Discussion on White-Paper: Lattice QCD Calculation of the HVP

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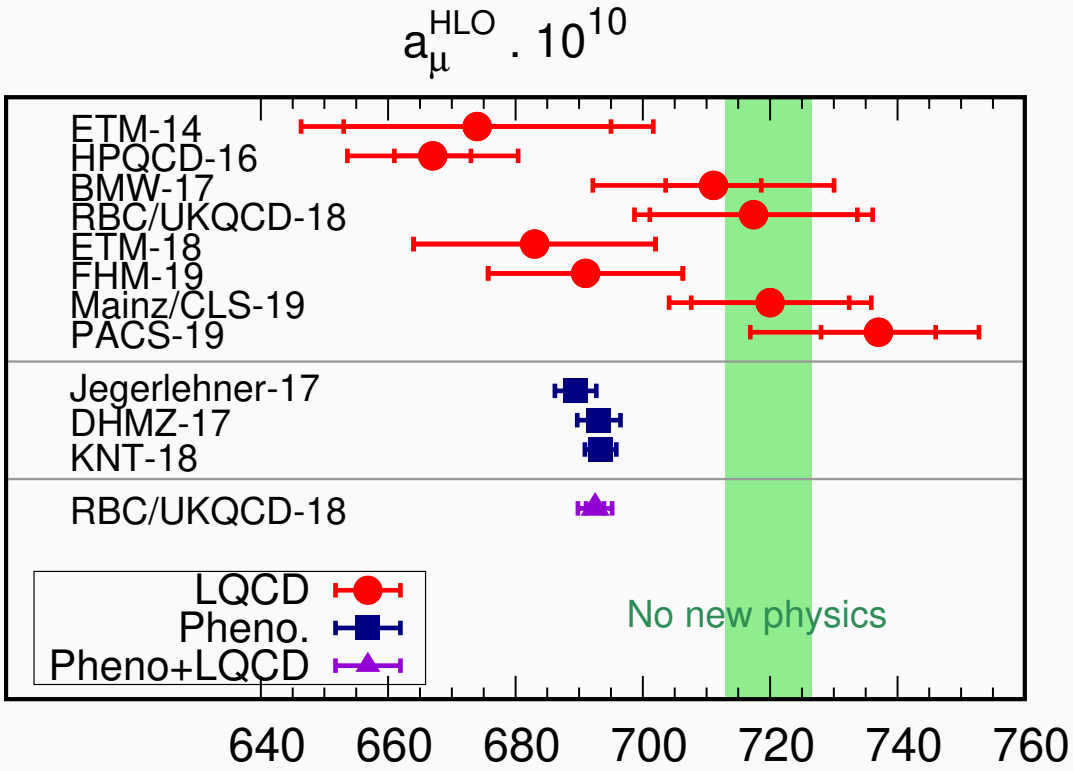
Muon g-2 Theory Initiative, INT-19-74W
September 11, 2019, Univ. of Washington, Seattle

Summary on $a_\mu^{\text{LO-HVP}}$

We shall discuss how we present/design our message from the latttice-HVP section in the white paper.



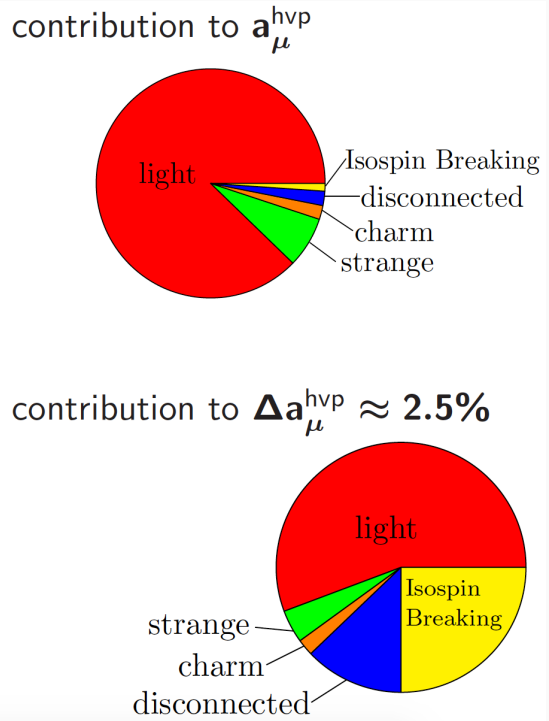


Figure: Left: From White Paper. Right: Vera Gülpers' Plenary Talk at Lattice2019.

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Lattice HVP WP organization

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Quoted from Talk by Aida El-Khadra on Monday Afternoon.

Discussion

1. How do we summarize the lattice QCD results for $a_\mu^{\text{LO-HVP}}$? One idea is take an average.
2. The largest uncertainty in $a_\mu^{\text{LO-HVP}}$ originates to the light connected contribution. Many progress in the long-distance control and FV corrections. However, need more discussion for the clear understandings.
3. Are the strange/charm-connected and disconnected contributions determined with high enough precision for FNAL E989? Need more discussion, particularly on the disconnected (c.f. Mainz/CLS-PRD2019).
4. The SIB/QED correction are investigated based on the lattice in various groups. Need to improve precision (c.f. ETM-PRD2019: $7.1(2.9) \times 10^{-10} \sim 0.4\%$).
5. The window method is explained in the comparison section. Include any figure? It may provide the most precise prediction on $a_\mu^{\text{LO-HVP}}$. However, RBC/UKQCD-PRL18 vs ABGJPT-19....
6. Comparison/Combination with dispersive method and experiments: MUonE, HVP from tau decay, and running $\Delta_{\text{HVP}}\alpha(Q^2)$ and $\Delta_{HVP}\sin^2\theta_W(Q^2)$. Include any results?

Lattice HVP Session on Friday

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Status/update talks:

Davide Giusti — ETMC

Antoine Gerardin — Mainz:group

Laurent Lellouch — BMWc

Christoph Lehner — RBC/UKQCD

Steve Gottlieb — FNAL/HPQCD/MILC

Tom Blum — Aubin et al

Connections:

Marina Marinkovic — Lattice QCD for MUonE (Tuesday)

Nils Hermandsson-Truedsson — FV effects QED corrections

Mattia Bruno — Tau/Isospin-breaking corrections

Marco Cé — HVP contribution to the running of α and $\sin^2\theta$

Figure: Quoted from Talk by Aida El-Khadra on Monday.

Backup

Lattice Average vs Pheno vs NoNewPhys I

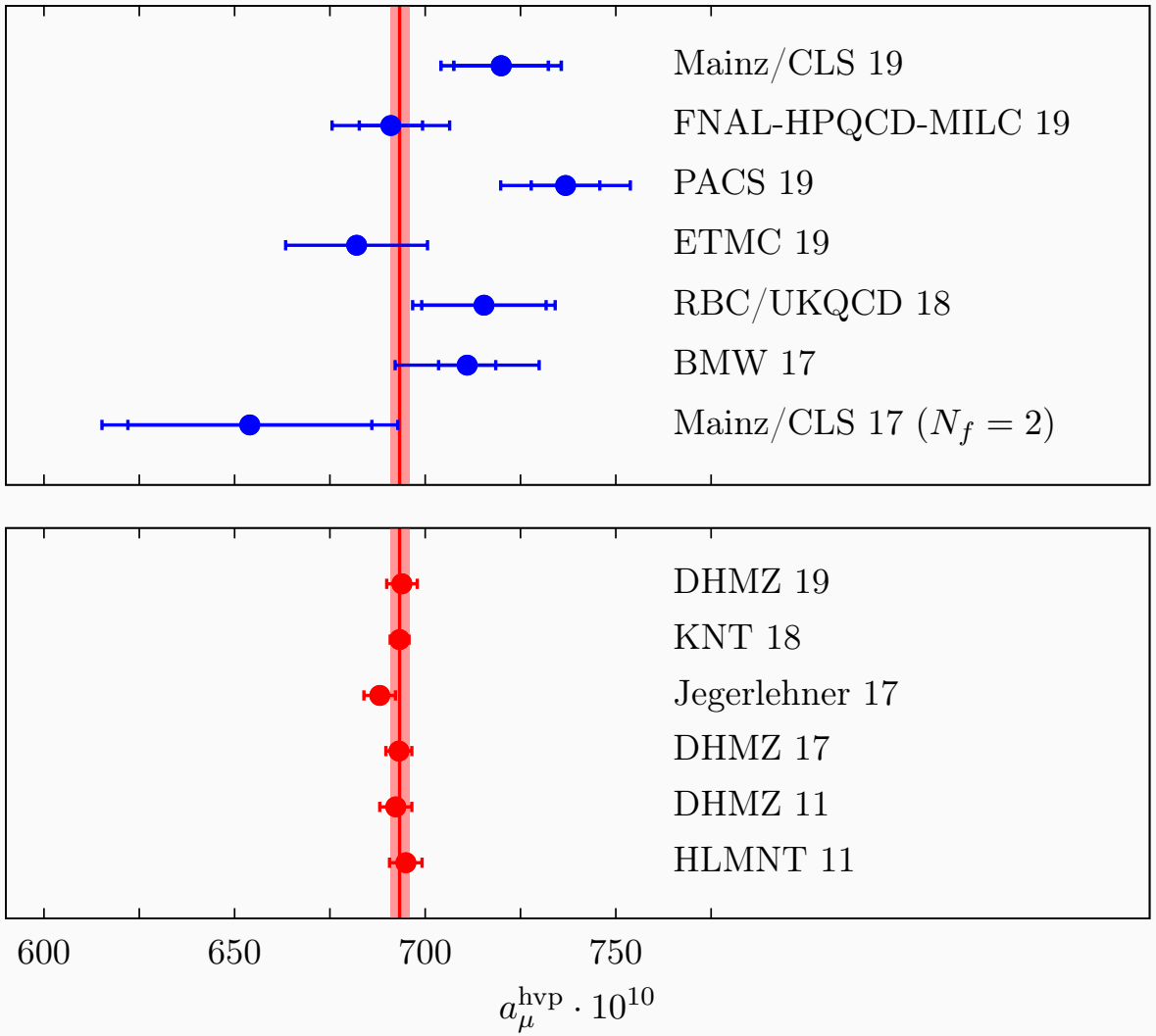


Figure: Talk by H. Wittig @ Capri2019

Lattice Average vs Pheno vs NoNewPhys II

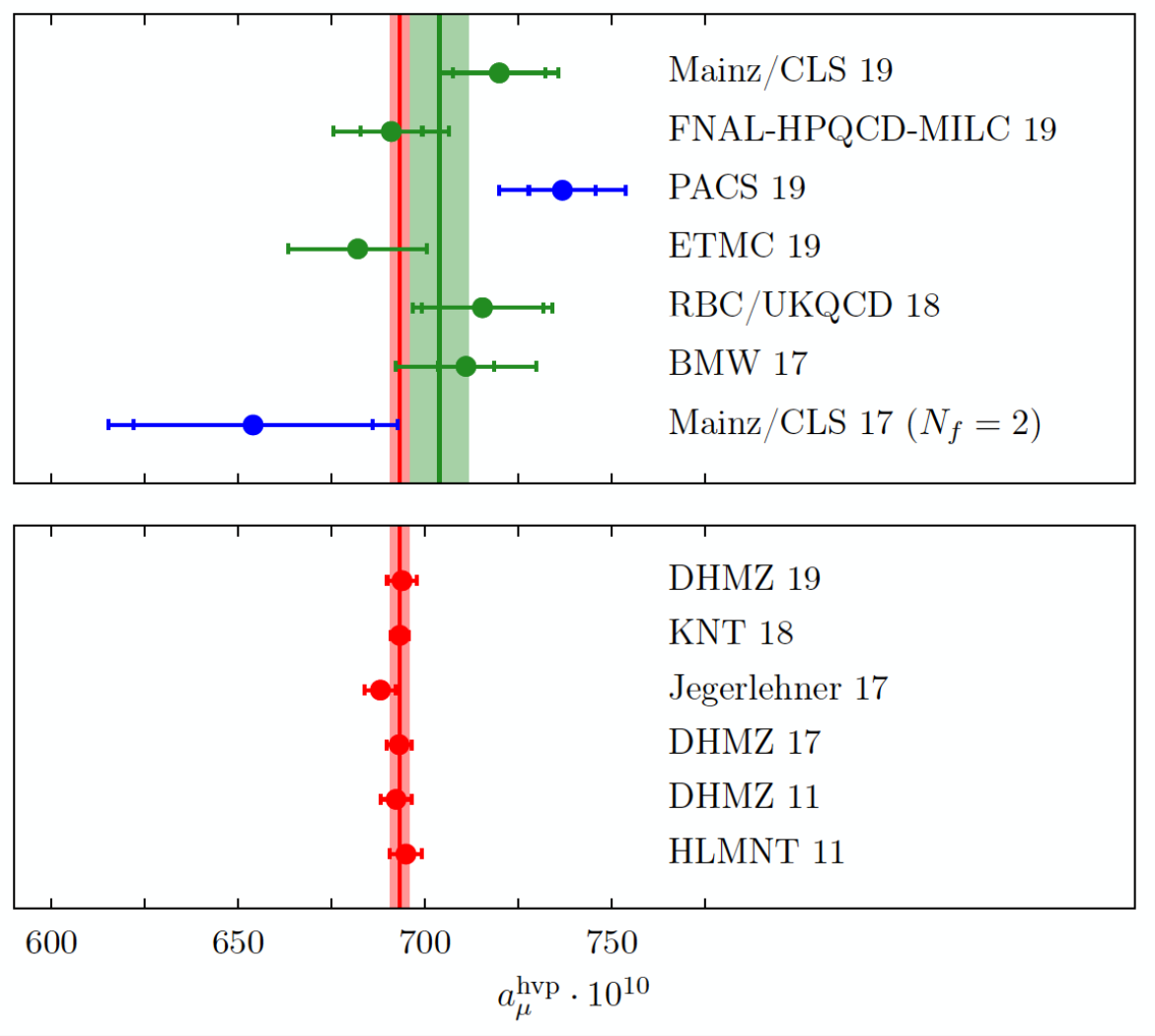


Figure: Talk by H. Wittig @ Capri2019

Lattice Average vs Pheno vs NoNewPhys III

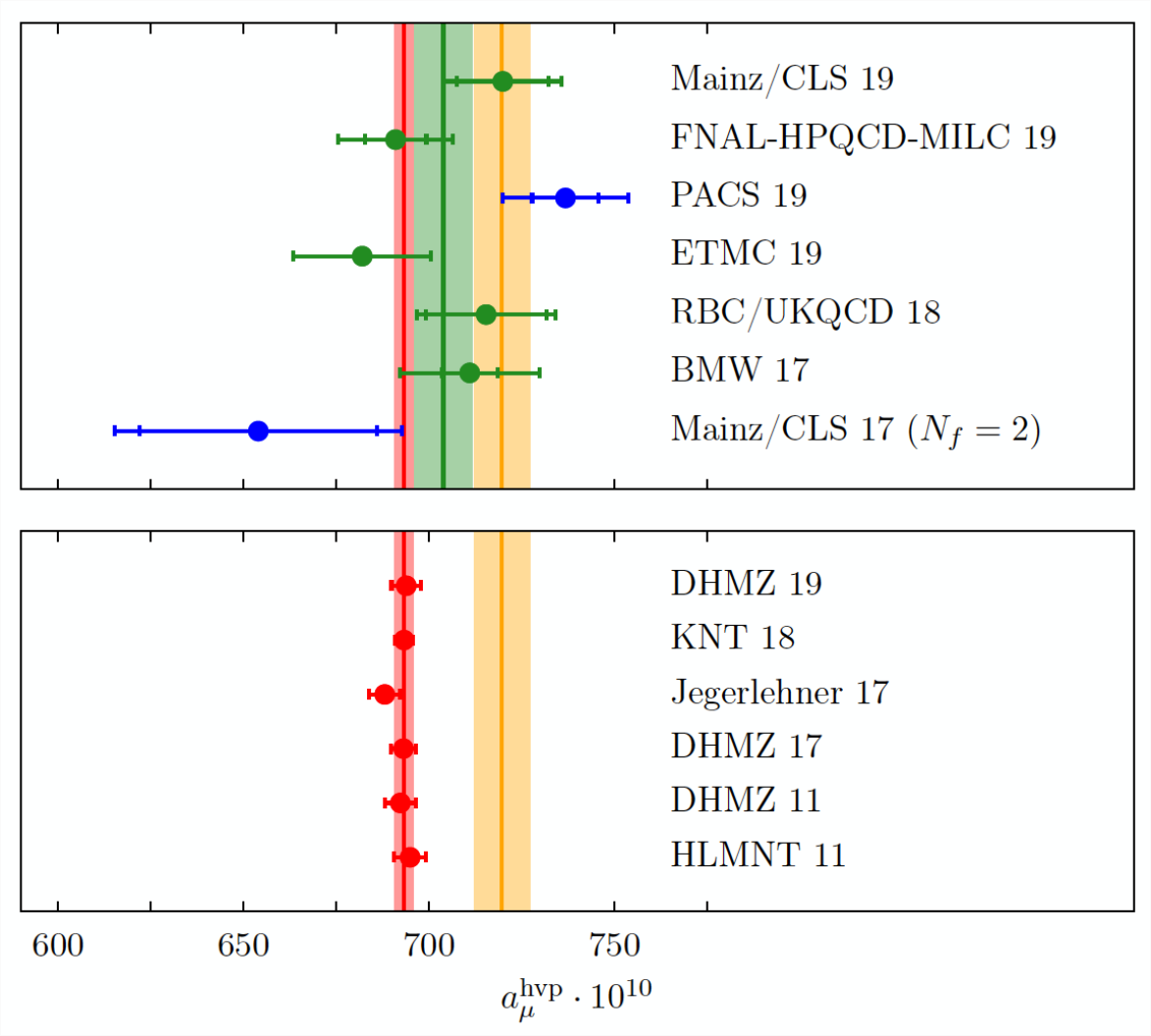
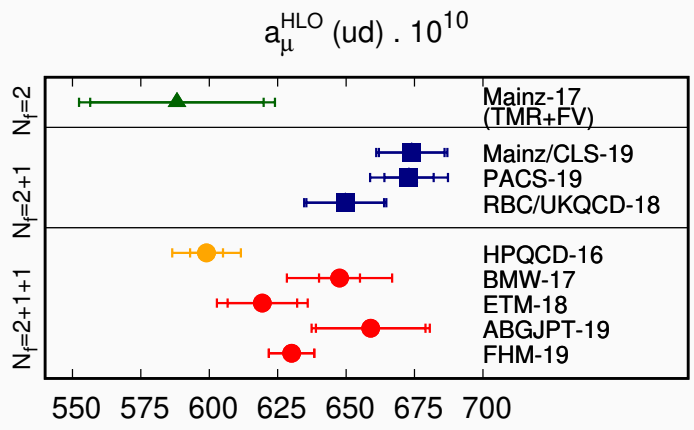
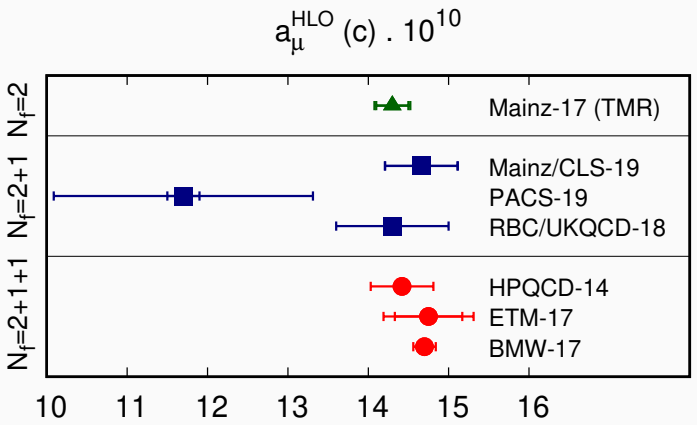
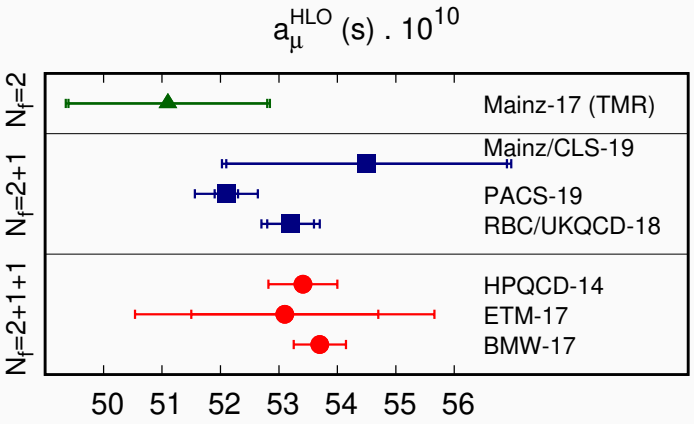


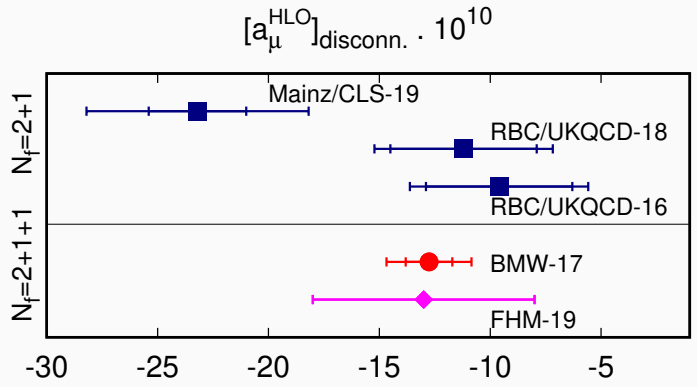
Figure: Talk by H. Wittig @ Capri2019

$a_\mu^{\text{LO-HVP}}$: flavor by flavor comparison







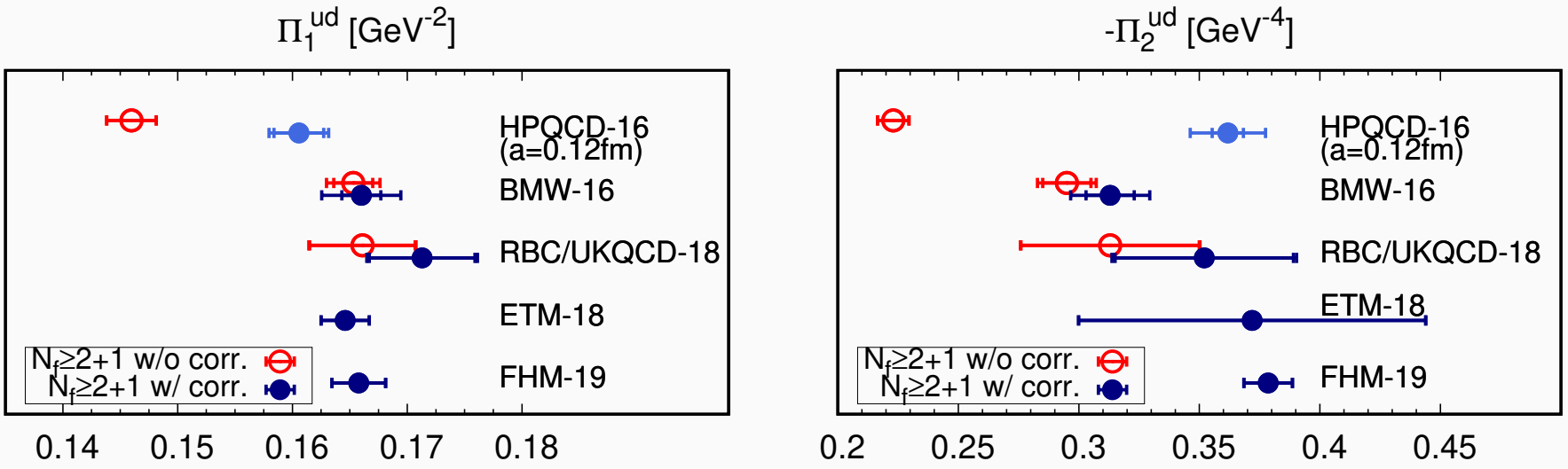


- **light-conn:** Clear understanding for the discrepancy? FHM-19 and ABGJPT-19 (sharing ensembles) do not necessarily agree. ETM-18 does not necessarlity get larger (w. sophisticated long-distance treatment).
- **s/c/disc:** Is it relevant to state that the srange/charm-connected and disconnected are determined with high enough precision for FNAL E989?

Moments

$$\Pi_n = \frac{1}{n!}\frac{d^n\hat{\Pi}(Q^2)}{(dQ^2)^n}\Big|_{Q^2\to 0} = \sum_x \frac{(-\hat{x}_\nu^2)^{n+1}}{(2n+2)!}\langle j_\mu(x) j_\mu(0)\rangle \ . \qquad (1)$$

Larger Π_1 and Smaller Π_2 gives Larger $a_\mu^{\text{LO-HVP}}$.



In Π_2, BMW vs. ETM and FHM. may be interesting to locate the source of the discrepancy of $a_\mu^{\text{LO-HVP}}$.

Window Method Systematics

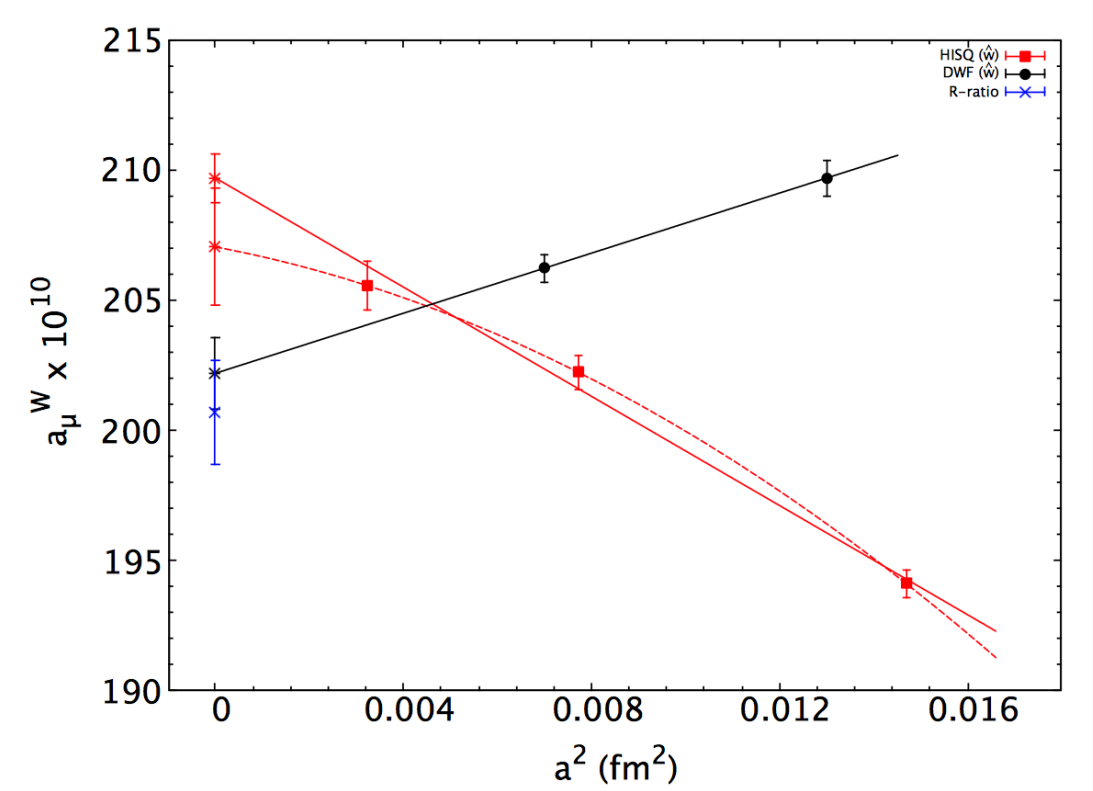


Figure: Quoted from Aubin et.al. 1905.09307. Comparison of the intermediate range contribution to $a_\mu^{\text{LO-HVP}}$.

Even using the same parameters, $\Delta = 0.15$, $t_0 = 0.4$ fm, $t_1 = 1$ fm, HISQ and DWF results show some tension in the intermediate range contribution.

HVP, Lattice vs Pheno

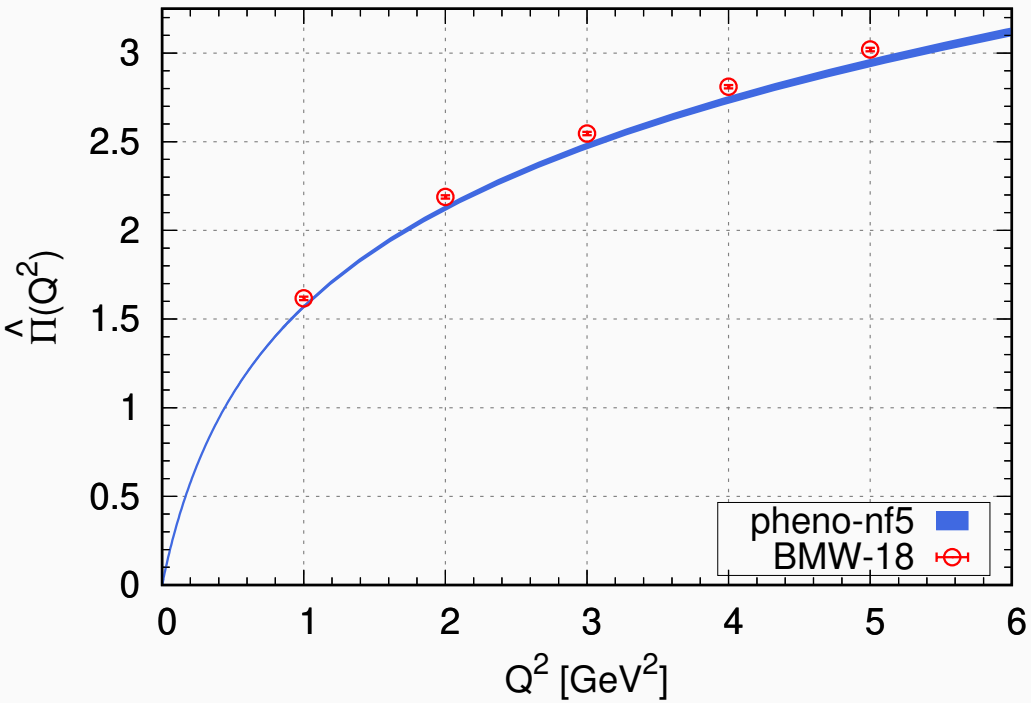


Figure: BMW PRL2018 vs. dispersive approach (updated version of alphaQEDc17, thanks to F. Jegerlehner) on the HVP.

The HVP from LQCD is larger than that from the dispersive method. If the lattice HVP is adopted (and $a_\mu^{\text{LO-HVP}}$ gets consistent with NoNewPhys.), EW grobal fit tends to show a lower M_{higgs} than the observed value (Passera-Marciano-Sirlin('08)).

c.f. Marco's Talk on Friday.

Lattice Average vs Pheno, w.r.t. MUonE

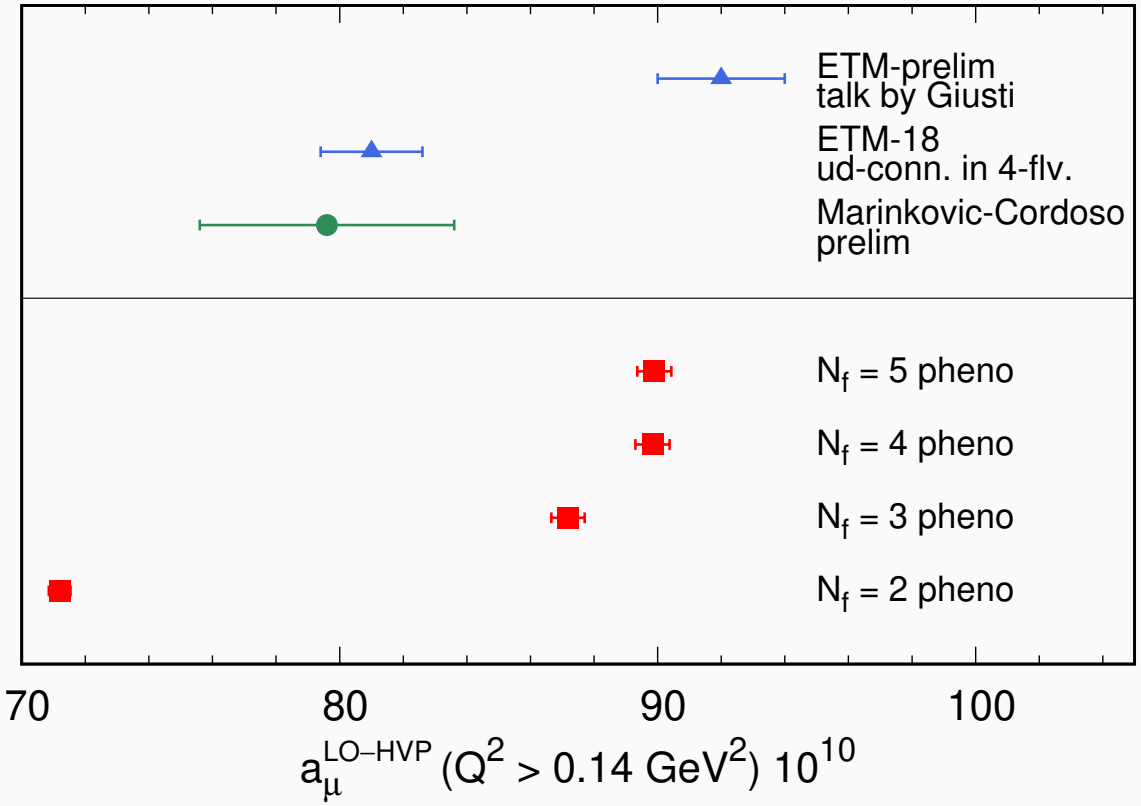


Figure: c.f. Talk by Marina and Davide. Pheno: Updated alphaQEDc17 thanks to F. Jegerlehner.

How we shall design the statement for MUonE?

XPT vs. More Sophisticated

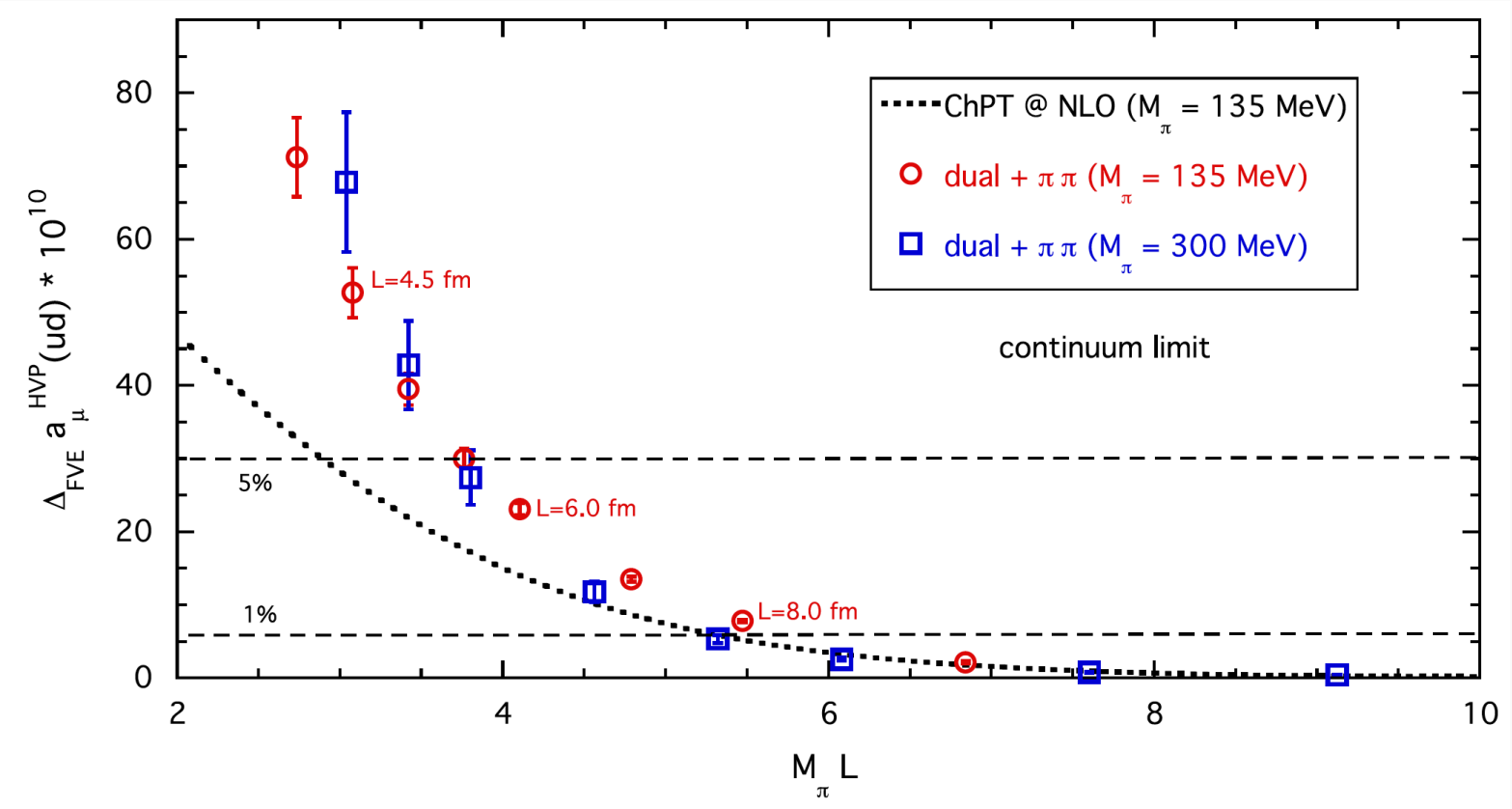


Figure: Quoted from ETM-PRD18.

Summary of FV

$$\Delta_{FV} a_{\mu,ud}(L_1, L_2) = a_{\mu,ud}^{\text{LO-HVP}}(L_2) - a_{\mu,ud}^{\text{LO-HVP}}(L_1)$$

- $\Delta_{FV} a_{\mu,ud}(4.66\text{fm}, 6.22\text{fm})$, physical M_π **[RBC/UKQCD Prelim.]**
  - XPT: 12.2×10^{-10} ,
  - LQCD: $21.6(6.3) \times 10^{-10}$,
  - GSL: $20(3) \times 10^{-10}$.
- $\Delta_{FV} a_{\mu,ud}(5.4\text{fm}, 10.8\text{fm})$, $M_\pi = 135[\text{MeV}]$ **[PACS-PRD19]**
  - LQCD: $\Delta_{FV}^{lat} a_{\mu,ud} / \Delta_{FV}^{XPT} a_{\mu,ud} = 1.74(71)$.
- $L_2 = \text{large}$, $M_\pi L_1 \sim 4$
  - XPT/RBCUK-PRL18: $16(4) \times 10^{-10}$,
  - GSL/RBCUK-Prelim: $22(1) \times 10^{-10}$,
  - XPT/BMW-PRL18: $15(15) \times 10^{-10}$,
  - GSL/Mainz-PRD19: $19.4(0.2) \times 10^{-10}$,
  - LQCD(w. GSL+dual)/ETM-PRD18: $23(1) \times 10^{-10}$.
  - XPT(LO+NLO)/ABGJPT-Prelim $(18.1 + 7.4) \times 10^{-10}$.

SIB/QED Diagrams

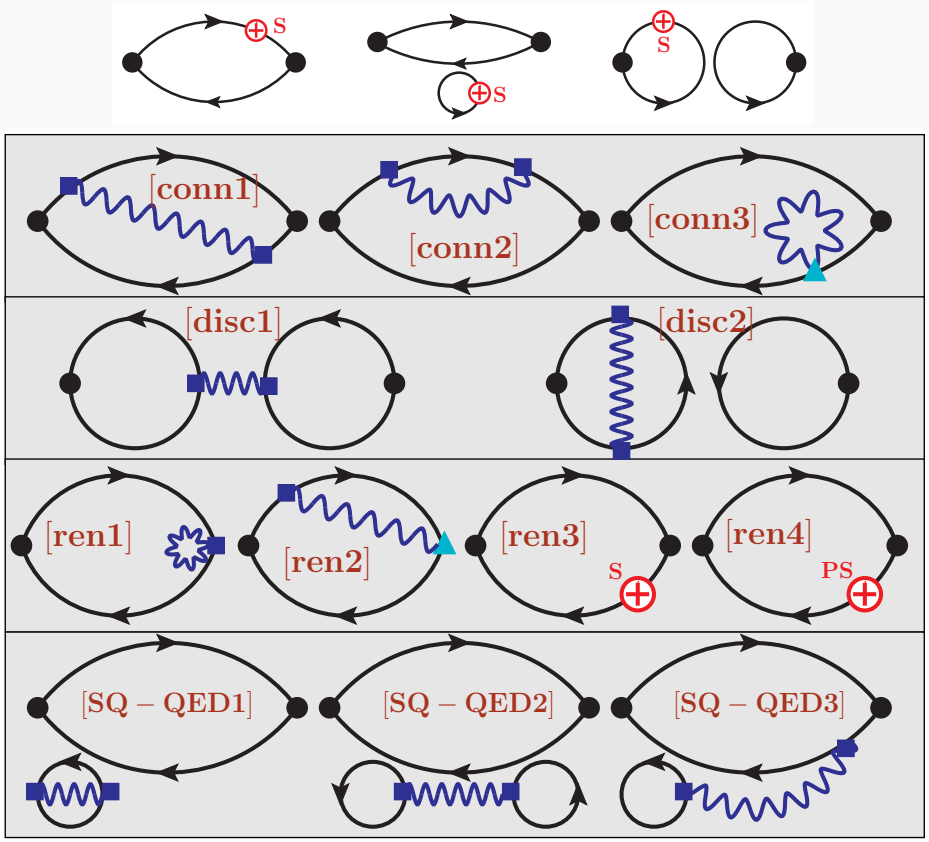


Figure: Upper: SIB corrections. ⊕ = $m_f \sum \bar{\psi}\psi$ insertion in the quark propagator. Lower: QED corrections in the perturbative method. The wavy lines = inserted photons, ■ = vector-current, ▲ = tadpole, ⊕ = (pseudo-)scalar insertions. From the first line, quark-connected, quark-disconnected, renormalization, and sea-quark QED corrections. In addition, the term "$-S_G[U]$" emerges from the derivative of the reweighting factor in terms of β.

SIB + QED Corrections, Summary

- **ETM-PRD19**
  $\delta a_\mu^{\text{LO-HVP}} \times 10^{10} = 7.1(2.9)$ (quark connected, quenched-QED).
- **BMW PRL2018**
  $\delta a_\mu^{\text{LO-HVP}} \times 10^{10} = 7.8(5.1)$ (pheno. ($\pi^0\gamma, \eta\gamma, \rho - \omega$ mix, $M_{\pi\pm}$)).
- **RBC/UKQCD PRL2018**
  $\delta a_\mu^{\text{LO-HVP}} \times 10^{10} = 9.5(10.2)$ (quark connected + one disconnected, quenched-QED.)
- **FNAL/HPQCD/MILC PRL2018**
  $\delta a_\mu^{\text{LO-HVP}} \times 10^{10} = 9.5(4.5)$ (Strong Isospin Breaking only).
- **QCDSF Prelim:**
  $\delta a_\mu^{\text{LO-HVP}} / a_\mu^{\text{LO-HVP}} \lesssim 1\%$ (Dynamical QED, $M_\pi \sim 400$[MeV])