Hadronic contributions to (g-2)_mu

INT, Seattle, September 12, 2019

Comments on Dispersive Approach to HLbL

Arkady Vainshtein

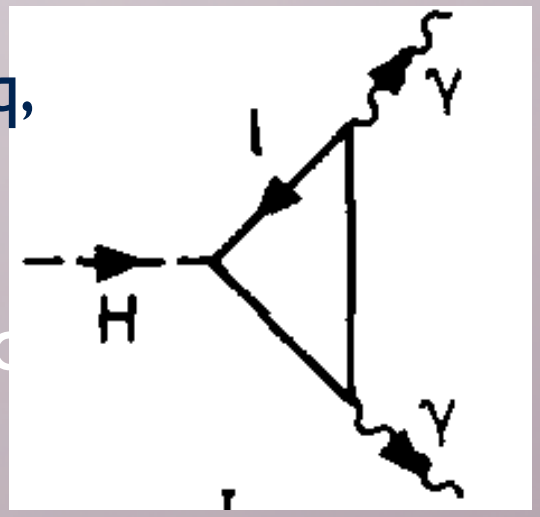
William Fine Theoretical Physics Institute
University of Minnesota
and
Kavli Institute for Theoretical Physics, UCSB

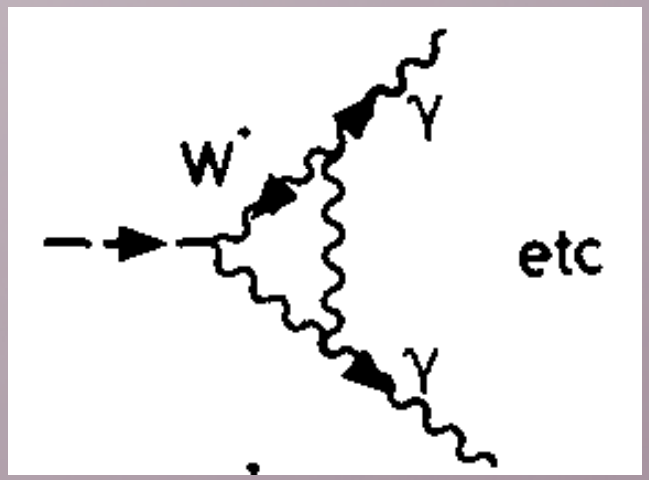
Higgs boson H was discovered on July 4, 2012.
Its two-photon decay $H \to \gamma\gamma$ represents a major discovery channel in spite of a small branching 2.3×10^{-3}.

This decay is also a major check of SM. Its strength in units of SM is 1.1 ± 0.1.

So theoretical attention to this decay is well justified.
Fermion and W-loops, sensitivity to BSM.

q,





The amplitude is

$$\mathcal{M} = \frac{\alpha}{4\pi v} F(m_H^2)(k_1^\mu \epsilon_1^\nu - k_1^\nu \epsilon_1^\mu)(k_{2\mu}\epsilon_{2\nu} - k_{2\nu}\epsilon_{2\mu})$$

where

$$F(s) = \sum_f N_c Q_f^2 F_f(\beta_f) + F_W(\beta_W)$$

$$\beta_f = \frac{4m_f^2}{s}, \qquad \beta_W = \frac{4m_W^2}{s}$$

Fermion loop gives

$$F_f(\beta) = -2\beta\left[1 + (1-\beta) f(\beta)\right]$$

$$f(\beta) = -\frac{1}{4}\left[\ln\frac{1+\sqrt{1-\beta}}{1-\sqrt{1-\beta}} - i\pi\right]^2$$

W-boson loops produce

$$F_W(s) = F_W^\infty + F_W^c(s),$$

$$F_W^c(s) = 3\beta\left[1 + (2-\beta) f(\beta)\right]$$

$$\boxed{F_W^\infty = F_W(s \to \infty) = 2}$$

No decoupling !

History of calculations of F_W :

1976: Ellis, Gaillard and Nanopoulos

For the light Higgs they found $\lim_{s\to 0} F_W = 7$.

1978: Ioffe and Khose reproduced the result.

1979: Shifman, A.V., Voloshin and Zakharov

extended result for an arbitrary Higgs mass and derived the low-energy theorem relating $\lim_{s\to 0} F_W$ to beta-function W-boson loops, $\lim_{s\to 0} F_W = b_W = 7$.

This implies $F_W^\infty = 2$.

2011: Gastmans, S. L. Wu and T. T. Wu claimed $F_W^\infty = 0$.

Blamed dimensional regularization and argued for decoupling at $m_W \to 0$.

Responding to this claim
Shifman, A.V., Voloshin and Zakharov argued against decoupling: Higgs boson interaction $2Hm_W^2 W_\mu^\dagger W^\mu/v$ does not vanish for the longitudinal polarization.
Marciano, Zhang and Willenbrock plus at least 5 more publications verified $F_W^\infty = 2$ in renormalizable gauges.

However, Christova and Todorov in 2015 used the dispersion approach to argue that $F_W^\infty = 0$.

Indeed, the imaginary part is unambiguous and could be calculated in the unitary gauge as well,

$$\mathrm{Im}\, F_W(s) = \frac{3\pi}{2}\,\theta(1-\beta)\,\beta(2-\beta)\ln\frac{1+\sqrt{1-\beta}}{1-\sqrt{1-\beta}}$$

Note that it does vanish at $m_W = 0$.

Then the full function $F_W(s)$ can be reconstructed using the unsubtracted dispersion relation in s,

$$F_W(s) = \frac{1}{\pi} \int_{4m_W^2}^{\infty} \frac{\mathrm{d}s_1 \mathrm{Im}[F_W(s_1)]}{s_1 - s - i0}$$

It leads to

$$F_W^{\infty} = F_W(s \to \infty) = 0$$

In 2016 Kirill Melnikov and I followed this argumentation to find out what goes wrong and when one can use unsubtracted dispersion integrals. The low-energy theorem guarantees nonvanishing $F_W^{\infty} = F_W(s \to \infty) = 2$

Longitudinal polarizations: toy model

At large energies $E \gg m_W$ we can substitute for longitudinal polarizations

$$W_\mu = \partial_\mu \phi / m_W$$

Then Higgs interaction $2(H/v) m_W^2 W_\mu^\dagger W^\mu$ becomes

$$S_{\rm int} = \int {\rm d}^4 x \, \frac{H}{v} \, \partial_\mu \partial^\mu (\phi^\dagger \phi)$$

which is nonrenormalizable interaction *per se*.

We will calculate now the loop of scalars for two-photon decay of Higgs by two methods.

Integrating by parts and using $\partial_\mu \partial^\mu H = -m_H^2 H$ we obtain

$$S_{\rm int} = -\frac{m_H^2}{v} \int {\rm d}^4 x \, H \, \phi^\dagger \phi$$

which is renormalizable and dispersion approach should work with unsubtracted integrals.

$$\langle\gamma\gamma|\phi^\dagger\phi|0\rangle = -\Phi(s)\cdot\frac{\alpha}{4\pi}\, f_1^{\mu\nu} f_{2,\mu\nu}$$

$$f_i^{\mu\nu} \;=\; k_i^\mu \epsilon_i^\nu \;-\; k_i^\nu \epsilon_i^\mu$$

The physical form factor is then

$$F_\phi(s) \;=\; m_H^2 \Phi(s) = s\,\Phi(s)$$

At large $s = (k_1 + k_2)^2 \gg m_\phi^2$ the scalar loop is well-known

$$\Phi(s) = \frac{2}{s}$$

Then multiplying by m_H^2 which is identified with s we get

$$\lim_{s\to\infty} F_\phi(s) = 2$$

It is straightforward to reproduce this in dispersion approach.

Indeed,

$$2\,\mathrm{Im}\,\langle\gamma\gamma|\phi^\dagger\phi|0\rangle = \int \mathrm{dLips}(p_1,p_2,K_{12})\,M^{\gamma\gamma}_{\phi\phi}$$

Here $M^{\gamma\gamma}_{\phi\phi}$ is the amplitude of $\phi(p_1)+\bar{\phi}(p_2)\to\gamma(k_1)+\gamma(k_2)$

$$M^{\gamma\gamma}_{\phi\phi} = 2e^2\Big\{(\epsilon_1\epsilon_2) - \frac{(p_1\epsilon_1)(p_2\epsilon_2)}{(p_1k_1)} - \frac{(p_1\epsilon_2)(p_2\epsilon_1)}{(p_1k_2)}\Big\}$$

After phase space integration

$$\mathrm{Im}\,\Phi(s) = -\pi\theta(1-\beta_\phi)\,\frac{\beta_\phi}{s}\ln\frac{1+\sqrt{1-\beta_\phi}}{1-\sqrt{1-\beta_\phi}}$$

and from unsubtracted dispersion integral

$$\Phi(s) = \frac{1}{\pi}\int_{4m_\phi^2}^{\infty}\frac{\mathrm{d}s_1\mathrm{Im}\,\Phi(s_1)}{s_1-s-i0} = \frac{2}{s}\left(1-\beta_\phi f(\beta_\phi)\right) \qquad \beta_\phi = 4m_\phi^2/s.$$

Multiplying by s we get

$$F_\phi(s) = s\Phi(s),$$

$$\mathrm{Im}\, F_\phi(s) = -\pi\, \theta(1-\beta_\phi)\beta_\phi \ln \frac{1+\sqrt{1-\beta_\phi}}{1-\sqrt{1-\beta_\phi}}\,,$$

$$F_\phi(s) = 2\big(1-\beta_\phi f(\beta_\phi)\big).$$

Thus, $F_\phi(s)$ does not vanish at $s \to \infty$, does not support decoupling. It satisfy the once-subtracted dispersion relation

$$F_\phi(s) = \frac{s}{\pi}\int_{4m_\phi^2}^{\infty} \frac{ds_1 \mathrm{Im}\, F_\phi(s_1)}{s_1(s_1-s-i0)}$$

and the condition $F_\phi(s=0)=0$. Rewriting this as

$$F_\phi(s) = \frac{1}{\pi}\int_{4m_\phi^2}^{\infty} \frac{ds_1 \mathrm{Im}\, F_\phi(s_1)}{(s_1-s-i0)} - \int \frac{\mathrm{d}s_1}{s_1}\mathrm{Im}[F_\phi(s_1)]$$

we see what is added to unsubtracted part,

$$-\int \frac{\mathrm{d}s_1}{s_1}\mathrm{Im}[F_\phi(s_1)] = 2.$$

We will now check that we can get the correct result for the form factor using the unsubtracted dispersion relations even if we work with the non-renormalizable interaction but regulate the theory in the UV, in spite of the fact that the final result turns out to be finite.

A simple form of the UV regularization is an introduction of Pauli-Villars charged scalar field with mass $m_{\rm PV}$. The regulator loop should be subtracted. Then,

$$\Delta_{\rm PV}[{\rm Im}\, F_\phi] = \pi\, \theta(1-\beta_R)\beta_R \ln \frac{1+\sqrt{1-\beta_R}}{1-\sqrt{1-\beta_R}} \qquad \beta_R = 4m_{\rm PV}^2/s$$

$$\Delta_{\rm PV} F_\phi(s) = \frac{1}{\pi}\int\limits_{4m_R^2}^{\infty} \frac{{\rm d}s_1 \Delta_{\rm PV}[{\rm Im}[F_\phi(s_1)]}{s_1 - s - i0} = 2\beta_R f(\beta_R)$$

We are interested in the limit $\beta_R = 4m_{\rm PV}^2/s \to \infty$; in that limit

$$\Delta_{\rm PV} F_\phi(s) = 2\beta_R f(\beta_R) \to 2$$

We will now demonstrate that the same result is obtained if dimensional regularization is used for the UV cut-off. In the frame where the photon momenta along z-axis choose polarizations as

$$\epsilon_{1,2} = (0, 1, \pm i, 0)\,/\sqrt{2}\,, \qquad \epsilon_1 \cdot \epsilon_2 = -1$$

$$2\,\mathrm{Im}\,\langle\gamma\gamma|H\rangle = \int \mathrm{dLips}(p_1, p_2, K_{12})\langle\phi\bar{\phi}\,|H\rangle\, M^{\gamma\gamma}_{\phi\phi}$$

We obtain

$$\mathrm{Im}\,F_\phi = (4\pi)^2\mu^{2\epsilon}\int \mathrm{dLips}(p_1, p_2, K_{12})\Big\{-1 + \frac{p_x^2 + p_y^2}{p_0^2 - p_z^2}\Big\}$$

Here μ is normalization point. At $d = 4 - 2\epsilon$ splitting 4-momenta p_μ into the four-dimensional part and the part p_ϵ living in remaining -2ϵ dimensions we get

$$\Delta_\epsilon \mathrm{Im}\,F_\phi = (4\pi)^2\mu^{2\epsilon}\int \mathrm{dLips}(p_1, p_2, K_{12})\,\frac{n_\epsilon^2}{\sin^2\theta} \approx 2\pi\epsilon\left(\frac{s}{\mu^2}\right)^{-\epsilon}$$

$$n_\epsilon = p_\epsilon/|p|$$

The correction to the imaginary part induces the following change in F_ϕ :

$$\Delta_\epsilon F_\phi(s) = \frac{1}{\pi}\int_{4m_R^2}^{\infty} \frac{\mathrm{d}s_1 \Delta_\epsilon \mathrm{Im}[F_\phi(s_1)]}{s_1 - s - i0} = 2\left(-\frac{s}{\mu^2}\right)^{-\epsilon}$$

It reproduces the constant 2 for $\sqrt{s}$ much smaller than the UV scale $\mu \exp(1/(2\epsilon))$

We made a similar exercise for the full amplitude. We also checked that in renormalizable gauges where unsubtracted dispersion relations are OK, an extra contribution comes from auxiliary fields whose masses goes to infinity.

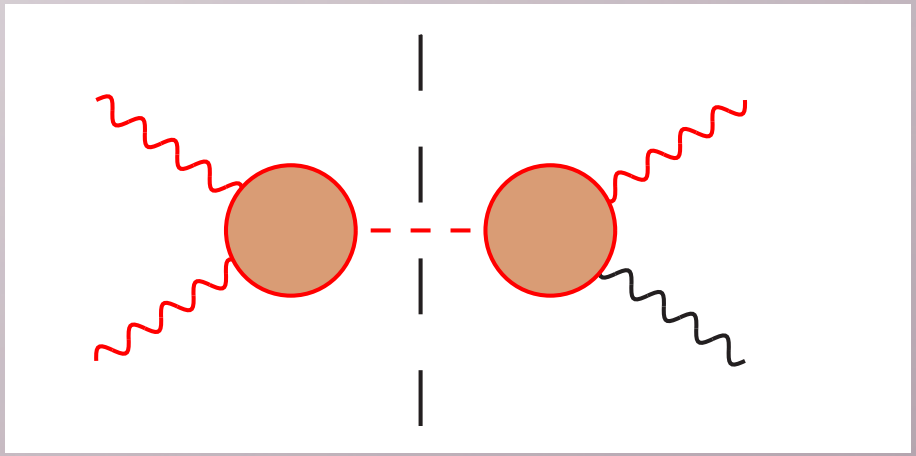
Conclusions on $H \to \gamma\gamma$

We have shown that the need to perform the subtraction in the dispersion integral for form factors computed in the unitary gauge is a consequence of the fact that the SM in the unitary gauge is not explicitly renormalizable. If one regularizes the (apparently finite) calculation by either introducing explicit UV regulator or starts from the formulation of the theory where the renormalizability is, in fact, apparent, one always obtains an additional contribution to the real part of the form factor. For values of s below the ultraviolet cut-off, this contribution is, essentially, a constant and can be interpreted as the subtraction term in the dispersion relation.

Pion exchange

The dispersive approach to this exchange I discussed at the Capri workshop in 2015 following Massimiliano Procura's talk. At the present meeting the issue was discussed by Gilberto Colangelo. What is the issue?

The one-pion exchange is represented by the diagram



$$F_{\gamma^*\gamma^*\pi}(q_1^2, q_2^2)\,\frac{1}{(q_3+q_4)^2 - m_\pi^2}\,F_{\gamma^*\gamma\pi}(q_3^2, 0)$$

In application to g-2 we take the limit $q_4 \to 0$, so come to

$$F_{\gamma^*\gamma^*\pi}(q_1^2, q_2^2)\,\frac{1}{q_3^2 - m_\pi^2}\,F_{\gamma^*\gamma\pi}(q_3^2, 0)$$

On the other hand, if we put $q_4 = 0$ at the beginning and use dispersion relations in the variable q_3^2 , we come to

$$F_{\gamma^*\gamma^*\pi}(q_1^2, q_2^2)\,\frac{1}{q_3^2 - m_\pi^2}\,F_{\gamma^*\gamma\pi}(m_\pi^2, 0)$$

where suppression due to the transitional form factor is absent. The difference

$$\frac{F_{\gamma^*\gamma\pi}(q_3^2, 0) - F_{\gamma^*\gamma\pi}(m_\pi^2, 0)}{q_3^2 - m_\pi^2}$$

is the smooth function near the pion pole but should be explained. I'll return to an explanation a bit later.

Short distance QCD constraints

To use OPE let us consider the range where

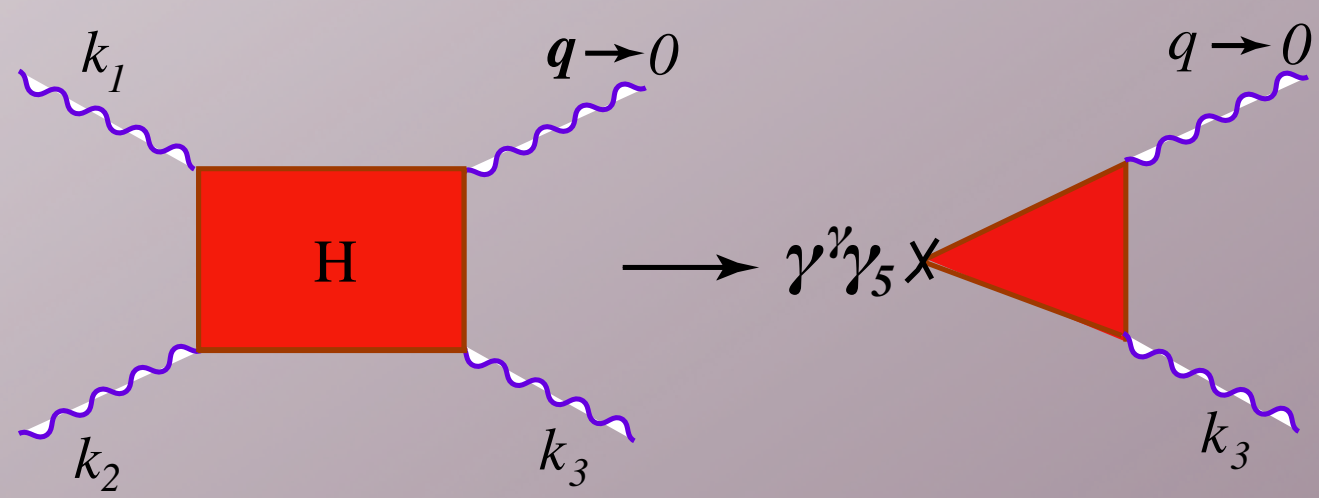
$$q_1^2 \sim q_2^2 \gg s, t, q_3^2, q_4^2$$

In this range the OPE for the product of two vector currents reads as

$$\int d^4x_1 \int d^4x_2 \, \mathrm{e}^{-iq_1\cdot x_1 - iq_2\cdot x_2} T\{j_\nu(x_1), j_\rho(x_2)\} =$$

$$\frac{2}{\hat{q}^2}\epsilon_{\nu\rho\delta\gamma}\hat{q}^\delta \int d^4z \, \mathrm{e}^{-i(q_1+q_2)\cdot z} \, j_5^\gamma(z) + \mathcal{O}\left(\frac{1}{\hat{q}^3}\right).$$

where $\hat{q} = (q_1 - q_2)/2 \approx q_1 \approx -q_2$



We discuss the isovector part of the axial current j_5^γ relevant to the pion exchange. We also implying the chiral limit $m_\pi^2 = 0$.

Then the longitudinal part of the axial current associated with pion exchange is fixed by the ABJ anomaly,

$$\partial_\gamma j_5^\gamma = C F_{\mu\nu} \tilde{F}^{\mu\nu}$$

It means that the longitudinal part of the axial current matrix element $\langle 0 | j_5^\gamma | \gamma(q_3), \gamma(q_4) \rangle$ is completely fixed at any q_3, q_4. No transition form factor present! It is just nonrenormalization of the axial anomaly.

Note, that another form factor $F_{\gamma^*\gamma^*\pi}(q_1^2, q_2^2)$ is represented by $\frac{2}{\hat{q}^2}$ factor in the OPE which is just the asymptotic of this form factor.

Now back to explanation of

$$\frac{F_{\gamma^*\gamma\pi}(q_3^2,0) - F_{\gamma^*\gamma\pi}(m_\pi^2,0)}{q_3^2 - m_\pi^2}$$

difference between different limits. The difference should be attributed to the transversal part of the axial current, i.e., it is a part of axial vector particles exchange. This, actually, was discussed long ago in our with Prades and de Rafael Glasgow white paper of 2008. In difference with the longitudinal part corrections to the transversal part of the AVV triangle do exist.

Conclusions

There are subtleties in dispersive approach even when applied for the fully finite quantities. Appearance of higher than 4 dimension operators could be a signal.

The absence of the transition form factor $F_{\gamma^*\gamma\pi}(q_3^2, 0)$ for the pion exchange is confirmed. Thus, the associated numerical enhancement of PS exchanges discovered in 2004 (Melnikov and A.V.) should not be neglected.