Chapter on "Analytic approaches to HLbL" Status report

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Outline

Introduction: structure of the chapter

Hadronic light-by-light contribution to $(g-2)_\mu$
- PS-pole contribution
- Two-pion contributions
- Higher hadronic intermediate states
- Short-distance constraints
- Summary

Experimental input

Conclusions

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Dispersive approaches

- model independent
- unambiguous definition of the various contributions
- makes a data-driven evaluation possible
  (in principle)
- if data not available: use theoretical calculations of subamplitudes, short-distance constraints etc.
- First attempts: GC, Hoferichter, Procura, Stoffer (14)
  Pauk, Vanderhaeghen (14)
- similar philosophy, with a different implementation:
  Schwinger sum rule Hagelstein, Pascalutsa (17)

The HLbL tensor

HLbL tensor:

$$\Pi^{\mu\nu\lambda\sigma} = i^3 \int dx \int dy \int dz \, e^{-i(x\cdot q_1 + y\cdot q_2 + z\cdot q_3)} \langle 0 | T\{j^\mu(x) j^\nu(y) j^\lambda(z) j^\sigma(0)\} | 0 \rangle$$

$$q_4 = k = q_1 + q_2 + q_3 \qquad k^2 = 0$$

General Lorentz-invariant decomposition:

$$\Pi^{\mu\nu\lambda\sigma} = g^{\mu\nu} g^{\lambda\sigma} \Pi^1 + g^{\mu\lambda} g^{\nu\sigma} \Pi^2 + g^{\mu\sigma} g^{\nu\lambda} \Pi^3 + \sum_{i,j,k,l} q_i^\mu q_j^\nu q_k^\lambda q_l^\sigma \Pi^4_{ijkl} + \dots$$

consists of 138 scalar functions $\{\Pi^1, \Pi^2, \dots\}$, but in $d=4$ only 136 are linearly independent

Eichmann *et al.* (14)

Constraints due to gauge invariance? (see also Eichmann, Fischer, Heupel (2015))

$\Rightarrow$ Apply the Bardeen-Tung (68) method+Tarrach (75) addition

Gauge-invariant hadronic light-by-light tensor

Applying the Bardeen-Tung-Tarrach method to $\Pi^{\mu\nu\lambda\sigma}$ one ends up with: GC, Hoferichter, Procura, Stoffer (2015)

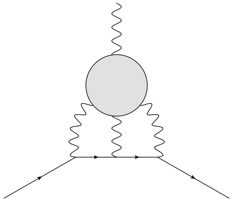
- 43 basis tensors (BT) in $d=4$: 41=no. of helicity amplitudes
- 11 additional ones (T) to guarantee basis completeness everywhere
- of these 54 only 7 are distinct structures
- all remaining 47 can be obtained by crossing transformations of these 7: manifest crossing symmetry
- the dynamical calculation needed to fully determine the LbL tensor concerns these 7 scalar amplitudes

$$\Pi^{\mu\nu\lambda\sigma} = \sum_{i=1}^{54} T_i^{\mu\nu\lambda\sigma} \Pi_i$$

Master Formula

$$a_\mu^{\text{HLbL}} = -e^6 \int \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_2}{(2\pi)^4} \frac{\sum_{i=1}^{12} \hat{T}_i(q_1, q_2; p) \hat{\Pi}_i(q_1, q_2, -q_1 - q_2)}{q_1^2 q_2^2 (q_1 + q_2)^2 [(p+q_1)^2 - m_\mu^2][(p-q_2)^2 - m_\mu^2]}$$

- $\hat{T}_i$: known kernel functions
- $\hat{\Pi}_i$: linear combinations of the Π_i
- the Π_i are amenable to a dispersive treatment: their imaginary parts are related to measurable subprocesses
- 5 integrals can be performed with Gegenbauer polynomial techniques



GC, Hoferichter, Procura, Stoffer (2015)

Master Formula

After performing the 5 integrations:

$$a_\mu^{\mathrm{HLbL}} = \frac{2\alpha^3}{48\pi^2} \int_0^\infty dQ_1^4 \int_0^\infty dQ_2^4 \int_{-1}^1 d\tau \sqrt{1-\tau^2} \sum_{i=1}^{12} T_i(Q_1, Q_2, \tau) \bar{\Pi}_i(Q_1, Q_2, \tau)$$

where Q_i^μ are the Wick-rotated four-momenta and τ the four-dimensional angle between Euclidean momenta:

$$Q_1 \cdot Q_2 = |Q_1||Q_2|\tau$$

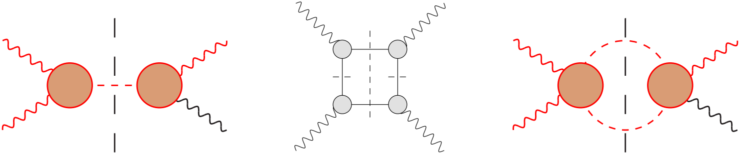
The integration variables $Q_1 := |Q_1|$, $Q_2 := |Q_2|$.

GC, Hoferichter, Procura, Stoffer (2015)

Setting up the dispersive calculation

The HLbL tensor is split as follows:

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\pi\text{-box}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \cdots$$



Last diagrams = all partial waves ⇔ scalars and tensors etc.

3π states are in . . . ⇒ axial vector resonances

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Pion-pole contribution

- The pion transition form factor completely fixes this contribution Knecht-Nyffeler (01)

$$\bar{\Pi}_1 = \frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) F_{\pi^0\gamma^*\gamma^*}(q_3^2, 0)}{q_3^2 - M_{\pi^0}^2}$$

- Both transition form factors (TFF) must be included: [dropping one bc short-distance not correct Melnikov-Vainshtein (04)]
- data on singly-virtual TFF available CELLO, CLEO, BaBar, Belle, BESIII
- several calculations of the transition form factors in the literature Masjuan & Sanchez-Puertas (17), Eichmann et al. (17), Guevara et al. (18)
- dispersive approach works here too Hoferichter et al. (18)
- quantity where lattice calculations can have a significant impact Gèrardin, Meyer, Nyffeler (16,19)

PS-pole contributions

B. Kubis and P. Sanchez Puertas

Philosophy adopted in the section:
The calculations must be model-independent and data-driven to as large an extent as possible (...)

Three criteria must be fulfilled:

1. TFF normalization given by the real-photon decay widths, and high-energy constraints must be fulfilled;
2. at least the space-like experimental data for the singly-virtual TFF must be reproduced;
3. systematic uncertainties must be assessed with a reasonable procedure.

Results above the bar

- Dispersive calculation of the pion TFF — Hoferichter et al. (18)

$$a_\mu^{\pi^0} = 63.0^{+2.7}_{-2.1} \times 10^{-11}$$

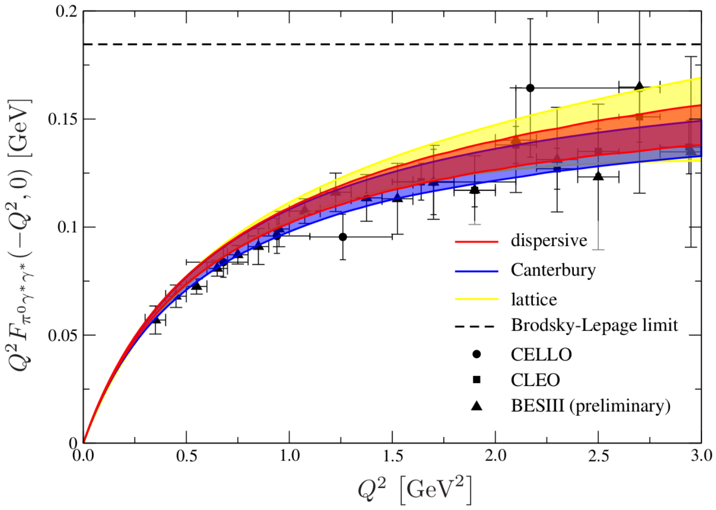
- Padé-Canterbury approximants — Masjuan & Sanchez-Puertas (17)

$$a_\mu^{\pi^0} = 63.6(2.7) \times 10^{-11}$$

- Lattice — Gérardin, Meyer, Nyffeler (19)

$$a_\mu^{\pi^0} = 62.3(2.3) \times 10^{-11}$$

Results above the bar



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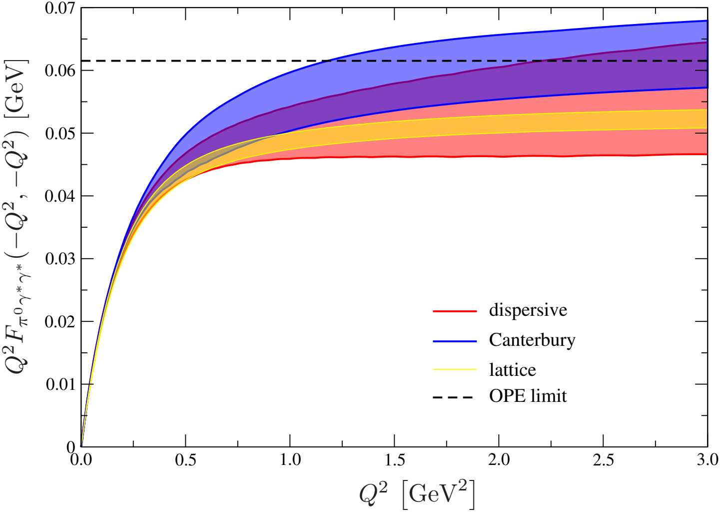
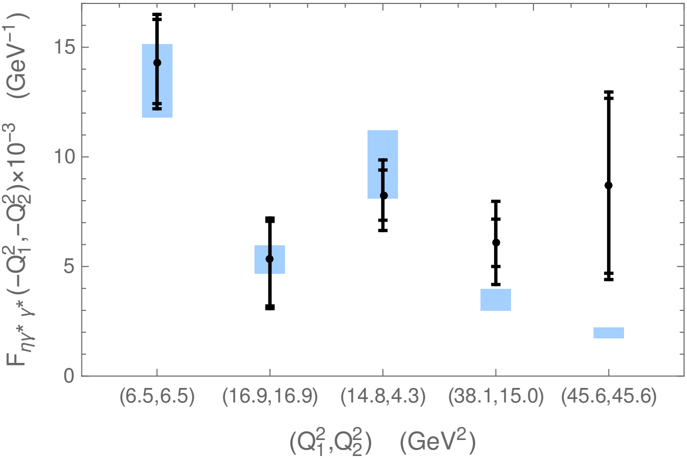


η- and η'-pole contribution

- Dispersive calculation not yet available (η-η' mixing, different isospin structure etc.) $\rightarrow$ talk by S. Holz
- Less data (BaBar)
- Canterbury approach:

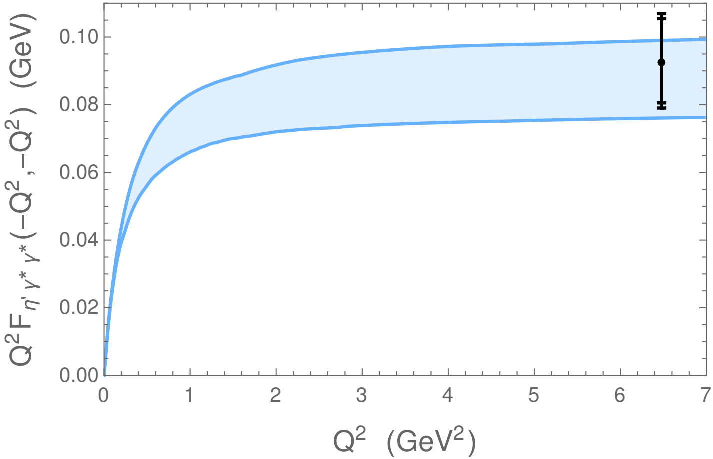
$$a_\mu^\eta = 16.3(1.0)_{\rm stat}(0.5)_{a_{P;1,1}}(0.9)_{\rm sys} \times 10^{-11} \rightarrow 16.3(1.4) \times 10^{-11}$$
$$a_\mu^{\eta'} = 14.5(0.7)_{\rm stat}(0.4)_{a_{P;1,1}}(1.7)_{\rm sys} \times 10^{-11} \rightarrow 14.5(1.9) \times 10^{-11}$$

η- and η'-pole contribution



Data points: BaBar. Blue band: Canterbury representation.

η- and η'-pole contribution



Data points: BaBar. Blue band: Canterbury representation.

PS-poles: conclusion

Dispersive (π^0) + Canterbury (η, η'):

$$a_\mu^{\pi^0+\eta+\eta'} = 93.8^{+4.0}_{-3.6} \times 10^{-11}$$

Canterbury:

$$a_\mu^{\pi^0+\eta+\eta'} = 94.3(5.3) \times 10^{-11}$$

Outlook:

Dispersive evaluation of the η, η' contributions will give two fully independent evaluations $\Rightarrow$ better control over systematics

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2π-contributions

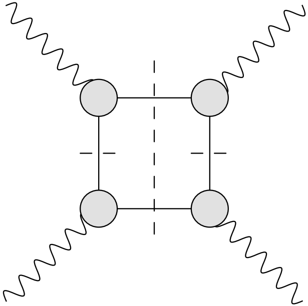
I. Danilkin & P. Stoffer

This can be split in several components

- π-box
- 2π *S*-wave below 1 GeV
- 2π *S*-wave above 1 GeV
- 2π *D*-wave
- 2π yet higher waves

Pion-box contribution

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\text{FsQED}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \cdots$$



Pion-box contribution

The only ingredient needed for the pion-box contribution is the vector form factor

$$\hat{\Pi}_i^{\pi\text{-box}} = F_\pi^V(q_1^2)F_\pi^V(q_2^2)F_\pi^V(q_3^2)\frac{1}{16\pi^2}\int_0^1 dx \int_0^{1-x} dy\, I_i(x,y),$$

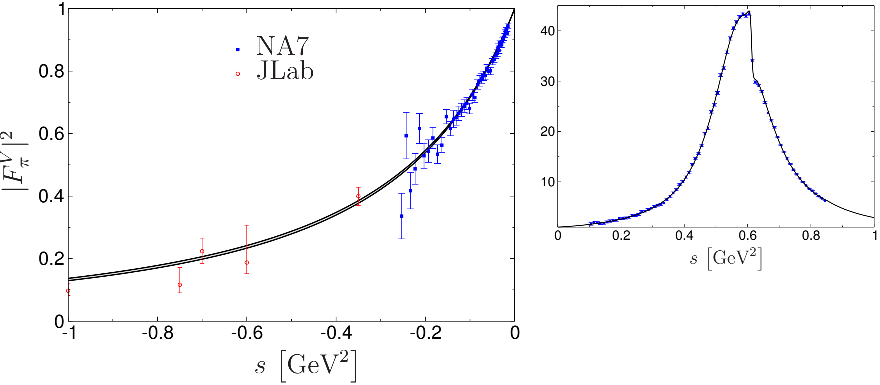
where

$$I_1(x,y) = \frac{8xy(1-2x)(1-2y)}{\Delta_{123}\Delta_{23}},$$

and analogous expressions for $I_{4,7,17,39,54}$ and

$$\Delta_{123} = M_\pi^2 - xyq_1^2 - x(1-x-y)q_2^2 - y(1-x-y)q_3^2,$$
$$\Delta_{23} = M_\pi^2 - x(1-x)q_2^2 - y(1-y)q_3^2$$

Pion-box contribution



Uncertainties are negligibly small:

$$a_\mu^{\text{FsQED}} = -15.9(2) \cdot 10^{-11}$$

Pion-box contribution

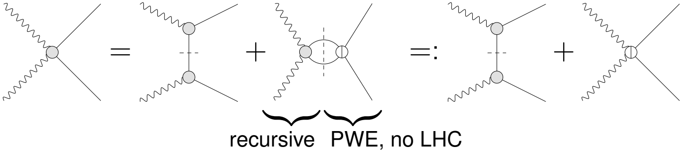
Contribution	BPaP(96)	HKS(96)	KnN(02)	MV(04)	BP(07)	PdRV(09)	N/JN(09)
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	–	114±13	99±16
π, K loops	−19±13	−4.5±8.1	–	–	–	−19±19	−19±13
" " + subl. in N_c	–	–	–	0±10	–	–	–
axial vectors	2.5±1.0	1.7±1.7	–	22± 5	–	15±10	22± 5
scalars	−6.8±2.0	–	–	–	–	−7± 7	−7± 2
quark loops	21± 3	9.7±11.1	–	–	–	2.3	21± 3
total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

Uncertainties are negligibly small:

$$a_\mu^{\text{FsQED}} = -15.9(2) \cdot 10^{-11}$$

First evaluation of S- wave 2π-rescattering

Omnès solution for $\gamma^*\gamma^* \to \pi\pi$ provides the following:



Based on:

- taking the pion pole as the only left-hand singularity
- $\Rightarrow$ pion vector FF to describe the off-shell behaviour
- $\pi\pi$ phases obtained with the inverse amplitude method

  [realistic only below 1 Gev: accounts for the $f_0(500)$ + unique and well defined extrapolation to ∞]
- numerical solution of the $\gamma^*\gamma^* \to \pi\pi$ dispersion relation

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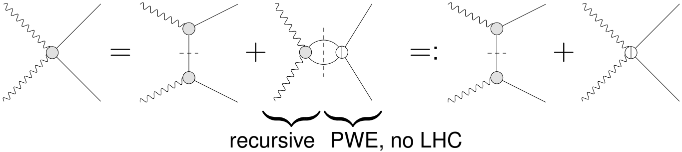

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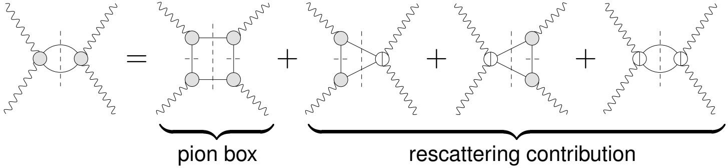
- numerical solution of the $\gamma^*\gamma^* \to \pi\pi$ dispersion relation

S-wave contributions : $a_{\mu,J=0}^{\pi\pi,\pi\text{-pole LHC}} = -8(1) \times 10^{-11}$

Two-pion contribution to $(g-2)_\mu$ from HLbL

Two-pion contributions to HLbL:



$$a_\mu^{\pi-\mathrm{box}} + a_{\mu,J=0}^{\pi\pi,\pi\text{-pole LHC}} = -24(1)\cdot 10^{-11}$$

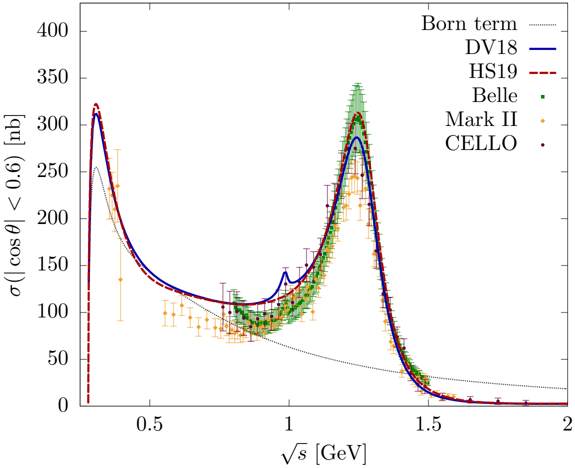
$\gamma^*\gamma^* \to \pi\pi$ contribution from other partial waves

- formulae get significantly more involved with several subtleties in the calculation
- in particular sum rules which link different partial waves must be satisfied by different resonances in the narrow width approximation Danilkin, Pascalutsa, Pauk, Vanderhaeghen (12,14,17)
- data and dispersive treatments available for on-shell photons e.g. Dai & Pennington (14,16,17)
- dispersive treatment for the singly-virtual case and check with forthcoming data is very important

$\longrightarrow$ talks by Danilkin & Stoffer

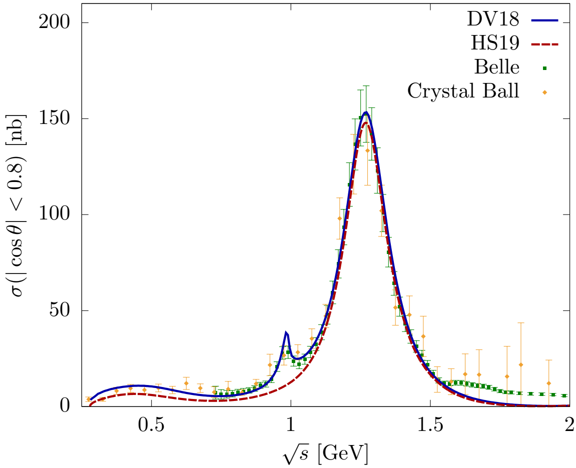
$\gamma^{(*)}\gamma^{(*)} \to \pi\pi$ cross-section: data vs theory

$\gamma\gamma \to \pi^+\pi^-$



$\gamma^{(*)}\gamma^{(*)} \to \pi\pi$ cross-section: data vs theory

$\gamma\gamma \to \pi^0\pi^0$



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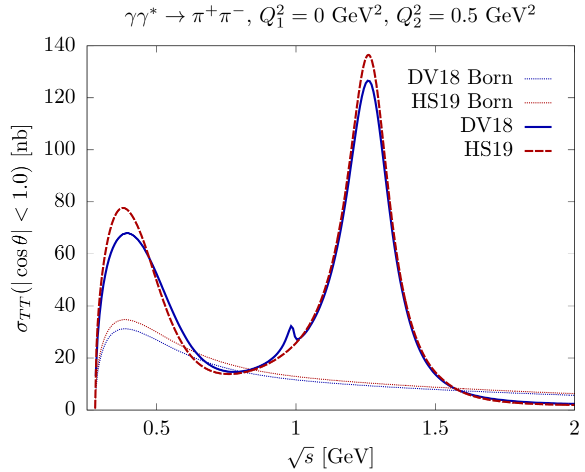

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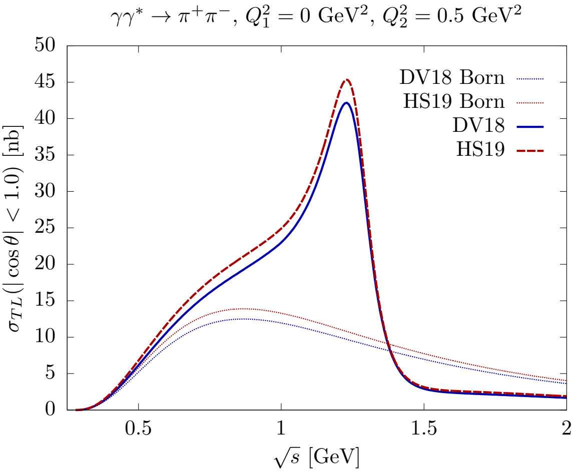

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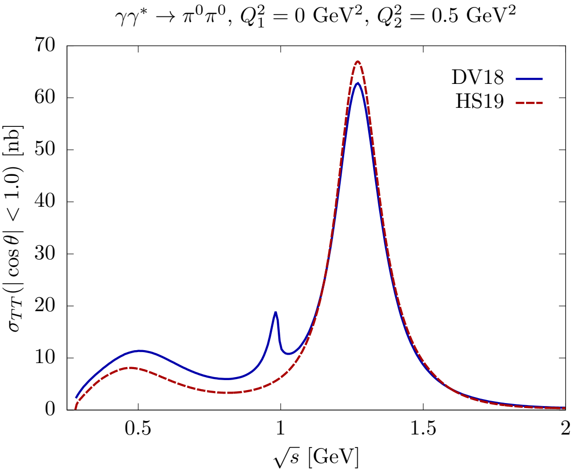

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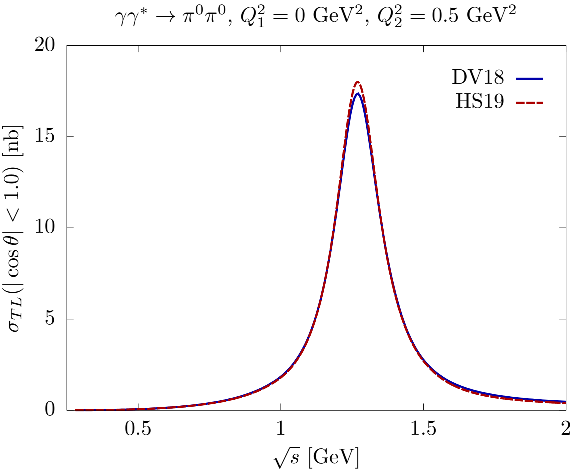

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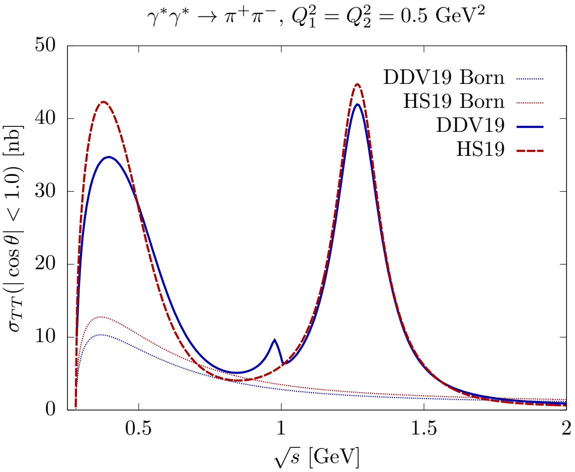

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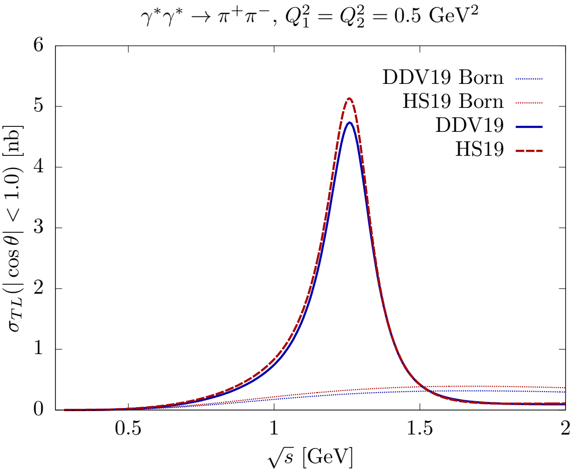

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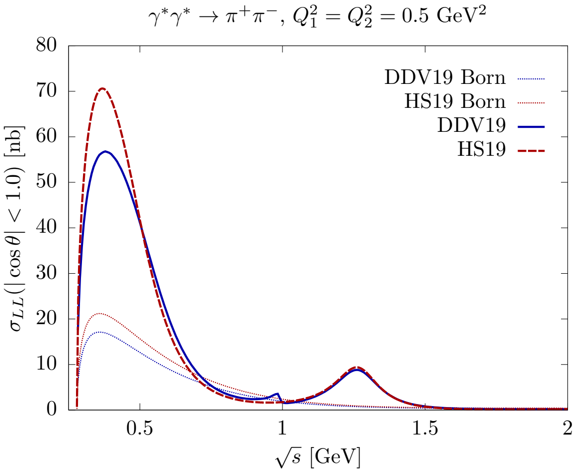

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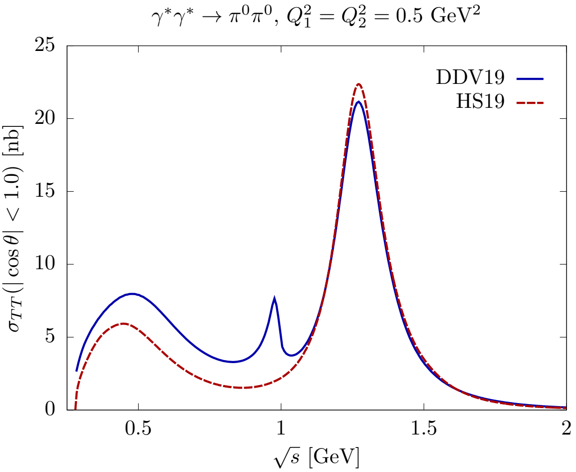

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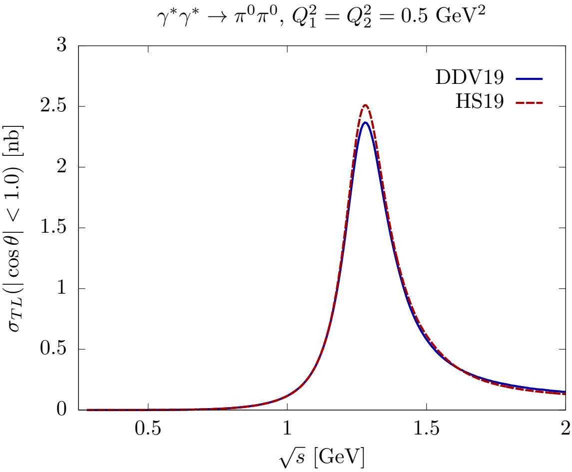

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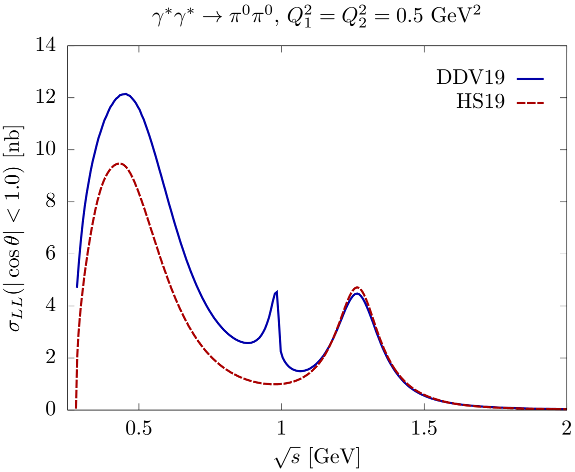

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Higher hadronic intermediate states

P. Stoffer & M. Vanderhaeghen

- Kaon-box: (based on a VMD description of F_V^K. VMD for F_V^π gives π-box within 3%)

$$a_\mu^{K-\text{box}} = -0.50 \times 10^{-11}$$

- Higher scalars

$$a_\mu^{\text{scalars}} = [-(3.1 \pm 0.8), -(0.9 \pm 0.2)] \times 10^{-11} \qquad \text{Pauk et al.(14)}$$
$$a_\mu^{\text{scalars}} = [-(2.2^{+3.2}_{-0.7}), -(1.0^{+2.0}_{-0.4})] \times 10^{-11} \qquad \text{Knecht et al.(18)}$$

- Tensors ($f_2(1270)$, $f_2(1565)$, $a_2(1320)$, and $a_2(1700)$)

$$a_\mu^{\text{tensors}} = 0.9(0.1)\times 10^{-11} \qquad \text{Danilkin et al.(16)}$$

- Axial vectors $\longrightarrow$ talks by Hoferichter, Kampf

$$a_\mu^{\text{axials}}[f_1, f_1'] = 6.4(2.0) \times 10^{-11} \qquad \text{Pauk et al.(14)}$$
$$a_\mu^{\text{axials}}[a_1, f_1, f_1'] = 7.6(2.7) \times 10^{-11} \qquad \text{Jegerlehner(17)}$$

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Short-distance constraints

J. Bijnens, M. Hoferichter

Two possible high-energy regimes for HLbL:

$$\text{a) } q_1^2 \sim q_2^2 \gg q_3^2, \qquad \text{b) } q_1^2 \sim q_2^2 \sim q_3^2$$

- Constraints in regime a) have been discussed by

Melnikov & Vainshtein (04)

$$\Pi_1^L(q_1^2, q_2^2, q_3^2) \overset{q_{1,2}^2 = q^2 \gg q_3^2}{\longrightarrow} -\frac{2N_C}{\pi^2 q^2 q_3^2} \sum_a C_a^2 + \ldots \overset{a=3}{\longrightarrow} -\frac{1}{6\pi^2 q^2 q_3^2}$$

to be compared with

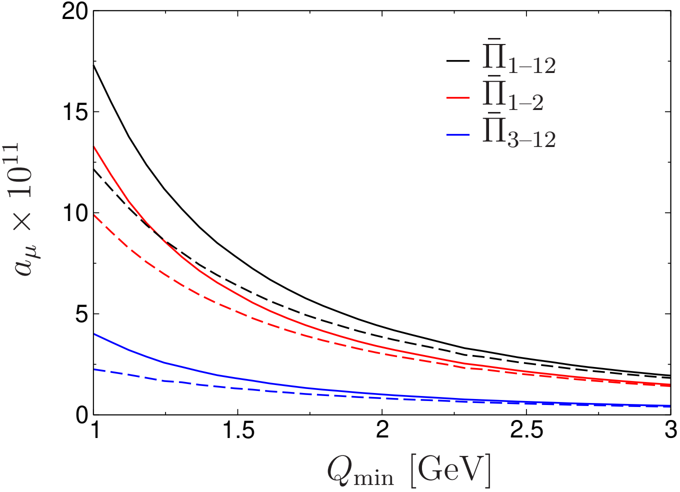
$$\Pi_1^{\pi-\text{pole}}(q_1^2, q_2^2, q_3^2) = \frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) F_{\pi^0\gamma^*\gamma^*}(q_3^2, 0)}{q_3^2 - M_{\pi^0}^2}$$

- Constraints in regime b) can be derived from the plain quark loop

$\longrightarrow$ talks by Bijnens & Hoferichter

Short-distance constraints

J. Bijnens, M. Hoferichter



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- Constraints in regime a) have been discussed by

  Melnikov & Vainshtein (04)

- Constraints in regime b) can be derived from the plain quark loop $\longrightarrow$ talks by Bijnens & Hoferichter
- In the dispersive approach, the sum of the contributions discussed so far does not satisfy these constraints
- $\Rightarrow$ add more ($\rightarrow$ infinitely many!) hadronic states to satisfy the SDC $\longrightarrow$ talks by Hoferichter & Laub

Outline

Summary of HLbL (as of May ’19, very preliminary!)

Contributions to $10^{11} \cdot a_\mu^{\text{HLbL}}$

- Pseudoscalar poles $= 93.8^{+4.0}_{-3.6}$
- pion box (kaon box ~ -0.5) $= -15.9(2)$
- S-wave $\pi\pi$ rescattering $= -8(1)$
- scalars and tensors with $M_R > 1$ GeV $\sim -2(3)$
- axial vectors $\sim 8(3)$
- short-distance contribution $\sim 10(10)$

Central value:	$85 \pm XX$
Uncertainties added in quadrature:	$XX = 12$
Uncertainties added linearly:	$XX = 21$

Improvements obtained with the dispersive approach

Contribution	PdRV(09)	N/JN(09)	J(17)	White Paper
π^0, η, η'-poles	114 ± 13	99 ± 16	95.45 ± 12.40	93.8 ± 4.0
π, K-loop/box	-19 ± 19	-19 ± 13	-20 ± 5	-16.4 ± 0.2
S-wave $\pi\pi$	–	–	–	-8 ± 1
scalars	-7 ± 7	-7 ± 2	-5.98 ± 1.20	$\left.\right\} -2 \pm 3$ (scalars + tensors)
tensors	–	–	1.1 ± 0.1	
axials	15 ± 10	22 ± 5	7.55 ± 2.71	8 ± 8
q-loops / SD	2.3	21 ± 3	22.3 ± 5.0	10 ± 10
total	105 ± 26	116 ± 39	100.4 ± 28.2	$85 \pm XX$

HLbL in units of 10^{-11}.

PdRV = Prades, de Rafael, Vainshtein (“Glasgow consensus”); N = Nyffeler;

J = Jegerlehner

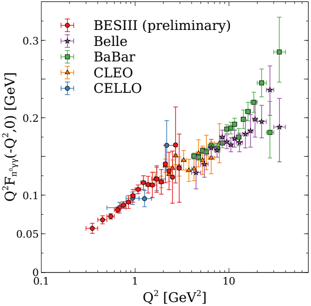
Exp. inputs and Monte Carlo studies

F. Curciarello, H. Czyż, E. Perez del Rio, C. Redmer

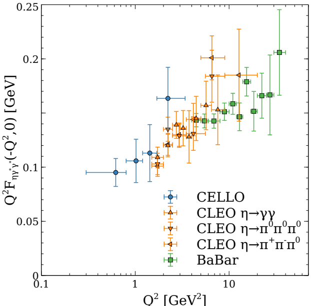
π, η, η' transition form factors (TFFs)

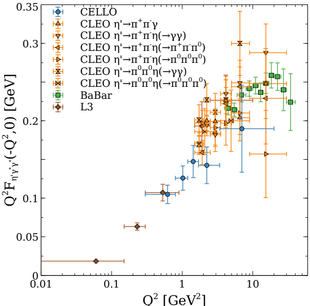
- Existing experimental data on single-virtual TFFs: spacelike regime from $\gamma^*\gamma$ collisions; timelike reg. from radiative production in e^+e^- annihil.
- Single Dalitz decays of pseudoscalars (slope of TFFs) Double Dalitz decay: no momentum dependence yet
- Very recently: first results from BaBar for double-virtual η' TFF for 7 intervals of rather large (Q_1^2, Q_2^2)
- TFFs also enter in Dalitz decays of vector mesons: $\omega \to \pi^0\mu^+\mu^-(\pi^0 e^+e^-)$ or $\phi \to e^+e^-\pi^0(e^+e^-\eta)$
- Update from BESIII: $\longrightarrow$ talk by Ch. Redmer

π^0, η, η' TFFs in spacelike region from $\gamma\gamma$-collisions

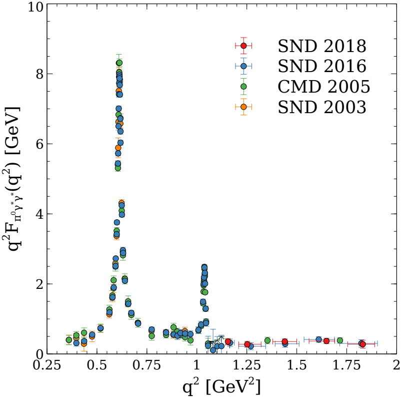


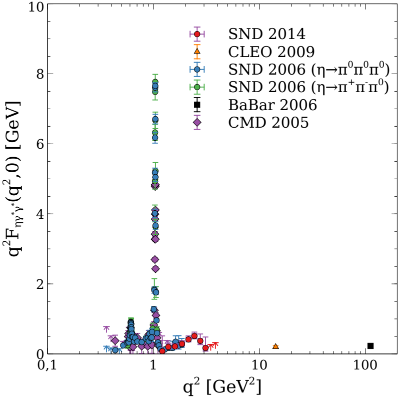
- Error bars indicate total uncertainties.
- For π^0 (η, η')-pole contributions to HLbL, double-virtual low-energy region $Q_i^2 \leq 1$ (4) GeV^2 most relevant.





π^0 and η TFFs in timelike region in e^+e^- annihilation





Error bars indicate total uncertainties.

$\pi^0 \to \gamma\gamma$ and $\gamma^{(*)}\gamma \to \pi\pi$ (and other PS pairs)

- $\pi^0 \to \gamma\gamma$ decay width (PrimEx-II)

  Related to normalization of $\mathcal{F}_{\pi^0\gamma\gamma}(0,0)$. Combined PrimEx-I and II result presented at PhiPsi 2019:

  $\Gamma(\pi^0 \to \gamma\gamma) = 7.802 \pm 0.52_{\text{stat.}} \pm 0.105_{\text{syst.}}\,\text{eV} = 7.802 \pm 0.117\,\text{eV}$

  1.5% accuracy, tension w/ ChPT at (N)NLO ? $\rightarrow$ talk by A. Gasparian

- $\gamma^{(*)}\gamma \to \pi\pi$ and other PS pairs

  Old data with real photons by DESY and SLAC, more precise recently by Belle, also for the first time $\gamma^*\gamma \to \pi^0\pi^0, K_s^0 K_s^0$, but at rather large $Q^2 \geq 3.0\,\text{GeV}^2$.
  Update from BESIII: $\longrightarrow$ talk by Ch. Redmer

Other relevant measurements and a wishlist

- Plans to measure $P \to \gamma\gamma$ and TFFs at low momenta at KLOE-2 and JLab (Primakoff program).
- BESIII: Feasibility studies for $\gamma^*\gamma^* \to \pi^0, \eta, \eta'$ in region $0.5\ \mathrm{GeV}^2 \leq Q_1^2, Q_2^2 \leq 2.0\ \mathrm{GeV}^2$.
- More processes (see wishlist below) should be measured at various experiments as input for DR approach to TFFs and for pion-loop.

issue	helpful experimental information
pseudoscalar TFF	$\gamma^*\gamma^* \to \pi^0, \eta, \eta'$ at arbitrary virtualities
pion loops	$\gamma^*\gamma^* \to \pi\pi$ at arbitrary virtualities, partial waves
dispersive analysis of π^0 TFF	high accuracy Dalitz plot $\omega \to \pi^+\pi^-\pi^0$
	$e^+e^- \to \pi^+\pi^-\pi^0$
	$\gamma\pi \to \pi\pi$
	$\omega \to \pi^0 l^+ l^-$ and $\phi \to \pi^0 l^+ l^-$ as cross check
dispersive analysis of η TFF	$\gamma\pi^- \to \pi^-\eta$
	$e^+e^- \to \eta\pi^+\pi^-$
	$\eta' \to \pi^+\pi^-\pi^+\pi^-$
	$\eta' \to \pi^+\pi^- e^+ e^-$
axial and tensor contributions	$\gamma^*\gamma^* \to 3$ or 4π
missing states	inclusive $\gamma^{(*)}\gamma^* \to \mathrm{hadrons}$ at 1-3 GeV

Dedicated discussion session on wishlist led by Andrzej Kupsc

Radiative corrections and MC event generators

- Strong tension between spacelike π^0 TFF data of BaBar at $Q^2 \geq 4\ \mathrm{GeV}^2$ and other exps. (CELLO, CLEO, Belle)
- Recent experiments used MC event generators that include radiative corrections in structure function method.
  Belle: TREPSPST — Uehara et al. (12, (13)
  BaBar: GGRESRC — Druzhinin et al. (14)
- Event generator EKHARA (Czyż et al. 06, 11) recently upgraded with exact QED corrections to $e^+e^- \to e^+e^- P$ Czyż and Kisza (19)
- Large rad. corrs. ($\sim 20\%$) found with EKHARA for BaBar sel. cuts, vs only $\sim 1\%$ in GGRESRC. Must be checked, also for TFF at lower momenta, e.g. at BESIII. Full detector simulation needed to judge final impact on TFF

$\longrightarrow$ talk by Henryk Czyż on Wednesday

Conclusions

- a lot of progress has happened in the last five years in the dispersive approach to HLbL
- this talk: status of this chapter as of the end of May 2019: for some contributions there has been a significant reduction in the theory uncertainties
- more work is needed for higher scalars, tensors and axial vectors as well as for the SDC
- this workshop: progress since last May
- this Friday $\Rightarrow$ where we will stand by end 2019