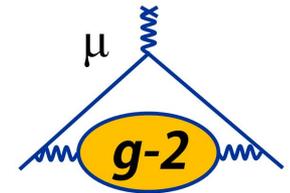


Summary HVP Dispersive*



Thomas Teubner



- Current Status of the White Paper chapter on HVP Disp.
- Experimental data (covered in detail by Simon)
- New developments
- Open questions & points for discussion sessions. Outlook

* A lot more detail about the physics in dedicated talks from experiments and theory this week

“Muon g-2 Theory Initiative” formed in June 2017

group photo from June 2018 workshop, <https://indico.him.uni-mainz.de/event/11/overview>



“map out strategies for obtaining the **best theoretical predictions for these hadronic corrections** in advance of the experimental results”

“Muon $g-2$ Theory Initiative”

- The main emphasis of this workshop is on the comprehensive white paper,
- which presents and discusses the current state-of-the-art for the SM prediction for $g-2$,
- traces and discusses differences between different approaches for the hadronic contributions,
- and, if possible, concludes with the best possible SM prediction

White Paper: status of HVP Dispersive (part I)

- So far the 'HVP Dispersive' chapter has 48 pages and 187 Refs
- However: some parts are still missing, others need further work and scrutiny
- Some difficulties to find agreement when opinions diverge
- In detail:
 - I. Introduction (TT, 4pp): needs polishing and some refs. missing ✓

Content (only short and introductory):

General formula and definitions

Hadronic cross section at low energies

Hadronic cross section at higher energies

Data treatment

Data combination

White Paper: status of HVP Dispersive (part II)

II. Hadronic Data: complete and remarks from different experiments taken into account



-- Part on e^+e^- (Michel & Simon, input from SND, CMD-3, KLOE ..)

Content: (17pp):

Experimental Approaches

The scan method

The ISR approach

Radiative Corrections and Monte Carlo simulation

Luminosity measurements

Input Data

Exclusive measurements (2pi, other 2-body, multihadronic)

Narrow resonances

Inclusive R measurements

Missing Channels

Major tensions in hadronic data (2pi, KK)

Short term perspectives

White Paper: status of HVP Dispersive (part II)

II. Hadronic Data (continued)

-- Part on tau (Zhiqing Zhang, 5pp) ✓

Q: Will the view (and conclusions) be shared by groups using the tau data in their evaluations? Interplay with FJ, Benayoun et al.?

-- Radiative corrections and MC generators (Henryk, 1p) ✓

Q: Integration with corresponding part in main e+e- data part?

III. Evaluations of the HVP

Plan: 1. Fred Jegerlehner

2. DHMZ

3. KNT

4. Maurice Benayoun et al. (HLS)

5. Other dispersive determinations – analyticity constraints
Martin Hoferichter et al., Caprini et al.

6. Comparison of different evaluations

White Paper: status of HVP Dispersive (part III)

III. Evaluations of the HVP Status: X / ✓

1. Fred Jegerlehner & 4. Maurice Benayoun et al. (BHLS₂)

Material received but difficult to fit in in current form, as large overlap with contribution from Maurice; more work needed

Q: Probably need to discuss the inherent model dependence as compared to other approaches which do impose no (or fewer) theoretical constraints.

Q: Use of tau data?!

HVP: new developments from other groups

Benayoun+DelBuono+Jegerlehner: arXiv:1903.11034 BLHS₂

- New analysis using effective theory based on (broken) Hidden Local Symmetry

Channel	BHLS	BLHS ₂ (BS) (excl. τ)	BLHS ₂ (RS) (incl. τ)	BLHS ₂ (RS) (excl. τ)	Exp. Value.
$\pi^+\pi^-$	493.73 ± 0.70	494.52 ± 0.92	494.51 ± 0.83	494.50 ± 1.04	497.82 ± 2.80
$\pi^0\gamma$	4.42 ± 0.03	4.48 ± 0.03	4.42 ± 0.03	4.42 ± 0.03	3.47 ± 0.11
$\eta\gamma$	0.63 ± 0.01	0.63 ± 0.01	0.64 ± 0.01	0.64 ± 0.01	0.55 ± 0.02
$\pi^+\pi^-\pi^0$	42.56 ± 0.54	43.03 ± 0.55	42.97 ± 0.55	43.12 ± 0.50	41.38 ± 1.28
K^+K^-	18.10 ± 0.14	18.05 ± 0.13	18.14 ± 0.16	18.11 ± 0.14	17.37 ± 0.55
$K_L K_S$	11.53 ± 0.08	11.70 ± 0.08	11.65 ± 0.09	11.65 ± 0.10	11.98 ± 0.36
HLS Sum	570.97 ± 0.92	572.42 ± 1.08	572.32 ± 1.03	572.44 ± 1.20	572.57 ± 3.15
χ^2/N_{pts}	949.1/1056	1062.2/1152	1128.0/1237	1038.2/1152	×
Probability	96.7%	91.6%	94.6%	96.7%	×

Table 7: HLS contributions to $10^{10} \times a^{\text{HVP-LO}}$ integrated up to 1.05 GeV, including FSR. The first data column displays the results using the former BHLS [25, 27] and, the second one, those derived from the Basic Solution for BHLS₂, the τ decay data being discarded. The next two data columns refer to the results obtained using the Reference Solution for BHLS₂ using the largest set of data samples, keeping or discarding the τ data. The last data column refers to the numerical integration for each channel of the same set of data which are used in the BHLS/BHLS₂ fits.

HVP: new developments from other groups

Benayoun+DelBuono+
Jegerlehner:

arXiv:1903.11034

- Their preferred mean value is for total LOHVP in (10^{-10}):
 $687.1 \pm 3.0 (+1.3-1.0)_{\text{sys}}$
[vs. KNT's 693.3 ± 2.5]
- Adding all contributions, they then $> \sim 4.2\sigma$.

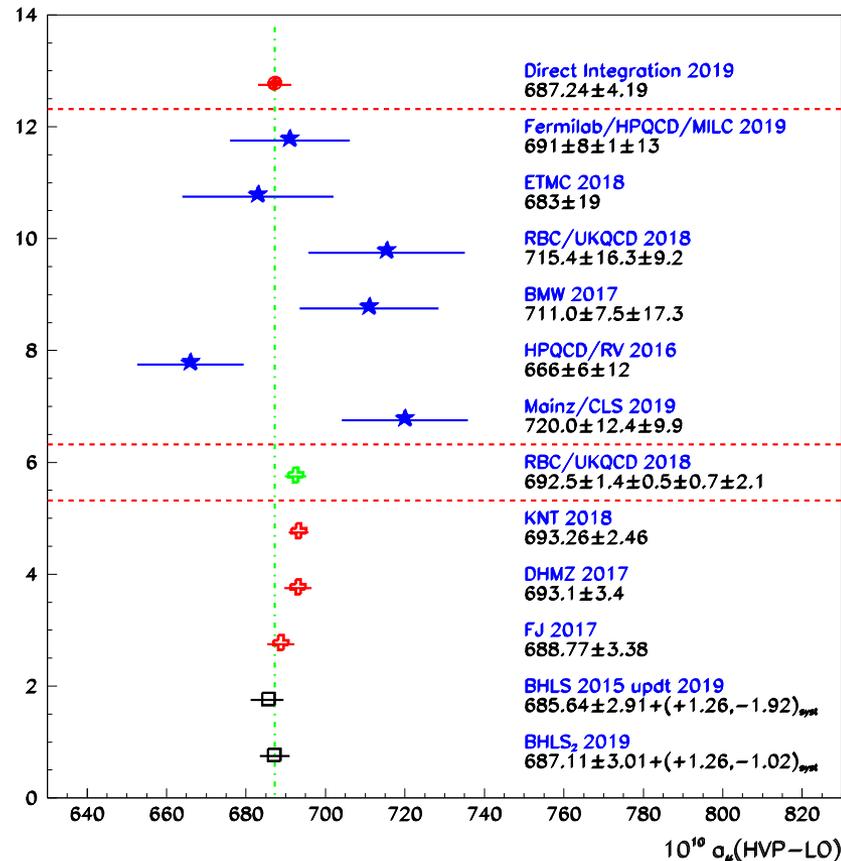


Figure 11: Recent evaluations of $10^{10} \times a_{\mu}^{\text{HVP-LO}}$: On top, the result derived by direct integration of the data combined with perturbative QCD; the next six points display some recent evaluations derived by LQCD methods and reported in resp. [124], [125], [126], [127], [11] and [128] with $N_f = 2 + 1 + 1$. The second point from [126] displayed has been derived by supplementing lattice data with some phenomenological information. These are followed by the evaluations from [71],[129] and [4]. The value derived using BHLS [27] – updated with the presently available data – and the evaluation from BHLS₂ are given with their full systematic uncertainties (see text).

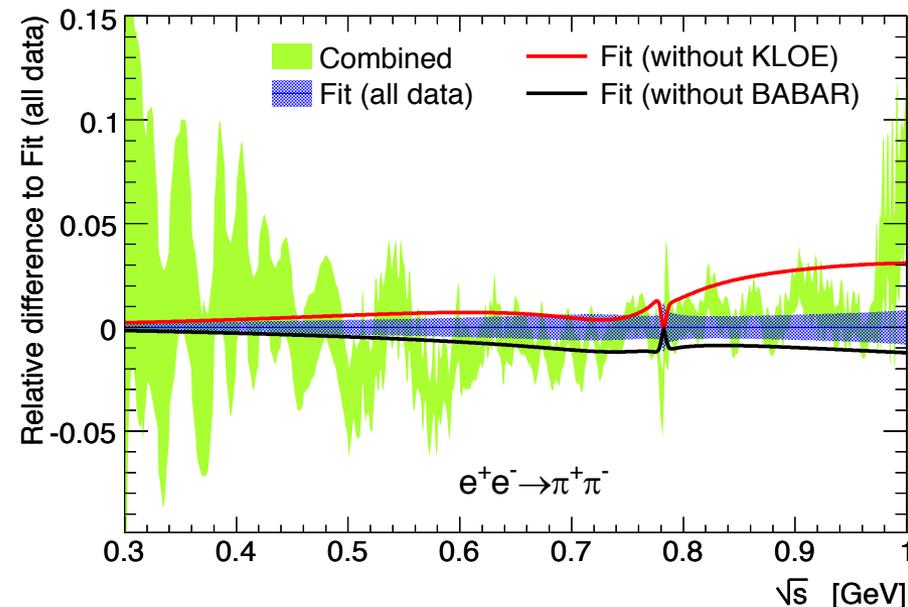
White Paper: status of HVP Dispersive (part III)

III. Evaluations of the HVP Status: 2. DHMZ ✓

--> see DHMZ talk for details

- Contribution (Bogdan, 4 pp) completed, emphasizing combination method, based on arXiv:1908.00921
- Add latest data. Use fit, based on analyticity&unitarity, somewhat similar to Colangelo et al. and Ananthanarayan + Caprini + Das, leading to stronger constraint/lower errors at low energies
- For 2π , based on difference between result w/out KLOE and BaBar, sizeable additional sys. error is applied and mean value adjusted
- inflation of the error beyond what a local error inflation of a combined fit does

- The resulting mean value is similar, total LOHVP in (10^{-10}): 693.9 ± 4.0 [vs. KNT's 693.3 ± 2.5]



arXiv:1908.00921 Figure 6

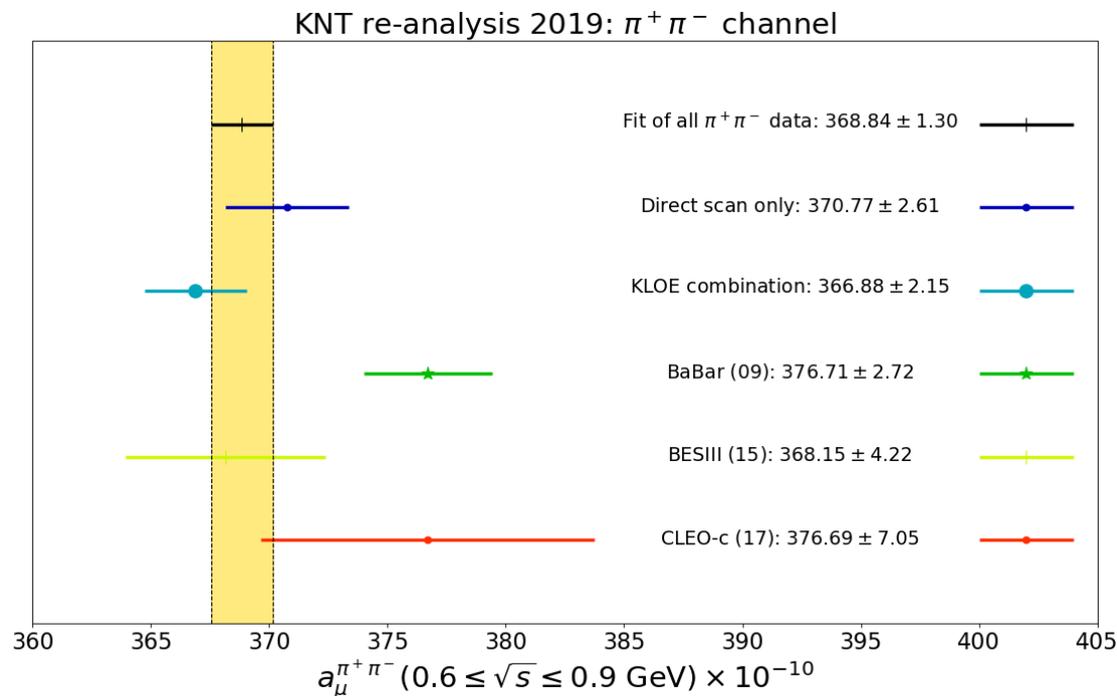
White Paper: status of HVP Dispersive (part III)

III. Evaluations of the HVP Status:

3. KNT X

--> see KNT talk for details

- Contribution not yet available
- KNT19 update in progress, with more data sets, but no change of method, i.e.
- no additional constraints or additional errors



White Paper: status of HVP Dispersive (part III)

III. Evaluations of the HVP Status:

5. Other dispersive determinations – analyticity constraints

Martin Hoferichter et al., Caprini et al. ✓

--> see talks on 2pi (Peter Stoffer) and 3pi (Bastian Kubis) for details

- Contribution from Martin on 2pi, covering Caprini et al. (2pp)
- In addition contribution from DHMZ (Zhiqing Zhang, 0.5p)
- BUT: no agreement on joint contribution due to different views
- So far nothing about the recent 3pi work

HVP: new developments from other groups

Colangelo+Hoferichter+Stoffer, JHEP1902 (2019) 006

- Comprehensive dispersive study of the 2π vector form factor, including space-like data and phase shift analysis,
- leading to stronger constraints compared to pure direct data fit and integration.
- For good fit quality, energy calibration for narrow resonances crucial.

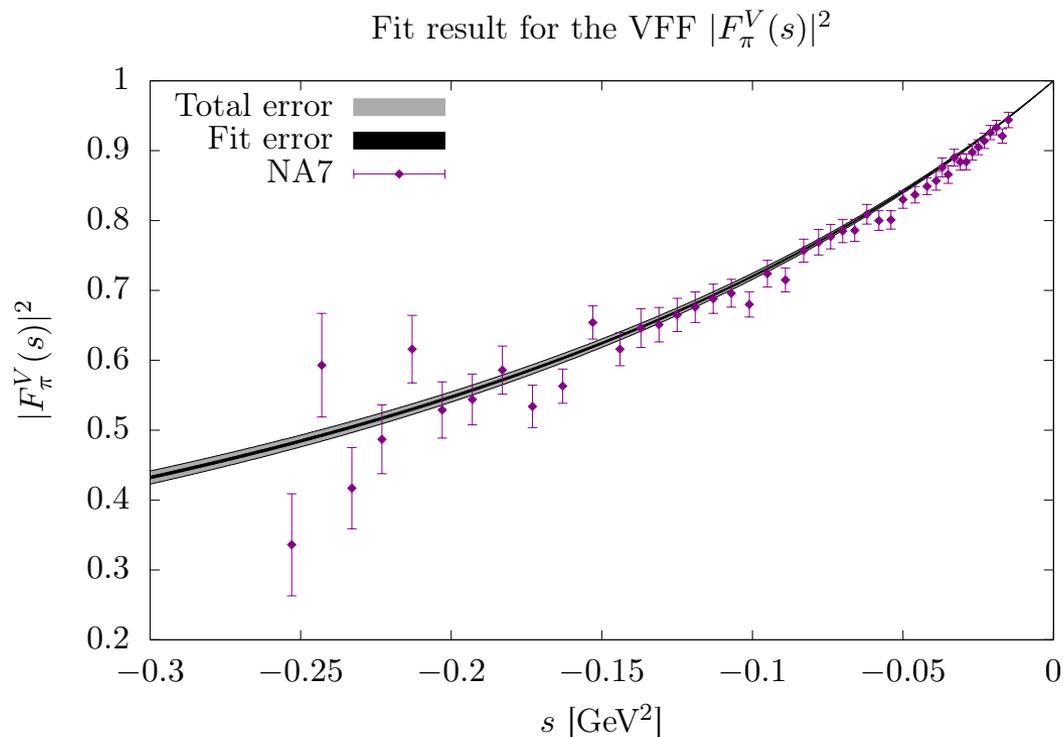


Figure 9: Fit result for the pion VFF in the space-like region, together with the NA7 data.

HVP: new developments from other groups

Colangelo+Hoferichter+Stoffer, JHEP1902 (2019) 006

- Detailed analysis and comparison with other work on a region-by-region basis,
 - > allows improved understanding of differences between different groups
 - > important input for Theory Initiative White Paper

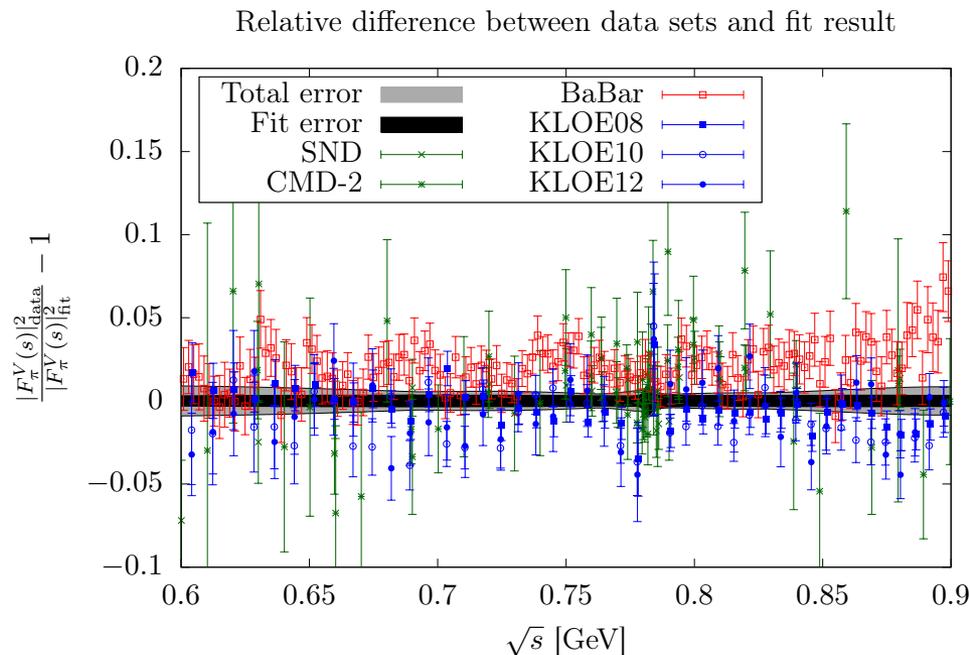


Figure 12: Relative difference between the data points (including the energy rescaling (4.10)) and the fit result for the VFF, normalized to the fit result for $|F_\pi^V(s)|^2$. As in all plots, we show fit errors and total uncertainties as two separate error bands. The total uncertainty is given by the fit error and the systematic uncertainty, added in quadrature.

White Paper: status of HVP Dispersive (part III)

III. Evaluations of the HVP Status:

6. Comparison of different evaluations (✓)

- A lot of work by Martin, in discussion with other groups
- Write-up available (Martin, 2.5pp), does not cover latest developments
- Detailed comparison in energy regions and clarifications where differences come from

White Paper: status of HVP Dispersive (part III)

III. Evaluations of the HVP Status:

6. Comparison of different evaluations

FJ17	DHMZ17	KNT18	BDJ19
HVP 688.1(4.1)	692.9(3.2)	693.3(2.5)	687.1(3.0)

TABLE I. Full evaluations of HVP from FJ17 [1], DHMZ17 [2] (2π [3] and K^+K^- [4] updated compared to published version), KNT18 [5], and BDJ19 [6], in units of 10^{-10} .

	DHMZ17	KNT18	Difference
$\pi^+\pi^-$	506.70(2.58)	503.74(1.96)	2.96
$\pi^+\pi^-\pi^0$	46.20(1.45)	47.70(89)	-1.50
$\pi^+\pi^-\pi^+\pi^-$	13.68(31)	13.99(19)	-0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(54)	18.15(74)	-0.12
K^+K^-	23.06(41)	23.00(22)	0.06
$K_S K_L$	12.82(24)	13.04(19)	-0.22
Sum of the above	620.49(3.06)	619.62(2.30)	0.87
[1.8, 3.7] GeV	33.45(65)	34.54(56)	-1.09
Total HVP	692.9(3.2)	693.3(2.5)	-0.4

TABLE II. Selected channels from DHMZ17 and KNT18, for the energy range ≤ 1.8 GeV, in units of 10^{-10} .

White Paper: status of HVP Dispersive (part III)

III. Evaluations of the HVP Status:

6. Comparison of different evaluations

Energy range	CHS18	KNT18	DHMZ17	ACD18
≤ 0.6 GeV	110.1(9)	108.7(9)	110.2(1.0)	
≤ 0.7 GeV	214.8(1.7)	213.0(1.2)	214.7(1.3)	
≤ 0.8 GeV	413.2(2.3)	411.6(1.7)	414.0(1.9)	
≤ 0.9 GeV	479.8(2.6)	478.1(1.9)	481.2(2.2)	
≤ 1.0 GeV	495.0(2.6)	493.4(1.9)	496.7(2.2)	
[0.6, 0.7] GeV	104.7(7)	104.3(5)	104.5(5)	
[0.7, 0.8] GeV	198.3(9)	198.6(8)	199.3(9)	
[0.8, 0.9] GeV	66.6(4)	66.5(3)	67.2(4)	
[0.9, 1.0] GeV	15.3(1)	15.3(1)	15.5(1)	
≤ 0.63 GeV	132.8(1.1)	131.2(1.1)	132.8(1.1)	132.9(8)
[0.6, 0.9] GeV	369.6(1.7)	369.4(1.3)	371.0(1.5)	
$[\sqrt{0.1}, \sqrt{0.95}]$ GeV	490.7(2.6)	489.0(1.9)	492.2(2.2)	

TABLE III. Comparison of the 2π contribution to HVP from CHS18 [7], KNT18 [5, 9], and DHMZ17 [2, 10]. All numbers in units of 10^{-10} . For the low-energy region ≤ 0.63 GeV the comparison is also shown to ACD18 [8].

White Paper: status of HVP Dispersive (part IV)

IV. Prospects to improve HVP further

- Uncertainties on uncertainties (Bogdan, 2pp) ✓

Qualitative discussion of features and possible problems in different combinations arising from use of correlations and limited knowledge of uncertainties.

Q: do groups using (global) correlations in fits have a differing viewpoint?

- Impact of future measurements on dispersive HVP (from existing experiments and Belle II) ✗

- The MUonE Project (Massimo Passera, Graziano Venanzoni, Carlo Carloni Calame, 3pp) ✓

White Paper: status of HVP Dispersive (part V)

V. Summary and Conclusions: nothing written up so far X

-- Current status executive summary of HVP Disp. Results

-- Expected progress in the next few years

clearly here we have many new e^+e^- data analyses in the pipeline; but how much they will improve the current situation can not be predicted

-- Lessons learned ?

Not discussed yet in detail, but e.g. need for more detailed information on **systematic uncertainties** needed for further progress w.r.t. to reliable combinations and error estimates.

Also possibly need for further improved MC generators with higher order corrections.

HVP Dispersive: main questions for WP discussions

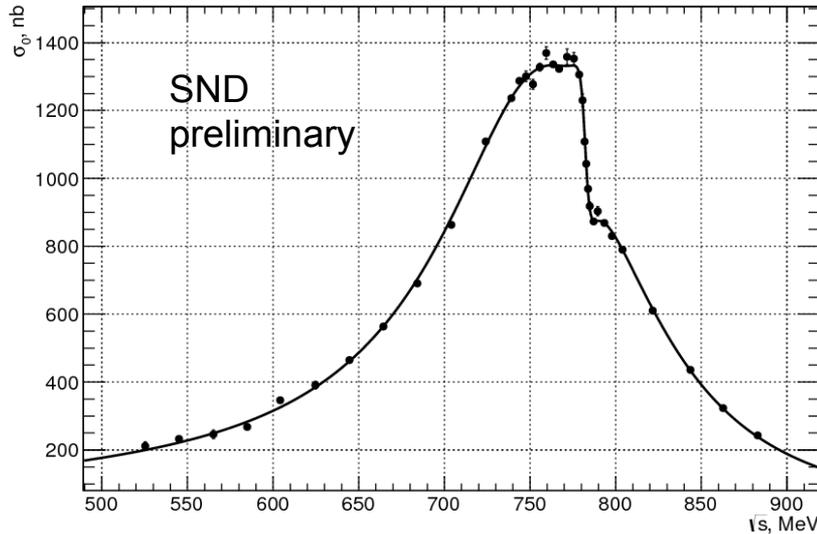
- At which stage should we open the document to scrutiny from outside the small number of authors?
- External assessment/refereeing?
- We have described the tensions in the data and the differences in the compilations. Can we achieve a `recommended' HVP Dispersion value or would that only hide unresolved disagreements/difficulties
- My personal assessment of the state of the HVP Disp. part:
 - Cause for serious concern?
 - A lot of good work has been done already, but
 - we have quite a way to go, though progress on the remaining missing parts can be achieved soon
 - More difficult to arrive at a `recommended' HVP Disp. value

HVP determinations: Outlook (beyond Disp.)

- Next big step as more data in the pipeline;
 - in the 2π channel from BaBar, CMD-3, SND,
 - in subleading channels, 3π , 4π , KK
 - in the inclusive region from BESIII and KEDR,
 - BELLE II will be able to contribute with ISR measurements.
- If new data produce no new tensions/puzzles, further improvement should be significant within few years
(but ultimately hit a limit with experimental systematics)
- Lattice expected to become a competitive alternative and check/challenge direct data-driven analyses;
- combined methods may provide the best HVP predictions
- Role global combined fits with with more TH input?

V. Druzhinin, EPS 2019

$$e^+e^- \rightarrow \pi^+\pi^-$$



Systematic uncertainty on the cross section (%)

Source	< 0.6 GeV	0.6 - 0.9 GeV
Trigger	0.5	0.5
Selection criteria	0.6	0.6
e/π separation	0.5	0.1
Nucl. interaction	0.2	0.2
Theory	0.2	0.2
Total	0.9	0.8

	SND @ VEPP-2000	SND @ VEPP-2M	PDG
$M_{\rho'}$, MeV	$775.4 \pm 0.5 \pm 0.4$	$775.6 \pm 0.4 \pm 0.5$	775.3 ± 0.3
$\Gamma_{\rho'}$, MeV	$145.7 \pm 0.7 \pm 1.0$	$146.1 \pm 0.8 \pm 1.5$	147.8 ± 0.9
$B_{\rho ee} \times 10^5$	$4.89 \pm 0.2 \pm 0.4$	$4.88 \pm 0.2 \pm 0.6$	4.72 ± 0.5
$B_{\omega\pi\pi}$, %	$1.77 \pm 0.08 \pm 0.02$	$1.66 \pm 0.08 \pm 0.05$	1.53 ± 0.06

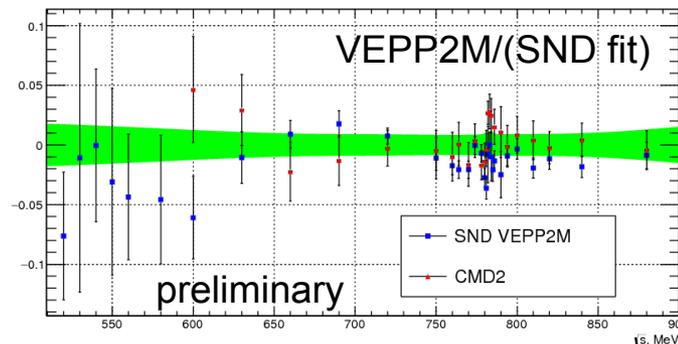
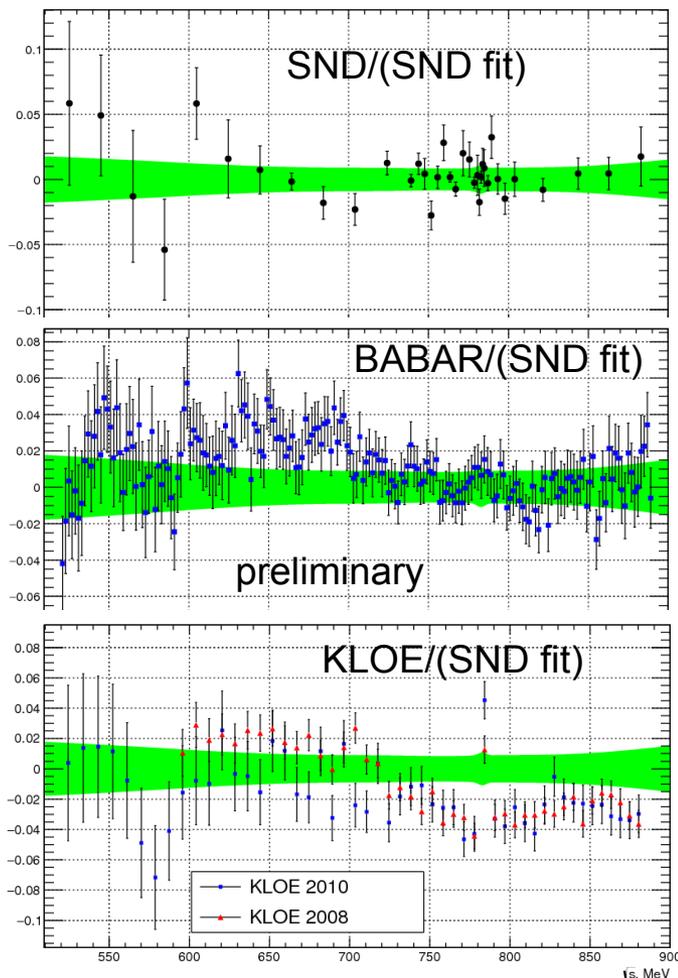
The analysis is based on 4.7 pb^{-1} data recorded in 2013, ~1/10 full SND data set.

Outlook: prel. news from SND (2)

Will this become a mediator?

V. Druzhinin, EPS 2019

$$e^+e^- \rightarrow \pi^+\pi^-$$



$$0.53 < \sqrt{s} < 0.88 \text{ GeV}$$

	$a_\mu(\pi^+\pi^-) \times 10^{10}$
SND & VEPP-2000	$411.8 \pm 1.0 \pm 3.7$
SND & VEPP-2M	$408.9 \pm 1.3 \pm 5.3$
BABAR	$414.9 \pm 0.3 \pm 2.1$