HVP contribution to a_μ

Status/Plan BMWc

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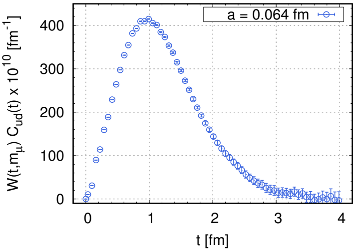
$a_\mu^{\text{LO-HVP}}$ from time-momentum current correlator

Bernecker et al. 11, BMWc 13, Feng et al 13, Lehner 14, ...

- Compute on $T \times L^3$ lattice

$$C_L(t) = \frac{a^3}{3} \sum_{i=1}^{3} \sum_{\vec{x}} \langle J_i(x) J_i(0) \rangle$$

w/ $J_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c + \cdots$



$(144 \times 96^3,\ a \sim 0.064\ \text{fm},\ M_\pi \sim 135\ \text{MeV})$

- Decompose ($C_L^{I=1} = \frac{9}{10} C_L^{ud}$)

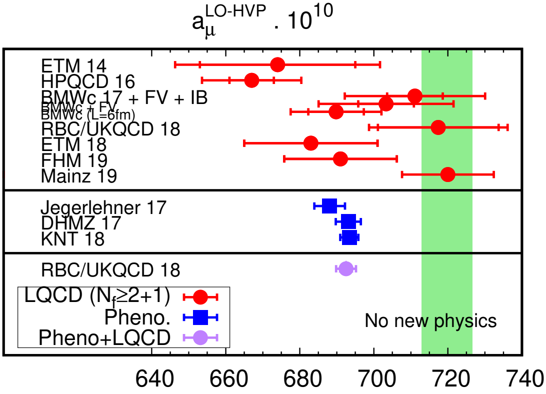
$$\begin{aligned} C_L(t) &= C_L^{ud}(t) + C_L^{s}(t) + C_L^{c}(t) + C_L^{\text{disc}}(t) \\ &= C_L^{I=1}(t) + C_L^{I=0}(t) \end{aligned}$$

- Obtain (BMWc 17)

$$a_{\mu,f}^{\text{LO-HVP}}(Q^2 \le Q^2_{\max}) = \lim_{a\to 0,\, L\to\infty,\, T\to\infty} \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{a}{m_\mu^2}\right) \sum_{t=0}^{T/2} W(tm_\mu, Q^2_{\max}/m_\mu^2)\ \mathrm{Re}C_L^f(t)$$

w/ $W(\tau, x_{\max})$ known kinematical function

Current situation: $a_\mu^{\text{LO-HVP}}$



- Some lattice results suggest new physics others not but all compatible with phenomenology
- Lattice errors $\gtrsim$ 2% vs phenomenology errors $\sim$ 0.4%
- Must reduce lattice error to $<$ 1% to have impact

BMWc @ end 2017: simulations and results

BMWc, Borsanyi' et al, PRL121 (2018) 022002 (Editors' Suggestion)

15 high-statistics simulations w/ N_f=2+1+1 flavors of 4-stout staggered quarks:

- Bracketing physical m_{ud}, m_s, m_c
- 6 a's: $0.134 \to 0.064\,\mathrm{fm}$
- $L = 6.1 \div 6.6\,\mathrm{fm}$, $T = 8.6 \div 11.3\,\mathrm{fm}$
- Conserved EM current
- Close to 9M / 39M conn./disc. measurements

β	a [fm]	$T \times L$	#conf-conn	#conf-disc
3.7000	0.134	64×48	1000	1000
3.7500	0.118	96×56	1500	1500
3.7753	0.111	84×56	1500	1500
3.8400	0.095	96×64	2500	1500
3.9200	0.078	128×80	3500	1000
4.0126	0.064	144×96	450	-

Contrib.	$a_\mu^{\text{LO-HVP}} \times 10^{10}$
$I = 1$	583(7)(7)(0)(0)(5)(13)
$I = 0$	121(3)(4)(0)(0)(1)
Total	711(8)(8)(0)(0)(6)(13)(5)

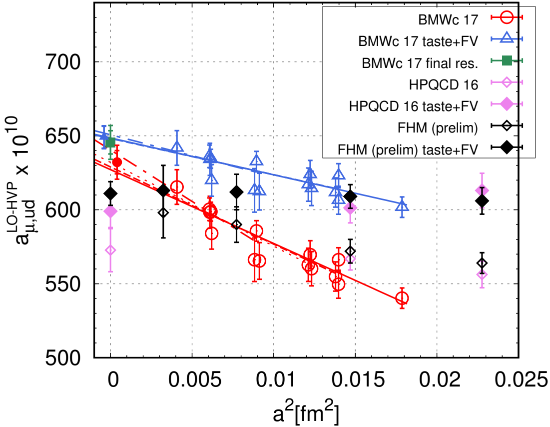
$$\sigma_{\text{FV}} > \sigma_{\text{stat}} \sim \sigma_{\text{ana}} > \sigma_{\text{a-syst}} \sim \sigma_{\text{IB}}$$

Error on total:

- Stat. = 1.1% (all from ud)
- $a \to 0$ syst. = 1.1%
- a syst. = 0.8%
- FV = 1.9%
- IB = 0.7%
- Total = 2.7%

Staggered continuum extrapolation of $a_{\mu,ud}^{\text{LO-HVP}}$

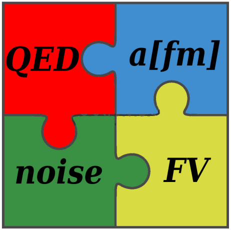
- Goldstone has more massive "taste" partners that dilute its contribution to $a_{\mu,ud}^{\text{LO-HVP}}$
- "Effective" pion mass larger at larger a, e.g. $M_\pi^{\text{RMS}} \simeq 310\,\text{MeV}$ for $a = 0.134\,\text{fm}$
- Effect dissappears in $a \to 0$ limit
- $a \to 0$ extrapolation includes $M_\pi^{\text{RMS}} \to M_\pi^{\text{PDG}}$ extrapolation and is quite pronounced



HPQCD 16 & FNAL/HPQCD/MILC prelim already include large t modeling of $\langle J_\mu(t) J_\nu(0) \rangle$

Path to HVP Nirvana

To reach sub-percent accuracy on lattice, must:



- correct FV effects reliably (preferably with LQCD)
- reduce statistical error on *ud* contribution significantly
- determine $m_u \neq m_d$ and EM corrections on lattice
- determine a very precisely: $\frac{\sigma_{a_\mu^{\text{LO-HVP}}}}{a_\mu^{\text{LO-HVP}}} \sim 2 \times \frac{\sigma_a}{a}$
- staggered: must improve continuum extrapolation of $a_{\mu,\,ud}^{\text{LO-HVP}}$

Phenomenology inspired FV corrections

Use (Meyer 11):

$$C^{I=1}_{L,\text{LO-}\chi\text{PT}}(t) = \frac{1}{3L^3}\sum_{\vec{p}_\text{free}}\left(\frac{\vec{p}_\text{free}}{E_p^\text{free}}\right)^2 e^{-2E_p^\text{free}t} \to C^{I=1}_{L,\text{LLGS}}(t) = \frac{1}{3L^3}\sum_i\sum_p |\langle 0|J_i|\pi^+(\vec{p})\pi^-(-\vec{p})\rangle|^2_L \; e^{-2E_p t}$$

with

- Lüscher to get $E_p = \sqrt{M_\pi^2 + p^2}$ in FV, where $p = |\vec{p}|$ is momentum carried by each of the two interacting π in FV
- Lellouch-Lüscher (LL) for interacting $|\langle 0|J_i|\pi^+(p)\pi^-(p)\rangle|_L$ in FV from free amplitude $\frac{p_i^\text{free}}{E_p^\text{free}}$
- 2-π, $\delta^{J=1}_{I=1}(p)$ from phenomenology

Then $C^{I=1}_{\infty,\text{LLGS}}(t) - C^{I=1}_{L,\text{LLGS}}(t)$ gives estimate of FV

$\to$ good for FV corrections: increase by $O(50\%)$ over LO χPT for $M_\pi \sim 135\,\text{MeV}$ and $L \sim 6\,\text{fm}$
(Della Morte et al 17, Shintani et al 19, Aubin et al 19 . . .)

Implementing simulations at different volumes to test model

Phenomenology inspired taste corrections

Here explore naive staggerization of phenomenological model

$$C^{I=1}_{L,\mathrm{LLGS}}(t) \to C^{I=1}_{L,\mathrm{SLLGS}}(t; a^2\Delta^{\mathrm{KS}}) = \frac{1}{3L^3}\sum_i \sum_p \sum_{j=0}^{4} w_j |\langle 0|J_i|\pi^+(\vec{p})\pi^-(-\vec{p})\rangle|^2_{j,L} \; e^{-2E_{p,j}t}$$

- Compare model to lattice data using a sliding window

$$a^{\mathrm{LO\text{-}HVP}}_{\mu,\,I=1}(t_{\mathrm{win}}, \Delta t, \Delta, Q^2 \le Q^2_{\mathrm{max}}) = \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{a}{m_\mu^2}\right) \sum_{t=0}^{T/2} [\Theta(t; t_{\mathrm{win}}, \Delta) - \Theta(t; t_{\mathrm{win}} + \Delta t, \Delta)]\, W(tm_\mu, Q^2_{\mathrm{max}}/m_\mu^2)\; \mathrm{Re}\, C^{I=1}_L(t)$$

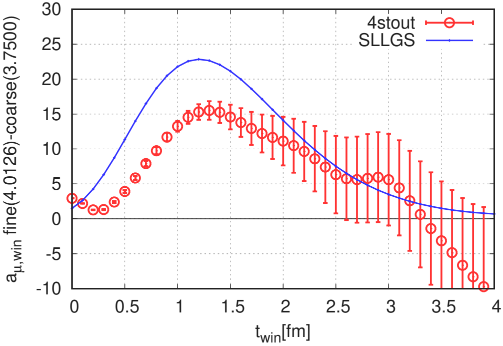
w/ $\Theta(t, t_0, \Delta) = [1 + \tanh[(t - t_0)]/\Delta]/2$ as in RBC/UKQCD 18

- Take $\Delta t = 0.5\,\mathrm{fm}$, $\Delta = 0.15\,\mathrm{fm}$ and slide window in steps of $0.1\,\mathrm{fm}$
- Implement using Gounaris-Sakurai (GS) model (Francis et al 13)

Sliding window: lattice vs SLLGS - PRELIMINARY

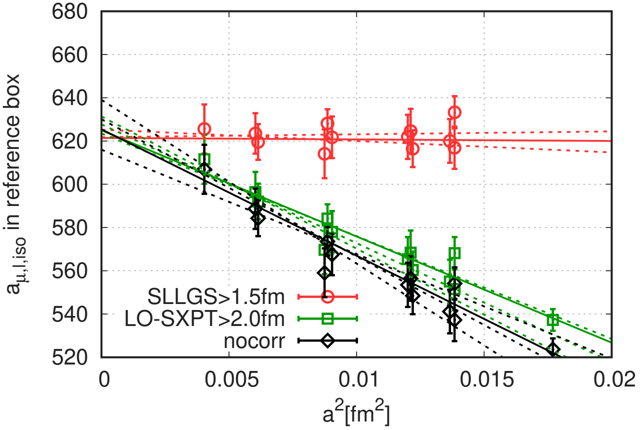
$$\Delta^{\text{lat}}_{\text{taste}}((t_{\text{win}}) = a^{\text{LO-HVP}}_{\mu,\,I=1,\text{lat}}(t_{\text{win}}, L, a_{\text{fine}}) - a^{\text{LO-HVP}}_{\mu,\,I=1,\text{lat}}(t_{\text{win}}, L, a_{\text{coarse}})$$

$$\Delta^{\text{SLLGS}}_{\text{taste}}(t_{\text{win}}) = a^{\text{LO-HVP}}_{\mu,\,I=1,\text{SLLGS}}(t_{\text{win}}, L, (a^2\Delta^{\text{KS}})_{\text{fine}}) - a^{\text{LO-HVP}}_{\mu,\,I=1,\text{SLLGS}}(t_{\text{win}}, L, (a^2\Delta^{\text{KS}})_{\text{coarse}})$$



- SLLGS describes taste-breaking corrections well for $t \gtrsim 1.5\,\mathrm{fm}$
- Correct taste breaking in simulations with SLLGS using either $t \geq 1.5\,\mathrm{fm}$ or $t \geq 2.0\,\mathrm{fm}$
- Use spread in continuum systematic error

Continuum extrapolations of $a_{\mu,ud}^{\text{LO-HVP}}$ - PRELIMINARY

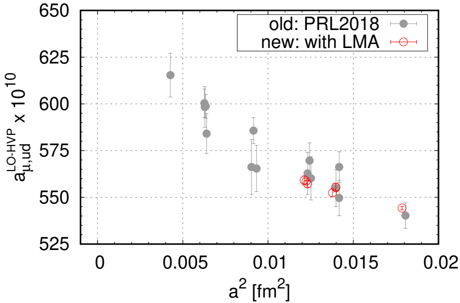


error [%] / correction	none	LO-SχPT	SLLGS
$\delta^{\text{stat}}(a_{\mu,ud}^{\text{LO-HVP}})$	1.5	1.5	1.6
$\delta^{a\text{-extrap}}(a_{\mu,ud}^{\text{LO-HVP}})$	2.2	1.0	0.6

Noise reduction - PRELIMINARY

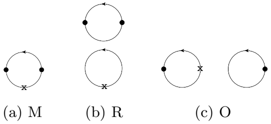
- Rigorous upper and low bounds on hardronic current correlator (BMWc, PRD96 (2017))
- W/ staggered, partial, reconstruction reconstruction of $\pi\pi$ states and improved bounds (Lehner, g-2 @ KEK 18; Géradin, PRD100 (2019); Meyer Lat19) not realistic

  $\rightarrow \sim 40$ staggered $\pi\pi$ states below the ρ on finest lattice for $L = 6\,\mathrm{fm}$ (vs 3 w/out taste breaking) !
- Have begun implementing low-mode-averaging

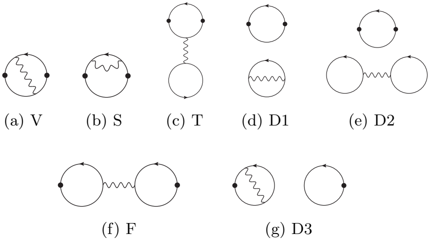


Very challenging and expensive part has yet to be done

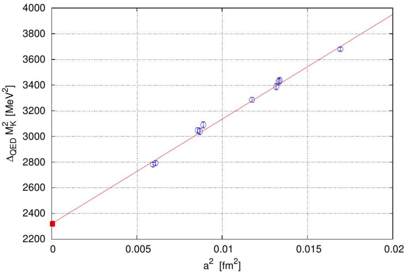
$m_u \neq m_d$ and QED corrections - PRELIMINARY



(RBC/UKQCD '18)



- Must implement for determination of
  - quantities that determine lattice spacing and physical point
  - hadronic current correlator
- Have implemented M, V & S
- QED corrections to $\Delta M_K^2 = M_{K^+}^2 - M_{K^0}^2 \rightarrow$



Agrees with earlier Wilson fermion result (BMW, Science (2015))

Conclusion

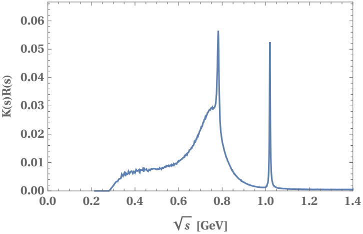
- Also working on determining sub-percent level scale setting with M_Ω (J. Guenther @ Lattice 2019)
- Huge amount of work to do
- Hope to have sub-percent results for $a_\mu^{\text{LO-HVP}}$ for publication of final FNAL experimental results ca. 2023
- 4 postdoctoral positions in lattice QCD open at CPT Marseille with starting date as early as January 1, 2020 (information: antoine.gerardin@cpt.univ-mrs.fr or laurent.lellouch@univ-amu.fr)

BACKUP

Leading systematics with staggered fermions

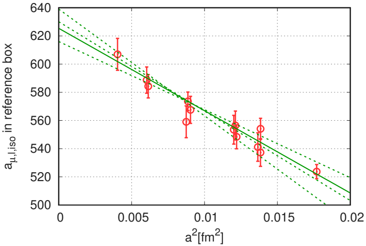
- $a_\mu^{\text{LO-HVP}}$ has strong dependence on 2-π states

$$a_\mu^{\text{LO-HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_{s_{th}}^{\infty} ds\, K(s) R_{e^+e^- \to \text{hadrons}}(s)$$



⇒ FV effects may be large (Golterman et al. 16), i.e. few % for $L \sim 6\,\text{fm}$

⇒ Taste-breaking effects are significant: effective $M_\pi \sim M_\pi^{\text{RMS}} > M_\pi^{\text{GB}}$



Continuum extrapolation is also a chiral extrapolation

⇒ large a^2-dependence ($\sim 20\%$ for $a \sim 0.131\,\text{fm}$ (BMWc 17))

⇒ possible non-linearities through $\delta_L a_{\mu,\,ud}^{\text{LO-HVP}} \sim \sum_{j=0}^{4} w_j \exp\left[-L\sqrt{(M_\pi^{\text{GB}})^2 + j \times a^2 \Delta^{\text{KS}}}\right]$

⇒ must be controlled to get $\delta_{\text{tot}} a_\mu^{\text{LO-HVP}} < 1\%$

Soln 1: LO χPT for FV effects

- FV effects are long-distance effects, determined by lightest states contributing to process
- Here $I = J = 1$, 2-π states
- Determine in χPT, to LO (Aubin et al 15), i.e.

$$C^{I=1}_{L,\text{LO-}\chi\text{PT}}(t) = \frac{1}{3L^3}\sum_{\vec{p}_{\text{free}}}\left(\frac{\vec{p}_{\text{free}}}{E_p^{\text{free}}}\right)^2 e^{-2E_p^{\text{free}}t}$$

with $E_p^{\text{free}} = \sqrt{M_\pi^2 + \vec{p}^{\,2}_{\text{free}}}$

- Then $C^{I=1}_{\infty,\text{LO-}\chi\text{PT}}(t) - C^{I=1}_{L,\text{LO-}\chi\text{PT}}(t)$ can be used to estimate FV effects
- Find, for $M_\pi \sim 135\,\text{MeV}$ and $L \sim 6\,\text{fm}$ (BMWc 17),

$$\Delta_{\text{FV}} a^{\text{LO-HVP}}_{\mu,\,I=1} \sim 2.3\% \times a^{\text{LO-HVP}}_{\mu,\,I=1}$$

$\rightarrow$ probably $O(50\%)$ too small (Della Morte et al 17, Shintani et al 19, Aubin et al 19 ...)

- Can do better

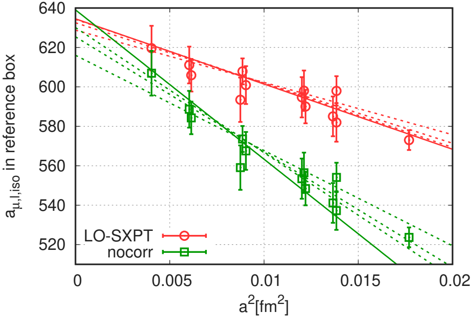
Soln 1: LO SχPT for taste effects

- Taste-breaking effects also mostly come from low-lying 2-π states
- Determine in SχPT, to LO (Aubin et al 15, HPQCD 17), i.e.

$$C^{I=1}_{L,\text{LO-S}\chi\text{PT}}(t, a^2\Delta^{KS}) = \frac{1}{3L^3}\sum_{\vec{p}_{\text{free}}}\sum_{j=0}^{4} w_j \left(\frac{\vec{p}_{\text{free}}}{E^{\text{free}}_{p,j}}\right)^2 e^{-2E^{\text{free}}_{p,j}t}$$

with $E^{\text{free}}_{p,j} = \sqrt{M^2_{\pi,j} + \vec{p}^2_{\text{free}}}$

- Then $C^{I=1}_{L,\text{LO-S}\chi\text{PT}}(t,0) - C^{I=1}_{L,\text{LO-S}\chi\text{PT}}(t, a^2\Delta^{KS})$ can be used to estimate taste effects



- Helps but is it possible to do better?

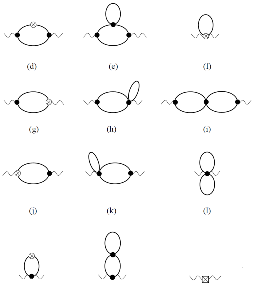
Soln 2: NLO SχPT

- At LO, the two π are free

  $\Rightarrow$ omits strong $\rho-\pi\pi$ coupling

  $\Rightarrow$ compute at NLO (Bijnens et al 99, Aubin et al 19)

- NLO includes LO 2-π rescattering and slope of $F_\pi(Q^2)$



- However NLO only obtained in continuum (Aubin et al 19)

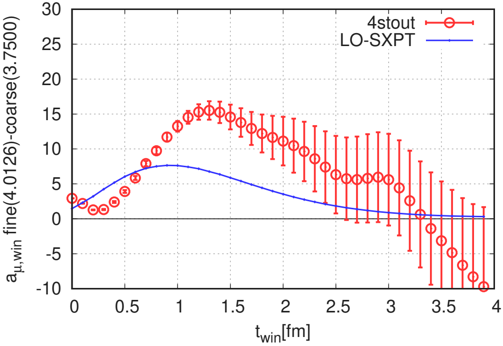
  $\Rightarrow$ helps FV corrections: increase by $O(50\%)$ for $M_\pi \sim 135\,\mathrm{MeV}$ and $L \sim 6\,\mathrm{fm}$

  $\Rightarrow$ does not improve taste corrections

Sliding window: lattice vs LO-SχPT

$$\Delta_{\text{taste}}^{\text{lat}}(t_{\text{win}}) = a_{\mu,\,I=1,\text{lat}}^{\text{LO-HVP}}(t_{\text{win}}, L, a_{\text{fine}}) - a_{\mu,\,I=1,\text{lat}}^{\text{LO-HVP}}(t_{\text{win}}, L, a_{\text{coarse}})$$

$$\Delta_{\text{taste}}^{\text{LO-S}\chi\text{PT}}(t_{\text{win}}) = a_{\mu,\,I=1,\text{LO-S}\chi\text{PT}}^{\text{LO-HVP}}(t_{\text{win}}, L, (a^2\Delta^{\text{KS}})_{\text{fine}}) - a_{\mu,\,I=1,\text{LO-S}\chi\text{PT}}^{\text{LO-HVP}}(t_{\text{win}}, L, (a^2\Delta^{\text{KS}})_{\text{coarse}})$$



- LO SXPT describes taste-breaking corrections well for $t \gtrsim 2.0\,\text{fm}$
- Correct taste breaking in simulations with LO SXPT using either $t \geq 2.0\,\text{fm}$ or $t \geq 2.5\,\text{fm}$
- Use spread in continuum systematic error