Progress in the Mainz effort to compute a_μ^{HLBL} from the lattice

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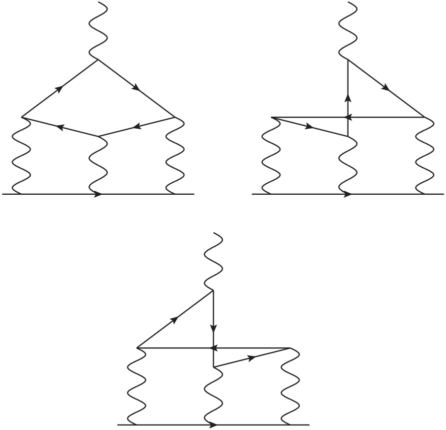
We see the hadronic computation as a **4-pt function in position space**

$$j_\mu(x)j_\nu(y)j_\sigma(z)j_\lambda(0) = (\bar{\psi}\gamma_\mu\psi)(x)(\bar{\psi}\gamma_\nu\psi)(y)(\bar{\psi}\gamma_\sigma\psi)(z)(\bar{\psi}\gamma_\lambda\psi)(x_0)$$

And the QED part as something that can be done in continuum

4	2+2	3+1	2+1+1	1+1+1+1
6	3	8	6	1

Table: Number of contractions needed for each type of diagram



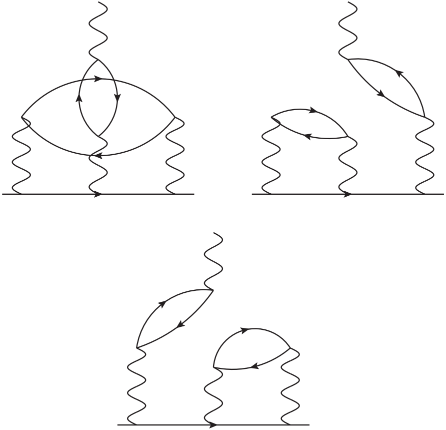
The three connected contributions we wish to compute. Corresponding to contractions 1,2, and 3. From top left, top right, and bottom.

$$a_\mu = \frac{-me^6}{3}\int d^4y \int d^4x \int d^4z\, \mathcal{L}^{(i)}_{\rho\sigma:\mu\nu\lambda}(x,y) z_\rho \langle j_\mu(x) j_\nu(y) j_\sigma(z) j_\lambda(0)\rangle$$

$$a_\mu = \frac{-me^6}{3} 2\pi^2 \int dy\, |y|^3 \underbrace{\int d^4x \int d^4z\, \mathcal{L}^{(i)}_{\rho\sigma:\mu\nu\lambda}(x,y) z_\rho \langle j_\mu(x) j_\nu(y) j_\sigma(z) j_\lambda(0)\rangle}_{f(|y|)}$$

Selecting diagram 2 as our reference ($\Pi^{(4)}_{\mu\nu\sigma\lambda}(x,y,z,0)$) we can perform a change of variables to rewrite the **connected part** of the integral:

$$a_\mu^{(c)} = \frac{-me^6}{3} 4\pi^2 \int d|y| |y|^3 \int d^4x \Big[\left(\mathcal{L}^{(i)}_{\rho\sigma;\mu\nu\lambda}(x,y) + \mathcal{L}^{(i)}_{\rho\sigma;\nu\mu\lambda}(y,x) - \mathcal{L}^{(i)}_{\rho\sigma;\lambda\nu\mu}(x,x-y)\right) \int d^4z\, z_\rho \Pi^{(4)}_{\mu\nu\sigma\lambda}(x,y,z,0) + \mathcal{L}^{(i)}_{\rho\sigma;\lambda\nu\mu}(x,x-y) x_\rho \int d^4z\, \Pi^{(4)}_{\mu\nu\sigma\lambda}(x,y,z,0)\Big]$$



The three $2+2$ diagrams that contribute to the HLBL (numbered 1,2,3 from top left, top right, and bottom).

Defining

$$\Pi^{(2)}_{\mu\nu}(x,y) = \text{Tr}[\bar{S}(x,y)\gamma_\mu S(x,y)\gamma_\nu] - \langle \text{Tr}[\bar{S}(x,y)\gamma_\mu S(x,y)\gamma_\nu]\rangle$$

The **2+2 contribution** to a_μ can be written as

$$a_\mu^{(2+2)} = \frac{-me^6}{3} 2\pi^2 \int dy |y|^3 \int d^4x\, \mathcal{L}^{(i)}_{\rho\sigma:\mu\nu\lambda}(x,y) \int d^4z$$
$$z_\rho \left(\Pi^{(2)}_{\sigma\nu}(z,y)\Pi^{(2)}_{\mu\lambda}(x,0) + \Pi^{(2)}_{\mu\nu}(x,y)\Pi^{(2)}_{\sigma\lambda}(z,0) + \Pi^{(2)}_{\sigma\mu}(z,x)\Pi^{(2)}_{\nu\lambda}(y,0) \right)$$

The third diagram is *unpleasant* so we rewrite...

$$a_\mu^{(2+2)} = \frac{-me^6}{3} 2\pi^2 \int_y dy |y|^3 \int_x d^4x$$
$$\left[\left(\mathcal{L}^{(i)}_{\rho\sigma:\mu\nu\lambda}(x,y) + \mathcal{L}^{(i)}_{\rho\sigma:\nu\mu\lambda}(y,x) \right) \Pi^{(2)}_{\mu\lambda}(x,0) \int_z d^4z\, z_\rho \Pi^{(2)}_{\sigma\nu}(z,y) \right.$$
$$\left. + \mathcal{L}^{(i)}_{\rho\sigma:\mu\nu\lambda}(x,y) \Pi^{(2)}_{\mu\nu}(x,y) \int_z d^4z\, z_\rho \Pi^{(2)}_{\sigma\lambda}(z,0) \right]$$

Have several choices for subtracted kernels [Blum et.al '17] all built from the infinite volume $\mathcal{L}^{(0)}$ **suppressing indices as they are all the same!**

$$\mathcal{L}^{(1)}(x,y) = \mathcal{L}^{(0)}(x,y) - \frac{1}{2}\mathcal{L}^{(0)}(x,x) - \frac{1}{2}\mathcal{L}^{(0)}(y,y)$$

$$\mathcal{L}^{(2)}(x,y) = \mathcal{L}^{(0)}(x,y) - \mathcal{L}^{(0)}(x,0) - \mathcal{L}^{(0)}(0,y)$$

$$\mathcal{L}^{(3)}(x,y) = \mathcal{L}^{(0)}(x,y) - \mathcal{L}^{(0)}(x,x) + \mathcal{L}^{(0)}(0,x) + \mathcal{L}^{(0)}(0,y)$$

Sanity check:

Test different kernels on the infinite volume lepton-loop

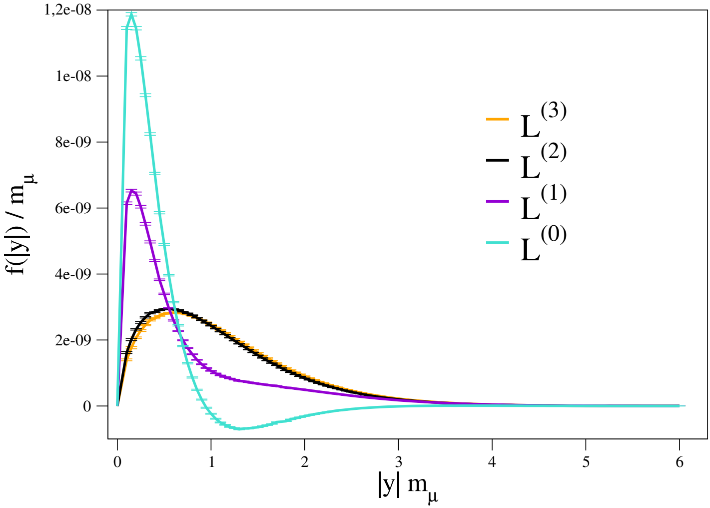


Figure: Lepton loop integrands for our various QED kernels

The **broadening** of the integrand for $\mathcal{L}^{(3/2)}$ is concerning
$\rightarrow$ *Typical lattice dimensions are of the order of a few fm.*

The **large peak** of the integrand for $\mathcal{L}^{(1/0)}$ at small $|y|$ is concerning
$\rightarrow$ *This could lead to significant discretisation effects*

Question

Can we keep the benefits of the $\mathcal{L}^{(2)}$ subtraction *whilst* making the integrand peak at low $|y|$?

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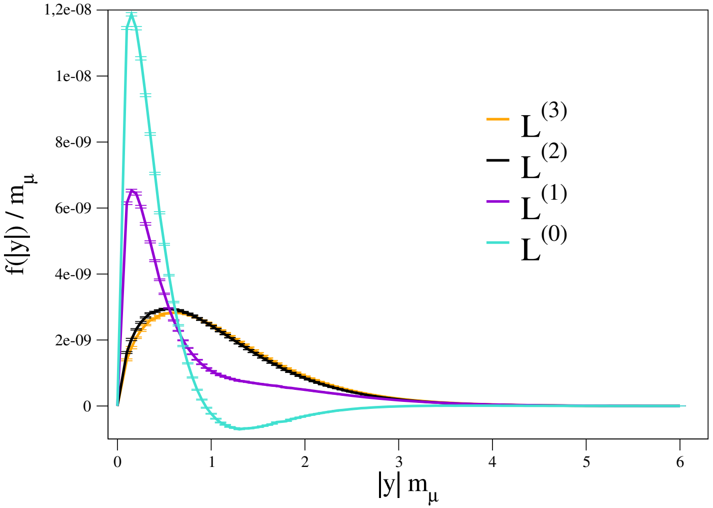
Question

Can we keep the benefits of the $\mathcal{L}^{(2)}$ subtraction *whilst* making the integrand peak at low $|y|$?

Yes! We are practically *free to do whatever we want* to terms like $\mathcal{L}(x,0)$ as they don't contribute to the integral

$$\mathcal{L}^{(2:\lambda)}(x,y) = \mathcal{L}^{(0)}(x,y) - \partial_\mu^{(x)}\left(x_\alpha e^{-\lambda m_\mu^2 x^2/2}\right) L^{(0)}_{\rho\sigma:\alpha\nu\lambda}(0,y) - \partial_\nu^{(y)}\left(y_\alpha e^{-\lambda m_\mu^2 y^2/2}\right) L^{(0)}_{\rho\sigma:\mu\alpha\lambda}(x,0)$$

λ is a dimensionless tuneable parameter $\lambda \to 0$ is $\mathcal{L}^{(2)}$ and $\lambda \to \infty$ is $\mathcal{L}^{(0)}$



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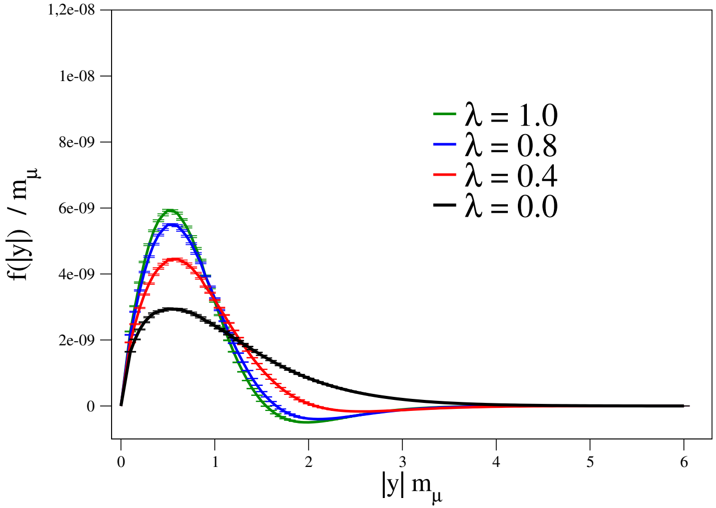


Figure: Lepton loop integrands for some choices of λ

- Integrals become discrete sums
  - Discretisation effects
  - Finite volume effects

- CLS $n_f = 2 + 1$ Wilson-Clover ensembles with open temporal boundary and 4 local currents
  - Focus on $SU(3)_f$ symmetric-point ensembles for fully-connected and 2+2 contributions
    ($m_\pi = m_K \approx 416$ MeV, $a = 0.0864(11)$ fm, $m_\pi L = 5.8$)

- Point sources along direction $(0, 2, 2, 2)$
  1. Two sources gives both $+y$ and $-y$
  2. Self averages per $|y|$ of $L/2$ exploiting periodicity in spatial directions
- Also investigate $(3, 1, 1, 1)$ direction for volume effects
  1. Larger $|y|$ available but fewer self-averages

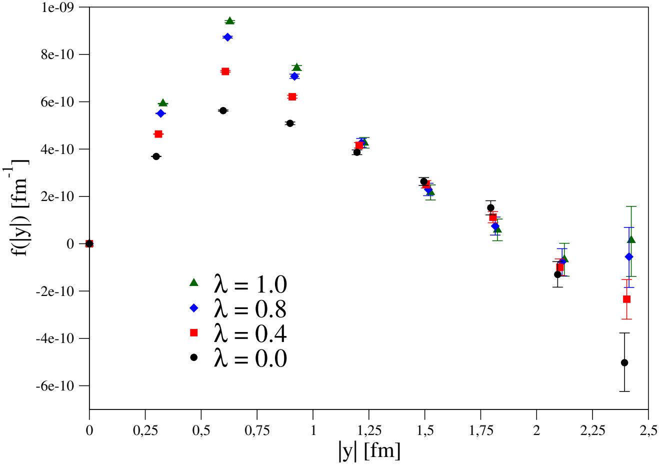


Figure: Fully-connected integrand for direction $(0, 2, 2, 2)$ (points shifted for clarity)

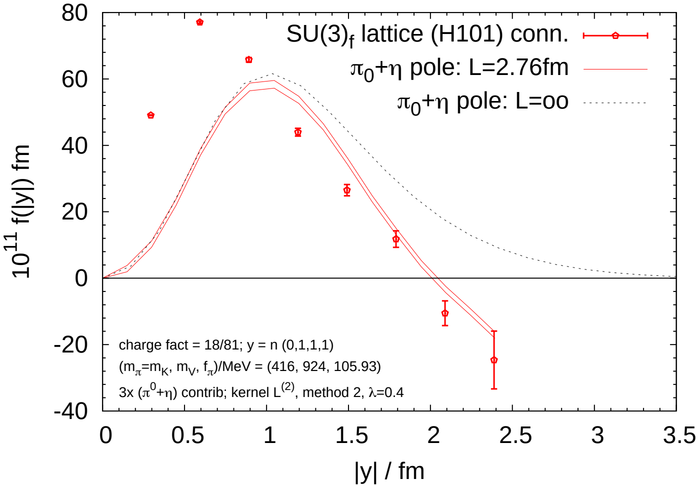


Figure: Fully-connected contribution to a_μ^{HLBL} and its corresponding finite-volume prediction

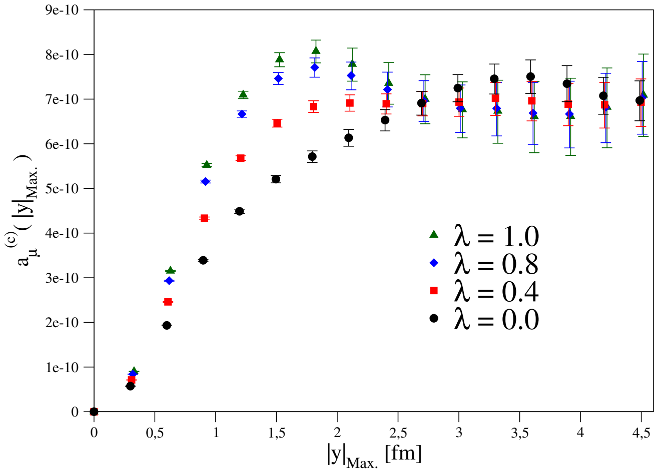


Figure: Integrated result up to a cut-off $|y|_{\text{Max.}}$ along direction $(3,1,1,1)$

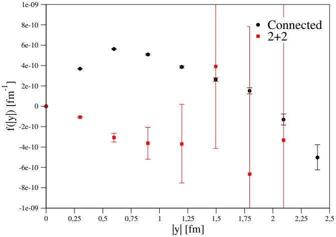
Compute our finite-volume corrected result at some y_c where the integrand is positive

$$a_\mu = a_\mu^{\text{lattice}}(|y| < y_c) + \underbrace{a_\mu^{\Pi_0}(\infty, |y|) - a_\mu^{\Pi_0}(L, |y| < y_c)}_{\text{FS}_{\text{Corr}}}$$

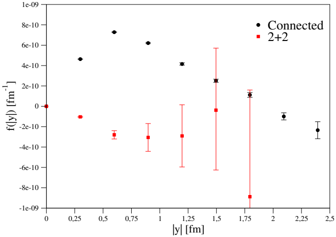
L/a	Direction	a [fm]	y_C [fm]	FS_{Corr}	$a_\mu \times 10^{11}$
32	$(0,2,2,2)$	0.0864	1.79	20.5	102.5(1.1)(5.1)
32	$(3,1,1,1)$	0.0864	3.0	8.8	79.7(4.4)(2.2)
32	$(3,1,1,1)$	0.0643	2.5	22.9	97.0(4.0)(5.7)
48	$(3,1,1,1)$	0.0643	3.0	5.1	97.2(4.2)(1.3)

Table: Fully-connected, finite-size-corrected $SU(3)_f$ symmetric-point results with $\lambda = 0.4$

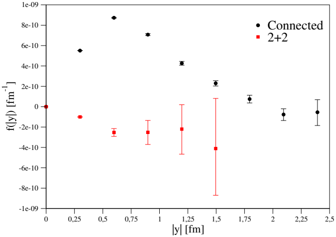
We achieve consistent finite-size-corrected results for our finest lattice



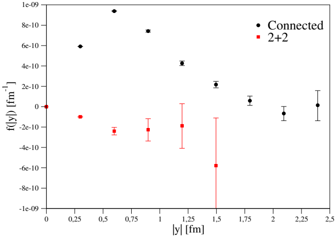
(a) $\lambda = 0.0$



(b) $\lambda = 0.4$



(c) $\lambda = 0.8$



(d) $\lambda = 1.0$

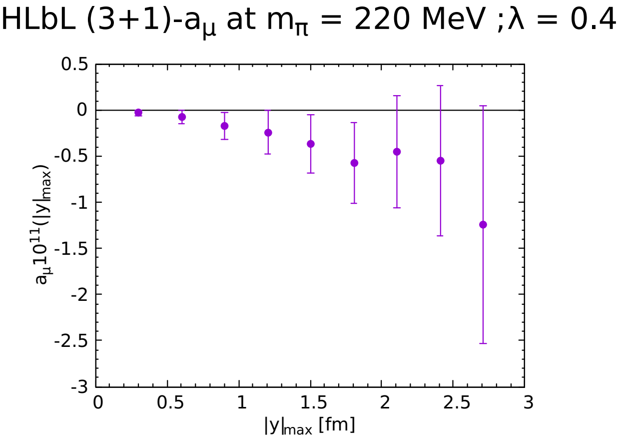


Figure: Preliminary Integrated 3+1 contribution from an ensemble with light pion mass, and direction (3,1,1,1)

- It is possible to have more choice for our QED kernel
- By considering different λ we can make the integrand narrower
  1. This helps to reduce finite volume effects
  2. This significantly reduces noise in the 2+2 contribution
- We believe that we have finite-size effects under control for the symmetric ensembles
- The error in the measurement is **currently dominated** by the 2+2 contribution
- The $3 + 1$ contribution appears small and consistent with zero within our statistical resolution