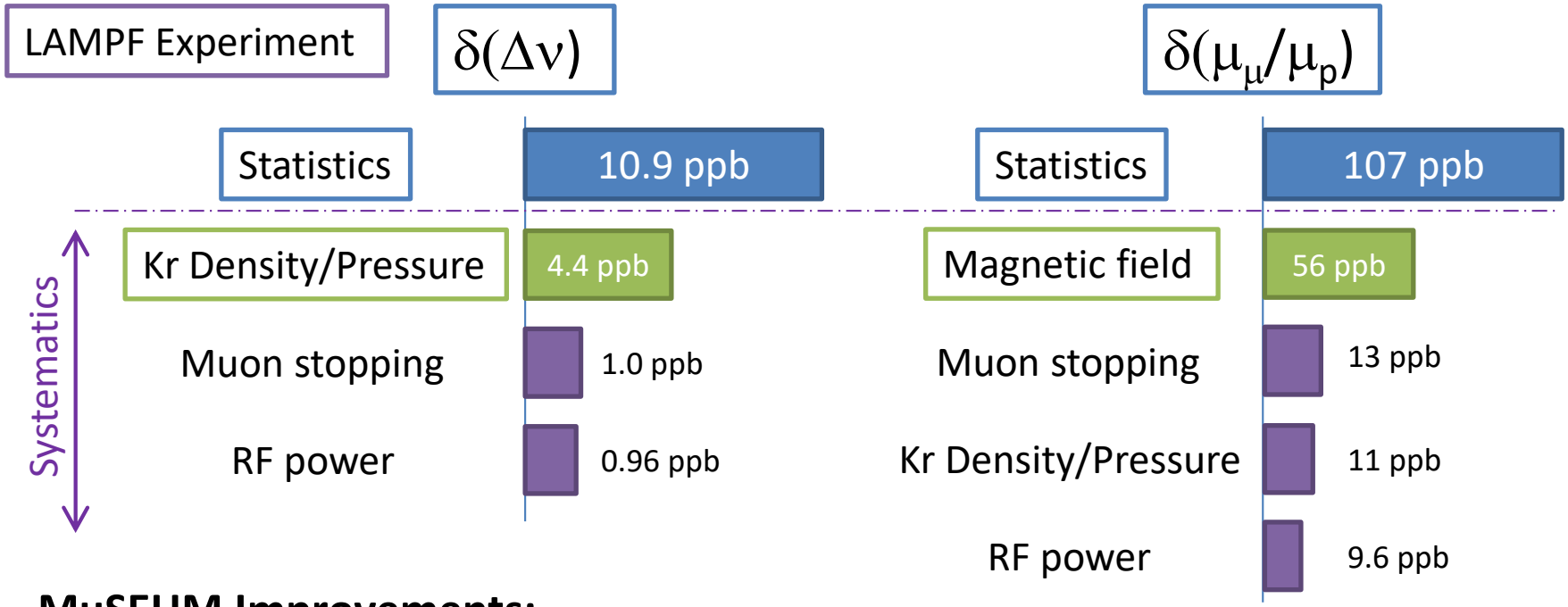
High field measurement at H Line



Two additional components are required.

- Permalloy magnetic shield
- RF cavity for ZF

Improvement of statistics



MuSEUM Improvements:

Statistics: LAMPF: DC 10^7/s, total 10^{13} → x200 → J-PARC/MUSE: **Pulsed 1×10^8**/s, H-Line, total 2×10^{15}

Systematics:
- magnetic field accuracy & uniformity
- pressure dependence (longer cavity lower pressure)
- muon stopping distribution measurement
- RF power stability

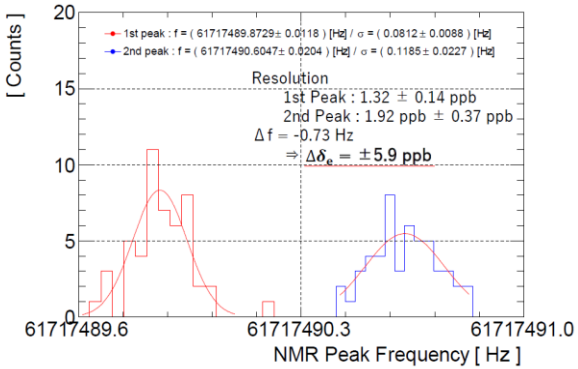
MRI Magnet for High-Field Experiment

Sasaki, Yamaguchi , T. Tanaka

Second-hand 2.9 T MRI magnet

CW-NMR Field Monitoring System

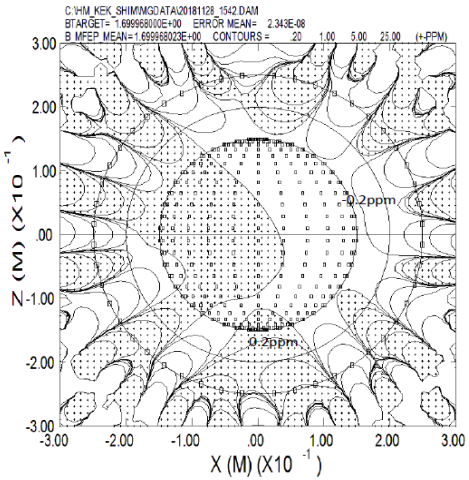
18 ppb→5.9 ppb (2017→2018)



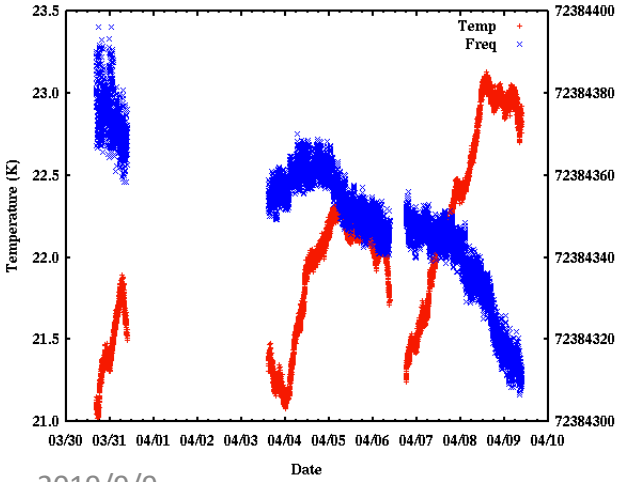
Field Homogeneity (after shimming)

Spheroid : r=100 mm, z=300 mm

1.4 ppm p-p→0.27 ppm p-p (2017→2018)



Long Term Stability

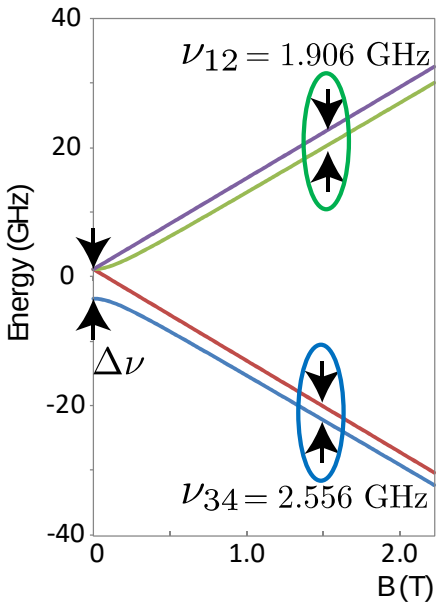


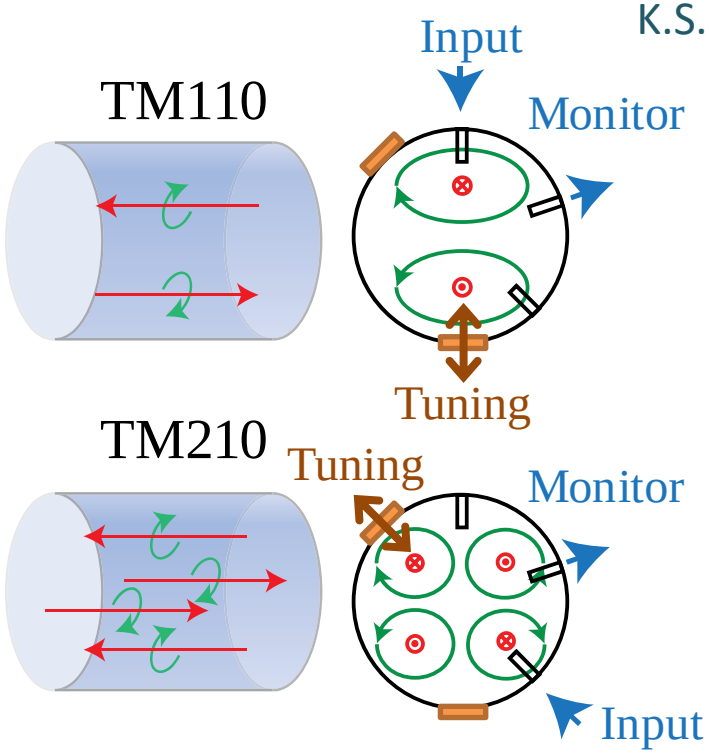
64 Hz / 9.7 days

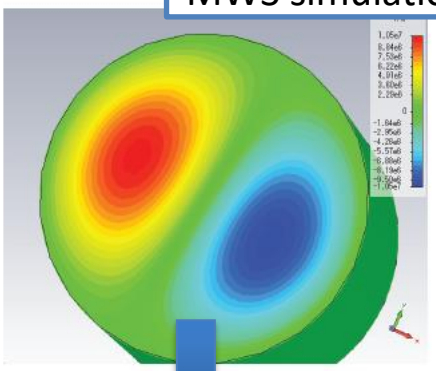
0.003 ppm /h

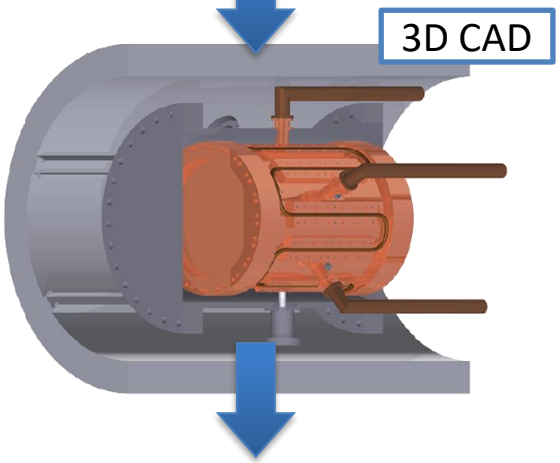
RF Cavity for High Field Experiment

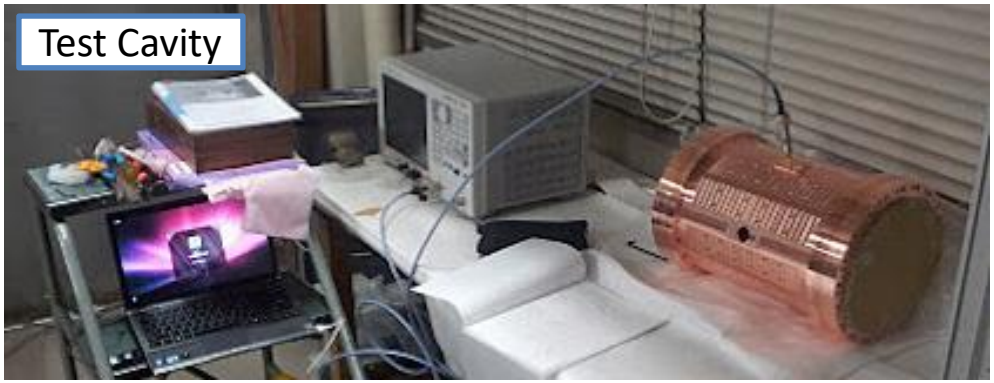
K.S. Tanaka, Seo











Q Value

Modes	Q (measured)	Q (simulation)
TM110	11,300	29,700
TM210	8,050	28,900

Preliminary Systematic Error (HF)

	Accuracy	ν_{12} and ν_{34}	$\delta(\Delta\nu_{HFS})$	$\delta(\mu_\mu/\mu_p)$
Magnetic Field*	30 ppb		0.0 ppb	15 ppb
RF power*	0.2 %	4 Hz	0.8 ppb	8 ppb
Kr gas temperature	0.2 deg.	< 2 Hz	0.4 ppb	4 ppb
Kr gas pressure	1 Pa	1 Hz	0.2 ppb	0 ppb
H impurity	<50 ppm	1 Hz	0.5 ppb	0 ppb
Quadratic dependence		5 Hz	1.0 ppb	5 ppb
Muonium position (x,y)	1 mm	3 Hz	0.6 ppb	6 ppb
Muonium position (z)	1 mm	< 1 Hz	0.2 ppb	2 ppb
Beamline	10(e-4)	< 1 Hz	0.2 ppb	2 ppb
Detector pile-up	w/ absorber	0.3 Hz	< 0.1 ppb	< 1 ppb

*should be re-estimated by latest progress and further MC simulation.

*should be re-estimated by latest progress of Time differential method.

Total systematic error of $\Delta\nu_{HFS}$~2 ppb, and μ_μ/μ_p ~ 20 ppb

MuSEUM Collaborators

(**Mu**onium **S**pectroscopy **E**xperiment **U**sing **M**icrowave)

KEK
M. Abe, Y. Fukao, Y. Ikedo, R. Kadono, N. Kawamura, A. Koda, T. Mibe, Y. Miyake, K. Nagamine, S. Nishimura, T. Ogitsu, N. Saito, K. Sasaki, Y. Sato, K. Shimomura, P. Strasser, A. Toyoda, K. Ueno, H. Yamaguchi, T. Yamazaki, A. Yamamoto, M. Yoshida

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S. Fukumura

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ICU
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E. Torikai

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T. U. Ito

Tohoku University
K. S. Tanaka

TRIUMF
K. M. Kojima

U. of Massachusetts Amherst
D. Kawall

Seoul National Univ.
S. Choi

Summary and Next Step

- Zero-field measurements at existing beamline (D-Line) in progress for engineering run of the apparatus.
  - Muonium HFS resonance clearly observed !
  - Soon new world record at zero field ! (data analysis in progress)
  - Several Analysis Methods are in progress (Old Muonium, Time differential)
  - Time-Differential Method promising to improve statistics and reduce RF power fluctuation systematics.
  - Need improvement of the RF power stability (systematics) !!!
  - 4 Doctor thesis 5 Master thesis
- New Precise muonium HFS measurements at high magnetic field will be carried out in a few years (H-Line).

- Present expected systematic error estimated as

HFS	~ 2 ppb (~8Hz)
Magnetic moment (μ_μ/μ_p)	~ 20 ppb

preliminary

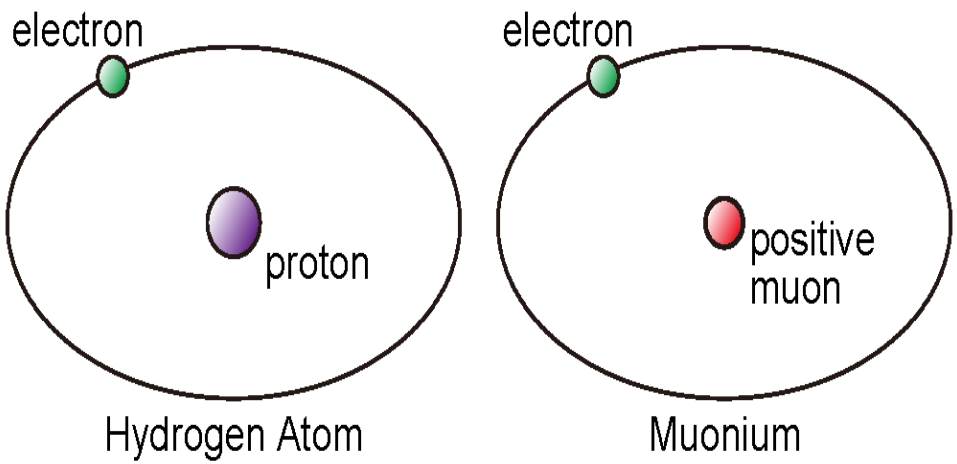
Stay tuned !

Backup Slides

What is Muonium ?

Muon:

- Elementary particle (lepton)
- 200 times heavier than an electron
- Lifetime of 2.2 microseconds.

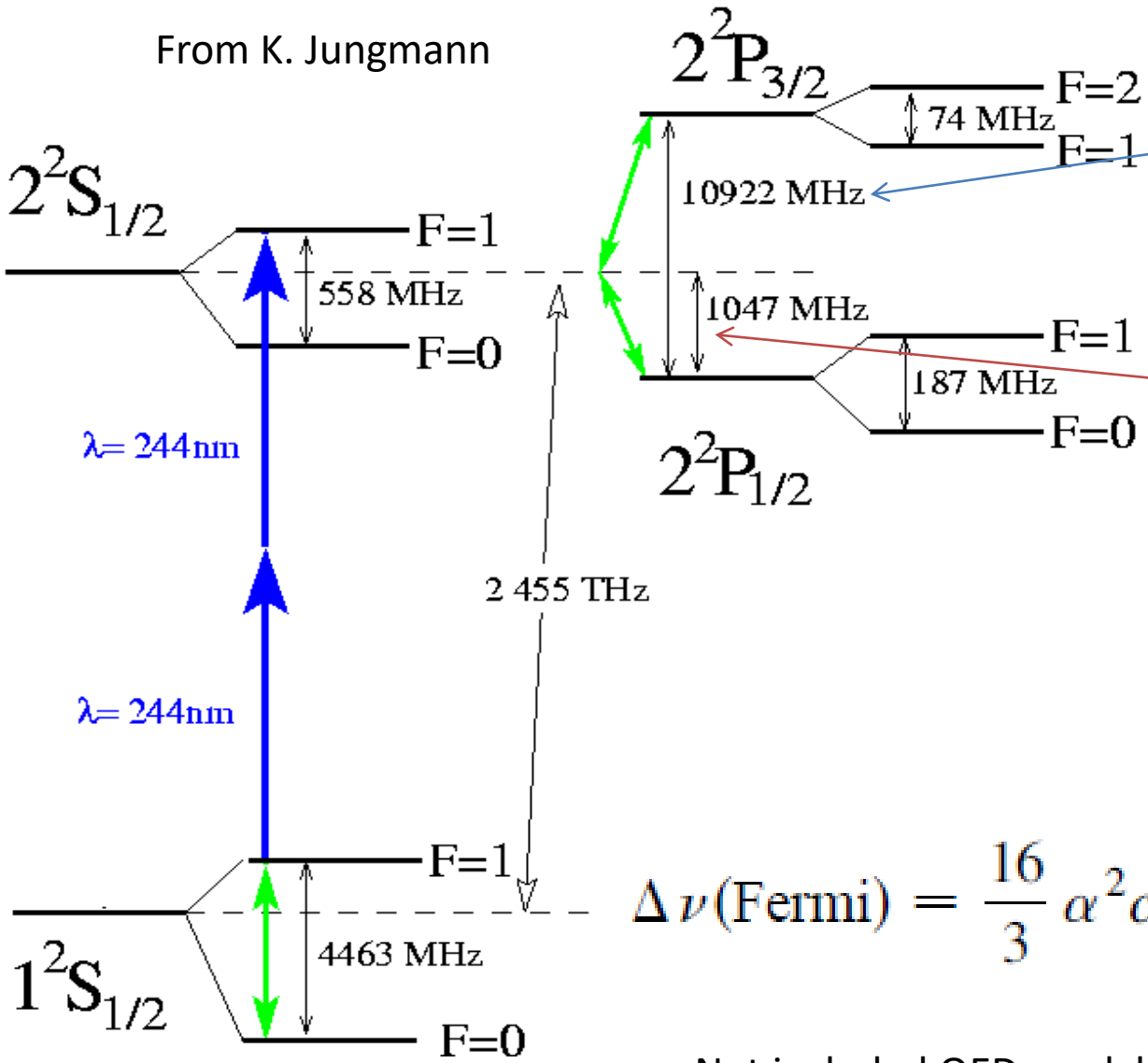


Muonium:

- Bound state of a positive muon and an electron.
- Hydrogen-like atom free from the finite size of the nucleon.
- Most suitable for validation of bound state quantum electrodynamics (QED).
- Theoretical and experimental precision of the hyperfine structure comparable.

Mu Energy Diagram

From K. Jungmann



Fine Structure explained by Dirac

Classical Lamb Shift
QED
Tomonaga, Shwinger, Feynman

$$\Delta\nu(\text{Fermi}) = \frac{16}{3}\alpha^2 c R_\infty \frac{m_e}{m_\mu}\left[1 + \frac{m_e}{m_\mu}\right]^{-3}.$$

Not included QED weak hadronic correction

Breit Rabi Diagram

$$\nu_{12} = -\frac{\mu_B^\mu g'_\mu H}{h} + \frac{\Delta\nu}{2}[(1+x) - \sqrt{1+x^2}],$$

$$\nu_{34} = +\frac{\mu_B^\mu g'_\mu H}{h} + \frac{\Delta\nu}{2}[(1-x) + \sqrt{1+x^2}],$$

$$x = (g_J \mu_B^e + g'_\mu \mu_B^\mu) H/(h\Delta\nu)$$

$$r'_e = \frac{g_J \mu_B^e}{\mu_p},$$

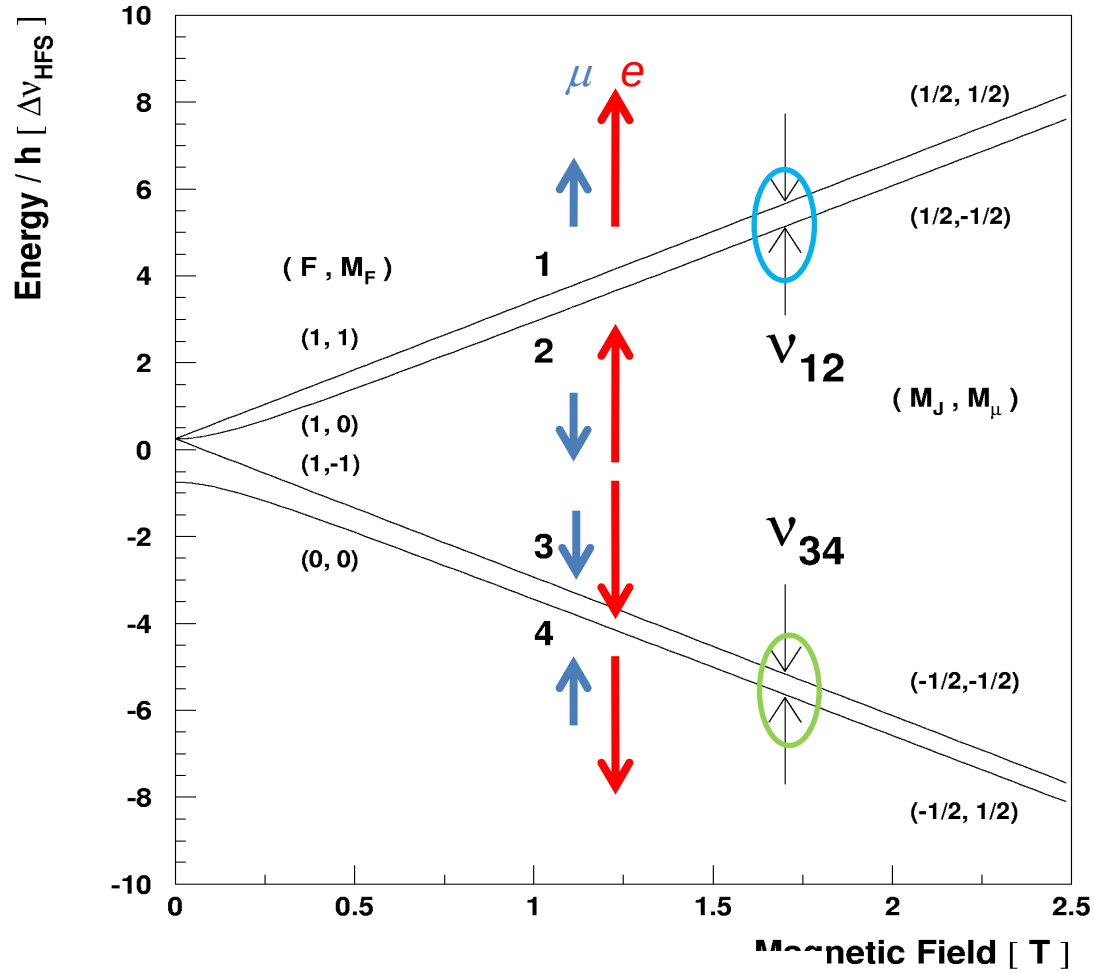
$$r'_\mu = \frac{g'_\mu \mu_B^\mu}{\mu_p},$$

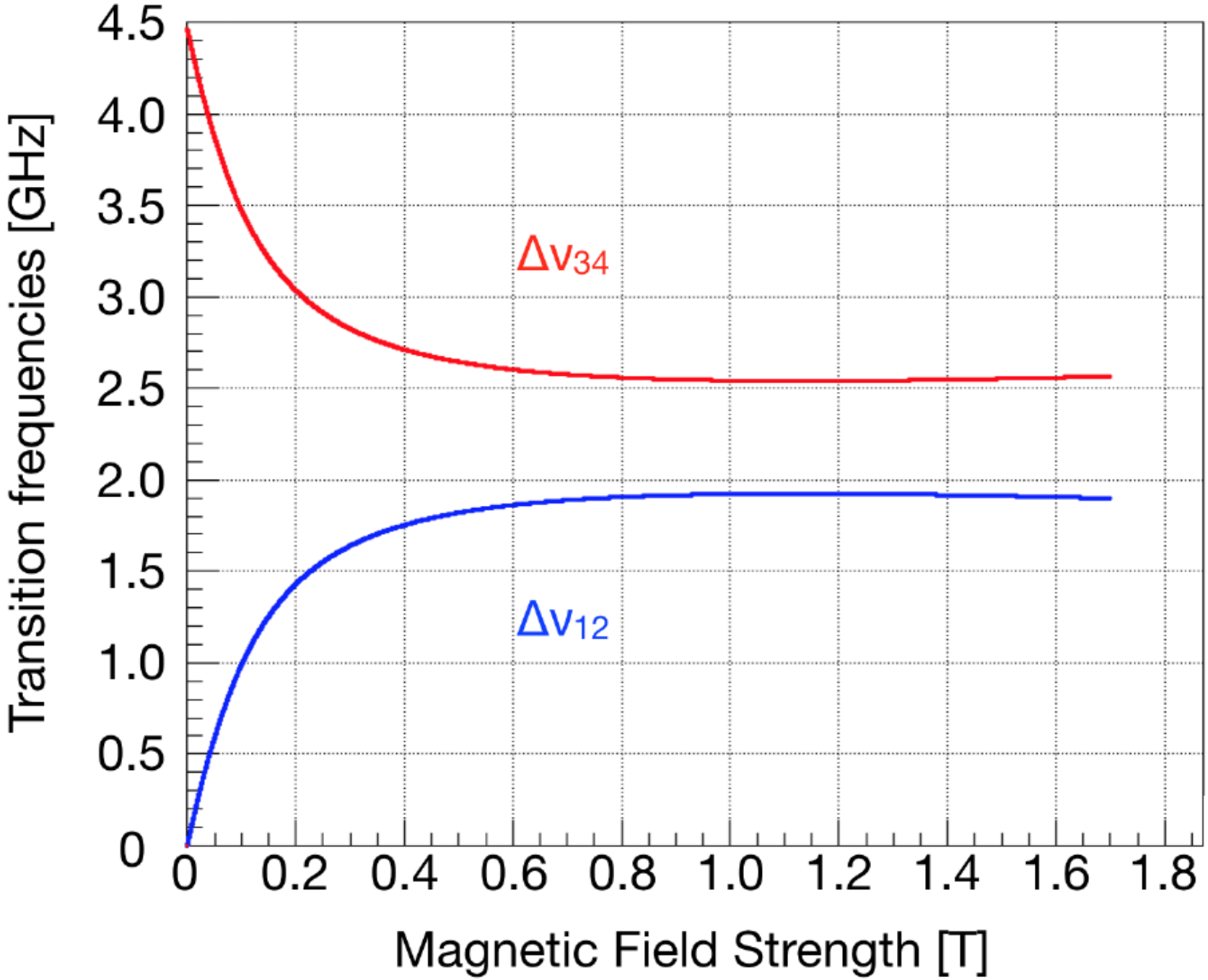
$$x = (r'_e + r'_\mu)\frac{\nu_p}{2\Delta\nu}.$$

$$\delta \equiv \nu_{34} - \nu_{12} = \frac{\mu_B^\mu g'_\mu \nu_p}{\mu_p} + \Delta\nu\left(\sqrt{1+x^2} - x\right).$$

$$\frac{\mu_\mu}{\mu_p} = \frac{r'_\mu}{2}\frac{g_\mu}{g'_\mu}$$

$$= \frac{1}{4}\frac{-(\Delta\nu)^2 + \nu_p r'_e \delta + \delta^2}{\nu_p\left(\nu_p r'_e + \delta\right)}\left(1 - \frac{\alpha^2}{3}\left(1 - \frac{3}{2}\frac{m_e}{m_\mu}\right) - \left(\alpha^2\left(\frac{m_e}{m_\mu}\right)^2 - \frac{\alpha^3}{12\pi}\frac{m_e}{m_\mu} + \frac{97}{108}\alpha^4\right)\right)^{-1}$$




Transition frequencies [GHz]
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
Magnetic Field Strength [T]
Δν34
Δν12

How to improve the accuracy of m_μ/m_e?

Comparison between theoretical and experimental value of $\Delta\nu$

$$\Delta\nu(\text{Fermi}) = \frac{16}{3}\alpha^2 c R_\infty \frac{m_e}{m_\mu}\left[1 + \frac{m_e}{m_\mu}\right]^{-3}.$$ ＋higher order

where recoil term 800kHz (120ppm) and so on are included.

R_∞＝10973731.568639(91)m^{-1}(0.09ppt)

（Cs atomic beam interferometory）

α^{-1}=137.03599958(52) (3.8ppb)

（from electron *g-2*）

m_μ/m_e=206.7682826(46) (22ppb), μ_μ/μ_p=3.183345396(94) (24ppb)

This value is used for the determination of *g-2*.

https://indico.cern.ch/event/687183/contributions/2928847/attachments/1627584/2592472/SWICH2018_Crivelli.pdf

Muon g-2
FNAL

- hadronic contribution
- hadronic lbl contribution
- New Physics

$$a_\mu = \frac{\omega_a/\omega_p}{\mu_\mu/\mu_p - \omega_a/\omega_p}$$

$$a_\mu = \frac{\omega_a}{\omega_p}\frac{m_\mu}{m_e}\frac{\mu_p}{\mu_B}$$

QED

μ_μ, α, g_μ

$$\mu_\mu = g_\mu \frac{e\hbar}{2m_\mu}$$

QED

m_μ

MUSEUM -HFS

$\Delta\nu_{HFS,\ n=1}$

- μ_μ
- α
- QED corrections
- weak contribution

QED

m_μ

Mu-MASS

$\Delta\nu_{1S\text{-}2S}$

- m_μ
- QED corrections
- Rydberg

From K. Jungmann

Time Integral Method

- Signal of all positrons

$$S_{\mathrm{int}} = \frac{\frac{aP}{2}\cos\theta}{1+\frac{\lambda}{\gamma}+\frac{aP}{2}\cos\theta}\,\frac{-2\left|b\right|^2\left(\gamma'^2+2\left|b\right|^2\right)}{\left(\gamma'^2+2\left|b\right|^2\right)^2+\gamma'^2\Delta\omega^2}$$

Δω / Detuning angular frequency

|b| / Microwave magnetic field intensity

λ / Spin relaxation rate

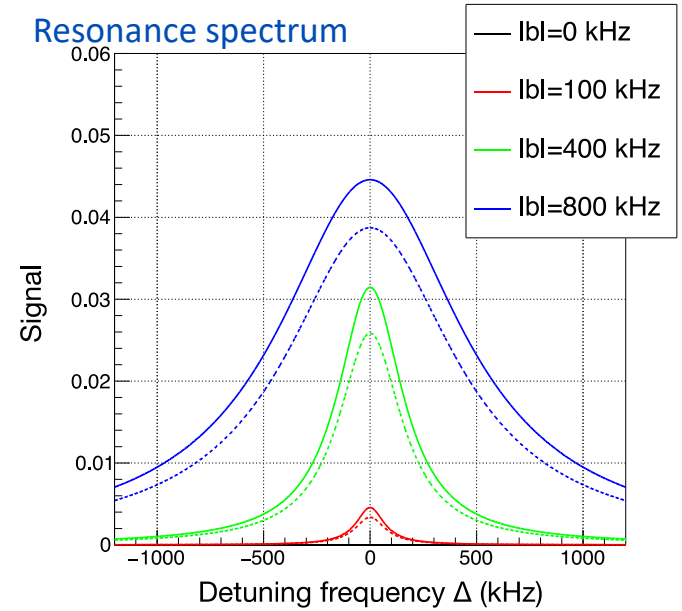
γ / Muon decay rate

P / Muon spin polarization

$$\gamma' = \gamma + \lambda$$

Resonance spectrum | Lorentzian function

- Peak of Lorentzian is equal to Mu HFS frequency
- Mu HFS is determined by multiple frequency data
- Width and height of spectrum is changed by microwave power



Time Differential Method

Time dependence of signal

$$dS_{\text{diff}} = \frac{aP}{2} \frac{\left(C(t) - 1\right) \cos\theta_s \mathrm{e}^{-(\lambda+\gamma)t}}{\left(1 + \frac{aP}{2}\mathrm{e}^{-\lambda t}\cos\theta_s\right)\mathrm{e}^{-\gamma t}}$$

$$C(t) = \frac{G_+}{\Gamma}\cos G_- t + \frac{G_-}{\Gamma}\cos G_+ t$$

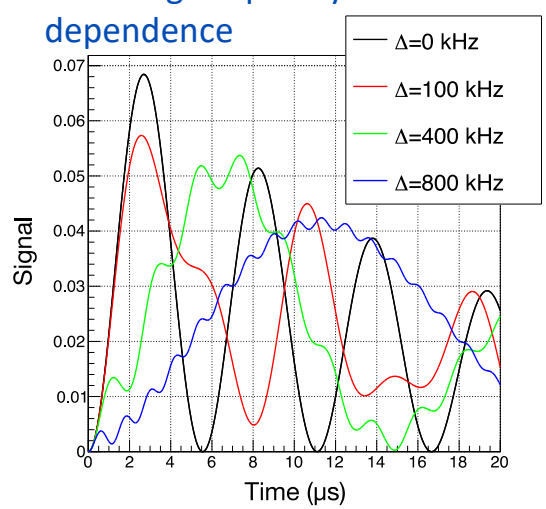
$$G_\pm = \frac{\Gamma \pm \Delta\omega}{2} \qquad \Gamma = \sqrt{\Delta\omega^2 + 8\left|b\right|^2}$$

Time spectrum |

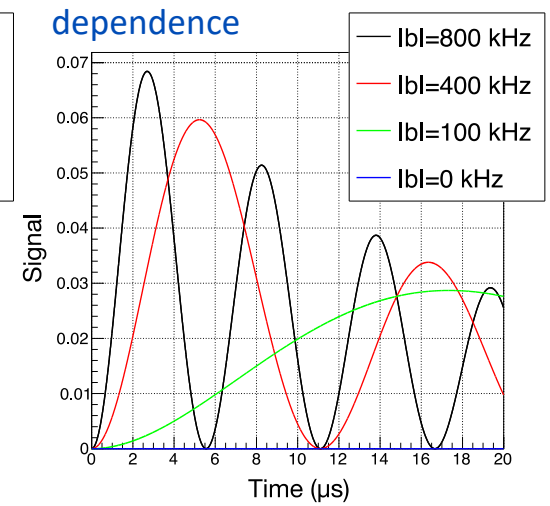
Summation of cos

- contains more information
  - Mu HFS frequency
  - Microwave power
  - Spin relaxation time
- can determine Mu HFS by only one detuning frequency data

Detuning frequency dependence

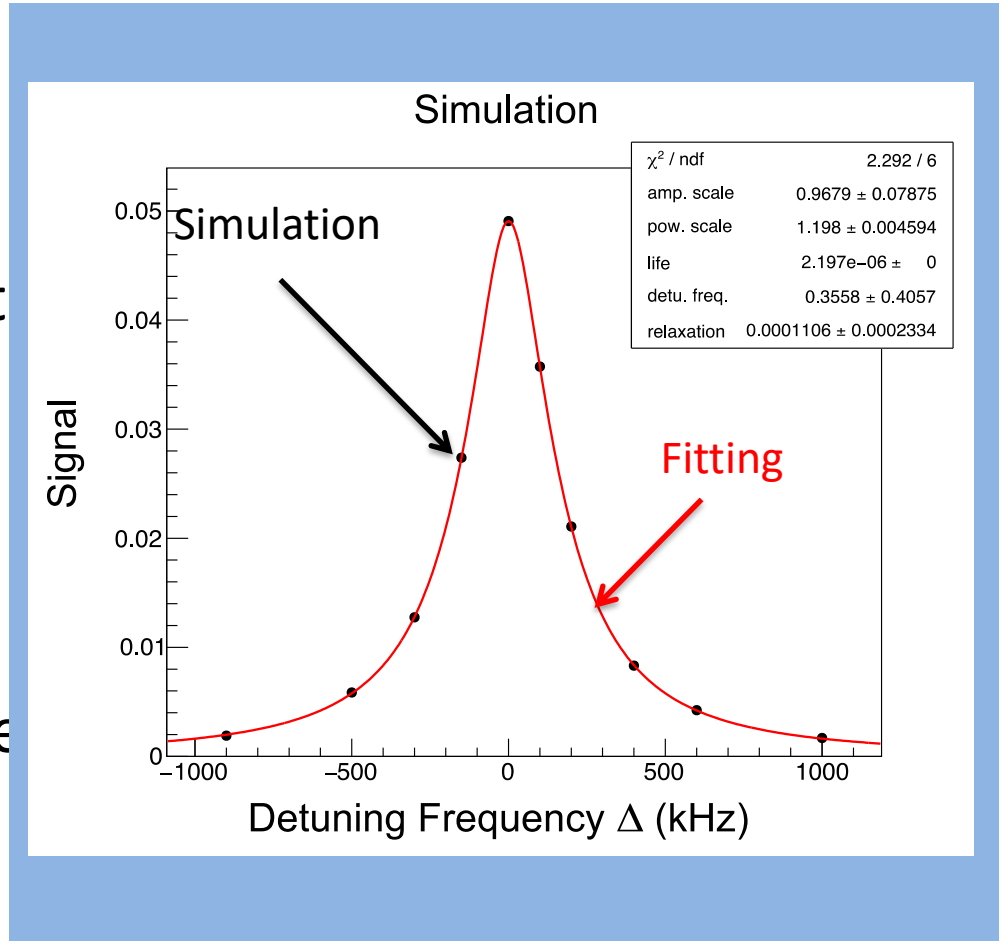


Microwave power dependence

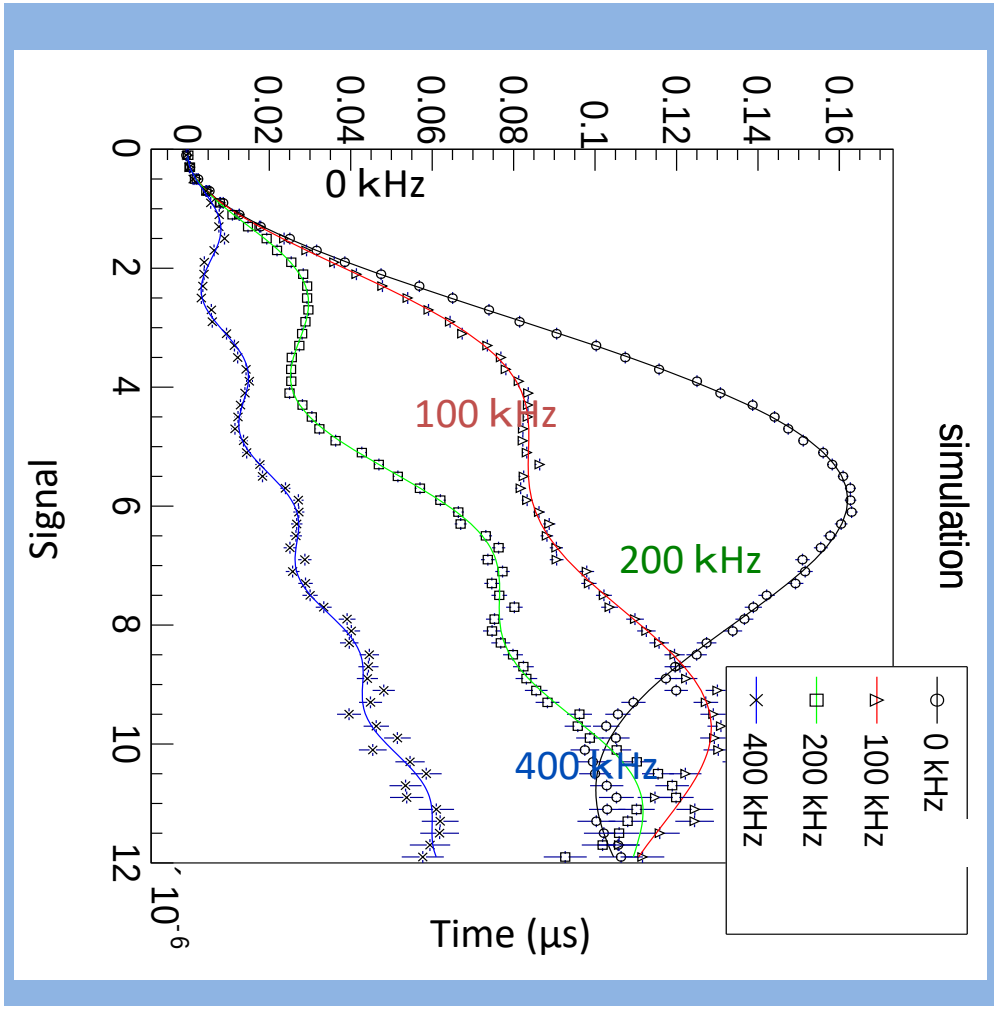


Time Integral Signal (Simulation)

- Simulation setup
  - Measurement point | 10 points
  - 7.8×10^{10} muon/point
- Fitting function for Time integral method
  - Summation of Lorentzian function considering microwave power distribution felt by muonium



Time Differential Signal （Simulation）

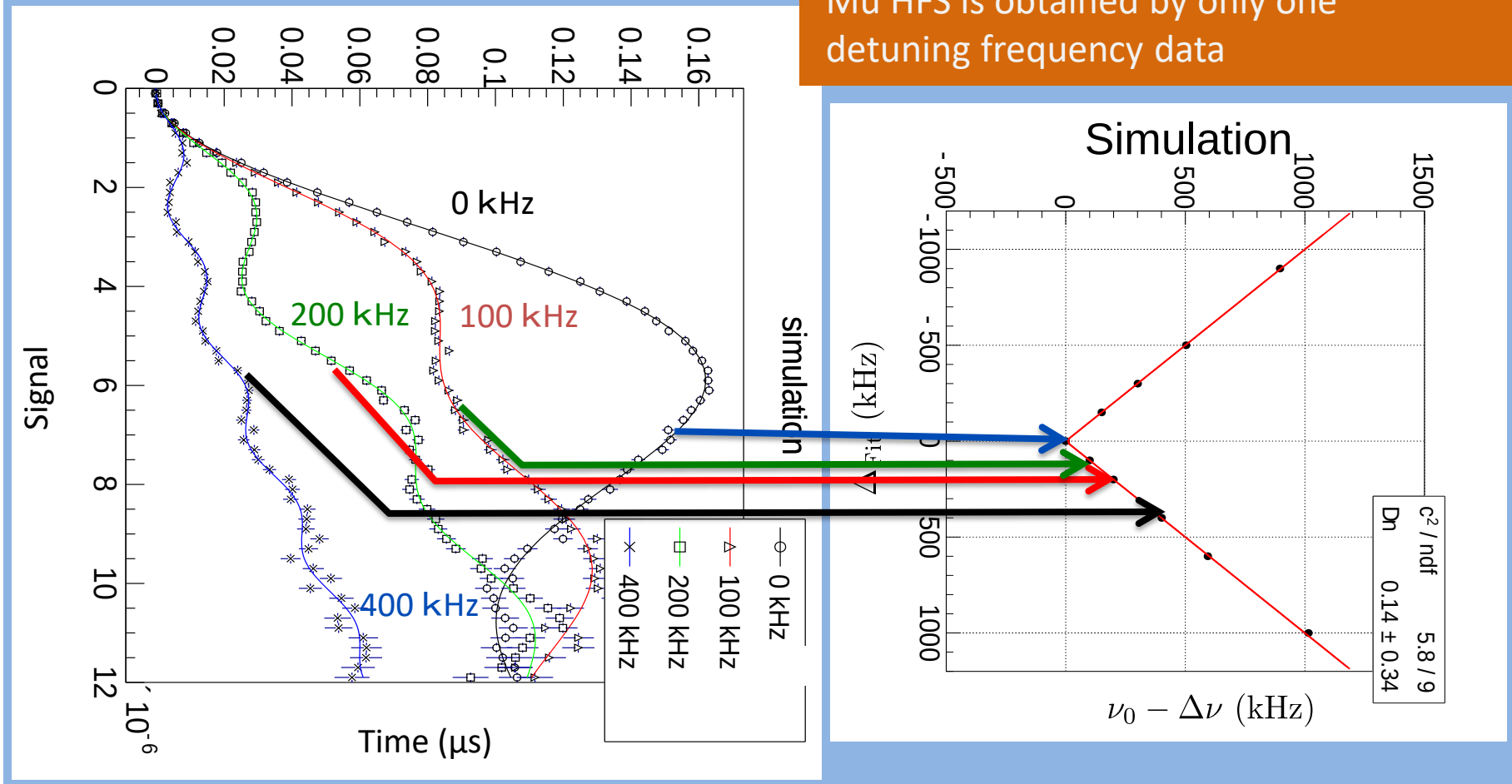


- Time differential signal | Same statistics as time integral method
- Time spectrum are changed by the detuning frequency

- Fitting function
  - Summation of cosine considering microwave power distribution felt by muonium

Time Differential Signal (Simulation)

Mu HFS is obtained by only one detuning frequency data



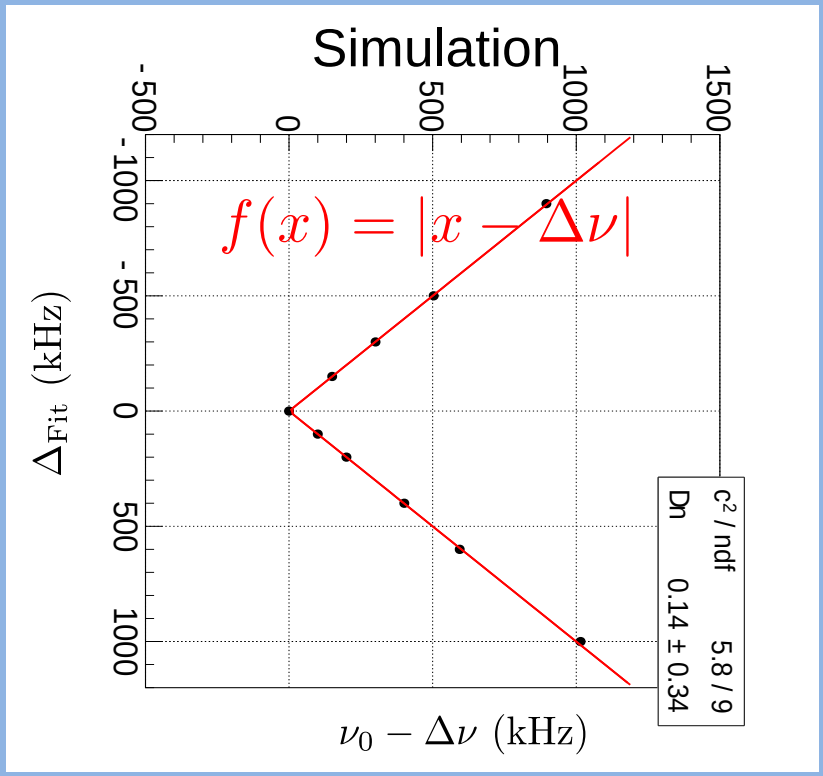
Multiple Time Differential Data

- Mu HFS is determined by multiple results of time differential method

Time Integral Method
| 0.41 kHz

Time Differential Method
| 0.34 kHz

15% improvement



More Efficient Measurement

- Concentrating one frequency is the most efficient method

Time Integral Method
| 0.41 kHz

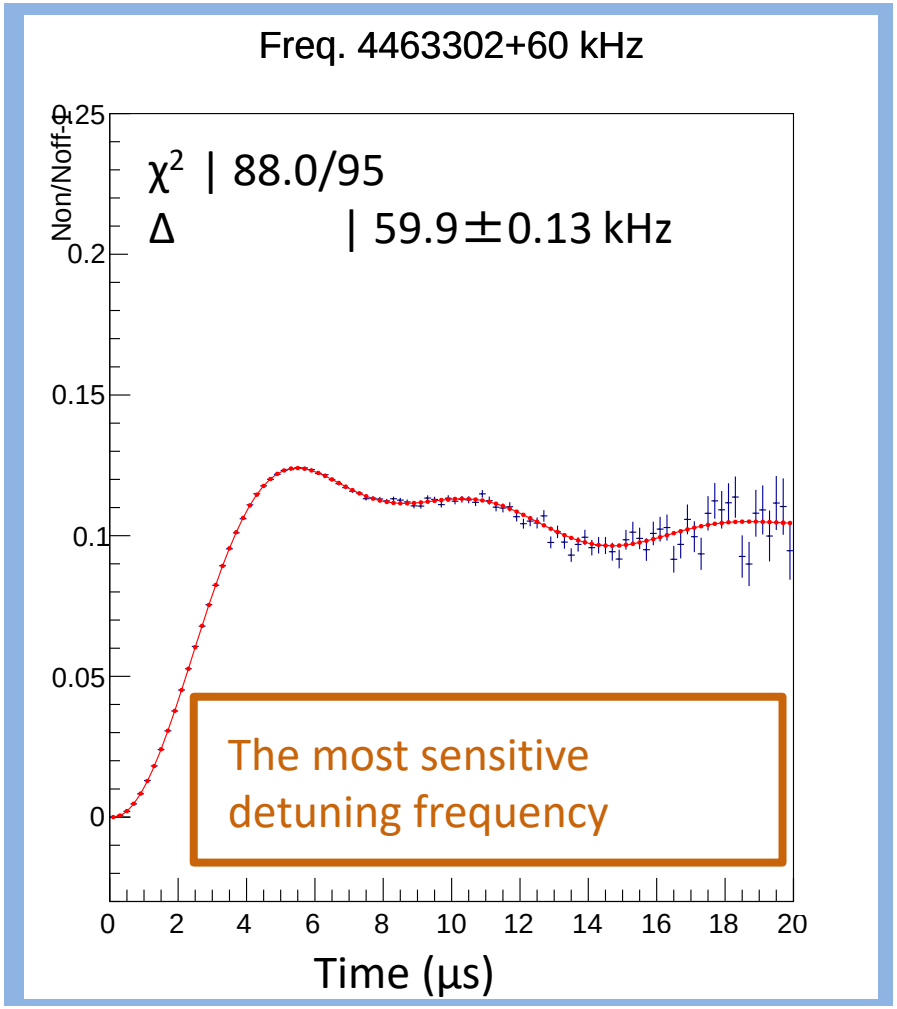
↓

Time Differential Method
| 0.34 kHz

↓

Time differential Method
(Concentrate on Δ=60 kHz)
| 0.13 kHz

3.2 times improvement compared to the time integral method



Plan for Measurement

FY2020 Zero Field Measurement H line
50 days？ Less than 12ppb uncertainty

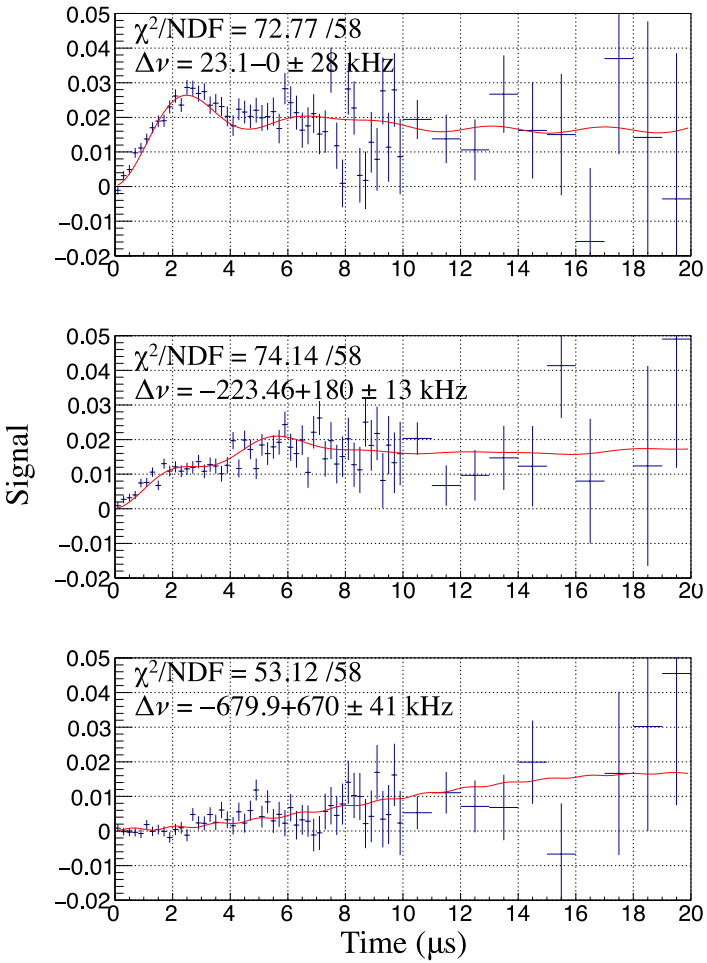
FY2021 High Field Measurement at H line (1.7T)
50 days 50 times statics

FY2022 High Field Measurement at H line (1.13T)
50 days 50 times statics
（magic field, rectangular cavity）

FY2023 High Field Measurement at H line (3T)
50 days 50 times statics
（Better condition for μ_μ determination, rectangular cavity）

Analysis by the Time Differential Method (2017)

- Time differential muon spin resonance signal was observed
  - Detuning frequency dependence is similar to the simulation results
  - Same fitting function as the simulation
  - Mu HFS was obtained from each detuning frequency data



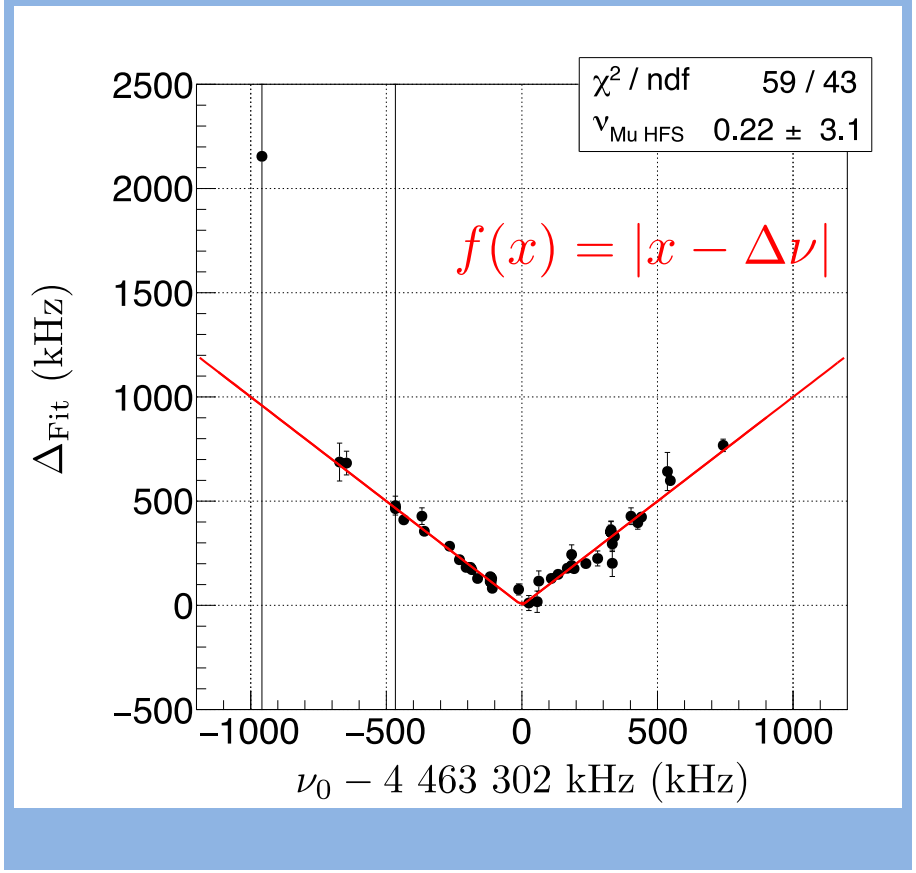
Mu HFS Measurement (2017)

- Mu HFS was obtained from multiple data
  - Obtained $\Delta\nu_{Mu}$

4 463 302.2 ± 3.1 kHz

  - It is consistent to the previous experiments

Previous experiment
ZF | 4 463 302.2(14) kHz
HF | 4 463 302.765(50)(17) kHz

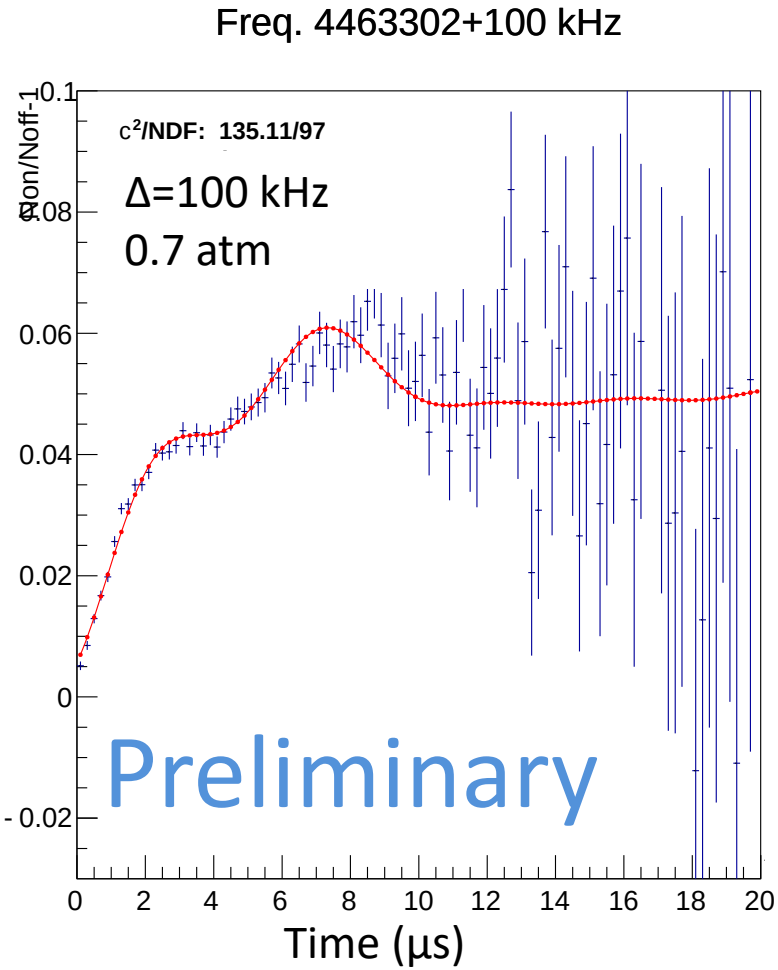


Phys. Lett. B59 (1975) 397-400 , Phys. Rev. Lett. 82 (1999) 711-714

Mu HFS Measurement in 2018

Nishimura

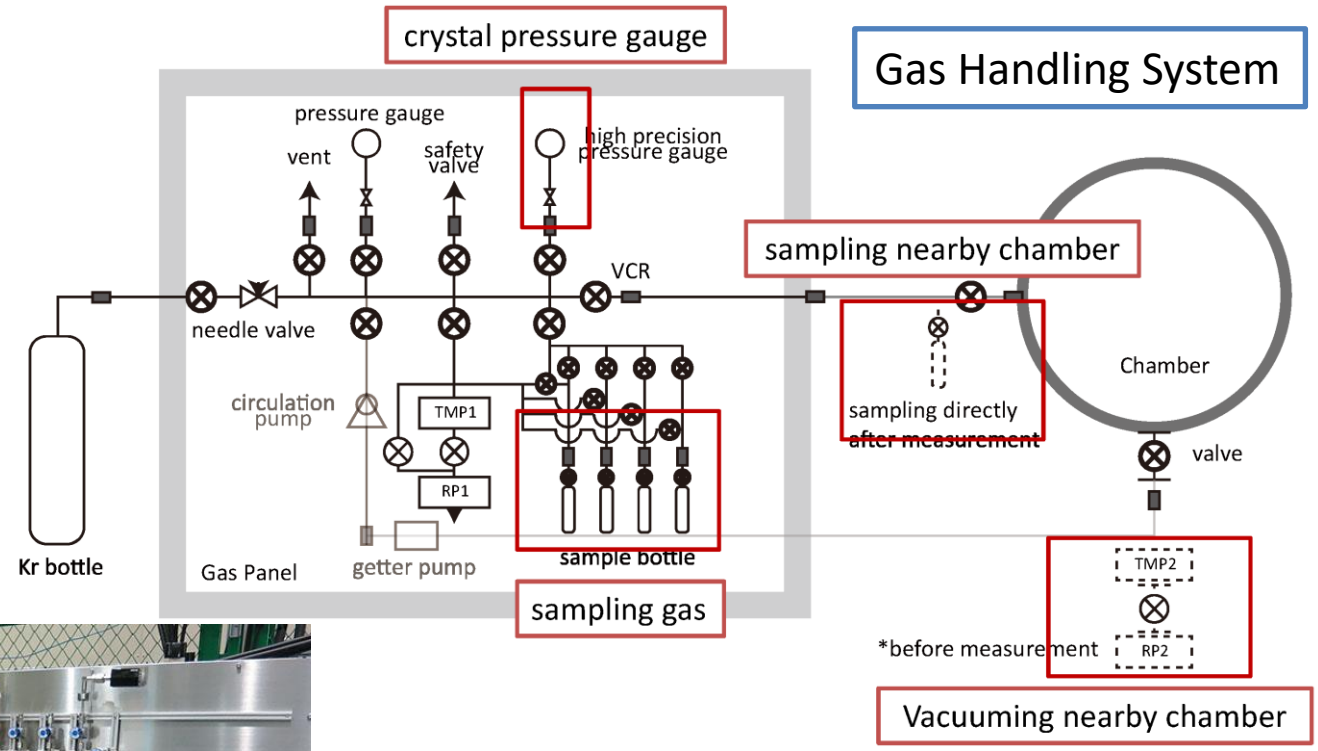
- Kr gas pressure shift
  - Resonance frequency is shifted due to collision of muonium & the Kr atom
  - Gas pressure in the experiment in 2018 | 0.3, 0.4, 0.7 atm
  - Spin flip resonance signal was obtained for each gas pressure
- Analysis is ongoing

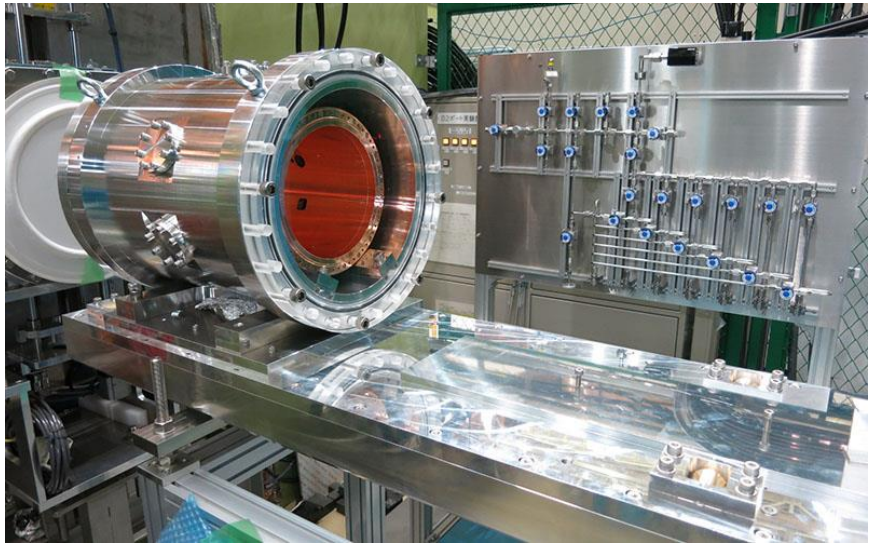


Precision of the hyperfine structure (HFS, $\Delta\nu$):

Hydrogen-like atom	Experiment	Theory	$(\Delta\nu_{theo} - \Delta\nu_{exp})/\Delta\nu_{exp}$
Hydrogen	0.2 ppt	1.2 ppm	(-0.45 ± 1.2) ppm
Positronium	3.3 ppm	2.0 ppm	(15 ± 4) ppm
Muonium (Zero-Field)	310 ppb	61 ppb	(150 ± 320) ppb
Muonium (High-Field)	12 ppb	61 ppb	(23 ± 62) ppb

RF Cavity & Gas Chamber



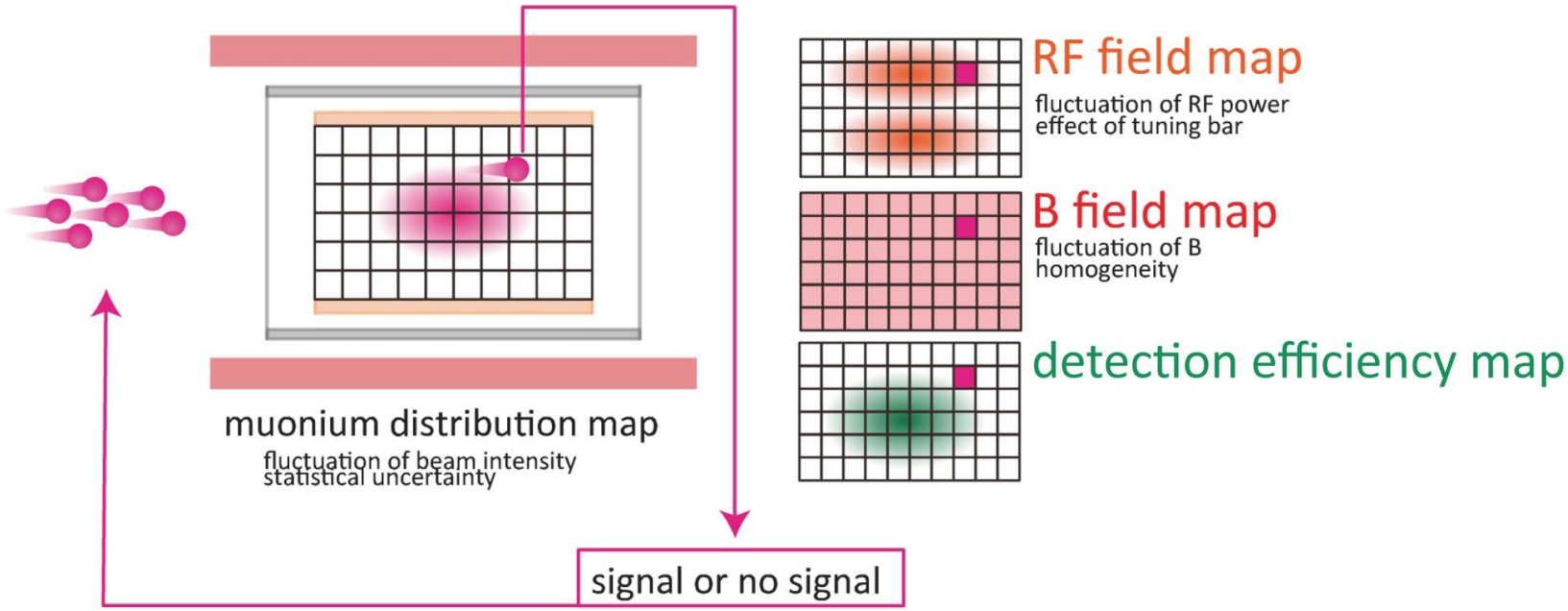


- Pressure: 0.5 – 1.5 atm
- Readout precision: several ppm (crystal gauge: 0.008% of full range)
- Contamination: below 1ppm.
- gas sampling before, during and after the experiment (several weeks).

Systematic Error

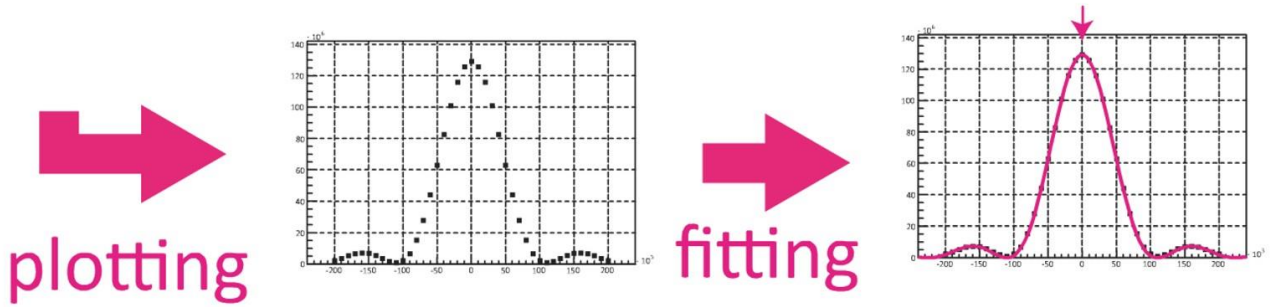
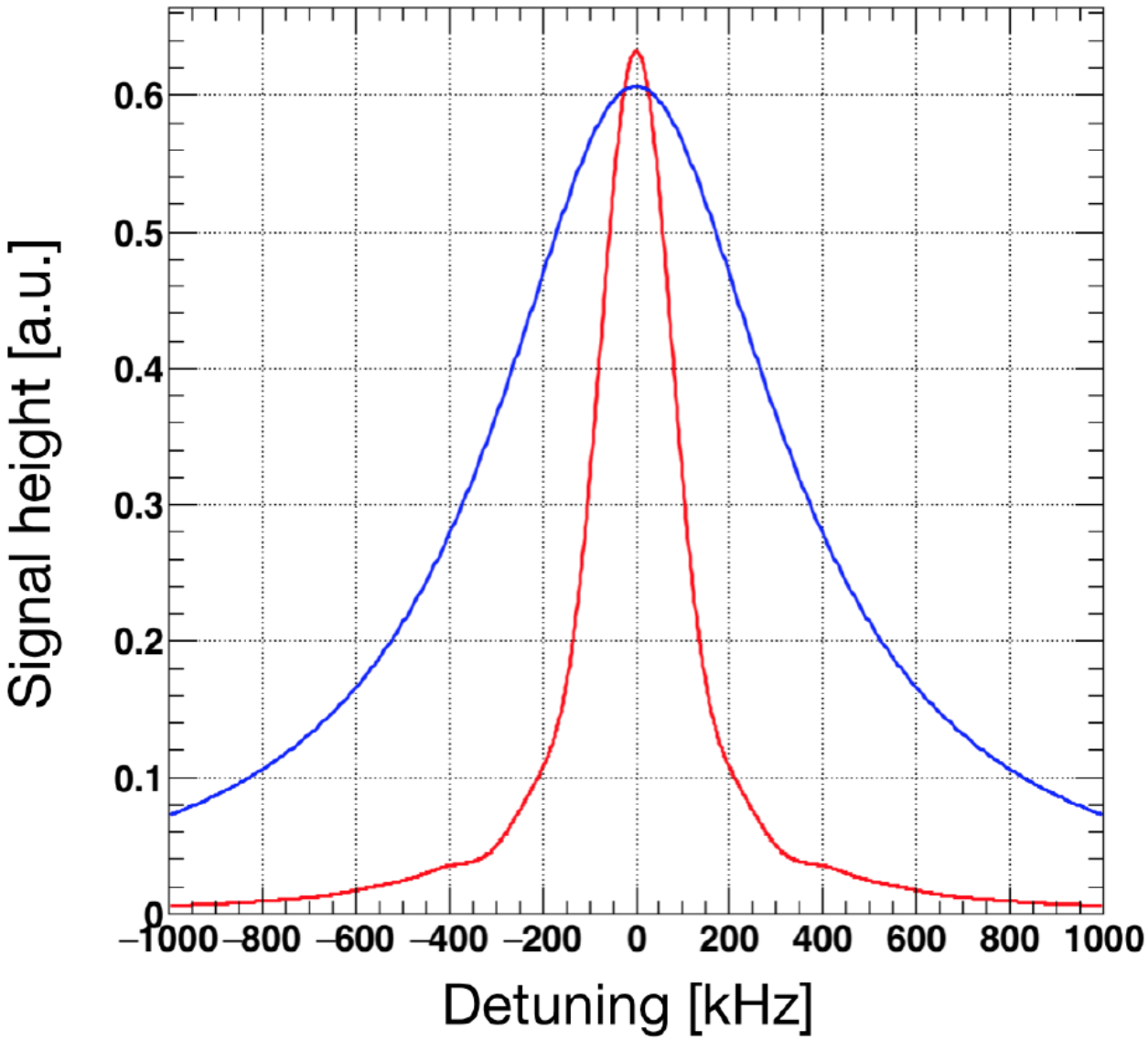
Tanaka, Kanda, Ishida

Calculate transition probability.



Repeat calculation for every muonium.

Center of the resonance line determined by fitting.



Old Muonium in ZF field