$e^+e^- \to \pi^+\pi^-$ cross section measurement with the SND detector at the VEPP-2000 collider

Kupich Andrey

Budker Institute of Nuclear Physics,
Novosibirsk State University
on behalf of the SND collaboration

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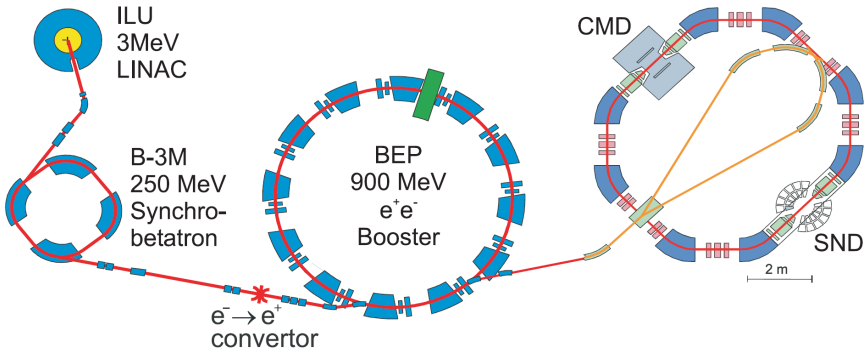
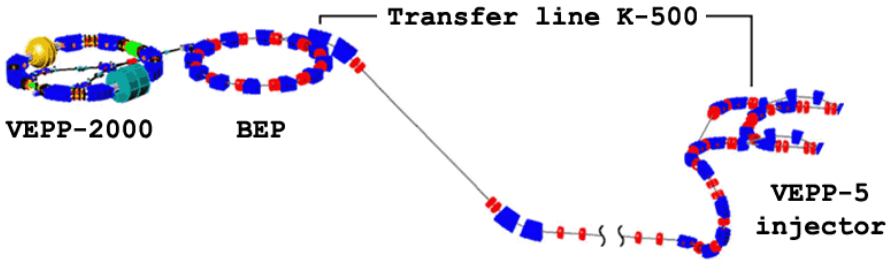
VEPP-2000 e^+e^- collider



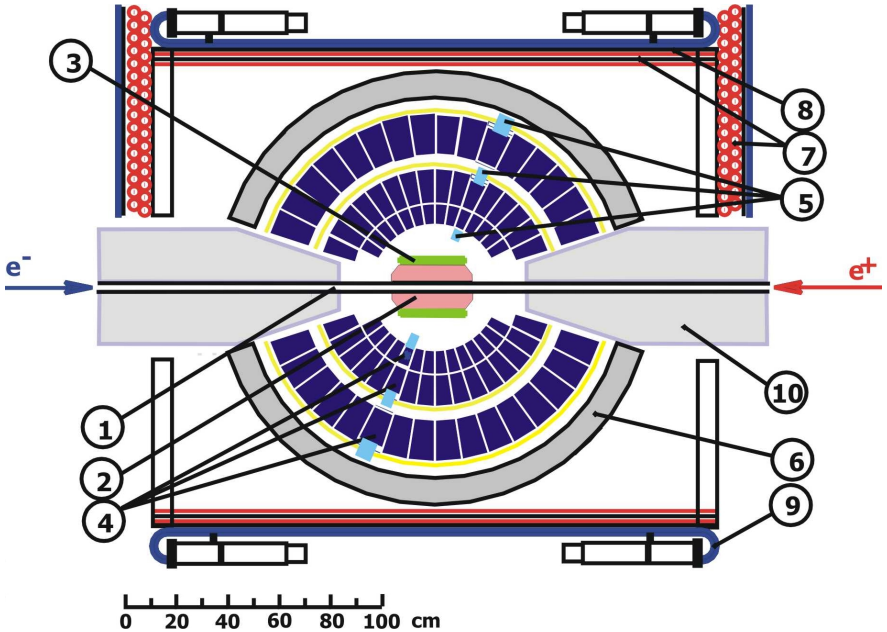


VEPP-2000 parameters

- c.m. energy E=0.3-2.0 GeV
- Luminosity at E=1.8 GeV
  $10^{32} cm^{-2} sec^{-1}$ (project)
  $4x10^{31} cm^{-2} sec^{-1}$ (achieved)
- Beam energy spread - 0.6 MeV at E=1.8 GeV

- 10 times more intense positron source
- Experiments at upgraded VEPP-2000 was restarted by the end of 2016
- About 50 pb^{-1} of integrated luminosity has been already collected during the 2017 run

SND detector



1-beam pipe, **2**-tracking system, **3**-aerogel Cherenkov counter , **4** - NaI(Tl) crystals, **5** - phototriodes, **6** - iron muon absorber, **7-9** - muon detector, **10** - focusing solenoids.

Main physics task of SND is study of all possible processes of e^+e^- annihilation into hadrons below 2 GeV.

- The total hadronic cross section, which is calculated as a sum of exclusive cross sections.
- Study of hadronization (dynamics of exclusive processes).

Analysis is based on the $\mathbf{4.6\ pb^{-1}}$ statistics, collected in 2012 – 2013, that corresponds to the $\mathbf{2.3 \times 10^6}$ collinear events, with $\mathbf{10^6}$ $e^+e^- \to \pi^+\pi^-, \mu^+\mu^-$ and $\mathbf{1.3 \times 10^6}$ $e^+e^- \to e^+e^-$

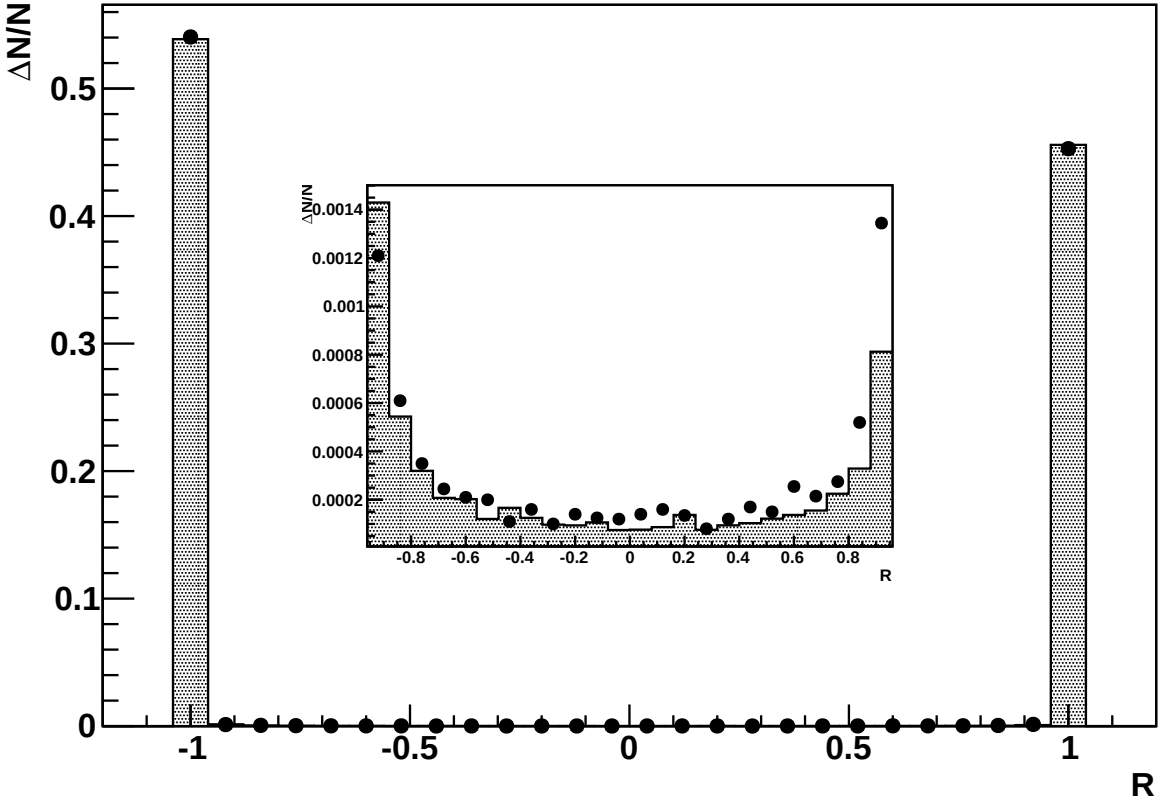
Events selection

1. $N_{ch} \geq 2$. The events can contain neutral particles due to nuclear interactions of charged pions with detector material or due to electromagnetic showers splitting
2. $|\Delta\theta| = |180^\circ - (\theta_1 + \theta_2)| < 12^\circ$ and $|\Delta\phi| = |180^\circ - |\phi_1 - \phi_2|| < 4^\circ$, where ϕ is the particle azimuthal angle
3. $E_{1,2} > 40$ MeV, where E_i is the ith particle $(i = 1, 2)$ energy deposition
4. $50^\circ < \theta_0 = (\theta_1 - \theta_2 + 180^\circ) \times 0.5 < 130^\circ$
5. $|d0_1| < 1$ cm , $|d0_2| < 1$ cm, where $|d0_i|$ is a distance between the ith particle track and the beam axis
6. $|z0_1| < 8$ cm , $|z0_2| < 8$ cm, where $|z0_i|$ is a distance from the center of the detector to the primary vertex of the ith particle track along the beam axis
7. The muon system $veto = 0$

e/π ID

The output signal of the trained BDT network R is a value in the interval from -1.0 to 1.0

The $e^+e^- \to e^+e^-$ events are located in the region $R < 0$, while $e^+e^- \to \pi^+\pi^-, \mu^+\mu^-$ events in $R > 0$.



Non-collinear and cosmic backgrounds

$$N_{cosm} = N_{exp}[veto = 1] \times \frac{N_{cosm}[veto = 0]}{N_{cosm}[veto = 1]}$$

$N_{exp}[veto = 1]$ – number of data events selected with 2π cuts but with veto=1; $N_{cosm}[veto = 0(1)]$ – number of special cosmic events

Two types of cosmic events are used:

- Non-central ($|d0_1| > 0.5$ cm , $|d0_2| > 0.5$ cm, $|z0_1| > 5$ cm and $|z0_2| > 5$ cm) events from the same data sample.
- Events from special cosmic runs without beams

Both give the **same 2.5%** ratio between $N_{cosm}[veto = 0]$ and $N_{cosm}[veto = 1]$ in the **whole energy spectrum**

Non-collinear and cosmic backgrounds

Background from the $e^+e^- \to \pi^+\pi^-\pi^0$ is subtracted directly in the $\omega(782)$ region, with a number of background events estimated according to the formula:

$$N_{3\pi} = N^{exp}[3\pi] \times \frac{N^{mc}_{3\pi}[2\pi]}{N^{mc}_{3\pi}[3\pi]}$$

$N^{exp}[3\pi]$ – number of $e^+e^- \to \pi^+\pi^-\pi^0$ events in the same data sample selected with a special 3π cuts:

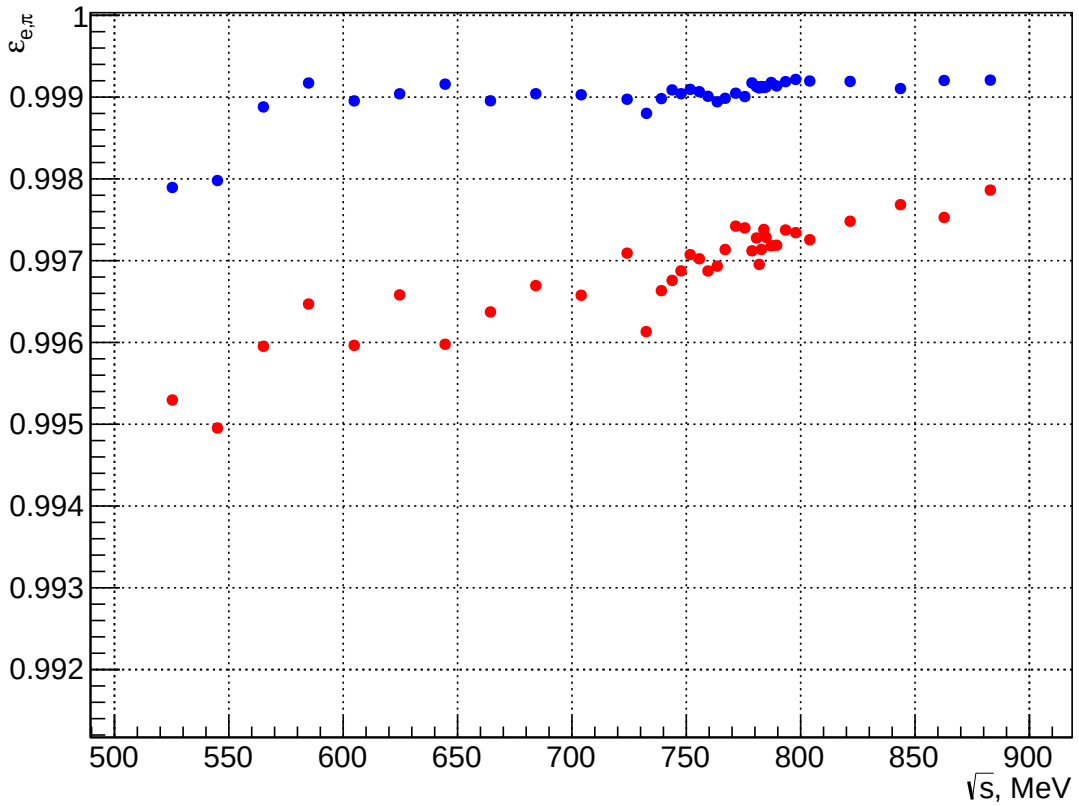
$$N_{cha} \geq 2,\ N_n \geq 2,\ |\Delta\theta| > 10^\circ,\ |\Delta\phi| > 10^\circ,$$
$$40^\circ < \theta_i < 140^\circ, \chi^2_{\pi^+\pi^-2\gamma} < 50,\ \chi^2_{\pi^+\pi^-\pi^0} < 30$$

Contribution of this background to the total $e^+e^- \to \pi^+\pi^-$ cross section **is less than 0.15%**, due to the strong suppression by $|\Delta\theta|$ and $|\Delta\phi|$ cuts.

ID efficiency

$$\varepsilon_e = \frac{N^{ee}(R \in [-1;0])}{N^{ee}(R \in [-1;1])}, \quad \varepsilon_\pi = \frac{N^{\pi\pi}(R \in [0;1])}{N^{\pi\pi}(R \in [-1;1])}$$

$N^{ee,\pi\pi}(R \in [a;b])$ are the numbers of $e^+e^- \to e^+e^-$ or $\pi^+\pi^-$ events with R in the interval $[a;b]$



Identification efficiencies for $e^+e^- \to e^+e^-$ and $e^+e^- \to \pi^+\pi^-$ simulated events

ID efficiency correction

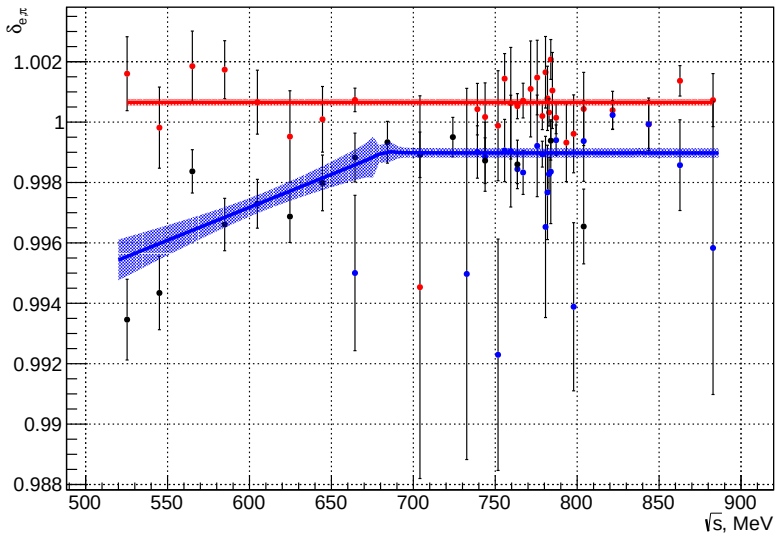
$$\delta_x = \frac{\epsilon_x^{exp}}{\epsilon_x^{mc}}$$

$x = e(\pi)$, ϵ_x^{exp} and ϵ_x^{mc} are identification efficienties for experimental and simulated pseudoevents respectively. The δ_e does not depend on energy, and its average value is 1.0006 ± 0.0001

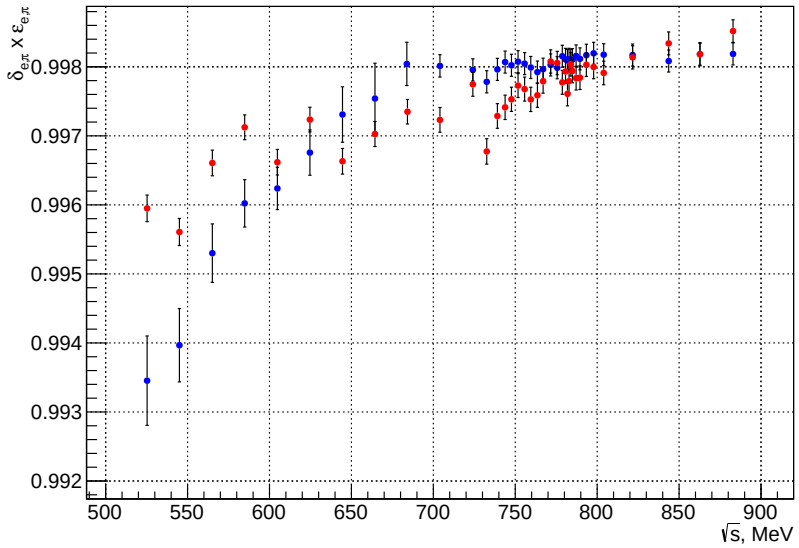
$$\delta_\pi(\sqrt{s}) = a\left(\sqrt{(\sqrt{s}-b)^2+10} - (\sqrt{s}-b)\right) + c$$

$\delta_\pi = 0.9990 \pm 0.0002$ at the energy region $\sqrt{s} > 0.65$ GeV and below δ_π changes upto 0.9950 ± 0.0006 at $\sqrt{s} = 0.52$ GeV

ID efficiency correction



Correction coefficients for ID efficiencies of the $e^+e^- \to e^+e^-$ and $e^+e^- \to \pi^+\pi^-$ events. δ_π obtained using pseudo $\pi\pi$ events constructed from $e^+e^- \to \pi^+\pi^-$ and $e^+e^- \to \omega, \phi \to \pi^+\pi^-\pi^0$ events. Lines are the fit results.

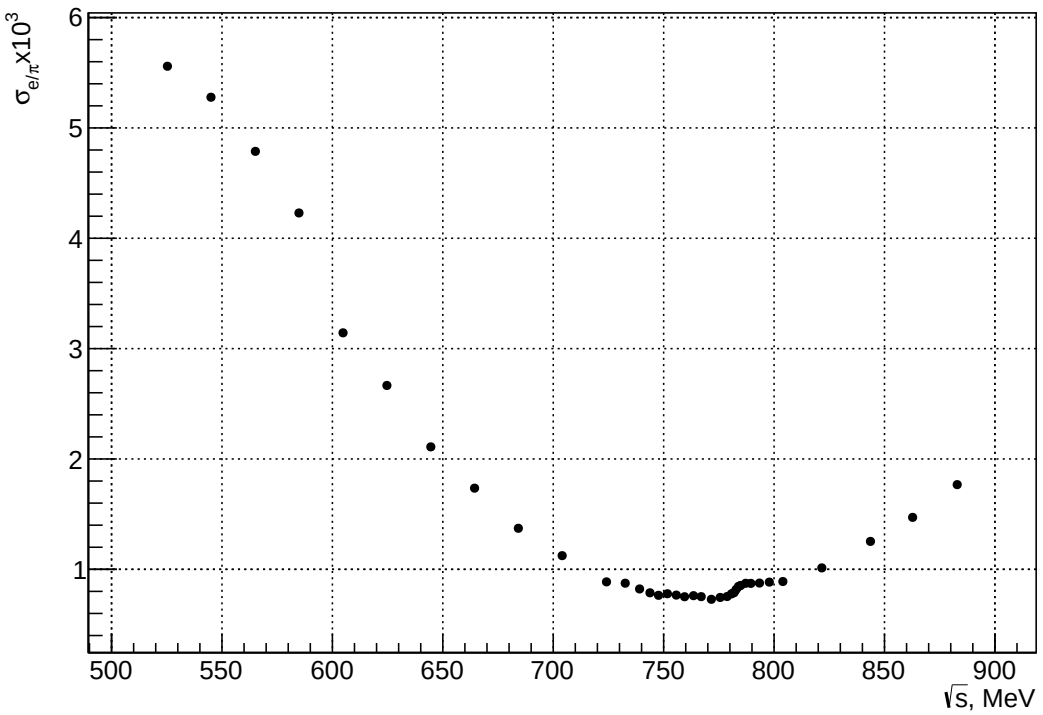


Corrected ID efficiencies for the $e^+e^- \to e^+e^-$ and $e^+e^- \to \pi^+\pi^-$ events

JINST T01002 (2017)

Contribution to the cross section uncertainty

Error	δ_e, %	δ_π at $\sqrt{s} > 0.65$ GeV, %	δ_π at $\sqrt{s} < 0.65$ GeV, %
σ_{stat}	0.01	0.02	0.02 – 0.06
σ_{ID}	0.02	0.01	0.02
σ_{bkg}	0.02	0.02	–
σ_{tot}	0.03	0.03	0.03 – 0.06

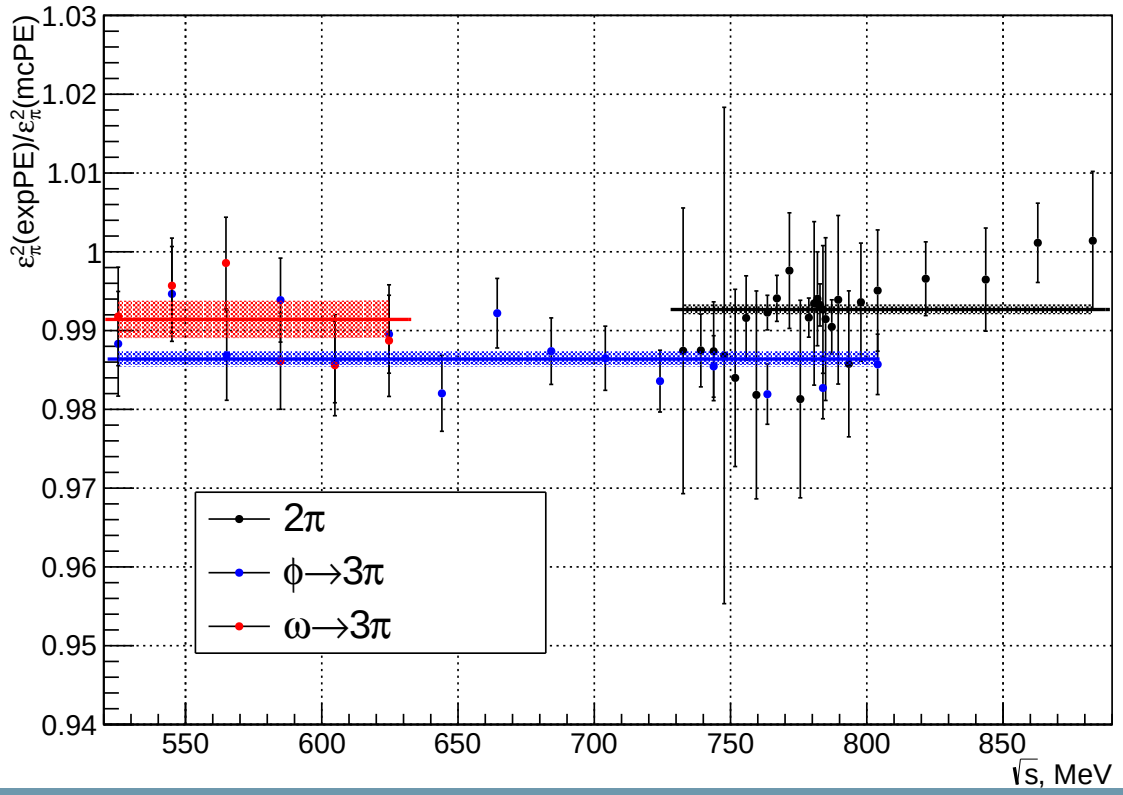


Contribution of the ID efficiencies to the relative error of $e^+e^- \to \pi^+\pi^-$ cross section is **less than 0.2%** for the most energy points

$E_{1,2} > 40$ MeV efficiency

The pseudo–$\pi\pi$ events are used to check the validity of efficiency for the $E_{1,2} > 40$ MeV cut, derived from the simulation

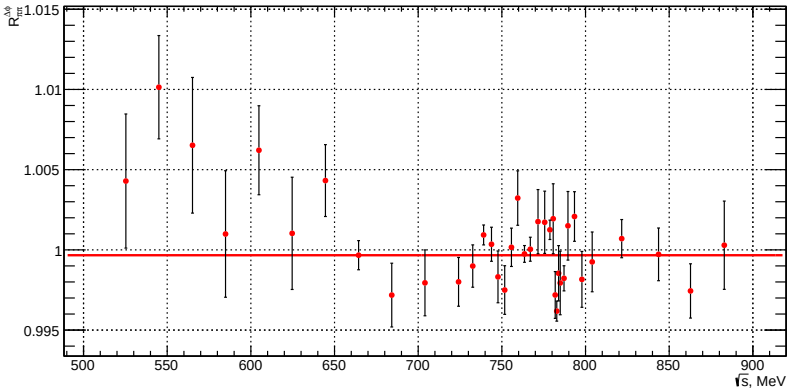
Obtained average correction is equal to **0.992**. The maximum difference between corrections derived from the different types of pseudo-events is **0.5%**



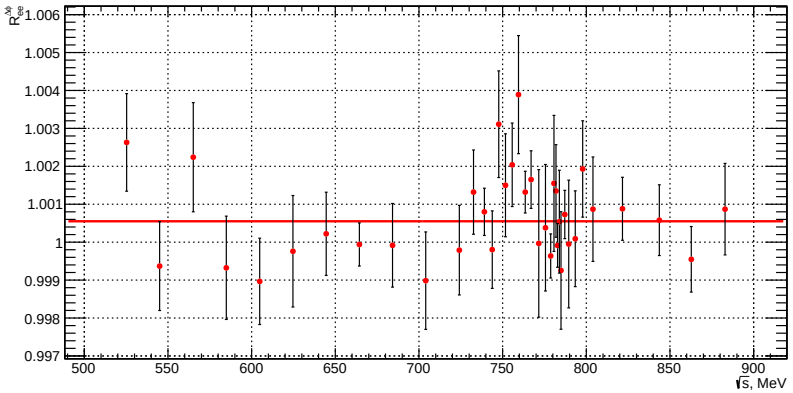
Efficiency of the $|\Delta\phi| < 4^\circ$ and $|\Delta\theta| < 12^\circ$ cuts

In order to study the differences between simulation and experimental data at each energy point, an efficiency correction is introduced:

$$R_i(x) = \frac{\epsilon_i^{exp}(x)}{\epsilon_i^{mc}(x)} \quad \epsilon_i^{exp} = \frac{N_i(x \in [A_x; B_x])}{N_i(x \in [C_x; D_x])} \quad \epsilon_i^{mc} = \frac{M_i(x \in [A_x; B_x])}{M_i(x \in [C_x; D_x])}$$

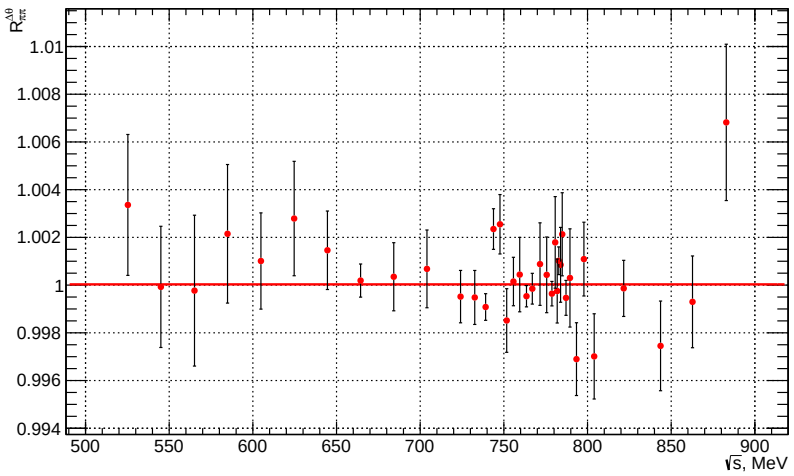


$e^+e^- \to \pi^+\pi^-$

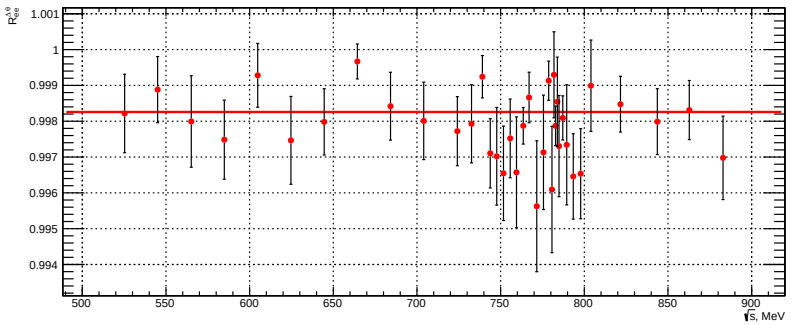


$e^+e^- \to e^+e^-$

Efficiency of the $|\Delta\phi| < 4^\circ$ and $|\Delta\theta| < 12^\circ$ cuts



$e^+e^- \to \pi^+\pi^-$



$e^+e^- \to e^+e^-$

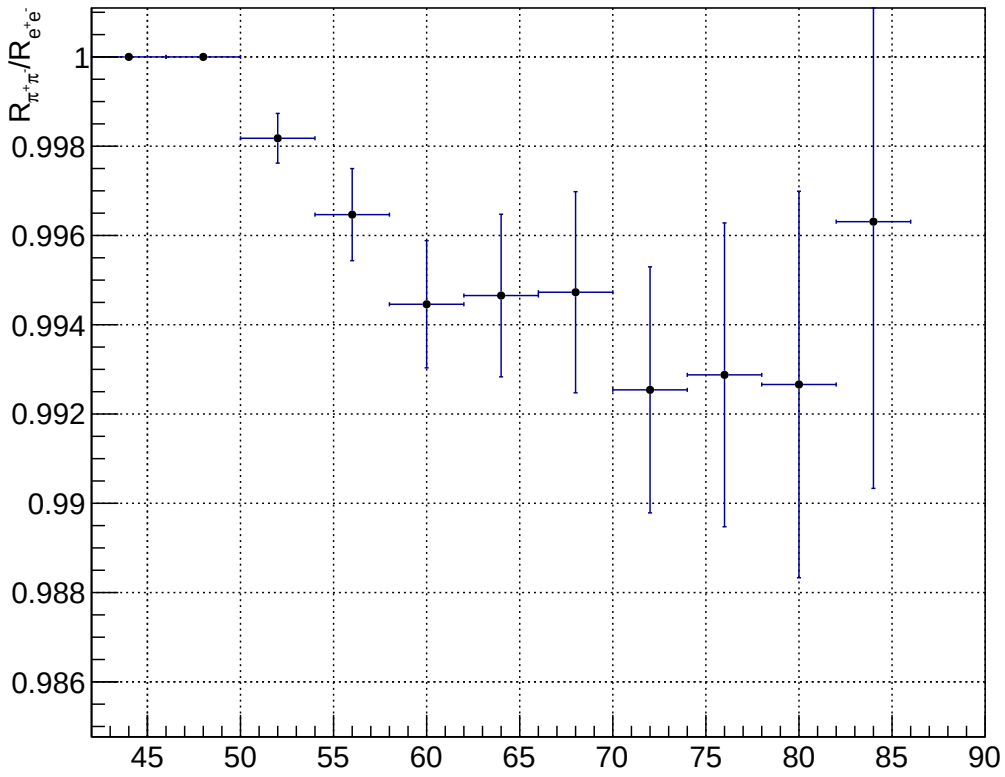
The average values of $\delta_{\Delta\phi} = R_{\pi\pi}(\phi)/R_{ee}(\phi)$ and $\delta_{\Delta\theta} = R_{\pi\pi}(\theta)/R_{ee}(\theta)$ differ from 1 by 0.1 % and 0.2 %, respectively

The overall contribution to the systematic uncertainty from the conditions on the $\Delta\phi$ and $\Delta\theta$ is equal to $0.001 \oplus 0.002 = 0.002$

Efficiency of the $50^\circ < \theta_0 < 130^\circ$ cut

$$R_i(z) = \frac{\varepsilon_i^{exp}(z)}{\varepsilon_i^{mc}(z)} \quad \varepsilon_i^{exp}(z) = \frac{N_i(\theta_0 \in [x; 180^\circ - z])}{N_i(\theta_0 \in [50^\circ; 130^\circ])} \quad \varepsilon_i^{mc}(z) = \frac{M_i(\theta_0 \in [x; 180^\circ - z])}{M_i(\theta_0 \in [50^\circ; 130^\circ])}$$

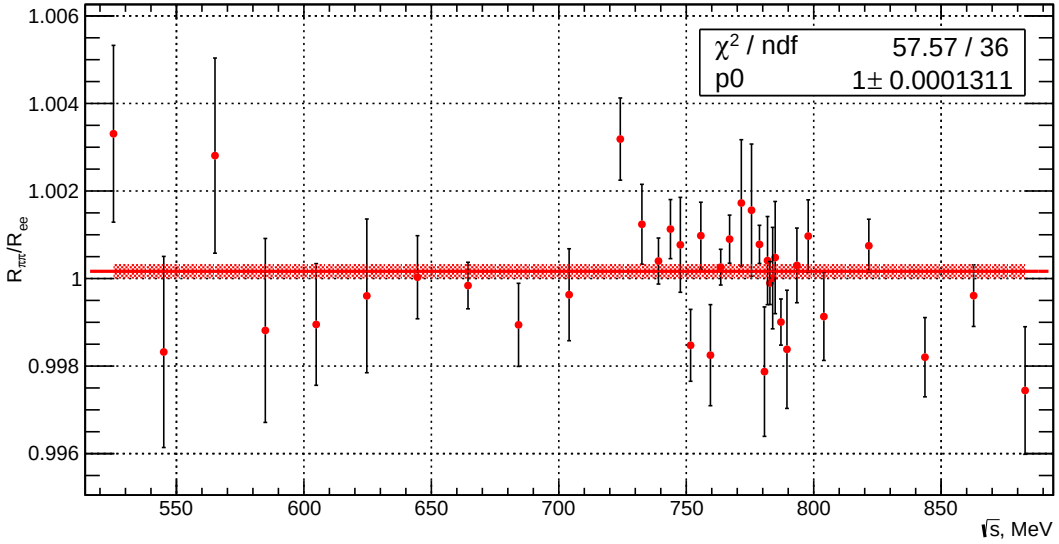
The statistically significant deviation of $R_{\pi\pi}/R_{ee}$ ratio from unity does not exceed 0.5 %



Track reconstruction efficiency

Probability of the π (e) track loss due reconstruction inefficiency is estimated from the $R_{\pi\pi}$ (R_{ee}):

the ratio of the number of events with one track, but the total number of particles > 1 and loosen $\Delta\phi$ and $\Delta\theta$ cuts, to the number of events with two or more tracks



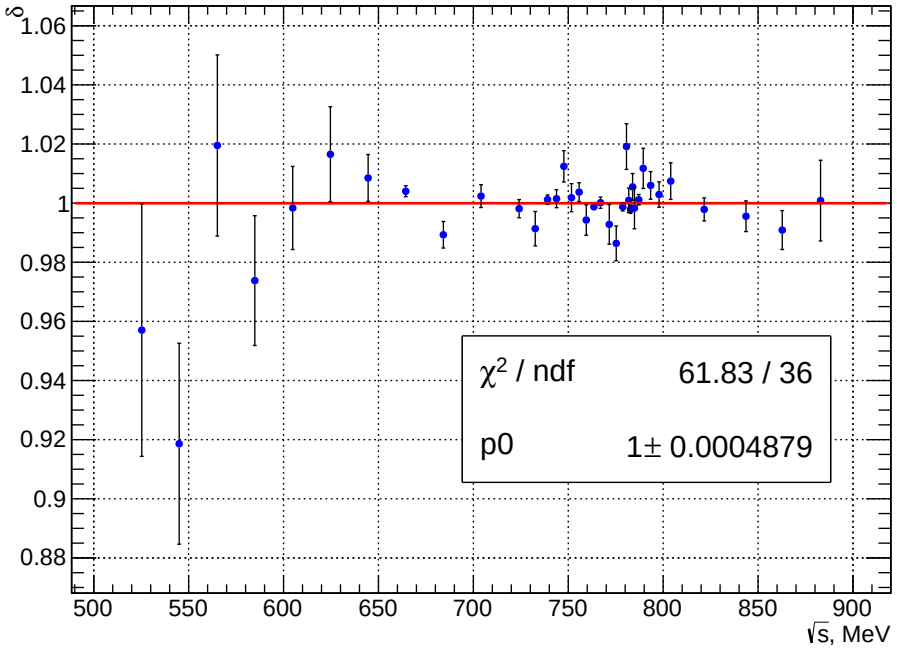
The ratio of $R_{\pi\pi}$ to R_{ee} is taken as a correction to the measured cross section

It does not show a significant deviation from unity

Muon system veto efficiency

$$\delta_{veto} = \frac{\sigma_{\pi\pi}((\phi_1 + \phi_2 - 180^\circ)/2 > 166^\circ or < 14^\circ; veto \geq 0)}{\sigma_{\pi\pi}((\phi_1 + \phi_2 - 180^\circ)/2 > 166^\circ or < 14^\circ; veto = 0)}$$

In case of the $veto \geq 0$ selection the certain number of the **residual cosmic background** events, derived from the fit of the $(z0_1 + z0_2)/2$ with a sum of uniform and normal distributions, is **subtracted** from the total number of the $e^+e^- \to \pi^+\pi^-$ events



δ_{veto} shows no significant energy dependence

Averaged value is applied and it is consistent with 1

Systematics

The main sources of systematic uncertainty

- $\Delta\theta$, $\Delta\phi$, θ_0 cuts: $0.001 \oplus 0.002 \oplus 0.005 = 0.55\%$
- $E_{1,2} > 40$ MeV condition: 0.5 %
- e/π–separation for the $\sqrt{s} \leq 600$ MeV: 0.5%

Additional sources of systematic uncertainty

- 0.2 % is taken as a systematic error from modeling of the pion loss due to nuclear interaction
- Contributions from the $N_{cha} \geq 2$ and $veto = 0$ cuts are considered to be negligible
- Calculation of the radiative correction gives 0.2 %

source	$\sigma_{E>40}$	σ_{nucl}	σ_{col}	σ_{θ_0}	$\sigma_{e/\pi}$	σ_{rad}	σ_{sys}
$\sqrt{s} > 600$ MeV	0.5 %	0.2 %	0.2 %	0.5 %	0.1 %	0.2 %	**0.8 %**
$\sqrt{s} \leq 600$ MeV	0.5 %	0.2 %	0.2 %	0.5 %	0.5 %	0.2 %	**0.9 %**

Calculating $e^+e^- \to \pi^+\pi^-$ cross section

$$N_a = L(\sigma_{\pi\pi}\varepsilon^a_{\pi\pi} + \sigma_{\mu\mu}\varepsilon^a_{\mu\mu} + \sigma_{ee}\varepsilon^a_{ee}) + N^a_{nc}$$

a = 1,2 correspond to the $R_{e/\pi} \in$ [0,1] and $R_{e/\pi} \in$ [- 1,0] respectively; σ_{jj} and ε^a_{jj}, with jj $= \pi^+\pi^-, \mu^+\mu^-, e^+e^-$ in the final state; N^a_{nc} is the number of non-collinear and cosmic background events; L is the IL collected at s_i.

From these equations $e^+e^- \to \pi^+\pi^-$ cross section and L can be deduced:

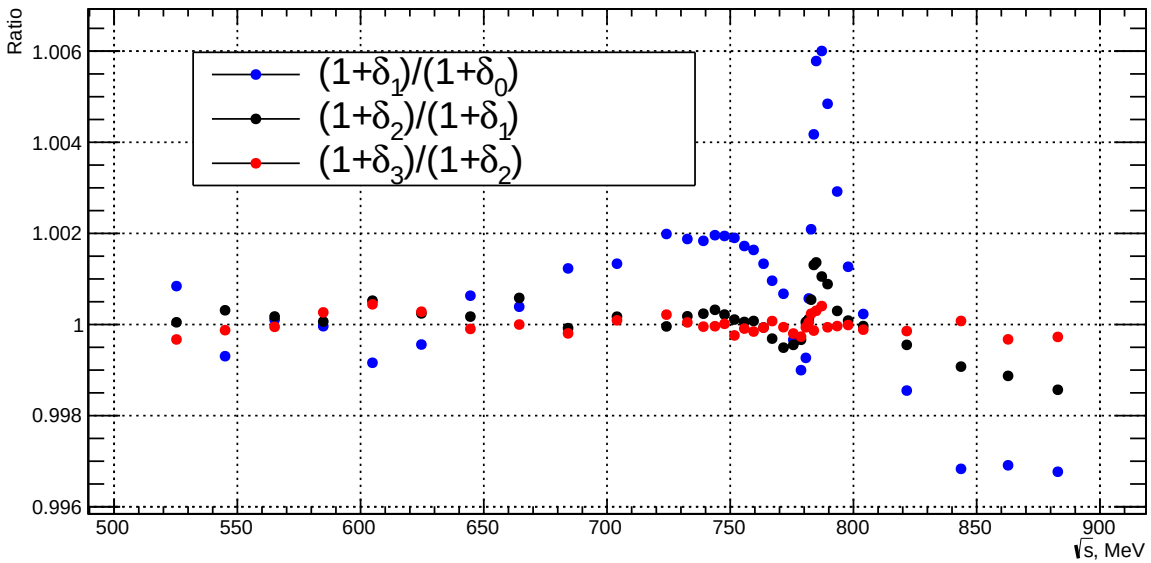
$$L(s_i) = \frac{(N_2 - N^2_{nc})\varepsilon^1_{\pi\pi} - (N_1 - N^1_{nc})\varepsilon^2_{\pi\pi}}{\sigma_{ee}(\varepsilon^2_{ee}\varepsilon^1_{\pi\pi} - \varepsilon^1_{ee}\varepsilon^2_{\pi\pi}) + \sigma_{\mu\mu}(\varepsilon^2_{\mu\mu}\varepsilon^1_{\pi\pi} - \varepsilon^1_{\mu\mu}\varepsilon^2_{\pi\pi})}$$

$$\sigma_{\pi\pi}(s_i) = \frac{N_1 - N^1_{nc} - L(s_i)\sigma_{\mu\mu}\varepsilon^1_{\mu\mu}(s_i) - L(s_i)\sigma_{ee}\varepsilon^1_{ee}}{L(s_i)\varepsilon^1_{\pi\pi}}$$

Born cross section

$$\sigma^0_{\pi\pi}(s_i) = \frac{\sigma_{\pi\pi}(s_i)}{1+\delta_{rad}(s_i)}$$

$1+\delta_{rad}(s_i)$ is a **radiative correction**, that accounts for radiation from the initial and final states, calculated using the **MCJPG** code.



JHEP **9710**, 006 (1997)

$\delta_{rad}(s_i)$ has to be calculated **iteratively**, by fitting measured cross sections with a model from the MCJPG

Fit model

$$\sigma_{\pi\pi}(s) = \frac{2}{3}\frac{\alpha^2}{s^{5/2}} P_{\pi\pi}(s)|A_{\pi\pi}(s)|^2$$

$$P_{\pi\pi}(s) = q_\pi^3(s), \qquad q_\pi(s) = \frac{1}{2}\sqrt{s - 4m_\pi^2}$$

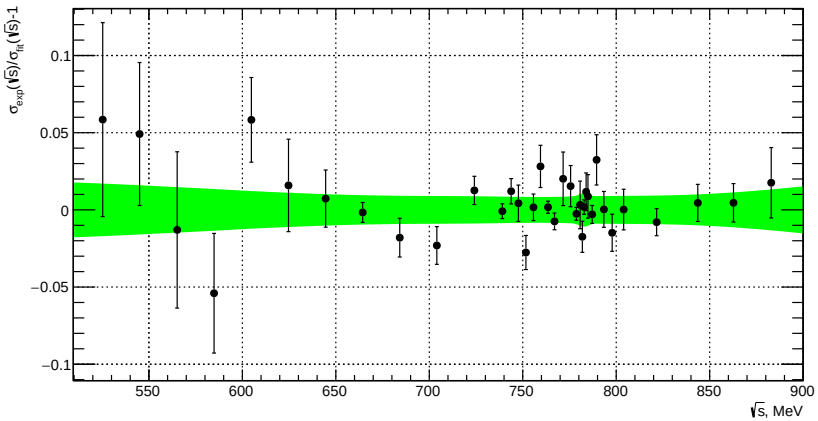
$$|A_{\pi\pi}(s)|^2 = \left| \sqrt{\frac{3}{2}}\frac{1}{\alpha} \sum_{V=\rho,\omega,\rho'} \frac{\Gamma_V m_V^3 \sqrt{m_V \sigma(V \to \pi^+\pi^-)}}{D_V(s)} \frac{e^{i\phi_{\rho V}}}{\sqrt{q_\pi^3(m_V)}} \right|^2$$

$$D_V(s) = m_V^2 - s - i\sqrt{s}\Gamma_V(s), \quad \Gamma_V(s) = \sum_f \Gamma(V \to f, s)$$

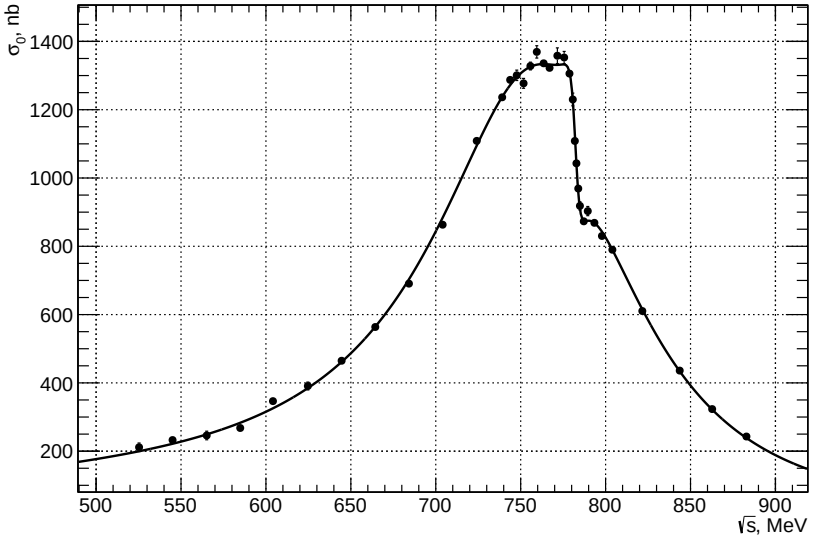
$$\Gamma_\omega(s) = \frac{m_\omega^2}{s}\frac{q_\pi^3(s)}{q_\pi^3(m_\omega)}\Gamma_\omega B_{\omega\to\pi^+\pi^-} + \frac{q_{\pi\gamma}^3(s)}{q_{\pi\gamma}^3(m_\omega)}\Gamma_\omega B_{\omega\to\pi^0\gamma} + \frac{W_{\rho\pi}(s)}{W_{\rho\pi}(m_\omega)}\Gamma_\omega B_{\omega\to 3\pi}$$

$$\Gamma_V(s) = \frac{m_V^2}{s}\frac{q_\pi^3(s)}{q_\pi^3(m_V)}\Gamma_V \quad (V = \rho, \rho')$$

Fit results



Cross section values in the $\sqrt{s}$ =352.02, 375.85 and 389.36 MeV energy points shows **non-statistical** deviation from the fit



The **relative difference** between the $e^+e^- \to \pi^+\pi^-$ cross section, measured by SND and fit of the SND experimental data. The **green bar** depicts **systematic** and **statistical** errors of the SND fit, folded quadratically.

Fit results

parameter	SND VEPP-2000	SND VEPP-2M
m_ρ, MeV	$775.4 \pm 0.5 \pm 0.4$	$774.6 \pm 0.4 \pm 0.5$
Γ_ρ, MeV	$146 \pm 0.7 \pm 1$	$146.1 \pm 0.8 \pm 1.5$
$\sigma(\rho \to \pi^+\pi^-)$, nb	$1189.25 \pm 4.6 \pm 9.5$	$1193 \pm 7 \pm 16$
$\sigma(\omega \to \pi^+\pi^-)$, nb	$31.3 \pm 1.3 \pm 0.3$	$29.3 \pm 1.4 \pm 1.0$
$\phi_{\rho\omega}$, deg.	110.6 ± 1.5	$113.7 \pm 1.3 \pm 2.0$
$\sigma(\rho' \to \pi^+\pi^-)$, nb	2.3 ± 0.6	1.8 ± 0.2
χ^2/ndf	$43/30$	–

parameters	SND VEPP-2000	SND VEPP-2M
$B_{\rho\to e^+e^-} \times B_{\rho\to\pi^+\pi^-}$	$(4.888 \pm 0.02 \pm 0.04) \times 10^{-5}$	$(4.876 \pm 0.02 \pm 0.06) \times 10^{-5}$
$B_{\omega\to e^+e^-} \times B_{\omega\to\pi^+\pi^-}$	$(1.311 \pm 0.06 \pm 0.01) \times 10^{-6}$	$(1.225 \pm 0.06 \pm 0.04) \times 10^{-6}$

Calculating bare $e^+e^- \to \pi^+\pi^-$ cross section

$$\sigma^{\text{bare}}_{\pi\pi}(s) = \sigma^0_{\pi\pi}(s) \times |1 - \Pi(s)|^2 \times (1 + \frac{\alpha}{\pi} a(s))$$

$$a(s) = \frac{1+\beta^2}{\beta}\left[4Li_2\left(\frac{1-\beta}{1+\beta}\right) + 2Li_2\left(-\frac{1-\beta}{1+\beta}\right) - \right.$$

$$\left. 3\ln\frac{2}{1+\beta}\ln\frac{1+\beta}{1-\beta} - 2\ln\beta\ln\frac{1+\beta}{1-\beta}\right] - 3\ln\frac{4}{1-\beta^2} - 4\ln\beta + \frac{1}{\beta^3}\left[\frac{5}{4}(1+\beta^2)^2 - 2\right]$$

$$\times \ln\frac{1+\beta}{1-\beta} + \frac{3}{2}\frac{1+\beta^2}{\beta^2}.$$

$$Li_2(x) = -\int_0^x dt \ln(1-t)/t, \quad \beta = \sqrt{1 - \frac{4m_\pi^2}{s}}$$

Contribution to a_μ

$$\mathbf{a_\mu(\pi\pi, s_{min} \leq \sqrt{s} \leq s_{max}) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{s_{min}}^{s_{max}} \frac{R(s)K(s)}{s^2} ds}$$

$K(s)$ is a known kernel (J. Phys. G **38**, 085003 2011) and

$$R(s) = \frac{\sigma_{\pi\pi}^{bare}}{\sigma(e^+e^- \to \mu^+\mu^-)}, \quad \sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$

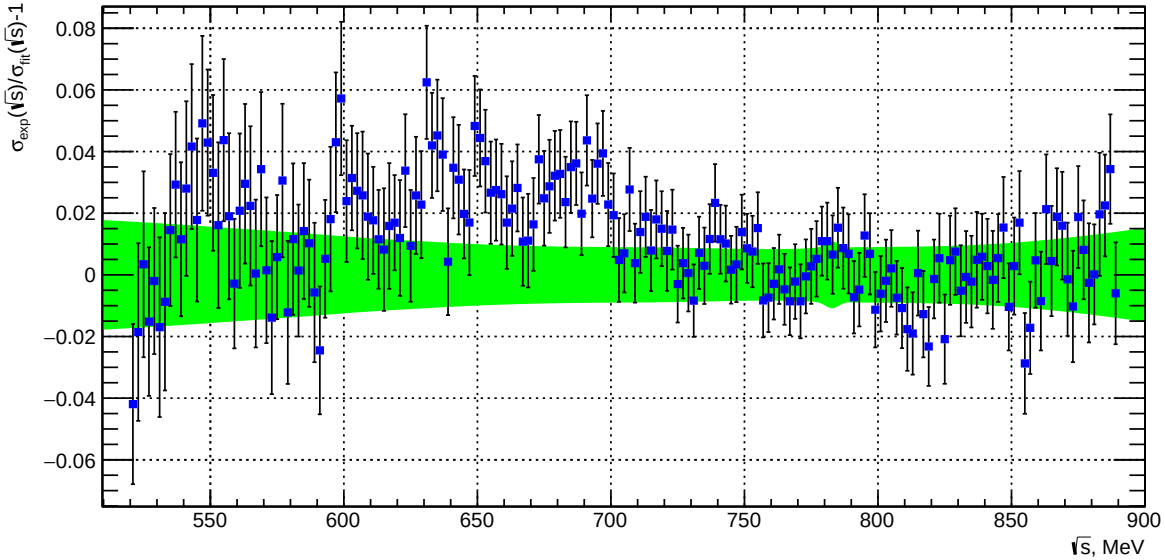
Trapezoid integration allows to compute $\mathbf{a_\mu}$ using measured cross sections

	SND VEPP-2000	SND VEPP-2M	BaBar
$a_\mu(\pi\pi) \times 10^{10}$	411.82 ± 1.02± 3.66	408.88±1.30±5.31	414.93±0.34±2.07

BaBar

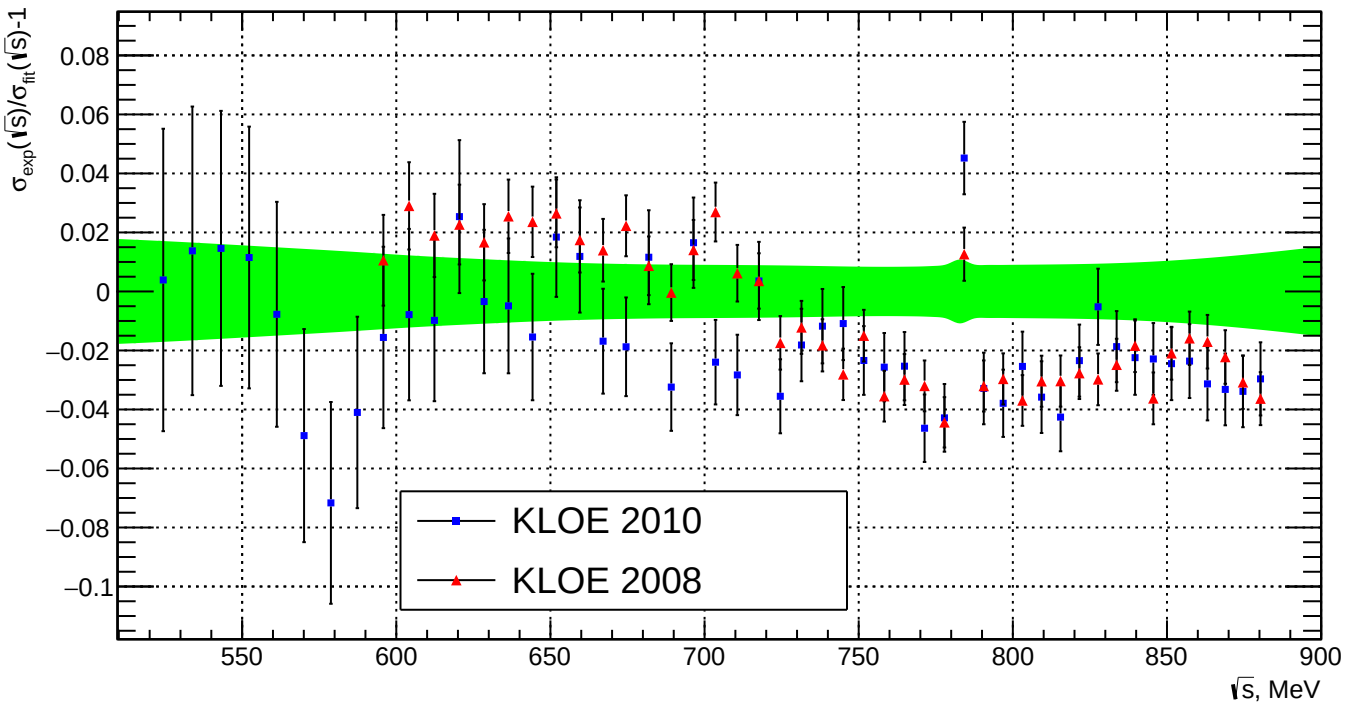
The relative difference between the $e^+e^- \to \pi^+\pi^-$ cross section, measured by BABAR and fit of the SND experimental data

- $\sigma_{sys} \oplus \sigma_{stat}$ errors are shown for the BABAR data
- The green bar depicts systematic and statistical errors of the SND fit, folded quadratically



Phys. Rev. 2012.Vol. 86D. 3,032013

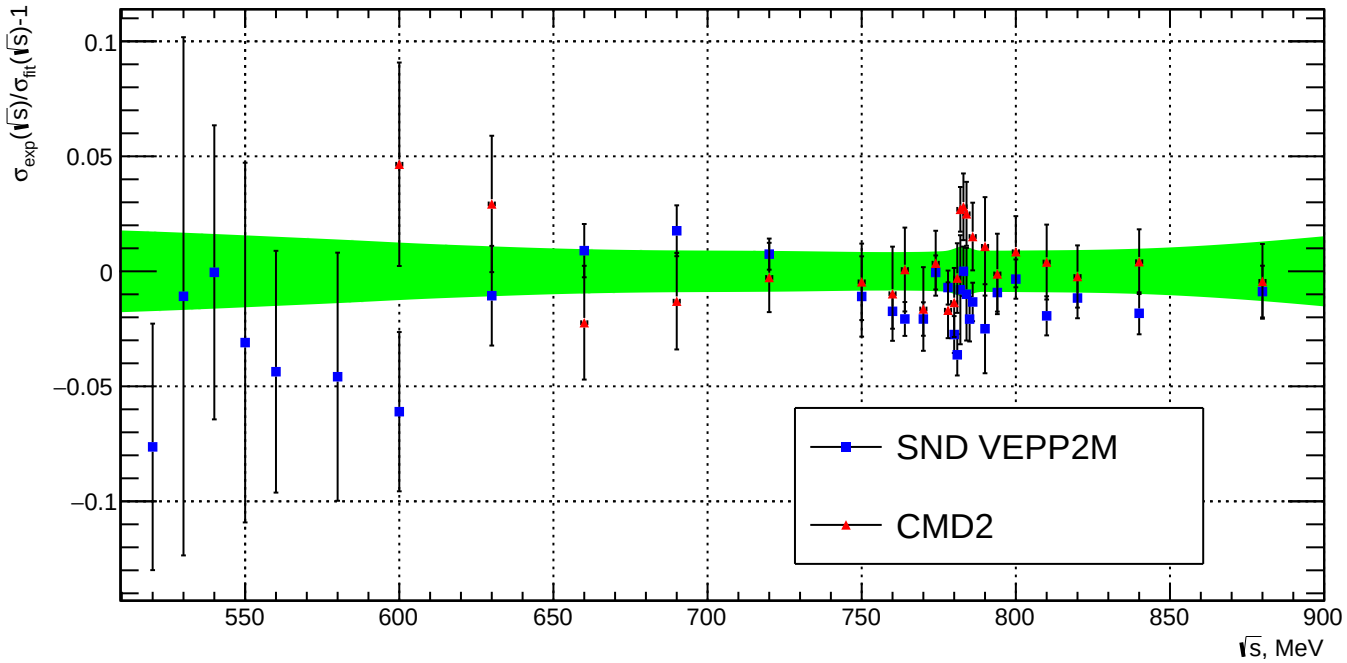
The relative difference between the $e^+e^- \to \pi^+\pi^-$ cross section, measured by KLOE and fit of the SND experimental data



Phys. Lett. 2009. Vol. 670B 4-5, P.285-291

Phys. Lett. 2011. Vol. 700B 2. P.102-110

The relative difference between the $e^+e^- \to \pi^+\pi^-$ cross section, measured in experiments at VEPP-2M and fit of the SND experimental data



JETP (2006) vol. 103, N3, pp 380-384.

Phys. Lett. 2007 Vol.648B 5. P.28-38

Summary

- The difference between the value of $a_\mu(\pi\pi, 525\text{MeV} \leq \sqrt{s} \leq 883\text{MeV})$ obtained from the SND data, and ones derived from the previous measurements $< 1\sigma$
- The parameters of the ρ and ω mesons in this analysis are consistent with ones obtained by SND in experiments at VEPP-2M
- Comparison with VEPP-2M results indicates no significant contradictions in the whole energy spectrum
- In the $0.6 \geq \sqrt{s} \leq 0.7$ GeV energy range there is a 3% discrepancy between the SND and BABAR data, but for the rest of the spectrum SND data is in agreement with the BABAR results
- There is 1–4 % deviation between KLOE and SND data for $\sqrt{s} \geq 0.7$ GeV