Systematic study of the odd sector...

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Outline:

- Theoretical part:
  - Chpt
  - Odd sector
  - Resonances
- Phenomenology
  - TFF
  - $\pi^0 \to \gamma\gamma$
  - $\pi^0 \to e^+e^-$
- Summary

Theoretical part

EFT $\rightarrow$ ChPT

EFT

- separated degrees of freedom (simplification)
- building the most general Lagrangian
- ordering principle (powercounting)

example: ChPT

- goldstone bosons (spontaneous symmetry breakdown of chiral symmetry)
- Lagrangian up to NNNLO
- dimensional counting

ChPT: effective theory of low-energy QCD

[Coleman, WZ, Callan '69]: nonlinear realization of SSB

$$u(\phi) \to g_R u h^\dagger = h u g_L^\dagger$$

$$u = \exp\left(\frac{i\phi}{F_0 2}\right), \quad \frac{\phi}{\sqrt{2}} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K^0} & \frac{-2}{\sqrt{6}}\eta \end{pmatrix}$$

$$u_\mu = i(u^\dagger \partial_\mu u - \partial_\mu u u^\dagger - i u^\dagger r_\mu u + i u l_\mu u^\dagger), \quad l_\mu(r_\mu) = v_\mu - (+) a_\mu, \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \quad \chi = 2B_0 (s + ip)$$

[Weinberg '79], [Gasser, Leutwyler '84]

$$\mathcal{L}_\chi = \mathcal{L}_\chi^{(2)} + \mathcal{L}_\chi^{(4)} + \mathcal{L}_\chi^{(6)} + \mathcal{L}_\chi^{(8)}$$

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$$\mathcal{L}_\chi^{(2)} = \frac{F_0^2}{4}\langle u_\mu u^\mu + \chi_+\rangle$$

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$$\begin{aligned} \mathcal{L}_\chi^{(4)} = {} & L_1 \langle u_\mu u^\mu \rangle^2 + L_2 \langle u_\mu u_\nu \rangle \langle u^\mu u^\nu \rangle + L_3 \langle u^\mu u_\mu u^\nu u_\nu \rangle + L_4 \langle u^\mu u_\mu \rangle \langle \chi_+ \rangle \\ & + L_5 \langle u^\mu u_\mu \chi_+ \rangle + L_6 \langle \chi_+ \rangle^2 + L_7 \langle \chi_- \rangle^2 + L_8 \langle \chi_+ \chi_- \rangle \\ & - i L_9 \langle F^R_{\mu\nu} u u^\mu u^\nu u^\dagger + F^L_{\mu\nu} u^\dagger u^\mu u^\nu u \rangle + L_{10} \langle F^R_{\mu\nu} U F^{L\mu\nu} U^\dagger \rangle \\ & + H_1 \langle F^R_{\mu\nu} F^{\mu\nu R} + F^L_{\mu\nu} F^{\mu\nu L} \rangle + H_2 \langle \chi_+^2 - \chi_-^2 \rangle / 4 \end{aligned}$$

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[Weinberg '79], [Gasser, Leutwyler '84]

$$\mathcal{L}_\chi = \mathcal{L}_\chi^{(2)} + \mathcal{L}_\chi^{(4)} + \mathcal{L}_\chi^{(6)} + \mathcal{L}_\chi^{(8)}$$

$$\mathcal{L}_\chi^{(6)} = C_1 \langle u \cdot u h_{\mu\nu} h^{\mu\nu} \rangle + \ldots \text{ (together 94 terms!)}$$

where $$f_\pm^{\mu\nu} = u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u, \quad \nabla_\lambda f_\pm^{\mu\nu}, \quad h_{\mu\nu} = \nabla_\mu u_\nu + \nabla_\nu u_\mu$$

$$\chi_{\pm\mu} = u^\dagger D_\mu \chi u^\dagger \pm u D_\mu \chi^\dagger u = \nabla_\mu \chi_\pm - \frac{i}{2}\{\chi_\mp, u_\mu\}$$

[Bijnens, Colangelo, Ecker '99]

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$\mathcal{L}_\chi^{(8)} = C_1^8 \langle \nabla_\mu u^\nu \nabla_\rho u^\sigma \rangle \langle \nabla_\nu u^\mu \nabla_\sigma u^\rho \rangle + \ldots$ (together 1254 terms!)

[Bijnens, Hermansson-Truedsson, Wang '19]

Odd sector

So far, the effective Lagrangian has a larger symmetry than QCD.

$\phi \leftrightarrow -\phi$ in this Lagrangian: only even number of GB

odd intrinsic parity is however not symmetry of original QCD

from phenomenology we know it exists: $K^+K^- \to 3\pi$

$\Rightarrow$ symmetry pattern of QCD must be studied more carefully

Odd sector

- first we need to add EM interaction:
  $\partial_\mu U \to D_\mu U = \partial_\mu U + i[U, v_\mu], \qquad v_\mu \sim Q A_\mu$
- and add by hands monomial to $\mathcal{L}$:

$$U F_{\mu\nu} \tilde{F}^{\mu\nu}$$

  U can be transformed out: we have to add (at least two) derivatives on U – vanishes in chiral limit (Sutherland theorem)

way out: anomaly, in fact two anomalies (non-trivial for $i = 0, 3, 8$, or for π^0, η, η' states):

$$\partial^\mu A^i_\mu = N_f \delta^{i0} \frac{\alpha_s}{4\pi} G^{\mu\nu a} \tilde{G}^a_{\mu\nu} + a^i \frac{\alpha}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

incorporated to the action by Wess, Zumino and Witten (WZW)
two-flavour case: [Kaiser'01]

Anomalies in SM: the good, the bad and the ugly

An "anomaly": a symmetry of the classical action violated by quantum corrections

the good: Sutherland theorem in contradiction with experiment for $\pi^0 \to \gamma\gamma$. Precluded by chiral anomaly.

the bad: in EW lepton sector we have an anomaly, the theory is internally inconsistent; solution "BIM" mechanism: lepton and quark must be treated together

the ugly: dubbed $U(1)_A$ problem by Weinberg: we have 8 not 9 goldstone bosons. η' is not protected by $U(1)_A$ as this is not a good symmetry (is affected by the anomaly): $\partial_\mu J_5^\mu \sim G_a^{\mu\nu} \tilde{G}_{a\mu\nu}$
however, this is a total derivative, but it contributes ['t Hooft '76]
it begets another problem: we should add to the Lagrangian $\theta G\tilde{G}$.
But θ (including chiral quark mass phase) is small, why? $\to$ strong CP problem

Calculation within 2-flavour ChPT: Anomaly

Wess-Zumino construction ['71]
The form of anomaly is determined by 'gauging' external fields, where v and a are gauge fields of

$$L = R = 1 + i\alpha, \quad \text{and} \quad L^\dagger = R = 1 + i\beta$$

$$\delta S\{v, a, s, p, \theta\} = -\frac{N_C}{16\pi^2}\int d^4x\, \epsilon^{\mu\nu\rho\sigma}\langle\hat{\beta}(\hat{v}_{\mu\nu} + i[\hat{a}_\mu, \hat{a}_\nu])\rangle\langle v_{\rho\sigma}\rangle$$

$$\mathcal{L}_{\mathrm{WZW}} = -\frac{N_C}{32\pi^2}\,\epsilon^{\mu\nu\rho\sigma}\big\{\langle U^\dagger\hat{r}_\mu U\hat{l}_\nu - \hat{r}_\mu\hat{l}_\nu + i\Sigma_\mu(U^\dagger\hat{r}_\nu U + \hat{l}_\nu)\rangle\langle v_{\rho\sigma}\rangle + \tfrac{2}{3}\langle\Sigma_\mu\Sigma_\nu\Sigma_\rho\rangle\langle v_\sigma\rangle\big\}$$

with $\Sigma_\mu = U^\dagger\partial_\mu U$, $\hat{r}_\mu = \hat{v}_\mu + \hat{a}_\mu$, $\hat{l}_\mu = \hat{v}_\mu - \hat{a}_\mu$.

Calculation within 2-flavour ChPT

NLO odd-intrinsic Lagrangian is given by [Bijnens, Girlanda, Talavera '02]

$$\mathcal{L}_6^W = \sum_{i=1}^{13} c_i^W o_i^W \,, \qquad c_i^W = c_i^{Wr} + \eta_i (c\mu)^{d-4} \Lambda \,,$$

monomial (o_i^W)	i 2-flavour	$384\pi^2 F^2 \eta_i$
$\epsilon^{\mu\nu\alpha\beta}\langle \chi_+ [f_{-\mu\nu}, u_\alpha u_\beta]\rangle$	1	0
$\epsilon^{\mu\nu\alpha\beta}\langle \chi_- \{f_{+\mu\nu}, u_\alpha u_\beta\}\rangle$	2	0
$i\epsilon^{\mu\nu\alpha\beta}\langle \chi_- f_{+\mu\nu} f_{+\alpha\beta}\rangle$	3	0
$i\epsilon^{\mu\nu\alpha\beta}\langle \chi_- f_{-\mu\nu} f_{-\alpha\beta}\rangle$	4	0
$i\epsilon^{\mu\nu\alpha\beta}\langle \chi_+ [f_{+\mu\nu}, f_{-\alpha\beta}]\rangle$	5	0
$\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle \chi_- u_\alpha u_\beta\rangle$	6	$-5N_C$
$i\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle f_{+\alpha\beta}\chi_-\rangle$	7	$4N_C$
$i\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle f_{+\alpha\beta}\rangle\langle\chi_-\rangle$	8	$-2N_C$
$i\epsilon^{\mu\nu\alpha\beta}\langle f_{+\gamma\mu}\rangle\langle h_{\gamma\nu} u_\alpha u_\beta\rangle$	9	$2N_C$
$i\epsilon^{\mu\nu\alpha\beta}\langle f_{+\gamma\mu}\rangle\langle f_{-\gamma\nu} u_\alpha u_\beta\rangle$	10	$-6N_C$
$\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle f_{+\gamma\alpha} h_{\gamma\beta}\rangle$	11	$4N_C$
$\epsilon^{\mu\nu\alpha\beta}\langle f_{+\mu\nu}\rangle\langle f_{+\gamma\alpha} f_{-\gamma\beta}\rangle$	12	0
$\epsilon^{\mu\nu\alpha\beta}\langle \nabla_\gamma f_{+\gamma\mu}\rangle\langle f_{+\nu\alpha} u_\beta\rangle$	13	$-4N_C$

n.b. it depends on the form of $\mathcal{L}_4$ [KK, Novotny 02], [Ananth.,Moussallam 02]

ChPT and resonances, even sector

[Ecker, Gasser, Pich, de Rrafael '89]: transformation properties as octet, singlet under $SU(3)_V$:

$$R \to hRh^\dagger$$
$$R_1 \to R_1$$

R in case of vector resonances is described by means of Proca or antisymmetric tensor fields

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R in case of vector resonances is described by means of Proca or antisymmetric tensor fields
LO interaction terms:

$$\mathcal{L} = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{\sqrt{2}}\langle V_{\mu\nu} u^\mu u^\nu\rangle$$

decays $\rho^0 \to e^+e^-$ and $\rho \to 2\pi$:

$$|F_V| = 154\,\text{MeV} \quad |G_V| = 69\,\text{MeV}$$

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$\to$ we can study effect on LECs L_i: whenever vector resonances contribute, they dominate

Odd basis construction

1. Partial integration
2. Equation of motion

$$\nabla^\mu u_\mu = \frac{\mathrm{i}}{2}\left(\chi_- - \frac{1}{N_F}\langle\chi_-\rangle\right)$$

3. Bianchi identities

$$\nabla_\mu\Gamma_{\nu\rho}+\nabla_\nu\Gamma_{\rho\mu}+\nabla_\rho\Gamma_{\mu\nu} = 0 \qquad \text{for} \qquad \Gamma_{\mu\nu} = \frac{1}{4}[u_\mu, u_\nu]-\frac{i}{2}f_{+\mu\nu}$$

4. Schouten identity

$$g_{\sigma\rho}\epsilon_{\alpha\beta\mu\nu} + g_{\sigma\alpha}\epsilon_{\beta\mu\nu\rho} + g_{\sigma\beta}\epsilon_{\mu\nu\rho\alpha} + g_{\sigma\mu}\epsilon_{\nu\rho\alpha\beta} + g_{\sigma\nu}\epsilon_{\rho\alpha\beta\mu} = 0$$

5. Identity

$$\nabla^\mu h_{\mu\nu} = \nabla_\nu h^\mu_\mu - 2\,[u^\mu, i\,\Gamma_{\mu\nu}] - \nabla^\mu f_{-\mu\nu}$$

Result

$$\mathcal{L}_{R\chi T}^{(6,\,\text{odd})} = \sum_X \sum_i \kappa_i^X \varepsilon^{\mu\nu\alpha\beta} \widehat{\mathcal{O}}^X_{i\,\mu\nu\alpha\beta}$$

where $X = V$, A, P, S, VV, AA, SA, SV, VA, PA, PV, VVP, VAS, AAP

and monomials are summarized in following tables.

It concludes work started in [Ruiz-Femenía, Pich, Portolés '03]

Result: one vector resonance field

i	$\widehat{\mathcal{O}}^V_{i\,\mu\nu\alpha\beta}$	i	$\widehat{\mathcal{O}}^V_{i\,\mu\nu\alpha\beta}$
1	$\mathrm{i}\langle V^{\mu\nu}(h^{\alpha\sigma}u_\sigma u^\beta - u^\beta u_\sigma h^{\alpha\sigma})\rangle$	11	$\langle V^{\mu\nu}\{f_+^{\alpha\rho}, f_-^{\beta\sigma}\}\rangle g_{\rho\sigma}$
2	$\mathrm{i}\langle V^{\mu\nu}(u_\sigma h^{\alpha\sigma}u^\beta - u^\beta h^{\alpha\sigma}u_\sigma)$	12	$\langle V^{\mu\nu}\{f_+^{\alpha\rho}, h^{\beta\sigma}\}\rangle g_{\rho\sigma}$
3	$\mathrm{i}\langle V^{\mu\nu}(u_\sigma u^\beta h^{\alpha\sigma} - h^{\alpha\sigma}u^\beta u_\sigma)\rangle$	13	$\mathrm{i}\langle V^{\mu\nu}f_+^{\alpha\beta}\rangle\langle\chi_-\rangle$
4	$\mathrm{i}\langle[V^{\mu\nu}, \nabla^\alpha\chi_+]u^\beta\rangle$	14	$\mathrm{i}\langle V^{\mu\nu}\{f_+^{\alpha\beta}, \chi_-\}\rangle$
5	$\mathrm{i}\langle V^{\mu\nu}[f_-^{\alpha\beta}, u_\sigma u^\sigma]\rangle$	15	$\mathrm{i}\langle V^{\mu\nu}[f_-^{\alpha\beta}, \chi_+]\rangle$
6	$\mathrm{i}\langle V^{\mu\nu}(f_-^{\alpha\sigma}u^\beta u_\sigma - u_\sigma u^\beta f_-^{\alpha\sigma})\rangle$	16	$\langle V^{\mu\nu}\{\nabla^\alpha f_+^{\beta\sigma}, u_\sigma\}\rangle$
7	$\mathrm{i}\langle V^{\mu\nu}(u_\sigma f_-^{\alpha\sigma}u^\beta - u^\beta f_-^{\alpha\sigma}u_\sigma)\rangle$	17	$\langle V^{\mu\nu}\{\nabla_\sigma f_+^{\alpha\sigma}, u^\beta\}\rangle$
8	$\mathrm{i}\langle V^{\mu\nu}(f_-^{\alpha\sigma}u_\sigma u^\beta - u^\beta u_\sigma f_-^{\alpha\sigma})\rangle$	18	$\langle V^{\mu\nu}u^\alpha u^\beta\rangle\langle\chi_-\rangle$
9	$\langle V^{\mu\nu}\{\chi_-, u^\alpha u^\beta\}\rangle$		
10	$\langle V^{\mu\nu}u^\alpha\chi_- u^\beta\rangle$		

Similarly for other resonances. Details in [KK,Novotny'11]. New on-going update with T.Kadavy.

Phenomenology part

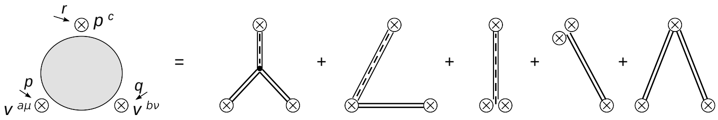
Transition formfactor $\mathcal{F}_{\pi\gamma\gamma}$

$\mathcal{F}_{\pi^*\gamma^*\gamma^*}$ can be obtained from QCD 3-point function

$$\Pi^{abc}_{\mu\nu}(p,q) = \int d^4x d^4y \mathrm{e}^{ip\cdot x + iq\cdot y} \langle 0 | T[V^a_\mu(x) V^b_\nu(y) P^c(0)] | 0 \rangle$$

- described by odd-sector in ChPT (C^W_7 and C^W_{22})
- can be enlarged by resonances [KK,Novotny'11]
- VVP can be connected with other processes: $\rho \to \pi\gamma$, $\pi(1300) \to \gamma\gamma$, $\pi(1300) \to \rho\gamma$

Applications: VVP in $R\chi T$



$$\frac{1}{B_0}\Pi^{\mathsf{R}\chi\mathsf{T}}(p^2,q^2,r^2) =$$

$$= -\frac{N_C}{16\pi^2 r^2} + \frac{4F_V^2\kappa_3^{VV}p^2}{r^2(p^2-M_V^2)(q^2-M_V^2)} - \frac{16\sqrt{2}d_m F_V\kappa_3^{PV}}{(p^2-M_V^2)(r^2-M_P^2)} - \frac{32 d_m\kappa_5^P}{r^2-M_P^2}$$

$$- \frac{8d_m F_V^2\kappa^{VVP}}{(p^2-M_V^2)(q^2-M_V^2)(r^2-M_P^2)} + \frac{2F_V^2}{(p^2-M_V^2)(q^2-M_V^2)}\left[8\kappa_2^{VV}-\kappa_3^{VV}\right]$$

$$- \frac{2\sqrt{2}F_V}{r^2(p^2-M_V^2)}\left[p^2(\kappa_{16}^V+2\kappa_{12}^V) - q^2(\kappa_{16}^V-2\kappa_{17}^V+2\kappa_{12}^V) - r^2(8\kappa_{14}^V+\kappa_{16}^V+2\kappa_{12}^V)\right]$$

$$+ (p \leftrightarrow q).$$

OPE $\Rightarrow$ only two constants left

Applications: VVP in $R\chi T$, $\pi\gamma\gamma$ formfactor

We can define the $\pi\gamma\gamma$ formfactor:

$$\mathcal{F}^{\mathsf{R}\chi\mathsf{T}}_{\pi^0\gamma\gamma}(p^2,q^2;r^2) = \frac{2}{3}\frac{1}{BF}r^2\Pi^{\mathsf{R}\chi\mathsf{T}}(p^2,q^2;r^2)\,,$$

For on-shell pion the κ^{VVP} drops out:

$$\mathcal{F}^{\mathsf{R}\chi\mathsf{T}}_{\pi^0\gamma\gamma}(p^2,q^2;0) = \frac{F}{3}\frac{(p^2+q^2)(1+32\sqrt{2}\frac{d_m F_V}{F^2}\kappa_3^{PV}) - \frac{N_C}{4\pi^2}\frac{M_V^4}{F^2}}{(p^2-M_V^2)(q^2-M_V^2)}$$

Only one constant!

the Brodsky-Lepage ['80] behaviour for a large momentum

$$\text{B-L cond.:} \qquad \lim_{Q^2\to\infty}\mathcal{F}_{\pi^0\gamma\gamma}(0,-Q^2;m_\pi^2) \sim -\frac{1}{Q^2}\,,$$

Then the last parameter is fixed:

$$\kappa_3^{PV} = -\frac{F^2}{32\sqrt{2}d_m F_V}$$

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Then the last parameter is fixed (generalized by δ_{BL}) in next:

$$\kappa_3^{PV} = -\frac{F^2}{32\sqrt{2}d_m F_V}(1+\delta_{\mathsf{BL}})$$

Applications: VVP in $R\chi T$, $\pi\gamma\gamma$ formfactor

short summary of possible formfactors (cf. [Moussallam'95], [Knecht,Nyffeler'01])

$$\mathcal{F}^{\text{VMD}}_{\pi^0\gamma\gamma}(p^2,q^2;0) = -\frac{N_C}{12\pi^2 F_\pi}\frac{M_V^2}{(p^2-M_V^2)}\frac{M_V^2}{(q^2-M_V^2)}\,,$$

$$\mathcal{F}^{\text{LMD}}_{\pi^0\gamma\gamma}(p^2,q^2;0) = \frac{F_\pi}{3}\,\frac{p^2+q^2-\frac{N_C}{4\pi^2}\frac{M_V^4}{F_\pi^2}}{(p^2-M_V^2)(q^2-M_V^2)}\,,$$

$$\mathcal{F}^{\text{LMD+V}}_{\pi^0\gamma\gamma}(p^2,q^2;0) = \frac{F_\pi}{3}\,\frac{p^2q^2(p^2+q^2)+h_1(p^2+q^2)^2+h_2p^2q^2+h_5(p^2+q^2)+h_7}{(p^2-M_{V_1}^2)(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)(q^2-M_{V_2}^2)}\,,$$

$$\mathcal{F}^{\text{R}\chi\text{T}}_{\pi^0\gamma\gamma}(p^2,q^2;0) = -\frac{F}{3}\,\frac{(p^2+q^2)\delta_{BL}+\frac{N_C}{4\pi^2}\frac{M_V^4}{F^2}}{(p^2-M_V^2)(q^2-M_V^2)}$$

note:

$$\mathcal{F}^{\text{R}\chi\text{T}}_{\pi^0\gamma\gamma}(p^2,q^2;0)\Big|_{\kappa_3^{PV}=0} = \mathcal{F}^{\text{LMD}}_{\pi^0\gamma\gamma}(p^2,q^2;0)$$

$$\mathcal{F}^{\text{R}\chi\text{T}}_{\pi^0\gamma\gamma}(p^2,q^2;0)\Big|_{\text{B-L}} = \mathcal{F}^{\text{VMD}}_{\pi^0\gamma\gamma}(p^2,q^2;0)\,.$$

Applications: VVP in $R\chi T$,π^0, $g-2$

π^0 exchange in the hadronic light-by-light contribution

model	$a_\mu^{\mathsf{LbyL};\pi^0} \times 10^{11}$
VMD	57.2
LMD	73.7
resonances	65.8 ± 1.2

not updated yet - without Belle data.

$\pi^0 \to \gamma\gamma$: chiral expansion   collaboration with B. Moussallam

- π^0 lightest hadron $\Rightarrow$ primary decay mode $\pi^0 \to \gamma\gamma$
- in chiral limit exact due to QCD axial anomaly:

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{m_{\pi^0}^3}{64\pi}\left(\frac{\alpha N_C}{3\pi F_\pi}\right)^2 = 7.73\,eV$$

Correction to the current algebra prediction:

- using [Pagels and Zepeda '72] sum rules in [Kitazawa '85]
- NLO corrections are hidden in $F \to F_{\pi^0}$ and $O(p^6)$ LECs [Donoghue, Holstein, Lin '85] [Bijnens, Bramon, Cornet '88]
- in 3-flavour case we can study π^0, η, η' mixing, resulting to [Goity, Bernstein, Holstein '02]:

$$\Gamma^{\text{NLO}} = 8.1 \pm 0.08\,\text{eV}$$

  in 2-flavour case EM corrections [Ananth., Moussallam '02]:

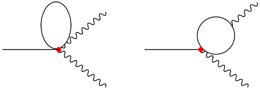
$$\Gamma^{\text{NLO}} = 8.06 \pm 0.02 \pm 0.06\,\text{eV}$$

- Quite recently another study based on dispersion relations, QCD sum rules, using only the value $\Gamma(\eta \to \gamma\gamma)$ gives [Ioffe, Oganesian '07]:

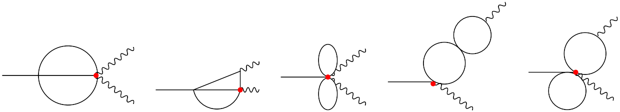
$$\Gamma^{\text{NLO}} = 7.93 \pm 0.11\,\text{eV}$$

$\pi^0 \to \gamma\gamma$ at NNLO in 2 flavour ChPT: technical part

- NLO: a) One-loop diagrams with one vertex from $\mathcal{L}^{WZ}$, b) tree diagrams with one vertex from $\mathcal{L}^{WZ}$ and one vertex from $O(p^4)$ Lagrangian, c) tree diagrams with one vertex from $O(p^6)$ anomalous-parity sector
- $O(p^6)$ anomalous-parity sector from [Bijnens, Girlanda, Talavera '02]
- representation of chiral field: $U = \sigma + i\frac{\tau.\pi}{F}$, $\sigma = \sqrt{1-\vec{\pi}^2/F^2}$ (no $\gamma 4\pi$ vertex at LO)
- one-loop

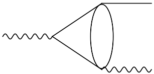


- two-loop



- verification of Z-factor, F_π/F [Bürgi '96], [Bijnens, Colangelo, Ecker, Gasser, Sainio '02]
- double log checked by Weinberg consistency rel. [Colangelo '95]

$\pi^0 \to \gamma\gamma$ technical part: some details on two-loop calculation



$$\mathcal{T} \sim \epsilon^{\mu\nu\rho\sigma} e^1_\alpha \, k^2_\rho \, e^2_\sigma \, p_\lambda$$
$$\times \int \frac{d^d l_1}{i(2\pi)^d} \frac{d^d l_2}{i(2\pi)^d} \frac{l_1^\lambda \, l_1^\mu \, l_2^\alpha \, l_2^\nu}{(l_1^2 - M^2)(l_2^2 - M^2)[(l_2 + k_1)^2 - M^2][(l_1 + l_2 - k_2)^2 - M^2]}$$

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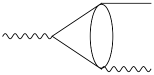

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[Bijnens, Colangelo, Ecker, Gasser, Sainio '97]

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$$J(t) = \int \frac{d^d l_1}{i(2\pi)^d} \frac{l_1^\lambda\, l_1^\mu}{(l_1^2 - M^2)((l_1 + t)^2 - M^2)} \quad \to \quad \frac{g_{\lambda\mu}}{4(2w+3)} \int_{4M^2}^\infty \frac{[d\sigma]}{\sigma - t^2} (4M^2 - \sigma)$$

$$\Rightarrow R^{\alpha\nu} = \frac{1}{4(2w+3)} \int \frac{d^d l_2}{i(2\pi)^d} \frac{l_2^\alpha\, l_2^\nu}{(l_2^2 - M^2)((l_2 + k_1)^2 - M^2)} \int_{4M^2}^\infty \frac{[d\sigma](4M^2 - \sigma)}{\sigma - t^2}$$

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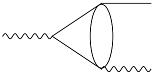

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[Bijnens, Colangelo, Ecker, Gasser, Sainio '97]

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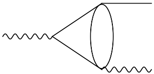
Divergences can be separated by Taylor expansion around $\sigma = \infty$

$$\int_{4M^2}^\infty [d\sigma](4M^2 - \sigma)\sigma^l = -\frac{2(2w+3)}{(4\pi)^{2+\omega}} \Gamma(-2-w-l) \frac{\Gamma(-l)}{\Gamma(-2l)} (M^2)^{w+2+l}, \qquad \text{conv. for } l < -2$$

$\pi^0 \to \gamma\gamma$ technical part: some details on two-loop calculation



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so far one would obtain:

$$R = \frac{3581}{8064} + \frac{\pi^2}{24} + R_1 + R_2\,, \qquad \text{with} \qquad R_1 = -\frac{1}{84}\int_4^\infty ds \sqrt{\frac{(s-4)^3}{s}} (\log(s) pol_1 + pol_2)$$

and R_2 can be expressed as double integral, where one integral comes from

$$I = \frac{M^6}{4} \int dx dy dz \int \frac{d^d l}{i(2\pi)^d} \frac{60\, x^3 y^2 z^3 (1-z)^4 \, l^2}{[A_z - x(1-y)B_z - l^2]^6} \quad \text{with} \quad A_z = z\sigma + (1-z)M^2, \quad B_z = z(1-z)M^2$$

n.b. possible expansion in $B_z/A_z \le 1/9$

$\pi^0 \to \gamma\gamma$ at NNLO, result

$$
\begin{aligned}
A_{NNLO} = \frac{e^2}{F_\pi}\Big\{ & \frac{1}{4\pi^2} \\
& + \frac{16}{3} m_\pi^2 (-4c_3^{Wr} - 4c_7^{Wr} + c_{11}^{Wr}) + \frac{64}{9} B(m_d - m_u)(5c_3^{Wr} + c_7^{Wr} + 2c_8^{Wr}) \\
& - \frac{M^4}{24\pi^2 F^4}\left(\frac{1}{16\pi^2} L_\pi\right)^2 + \frac{M^4}{16\pi^2 F^4} L_\pi \left[\frac{3}{256\pi^4} + \frac{32F^2}{3}\left(2c_2^{Wr} + 4c_3^{Wr} + 2c_6^{Wr} + 4c_7^{Wr} - c_{11}^{Wr}\right)\right] \\
& + \frac{32M^2 B(m_d - m_u)}{48\pi^2 F^4} L_\pi \left[-6c_2^{Wr} - 11c_3^{Wr} + 6c_4^{Wr} - 12c_5^{Wr} - c_7^{Wr} - 2c_8^{Wr}\right] \\
& + \frac{M^4}{F^4}\lambda_+ + \frac{M^2 B(m_d - m_u)}{F^4}\lambda_- + \frac{B^2(m_d - m_u)^2}{F^4}\lambda_{--} \Big\}
\end{aligned}
$$

$$
\begin{aligned}
\lambda_+ &= \frac{1}{\pi^2}\left[-\frac{2}{3} d_+^{Wr}(\mu) - 8c_6^r - \frac{1}{4}(l_4^r)^2 + \frac{1}{512\pi^4}\left(-\frac{983}{288} - \frac{4}{3}\zeta(3) + 3\sqrt{3}\,\mathrm{Cl}_2(\pi/3)\right)\right] \\
&\quad + \frac{16}{3} F^2 \left[8 l_3^r (c_3^{Wr} + c_7^{Wr}) + l_4^r(-4c_3^{Wr} - 4c_7^{Wr} + c_{11}^{Wr})\right] \\
\lambda_- &= \frac{64}{9}\left[d_-^{Wr}(\mu) + F^2 l_4^r (5c_3^{Wr} + c_7^{Wr} + 2c_8^{Wr})\right] \\
\lambda_{--} &= d_{--}^{Wr}(\mu) - 128F^2 l_7 (c_3^{Wr} + c_7^{Wr})\,.
\end{aligned}
$$

- 4 LECs in 2 combinations of NLO
- additional 4 LECs in 3 combinations of NNLO

Is it at all possible to make some reliable prediction?

$\pi^0 \to \gamma\gamma$: modified counting

- Use of $SU(3)$ phenomenology via $c_i^{Wr} \leftrightarrow C_i^{Wr}$ connection (based on [Gasser, Haefeli, Ivanov, Schmid '07,'08])

$$c_i^{Wr} = \frac{\alpha_i}{m_s} + \left(\beta_i + \gamma_{ij} C_j^{Wr} + \delta_i \ln \frac{B_0 m_s}{\mu^2}\right) + O(m_s)$$

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- implementation of modified counting

$$m_u, m_d \sim O(p^2) \quad \text{and} \quad m_s \sim O(p)$$

Result:

$$A_{NNLO}^{mod} = \frac{e^2}{F_\pi}\left\{\frac{1}{4\pi^2} - \frac{64}{3} m_\pi^2 C_7^{Wr} + \frac{1}{16\pi^2}\frac{m_d - m_u}{m_s}\left[1 - \frac{3}{2}\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\right] \right.$$
$$+ 32B(m_d - m_u)\left[\frac{4}{3} C_7^{Wr} + 4 C_8^{Wr}\left(1 - 3\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\right)\right.$$
$$\left.\left. - \frac{1}{16\pi^2 F_\pi^2}\left(3L_7^r + L_8^r - \frac{1}{512\pi^2}(L_K + \frac{2}{3} L_\eta)\right)\right] - \frac{1}{24\pi^2}\left(\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\right)^2\right\}$$

$\pi \to \gamma\gamma$: Phenomenology

- $F_\pi = 92.22 \pm 0.07$ MeV (using updated value of V_{ud} [Towner, Hardy'08]).
  rem.: if SM violated: $F_\pi \to \hat{F}_\pi$ [Bernard, Oertel, Passemar, Stern '08]

  using quark mass ratio (from lattice), pseudo-scalar meson masses, R from $\eta \to 3\pi$
- $\frac{m_d - m_u}{m_s} = (2.29 \pm 0.23)\, 10^{-2}$
- $B(m_d - m_u) = (0.32 \pm 0.03)\, M_{\pi^0}^2$
- $3L_7 + L_8^r(\mu) = (0.10 \pm 0.06)\, 10^{-3} \quad (\mu = M_\eta)$ (from pseudo-scalar meson masses formula [Gasser, Leutwyler '85])

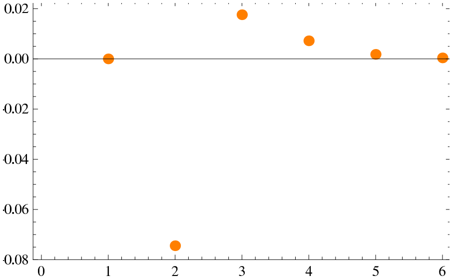
- $C_7^W = 0$ (more precisely $C_7^W \ll C_8^W$, motivated by simple resonance saturation)
- $C_8^W = (0.58 \pm 0.2) 10^{-3}\text{GeV}^{-2}$ (from $\eta \to 2\gamma$)

result $\Gamma_{\pi^0 \to 2\gamma} = (8.09 \pm 0.11)\text{eV}$

see Ashot's talk on Monday and Andreas' this morning

$\pi^0 \to \gamma\gamma$: place to improve

Leading logarithm contribution of individual orders in percent of the leading order: [Bijnens,KK,Lanz'12]



Possible week points of $\pi^0 \to \gamma\gamma$: C_8 $(\eta \to \gamma\gamma)$, $m_d - m_u$, F_π

$\hat{F}_\pi = 92.22(7)$, obtained from π_{l2} based on SM.
Other way round: from $\pi^0 \to \gamma\gamma$: $F_\pi \approx F_{\pi^0} = 93.85 \pm 1.4\,\text{MeV}$

$\pi^0 \to e^+e^-$

- first studied by [S. Drell '59]
- radiative corrections: [L.Bergström '83]
- most recent experiment: KTeV E799-II [Abouzaid'07]
- radiative corrections play important role

$\pi^0 \to e^+e^-$

KTeV's measurement:

$$\frac{\Gamma(\pi^0 \to e^+e^-,\, x > 0.95)}{\Gamma(\pi^0 \to e^+e^-\gamma,\, x > 0.232)} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}\,.$$

by extrapolating the Dalitz branching ratio to the full range of x

$$B(\pi^0 \to e^+e^-(\gamma),\, x > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}\,.$$

Extrapolating the radiative tail using Bergström:

$$B^{\text{no-rad}}_{\text{KTeV}}(\pi^0 \to e^+e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}\,.$$

Theoretical prediction [Dorokhov, Ivanov '07, '10]

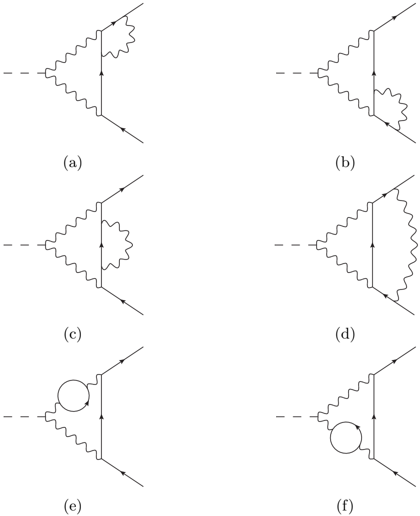
$$B^{\text{no-rad}}_{\text{SM}}(\pi^0 \to e^+e^-) = (6.23 \pm 0.09) \times 10^{-8}\,. \quad (1)$$

3.3 $\sigma \Rightarrow$ New physics?

In any case, radiative corrections play an important role in the analysis

$\pi^0 \to e^+e^-$

Radiative corrections $\to$ two-loop graphs



$\pi^0 \to e^+e^-$

- two-loop contributions, together with Bremsstrahlung (= Dalitz) [Dorokhov et al. '08], [Vasko,Novotny '11], [Husek,KK,Novotny'14]
- counter-term chiral Lagrangian for $\pi^0 l\bar{l}$ [Savage et al'92]
- modelled using the resonances [Knecht '99]

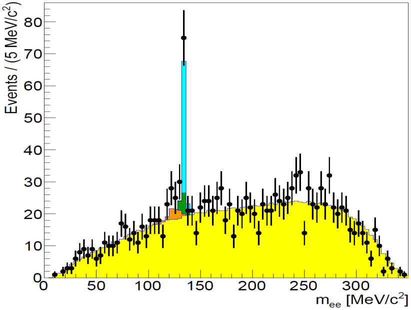
$$\chi^{(r)}_{\mathsf{LMD}}(M_\rho) = 2.2 \pm 0.9$$

- rem.: different models possible, see e.g. [Masjuan,Sanchez-Puertas '15], for $\chi = 2.76(23)$
- KTeV implies [Husek,KK,Novotny'14]

$$\chi^{(r)}_{\mathsf{KTeV}}(M_\rho) = 4.5 \pm 1.0$$

- original discrepancy down to 2σ level
- note: weak contributions mediated via $\pi^0 \to Z^* \to e^+e^-$ three orders of magnitude smaller than EM

On-going study of $\pi^0 \to e^+e^-$ at NA62



- based on 10-15% dataset of 2016-2018
- the sample smaller than had KTeV (2-3 times)
- independent cross-check of KTeV results
- EM corrections directly included [Husek,KK,Novotny'14]
- continuation after LS2 (2021-), expecting much higher statistic

Conclusion

- odd sector phenomenologically rich
- many interesting topics not covered in this talk, e.g. Dalitz [KK,Knecht,Novotny'06], [Husek, Goudzovski, KK,'18], double Dalitz [KK, Novotny, S.-Puertas], $\pi\gamma \to \pi\pi$ [Bijnens,KK,Lanz'12], [Hoferichter, Kubis, Sakkas'12], eta physcs ($\eta \to \gamma\gamma$ with J.Bijnens, [Guo,Kubis,Wirzba'12], [Kupsc et.al '12], Simon's talk this morning)
- what was briefly mentioned:
  - resonances: new study soon
  - $\pi^0 \to \gamma\gamma$: possible update via $\eta \to \gamma\gamma$
  - $\pi^0 \to e^+e^-$: new independent measurement soon
- $g-2$ connection: KTeV measurement would imply much smaller value for the pion-pole LBL contribution $\sim 40 \times 10^{-11}$
  cf. Francisca's value this morning
- new experimental cross-check coming (∼1 year)

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Thank you...