Axial vectors and transversal short-distance constraints

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Motivation

- Short-distance constraints for mixed region: OPE, VVA anomaly Melnikov, Vainshtein 2004
  - Mapping onto BTT see my talk from Mainz meeting
  - Longitudinal constraints: $\hat{\Pi}_{1-3}$, related to pseudoscalar poles see talk by L. Laub
  - Transversal constraints: all other $\hat{\Pi}_i$
- Status of the axial vectors $a_1(1260)$, $f_1(1285)$, $f_1'(1420)$
  - Large in MV: $a_\mu^{a_1+f_1+f_1'}\big|_{\text{MV}} = 22 \times 10^{-11}$ (used to saturate transversal SDCs)
  - Jegerlehner 2017: MV model violates Landau–Yang theorem

    $\hookrightarrow$ introduces antisymmetrization by hand $\Rightarrow a_\mu^{a_1+f_1+f_1'}\big|_{\text{J}} = 8 \times 10^{-11}$
  - Pauk, Vanderhaeghen 2014: Lagrangian model, $a_\mu^{f_1+f_1'}\big|_{\text{PV}} = 6 \times 10^{-11}$
- This talk:
  - BTT decomposition for axials
  - Mapping of MV model onto BTT

$\hookrightarrow$ clarify Landau–Yang, explain why MV number is so large

Axial vectors: matrix element

- Decomposition of $A \to \gamma^*\gamma^*$ amplitude

$$\langle\gamma^*(q_1,\lambda_1)\gamma^*(q_2,\lambda_2)|A(p,\lambda_A)\rangle = i(2\pi)^4\delta^{(4)}(q_1+q_2-p)e^2\epsilon_\mu^{\lambda_1 *}\epsilon_\nu^{\lambda_2 *}\epsilon_\alpha^{\lambda_A}\mathcal{M}^{\mu\nu\alpha}(q_1,q_2)$$

$$\mathcal{M}^{\mu\nu\alpha}(q_1,q_2) = \frac{i}{m_A^2}\sum_{i=1}^{3} T_i^{\mu\nu\alpha}\mathcal{F}_i(q_1^2,q_2^2)$$

$\hookrightarrow$ three form factors $\mathcal{F}_i(q_1^2,q_2^2)$

- Lorentz structures from BTT recipe

$$\begin{aligned} T_1^{\mu\nu\alpha} &= \epsilon^{\mu\nu\beta\gamma}q_{1\beta}q_{2\gamma}(q_1^\alpha - q_2^\alpha) \\ T_2^{\mu\nu\alpha} &= \epsilon^{\alpha\nu\beta\gamma}q_{1\beta}q_{2\gamma}q_1^\mu + \epsilon^{\alpha\mu\nu\beta}q_{2\beta}q_1^2 \\ T_3^{\mu\nu\alpha} &= \epsilon^{\alpha\mu\beta\gamma}q_{1\beta}q_{2\gamma}q_2^\nu + \epsilon^{\alpha\mu\nu\beta}q_{1\beta}q_2^2 \end{aligned}$$

- Crossing properties

$$\mathcal{C}_{12}\big[T_1^{\mu\nu\alpha}\big] = -T_1^{\mu\nu\alpha} \qquad \mathcal{C}_{12}\big[T_2^{\mu\nu\alpha}\big] = -T_3^{\mu\nu\alpha}$$

$$\mathcal{F}_1(q_1^2,q_2^2) = -\mathcal{F}_1(q_2^2,q_1^2) \qquad \mathcal{F}_2(q_1^2,q_2^2) = -\mathcal{F}_3(q_2^2,q_1^2)$$

$$\mathcal{F}_1(0,0) = 0 \qquad \mathcal{F}_2(0,0) = -\mathcal{F}_3(0,0)$$

Axial vectors: phenomenology

- Landau–Yang in action:

$$H_{++}(q_1^2,q_2^2)=\frac{\lambda(m_A^2,q_1^2,q_2^2)}{2m_A^3}\mathcal{F}_1(q_1^2,q_2^2)-\frac{q_1^2(m_A^2-q_1^2+q_2^2)}{2m_A^3}\mathcal{F}_2(q_1^2,q_2^2)-\frac{q_2^2(m_A^2+q_1^2-q_2^2)}{2m_A^3}\mathcal{F}_3(q_1^2,q_2^2)$$

$$\to 0 \qquad \text{for} \qquad q_1^2,q_2^2\to 0$$

- **Equivalent two-photon photon width**

$$\tilde{\Gamma}_{\gamma\gamma}=\lim_{q_1^2\to 0}\frac{m_A^2}{q_1^2}\frac{1}{2}\Gamma(A\to\gamma_L^*\gamma_T)=\frac{\pi\alpha^2 m_A}{12}\big|\mathcal{F}_2(0,0)\big|^2$$

- Experimental input from $e^+e^-\to e^+e^- f_1(')$ L3 2002, 2007

$$\tilde{\Gamma}_{\gamma\gamma}(f_1)=3.5(8)\,\text{keV} \qquad \tilde{\Gamma}_{\gamma\gamma}(f_1')\text{BR}(f_1'\to KK\pi)=3.2(9)\,\text{keV}$$

$$\Lambda_D(f_1)=1.04(8)\,\text{GeV} \qquad \Lambda_D(f_1')=0.93(8)\,\text{GeV}$$

assuming Schuler et al. 1998

$$\frac{\mathcal{F}_2(-Q^2,0)}{\mathcal{F}_2(0,0)}=\left(1+\frac{Q^2}{\Lambda_D^2}\right)^{-2} \qquad \mathcal{F}_1(-Q^2,0)=0$$

Axial vectors: mixing and $SU(3)$

- Mixing of f_1 and f_1'

$$\begin{pmatrix} f_1 \\ f_1' \end{pmatrix} = \begin{pmatrix} \cos\theta_A & \sin\theta_A \\ -\sin\theta_A & \cos\theta_A \end{pmatrix} \begin{pmatrix} f_1^0 \\ f_1^8 \end{pmatrix}$$

- Mixing angle

$$\frac{\tilde{\Gamma}_{\gamma\gamma}(f_1)}{\tilde{\Gamma}_{\gamma\gamma}(f_1')} = \frac{m_{f_1}}{m_{f_1'}} \cot^2(\theta_A - \theta_0) \qquad \theta_0 = \arcsin\frac{1}{3} \qquad \theta_A = 62(5)^\circ$$

- Assume $SU(3)$ symmetry for axial nonet ϕ

$$\text{Tr}(\mathcal{Q}^2\phi) = \frac{1}{9}\Big(3a_1 + 2\sqrt{6}f_1^0 + \sqrt{3}f_1^8\Big)$$

$$\tilde{\Gamma}_{\gamma\gamma}(a_1) = \frac{\tilde{\Gamma}_{\gamma\gamma}(f_1)}{3\cos^2(\theta_A - \theta_0)}\frac{m_{a_1}}{m_{f_1}} = \frac{\tilde{\Gamma}_{\gamma\gamma}(f_1')}{3\sin^2(\theta_A - \theta_0)}\frac{m_{a_1}}{m_{f_1'}} = 2.1\,\text{keV}$$

BTT projection of MV constraints

- MV constraint for $q_3^2 \ll q_1^2 \sim q_2^2$, $\hat{q} = (q_1 - q_2)/2$

$$\hat{\Pi}_1 = 2w_L(q_3^2)f(\hat{q}^2)$$

$$\hat{\Pi}_5 = \hat{\Pi}_6 = w_T(q_3^2)f(\hat{q}^2)$$

$$\hat{\Pi}_{10} = \hat{\Pi}_{14} = -\hat{\Pi}_{17} = -\hat{\Pi}_{39} = -\hat{\Pi}_{50} = -\hat{\Pi}_{51} = \frac{1}{q_1 \cdot q_2} w_T(q_3^2)f(\hat{q}^2)$$

$$\hat{\Pi}_i = 0 \qquad i \in \{2,3,4,7,8,9,11,13,16,54\}$$

  where

$$f(\hat{q}^2) = -\frac{1}{2\pi^2\hat{q}^2}\sum_{a=0,3,8} C_a^2 = -\frac{1}{18\pi^2\hat{q}^2} \qquad C_3 = \frac{1}{6} \qquad C_8 = \frac{1}{6\sqrt{3}} \qquad C_0 = \frac{2}{3\sqrt{6}}$$

- Non-renormalization theorems and anomaly condition in **chiral limit**
  Vainshtein 2003, Czarnecki et al. 2003, Knecht et al. 2004, ...

$$w_L(q^2) = 2w_T(q^2) = \frac{6}{q^2}$$

- Transversal relation receives **non-perturbative corrections**

Matching onto MV model

- Saturate transversal constraint from axial exchange, drop longitudinal amplitudes

$$\frac{8}{\hat{q}^2}\sum_{a=0,3,8} C_a^2 w_T(q_3^2) = \sum_{A=a_1,f_1,f_1'} \frac{1}{m_A^4}\frac{\hat{q}^2}{q_3^2-m_A^2}\phi_A(q_1^2,q_2^2)\mathcal{F}_2^A(q_3^2,0)$$

$$\phi_A(q_1^2,q_2^2) = \mathcal{F}_2^A(q_1^2,q_2^2)+\mathcal{F}_2^A(q_2^2,q_1^2) = 2\mathcal{F}_2^A(0,0)\frac{\Lambda_A^4}{(\Lambda_A^2-q_1^2)(\Lambda_A^2-q_2^2)}$$

- Conclusions
  - $\mathcal{F}_2(q_1^2,q_2^2) = -\mathcal{F}_3(q_2^2,q_1^2)$, but $\phi(q_1^2,q_2^2)$ indeed symmetric

    $\hookrightarrow$ **additional antisymmetrization** in Jegerlehner 2017 **incorrect**
  - Scaling matches for $\phi(q_1^2,q_2^2)\sim 1/\hat{q}^4$ and $\mathcal{F}_2(q_3^2,0)\to\mathcal{F}_2(0,0)$

$$1 = 9\sum_{a=0,3,8} C_a^2 \overset{?}{=} 9\sum_{A=a_1,f_1,f_1'}\frac{\tilde{\Gamma}_{\gamma\gamma}(A)}{\pi\alpha^2 m_A}\left(\frac{\Lambda_A}{m_A}\right)^4 \overset{\Lambda_A=0.77\,\text{GeV}}{=} 0.04$$

$\hookrightarrow$ axial vectors with VMD **not enough to saturate constraint**, need $\Lambda_A\sim 1.7\,\text{GeV}$

Consequences

- Original MV estimates for different mixing scenarios

$$a_\mu^{\text{ideal}}\big|_{\text{MV}} = (5.7 + 15.6 + 0.8) \times 10^{-11} = 22 \times 10^{-11}$$

$$a_\mu^{\text{octet/singlet}}\big|_{\text{MV}} = (5.7 + 1.9 + 9.7) \times 10^{-11} = 17 \times 10^{-11}$$

- Comparison in BTT
  - Model only well defined in OPE limit, need to pick kinematics in $\hat{\Pi}_{4-6}$

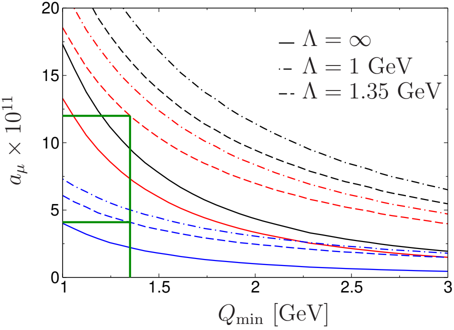
    $\hookrightarrow$ key difference to pseudoscalar poles, which are already the proper residues
  - Axial propagators modified to enforce $w_L(q^2) = 2w_T(q^2)$ at $\mathcal{O}(1/q^4)$

    $\hookrightarrow$ depends on mixing scheme not only for axials, but also for pseudoscalars
  - For VMD find similar numbers as MV
  - Increasing the VMD scale to correct $\tilde{\Gamma}_{\gamma\gamma}$ decreases a_μ^{axials} by about a factor 3

Conclusions

- MV model **does not violate the Landau–Yang theorem**, the critical combination of axial form factors is indeed symmetric
- MV model implies **significantly too large two-photon widths** $\tilde{\Gamma}_{\gamma\gamma}$
- Changing the VMD scale in the model to fix the widths decreases a_μ^{axials}
- All existing estimates for axial vectors are based on Lagrangian assumptions
  $\hookrightarrow$ need to isolate the **residues** and study the **sum rules**
- **Transversal OPE constraint** will be helpful for the **mixed regions**, just as the longitudinal one for the pseudoscalars

Outlook: matching to the quark loop



- Red: longitudinal $\hat{\Pi}_{1-3}$, blue: transversal, black: all
- Integration region

$$\theta(Q_1 - Q_{\min})\theta(Q_2 - Q_{\min})\theta(Q_3 - Q_{\min})$$
$$+\,\theta(Q_1 - Q_{\min})\theta(Q_2 - Q_{\min})\theta(Q_{\min} - Q_3)\frac{Q_3^2}{Q_3^2+\Lambda^2}$$
$$+\text{ crossed}$$

- Regge implementation of longitudinal SDCs see talk by L. Laub

$$\frac{\Delta a_\mu^{\eta} + \Delta a_\mu^{\eta'}}{\Delta a_\mu^{\pi^0}} \sim \frac{C_0^2 + C_8^2}{C_3^2} = 3 \qquad a_\mu^{\text{LSDC}} = \sum_{P=\pi^0,\eta,\eta'} \Delta a_\mu^P \sim 12 \times 10^{-11}$$

- Naive matching to the quark loop for scale $\Lambda \sim Q_{\min} \sim 1.35$ GeV
  $\hookrightarrow$ would imply transversal SDCs $a_\mu^{\text{TSDC}} \sim 4 \times 10^{-11}$
- But: axials resonances close to this scale

Encore: the charm loop

- **Perturbative QCD quark loops** with PDG masses

$$a_\mu^{c\text{-loop}} = 3.1 \times 10^{-11} \qquad a_\mu^{b\text{-loop}} = 2 \times 10^{-13} \qquad a_\mu^{t\text{-loop}} = 2 \times 10^{-15}$$

$\hookrightarrow$ charm loop borderline relevant

- What about **non-perturbative effects**?
  - Lowest-lying $c\bar{c}$ resonance: the $\eta_c(1S)$

$$m_{\eta_c(1S)} = 2.9839(5)\,\text{GeV} \qquad \Gamma(\eta_c(1S) \to \gamma\gamma) = 5.0(4)\,\text{keV}$$

  - Should couple to J/Ψ, since $\text{BR}(J/\Psi \to \eta_c(1S)\gamma) = 1.7(4)\%$ significant
  - VMD with $\Lambda = m_{J/\Psi}$ gives see talk by P. Roig at Mainz meeting

$$a_\mu^{\eta_c(1S)} = 0.8 \times 10^{-11}$$

- To avoid double counting take this as the error estimate

$$a_\mu^{c\text{-quark}} = 3(1) \times 10^{-11}$$