Dispersive sum rules for *g*—*2* and HLbL

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with
Franziska Hagelstein, PRL (2018); PoS (2019)
Volodymyr Biloshytskyi, MSc Thesis (2019)
Marc Vanderhaeghen, in prep.

@ g-2 Theory Initiative
INT UW, Seattle, USA
Sep 9—13, 2019

What is a dispersive "data-driven" approach?

- Existence of a general (dispersion) relation of the following type

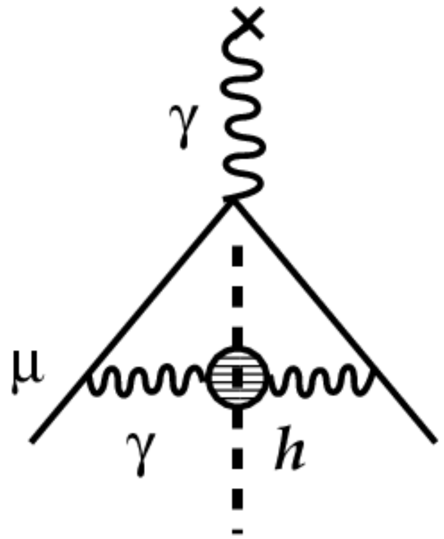
LHS
quantity of interest,
e.g., muon *g-2*

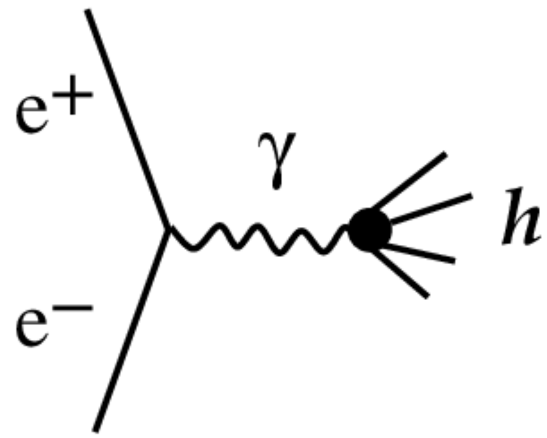
RHS
integral of an experimental observable
over energy, e.g., a cross section

$$a_\mu = \int_0^\infty ds\, \mathcal{K}(s)\, \sigma(s)$$

- Empirical knowledge of the experimental observable over the relevant energy range

HVP formula





Reviews:

F. Jegerlehner, Springer Tracts Mod. Phys. 274 (2017).

M. Davier, Nucl. Part. Phys. Proc. 287-288, 70 (2017).

- from causality/analyticity and field renormalization, one had a subtracted DR:

$$\Pi(q^2) = \frac{q^2}{\pi}\int_{s_0}^{\infty} \frac{ds}{s} \frac{\mathrm{Im}\,\Pi(s)}{s-q^2}$$

- substituted in the HVP diagram:

$$a^{\mathrm{HVP}} = \frac{\alpha}{\pi^2}\int_{s_0}^{\infty} \frac{ds}{s} K(s)\, \mathrm{Im}\,\Pi^{\mathrm{had}}(s)$$

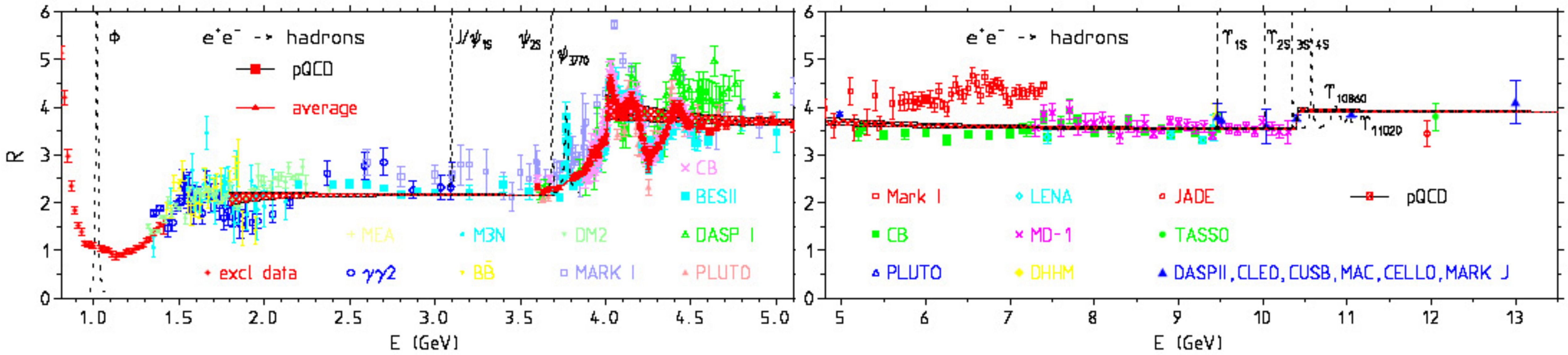
- unitarity:

$$\mathrm{Im}\,\Pi^{\mathrm{had}} = \frac{\alpha}{3} R(s) + O(\alpha^2), \qquad R(s) \equiv \frac{\sigma(e^+e^- \to \mathrm{had})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Data-driven evaluation at LO

- dispersion relation for the hadronic contribution, to leading order in the fine-structure constant:

$$a_\mu^{\text{had}} = \frac{\alpha^2}{3\pi^2}\int_{s_0}^{\infty}\frac{ds}{s}\,K(s)\,R(s) \;+\; O(\alpha^3)$$



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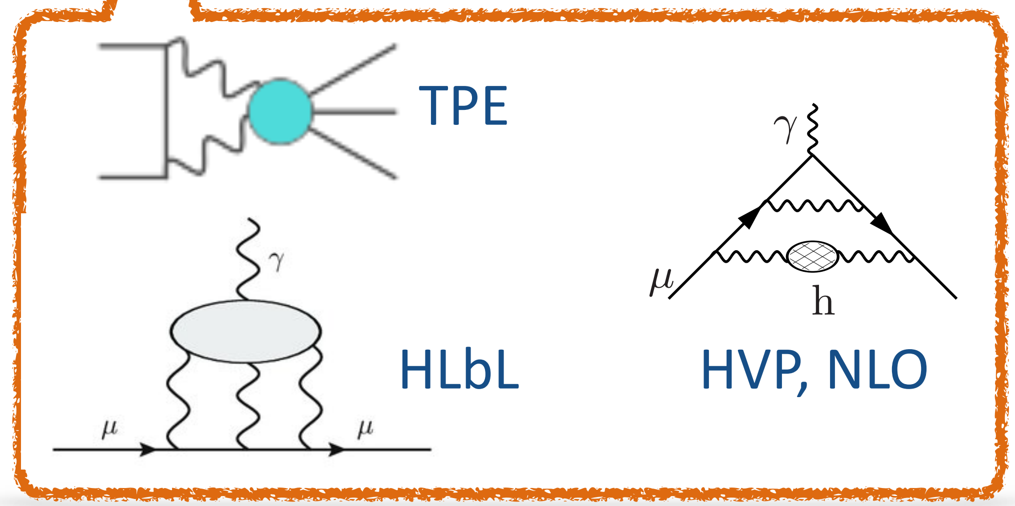
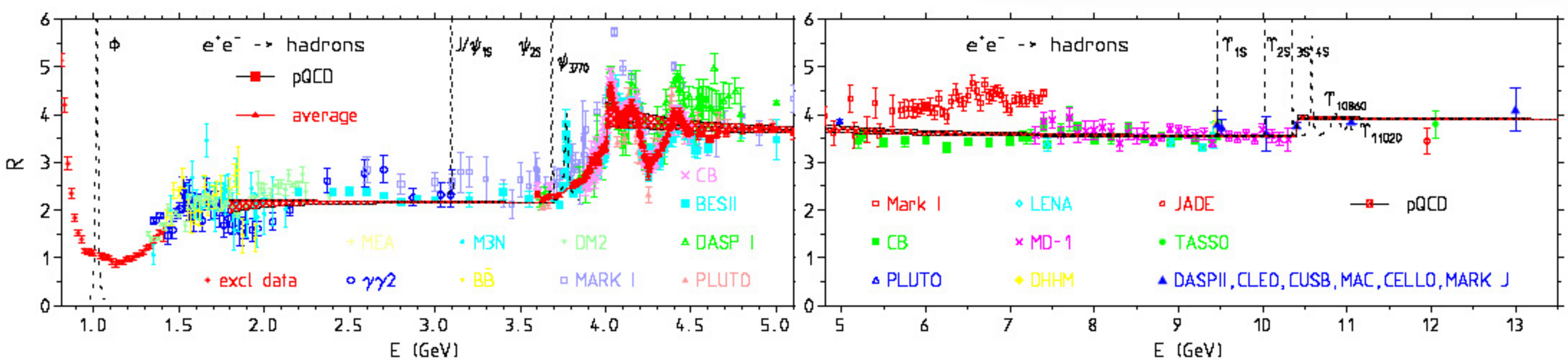
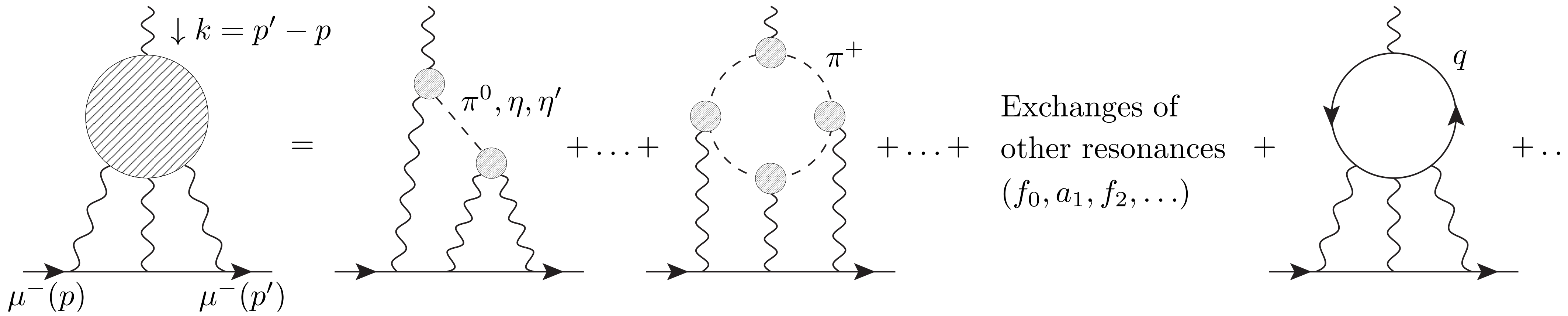




State-of-art analytic HLbL evaluation



WP (prelim.)

Contribution	PdRV(09) [5]	N/JN(09) [6, 124]	J(17) [37]	Our estimate eq. (8.1)
π^0, η, η'-poles	114 ± 13	99 ± 16	95.45 ± 12.40	93.8 ± 4.0
π, K-loops/boxes	-19 ± 19	-19 ± 13	-20 ± 5	-16.4 ± 0.2
S-wave $\pi\pi$ rescattering	–	–	–	-8 ± 1
scalars	-7 ± 7	-7 ± 2	-5.98 ± 1.20	$\Big\}\ -2 \pm 3$
tensors	–	–	1.1 ± 0.1	
axial vectors	15 ± 10	22 ± 5	7.55 ± 2.71	8 ± 8
quark-loops / short-distance	2.3	21 ± 3	22.3 ± 5.0	10 ± 10
total	105 ± 26	116 ± 39	100.4 ± 28.2	85 ± 17

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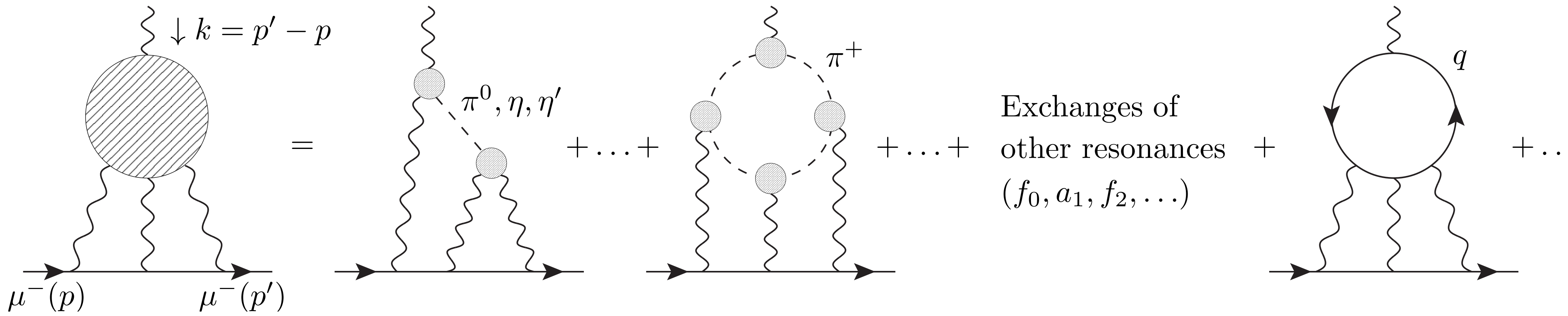

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$$a_\mu \neq \int_0^\infty ds\, \mathcal{K}(s)\, \sigma(s)$$

Exact dispersive formulas

$$a_\mu^2 = -\frac{m_\mu^2}{\alpha\pi^2}\int_{\nu_0}^{\infty}\frac{d\nu}{\nu}\sigma_{TT}(\nu)$$

GDH sum rule

M. Gell-Mann, M. L. Goldberger, and W. E. Thirring, Phys. Rev. 95, 1612(1954).

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J. S. Schwinger, Proc. Nat. Acad. Sci. 72, 1(1975); ibid. 72, 1559 (1975).

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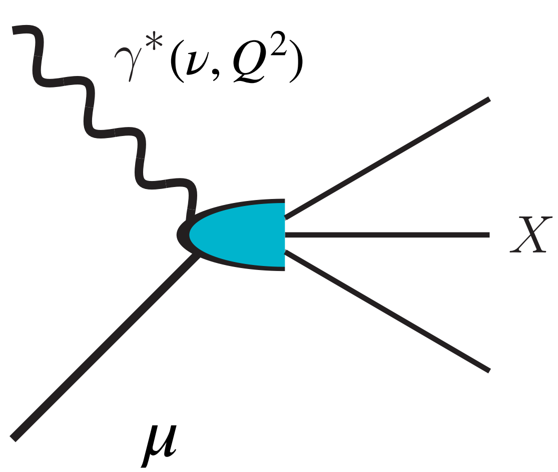
- σ_{TT} and σ_{LT} inclusive cross sections of polarized photo-absorption on polarized muon ($\gamma_\uparrow\mu_\uparrow \to X$):



$= \gamma\mu, \gamma\gamma\mu, \pi^0\mu, \pi^+\pi^-\mu, \ldots$

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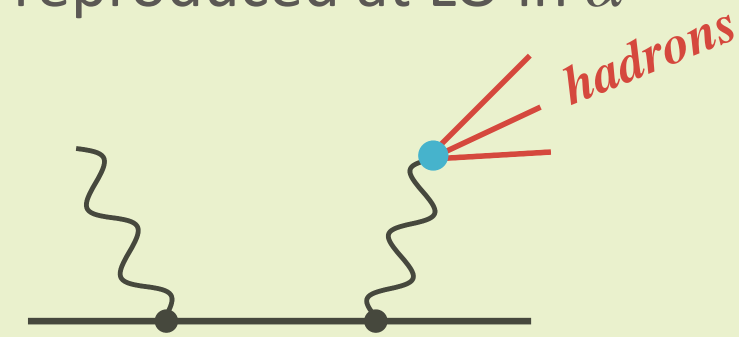
HVP dispersive formula reproduced at LO in α



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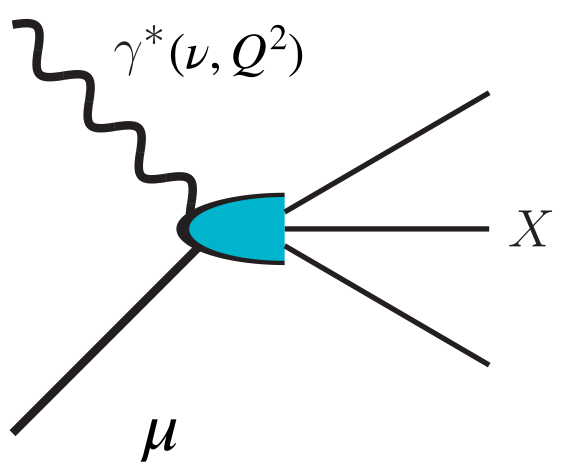

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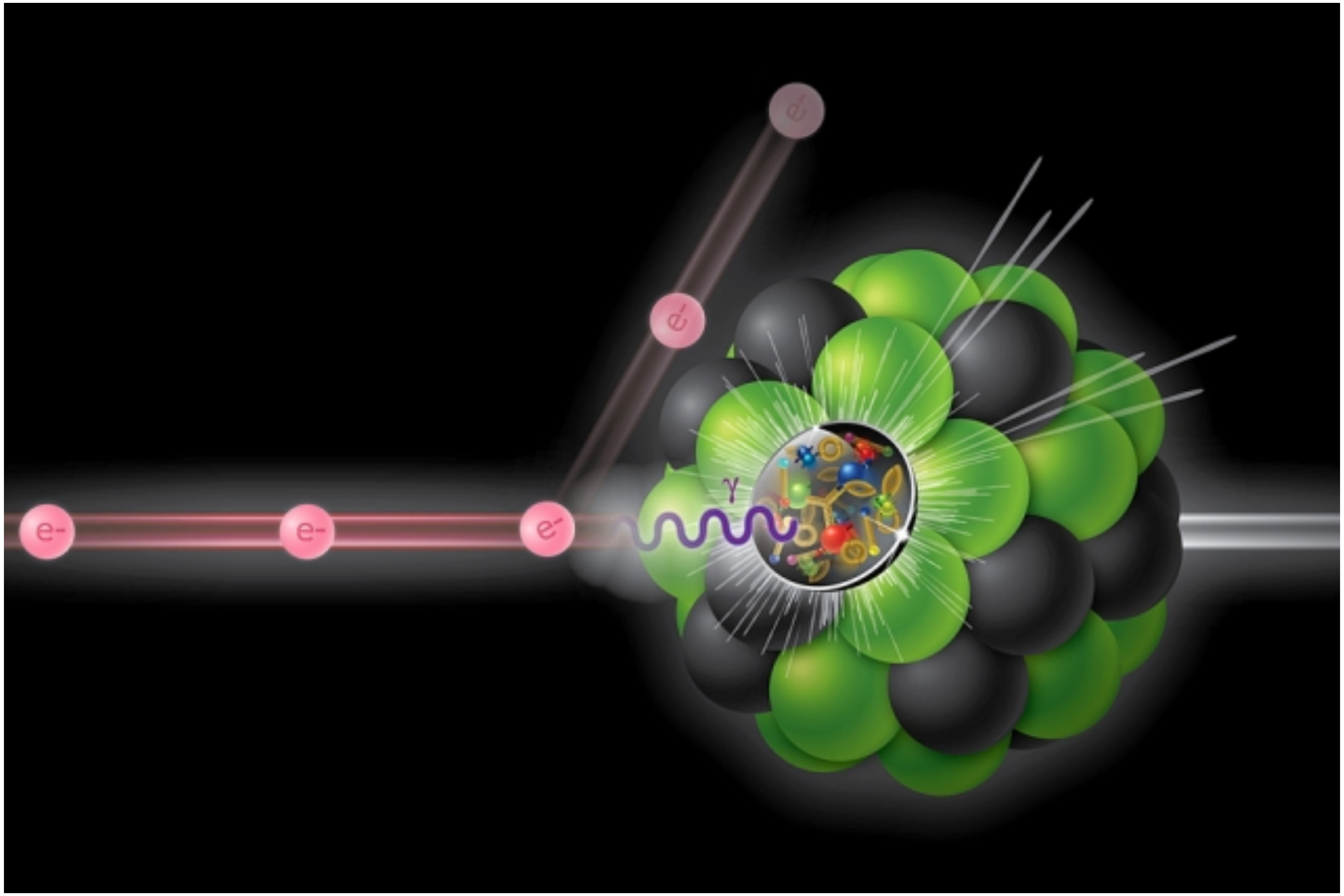
DIS formalism



$$a_\mu = \lim_{Q^2 \to 0} \frac{8m_\mu^2}{Q^2} \int_0^{x_0} dx \left[g_1^{(\mu)}(x, Q^2) + g_2^{(\mu)}(x, Q^2) \right]$$

Schwinger sum rule

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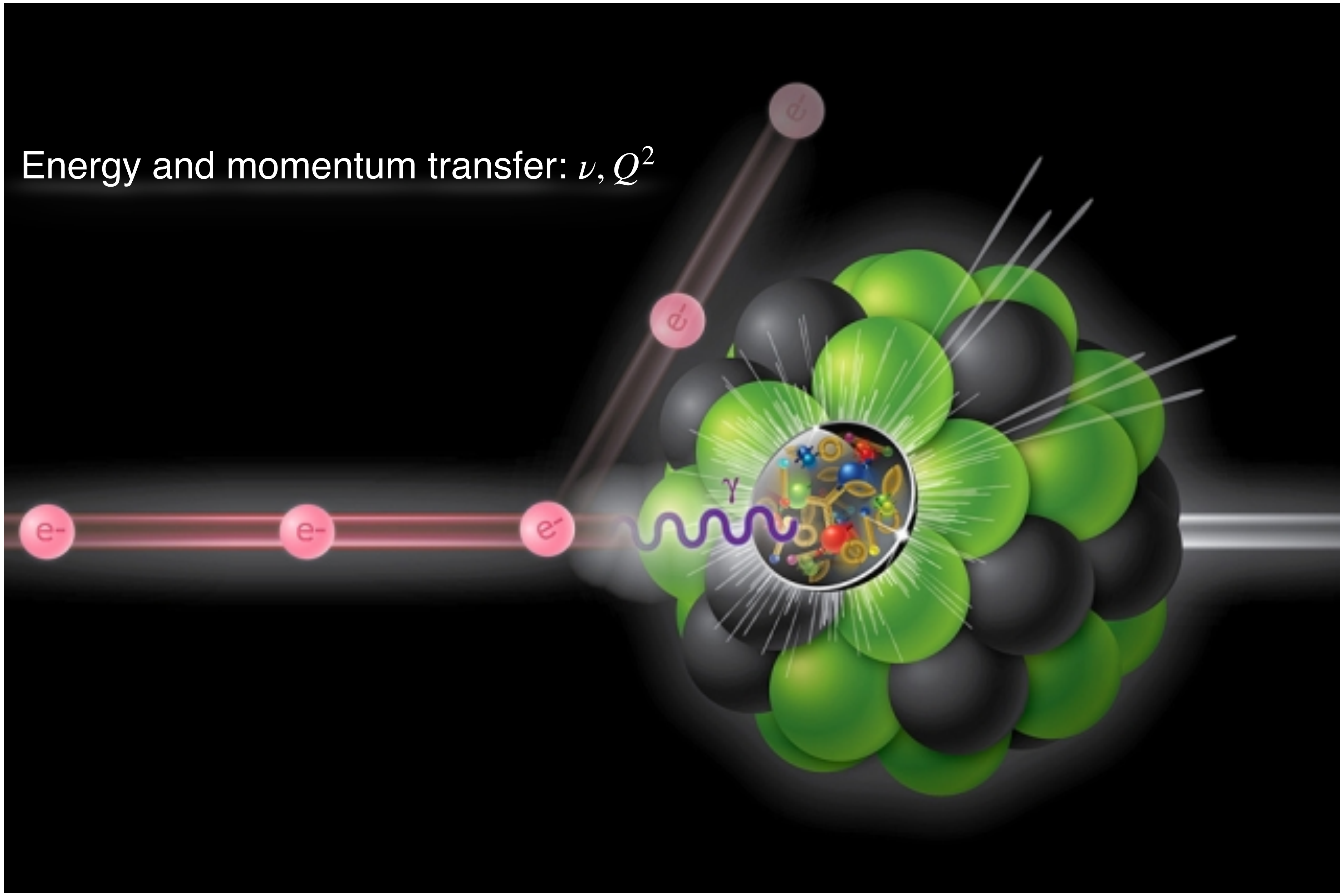

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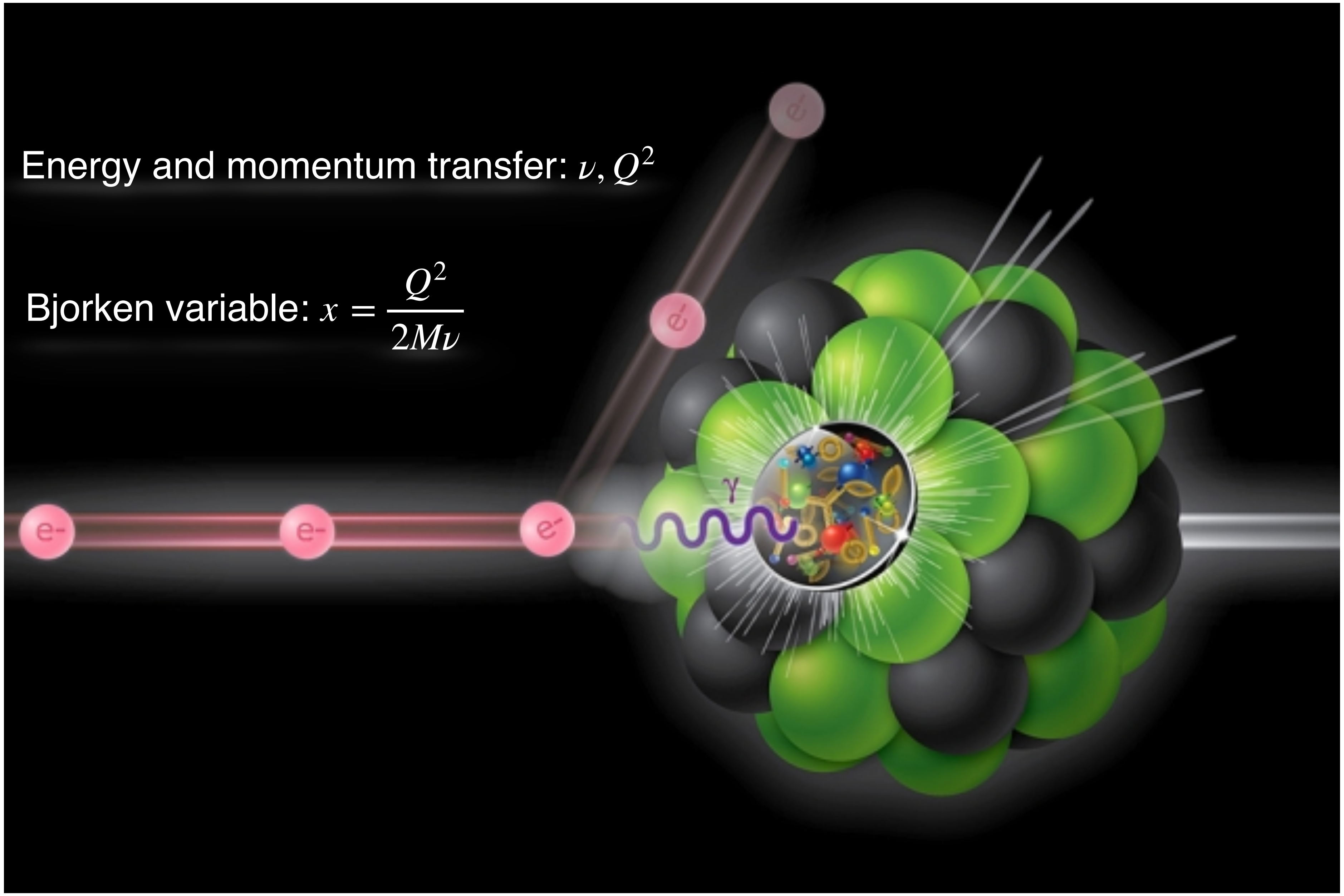

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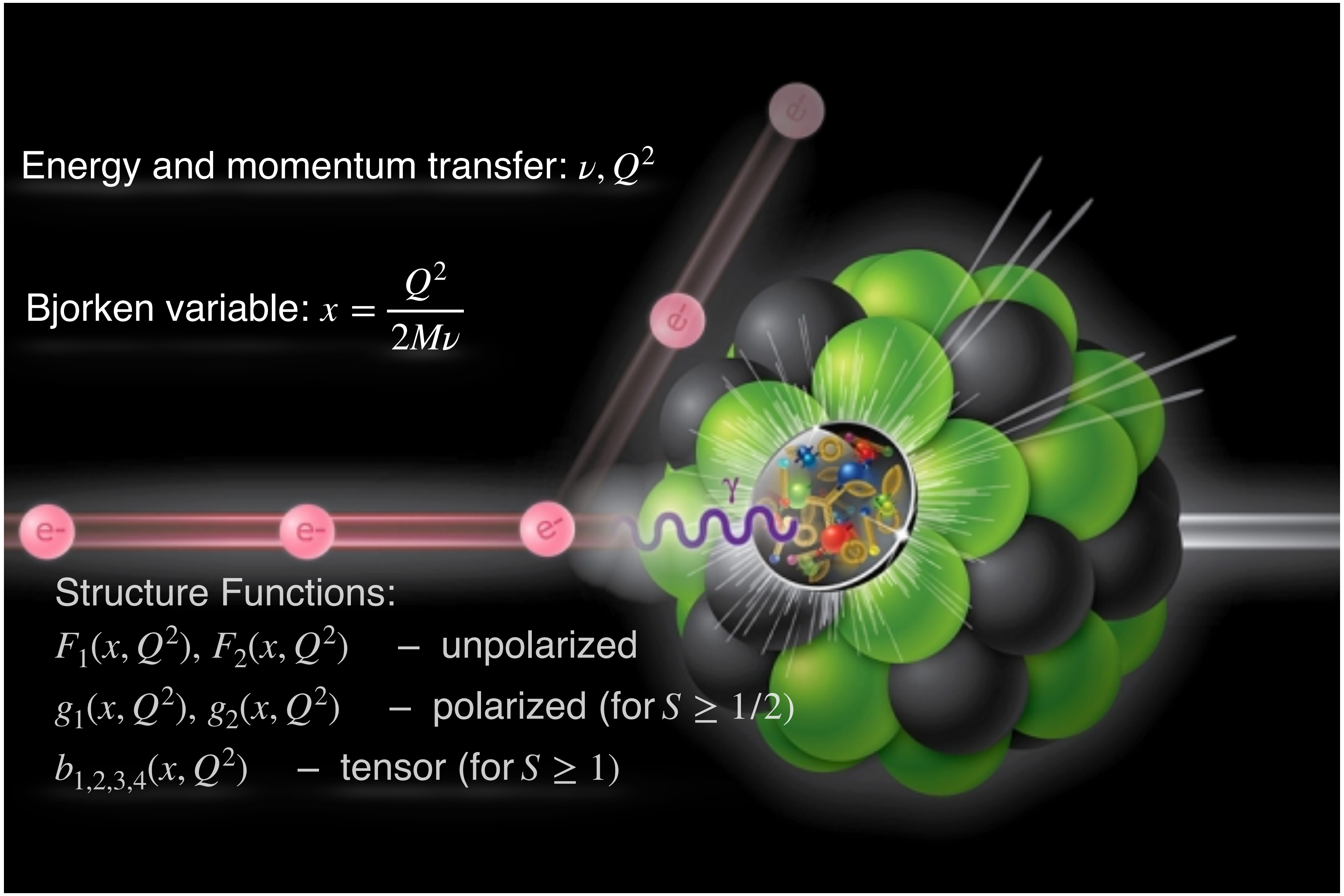

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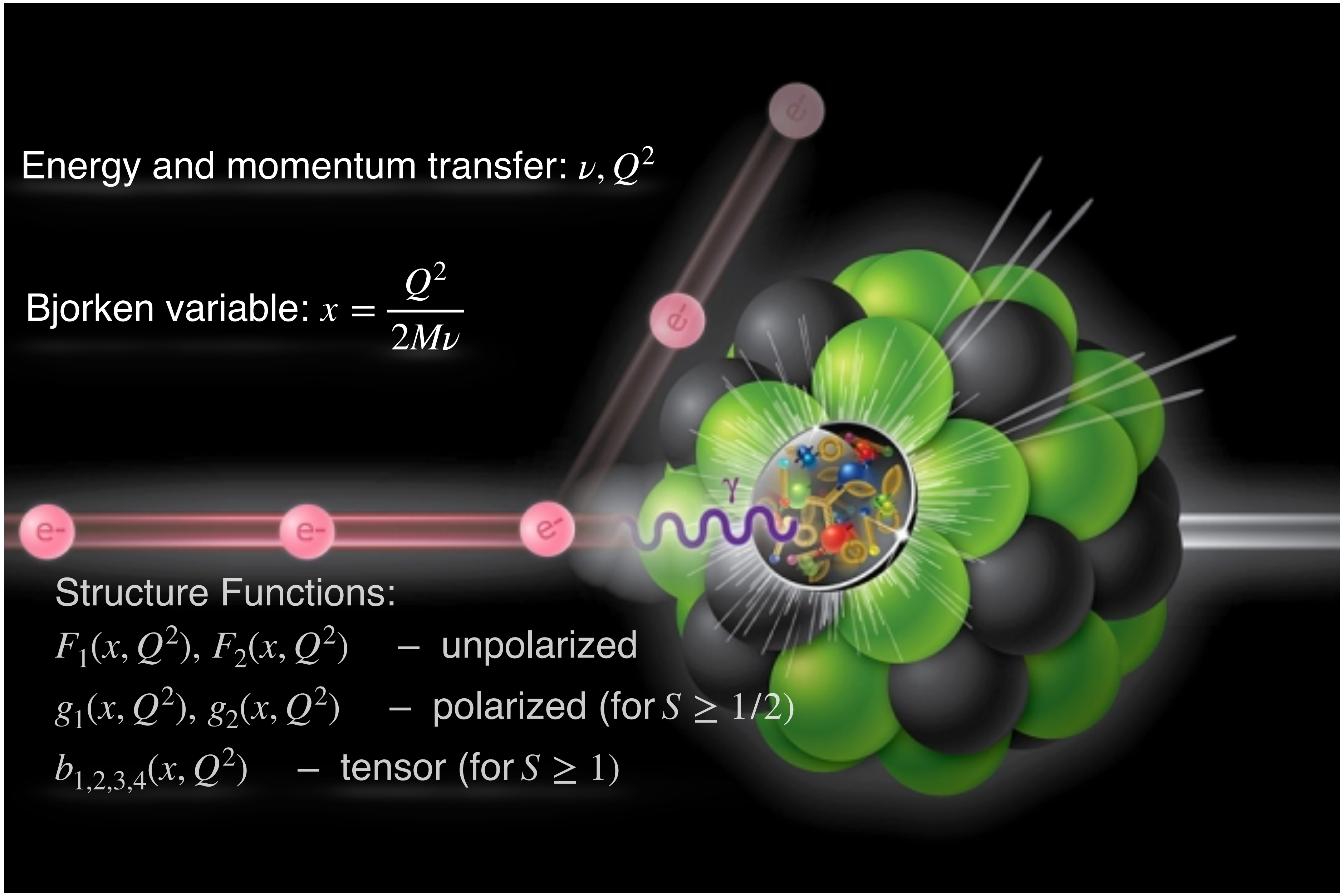

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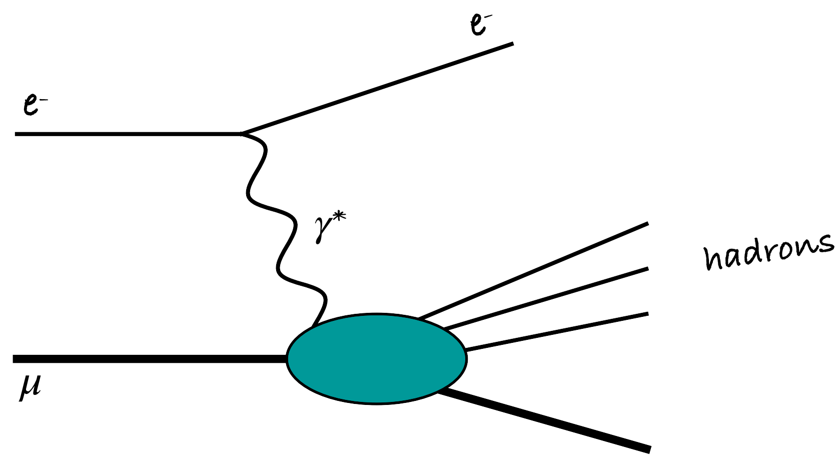
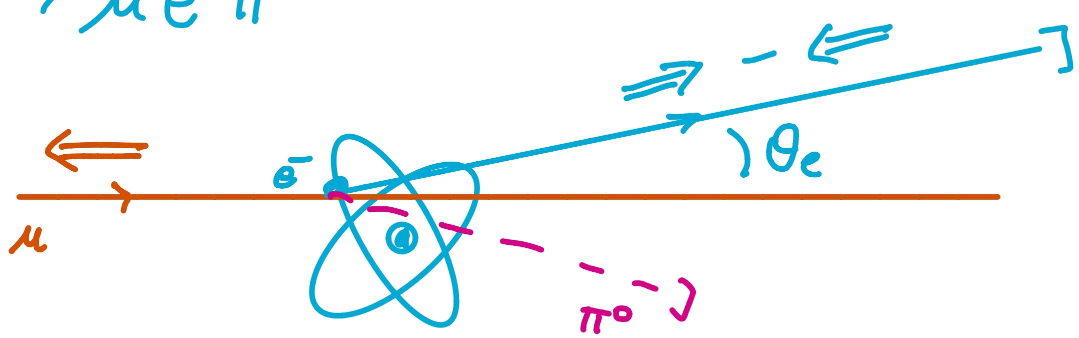
Measuring the muon structure functions



- MUonE setup (with recoil polarization!)

$\mu e \to \mu e \pi^0$



$E_\mu = 150,\ 200$ GeV

$E_e' \simeq 1$ GeV

$\theta_e \simeq 10$ mrad

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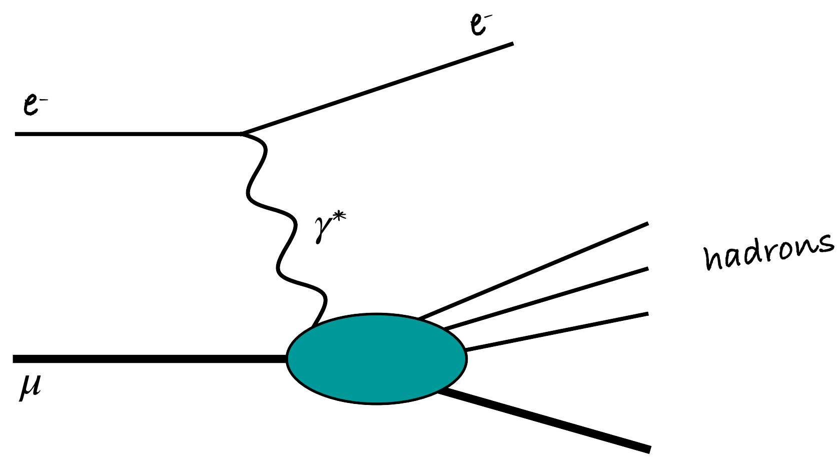

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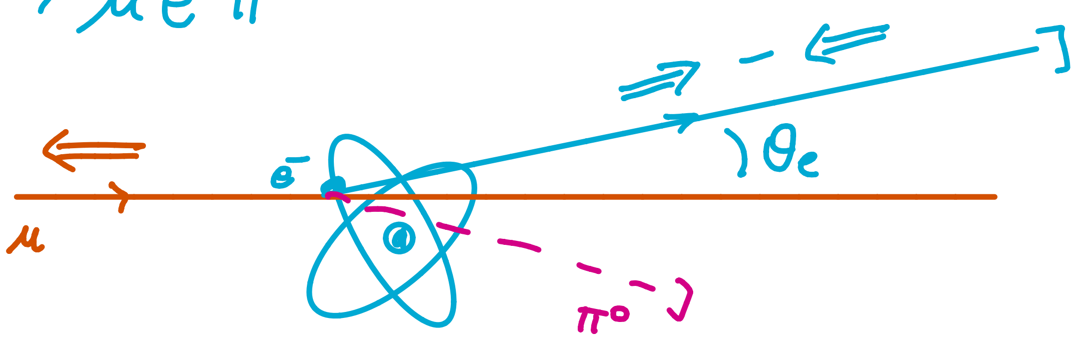

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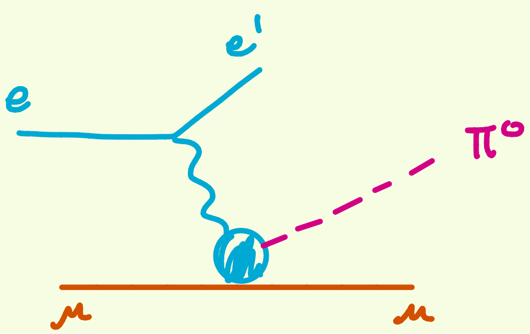
- inelastic kinematics



$Q^2 \simeq 2 m_e E_e' \simeq 10^{-3}\ \mathrm{GeV}^2$

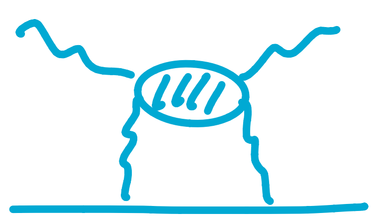
$\nu \simeq \frac{m_e E_\mu}{m_\mu}\left(1 - 2\frac{E_e'}{m_e}\sin^2\frac{\theta}{2}\right) = (\nu_{\pi^0},\ 1\ \mathrm{GeV})$

$\nu_{\pi^0} = \frac{m_{\pi^0}}{m_\mu}\left(\frac{1}{2}m_{\pi^0} + m_\mu\right) \simeq 230\ \mathrm{MeV}$

Hadronic contribution to electromagnetic channels

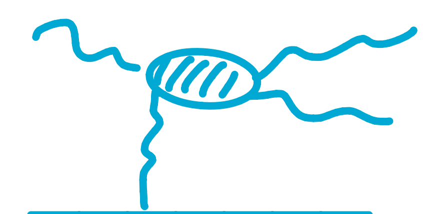
$$a_\mu^{\text{had}} = \frac{m_\mu^2}{\alpha\pi^2}\int_{\nu_0}^{\infty} d\nu \left[\frac{1}{Q}\sigma_{LT}^{\gamma\mu\to\mu+\text{hadrons}}(\nu,Q^2) + \frac{1}{Q}\sigma_{LT}^{\gamma\mu\to\mu\gamma,\,\mu\gamma\gamma}(\nu,Q^2)\right]_{Q^2=0} + O(\alpha^4)$$

$\gamma\mu \to \gamma\mu$



very small for π^0:
$(5 \pm 3)\times 10^{-11}$

$\gamma\mu \to \gamma\gamma\mu$



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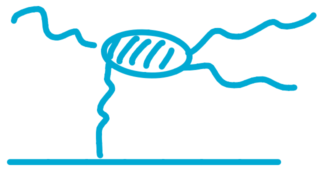
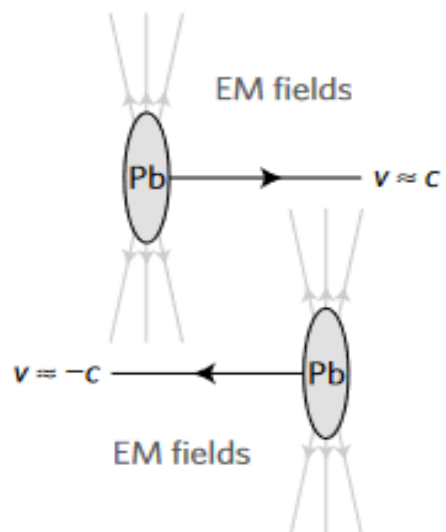


Quasireal LbL process at the LHC

ultra-peripheral collisions of two lead ions



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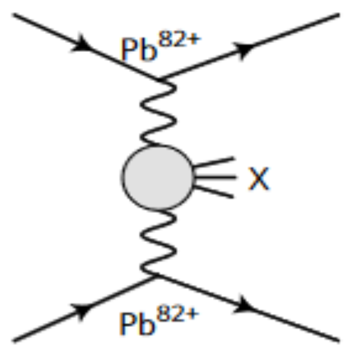
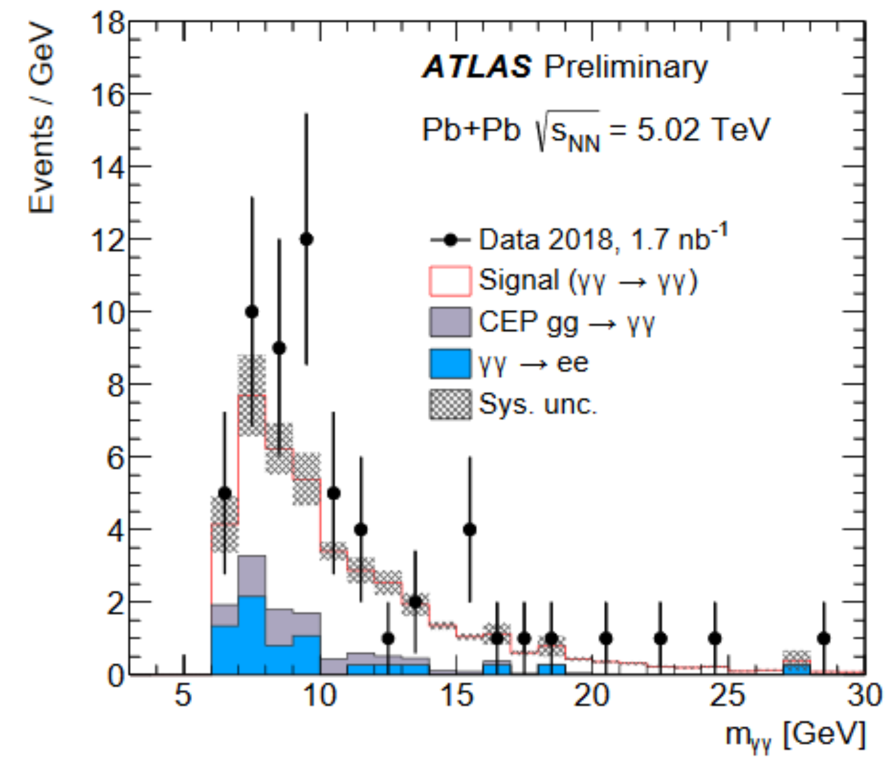


$\sqrt{s_{NN}} = 5.02\ \text{TeV}$



ATLAS Collaboration,
Evidence for light-by-light scattering in heavy-ion collisions with the ATLAS detector at the LHC,
Nature Physics 13, 852–858 (2017)

Significance: 4.4σ (3.8σ)
$\sigma_{\text{fid}} = 70 \pm 24(\text{stat.}) \pm 17(\text{syst.})$ nb
Predicted: 45 ± 9 nb, 49 ± 10 nb

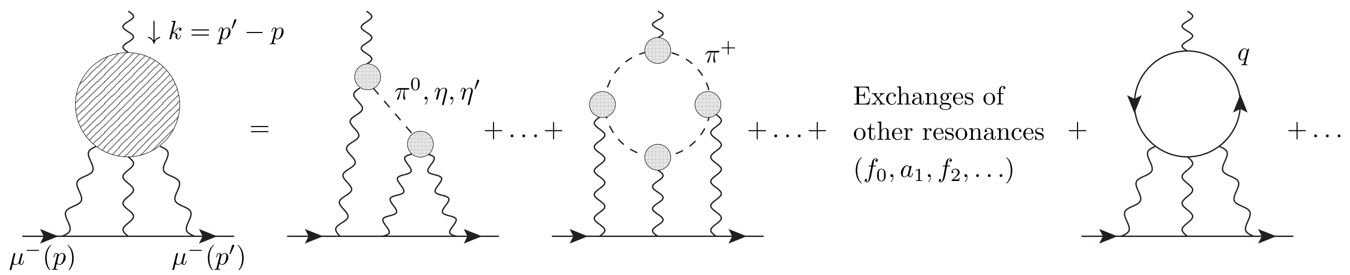
CMS Collaboration,
Evidence for light-by-light scattering and searches for axion-like particles in ultraperipheral PbPb collisions at $\sqrt{s_{NN}}$= 5.02 TeV,
arXiv:1810.04602 (2018)

Significance: 4.1σ (4.4σ)
$\sigma_{\text{fid}} = 120 \pm 46(\text{stat.}) \pm 28(\text{syst.}) \pm 4(\text{theo.})$nb
Predicted: 138 ± 14 nb

ATLAS Collaboration,
Observation of light-by-light scattering in ultraperipheral Pb+Pb collisions with the ATLAS detector,
arXiv:1904.03536 (2019)

Significance: 8.2σ (6.2σ)
$\sigma_{\text{fid}} = 78 \pm 13(\text{stat.}) \pm 7(\text{syst.}) \pm 3(\text{lumi.})$ nb
Predicted: 49 ± 5 nb, 48 ± 5 nb

Bringing HLbL together



Contribution	WP (prelim.) Our estimate eq. (8.1)
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π, K-loops/boxes	-16.4 ± 0.2
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scalars	$\Big\} - 2 \pm 3$
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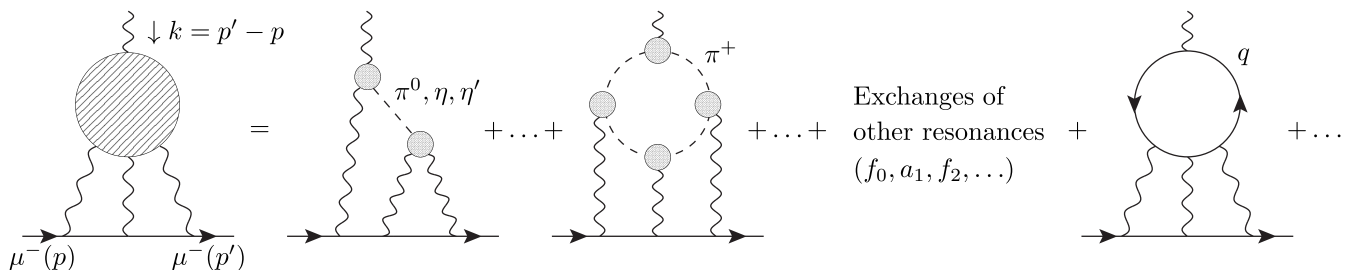

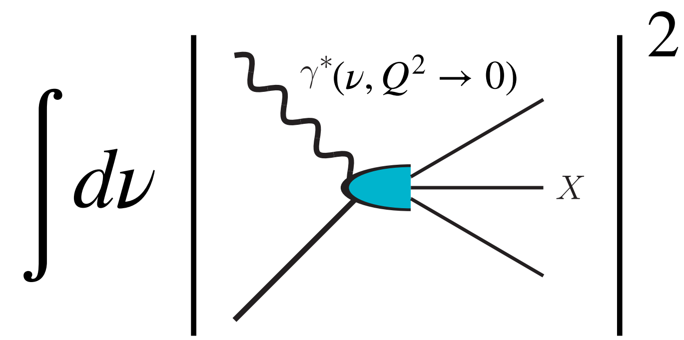

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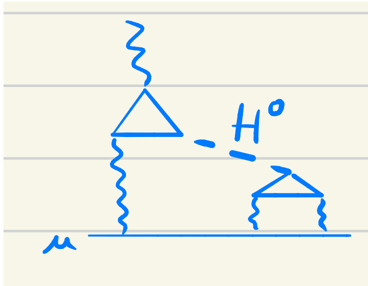

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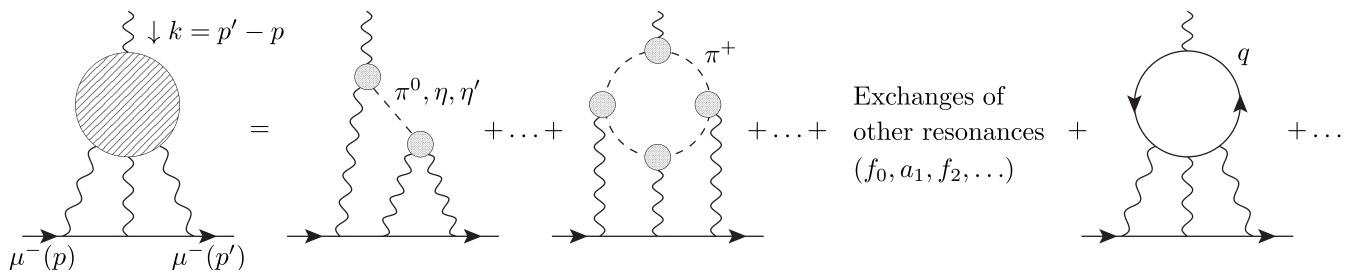

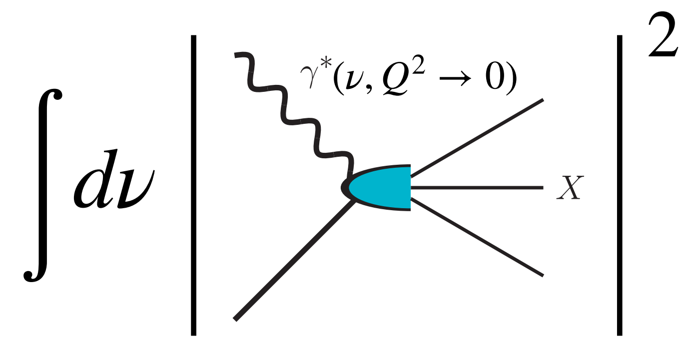

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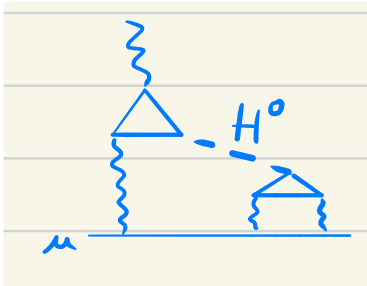

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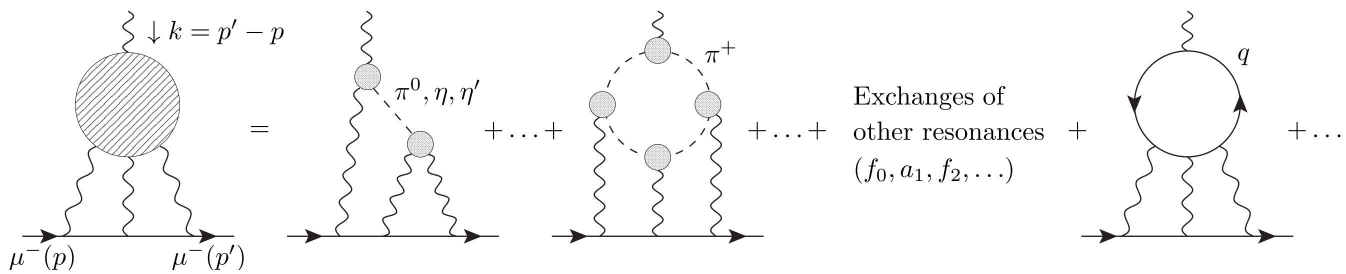

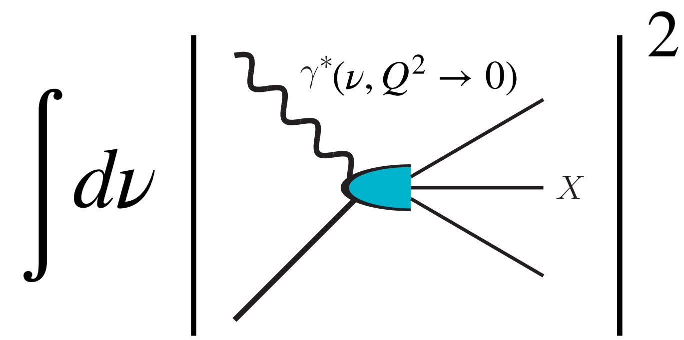

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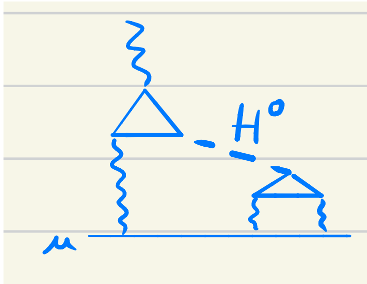
Schwinger sum rule

$\gamma\mu \to \mu(\pi^0, \eta, \eta')$	261 ± 47
$\gamma\mu \to \mu\gamma(\pi^0, \eta, \eta')$	??
pQCD, Regge	??



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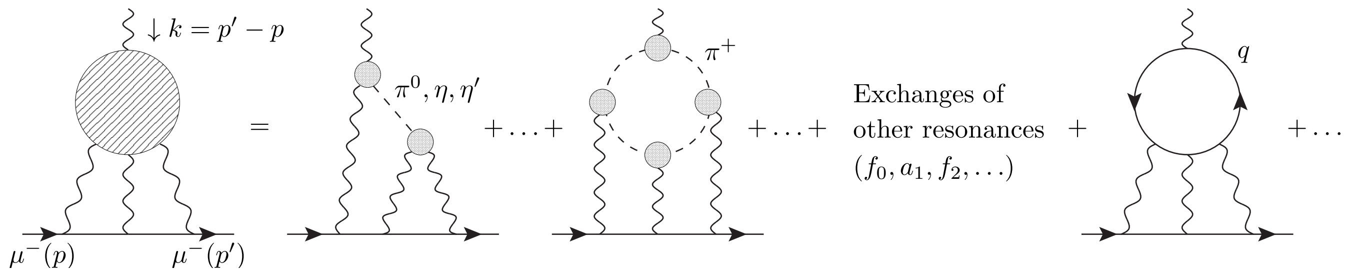

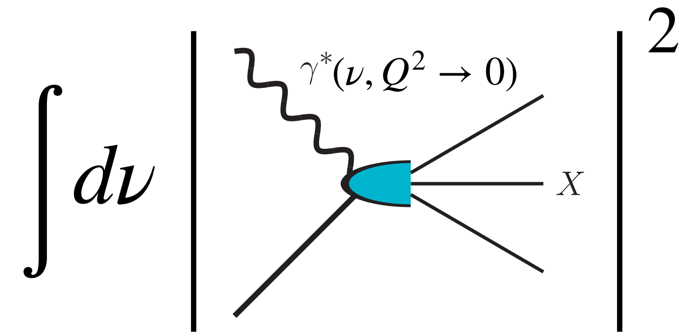

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pQCD, Regge	??

Conclusions:

- Schwinger sum rule is a dispersive formula covering HVP and (partially) HLbL
- Asymptotic behavior of the hadronic contribution to the photoabsorpion cross section is crucial for a quantitative interpretation

3rd workshop on Hadronic contributions to New Physics Searches (HC2NP 2020)

Crete, September 24—30, 2020