Lattice QCD for $|V_{cb}|$

Shoji Hashimoto (KEK, SOKENDAI)

@ "Mackenzie Fest"

Fermilab, Nov 8, 2019

Brodsky, Lepage, Mackenzie (1983)

I am a big fan of BLM

- Amusing example: position-space current correlator
  - Resumming a class of diagram by choosing
    $\tilde{\mu}^* = \tilde{\mu}\exp\left(-11/6 + 2\zeta(3)\right) \simeq 1.8\tilde{\mu}$ → ~ 2/x

    and rearranging the perturbative expansion.

$$\Pi_{SS}^{\widetilde{MS}}(x,\tilde{\mu}) = \frac{3}{\pi^4 x^6}(1 + 0.67\tilde{a}_s - 16.3\tilde{a}_s^2 - 31\tilde{a}_s^3 + 497\tilde{a}_s^4)$$

$$\Pi_{SS}^{\widetilde{MS}}(x,\tilde{\mu}^*) = \frac{3}{\pi^4 x^6}(1 + 2.9\tilde{a}_s^* + 1.1\tilde{a}_s^{*2} - 42\tilde{a}_s^{*3} + 24\tilde{a}_s^{*4})$$

$$\Pi_{VV}^{\widetilde{MS}}(x) = \frac{6}{\pi^4 x^6}(1 + \tilde{a}_s - 4\tilde{a}_s^2 - 1.9\tilde{a}_s^3 + 94\tilde{a}_s^4)$$

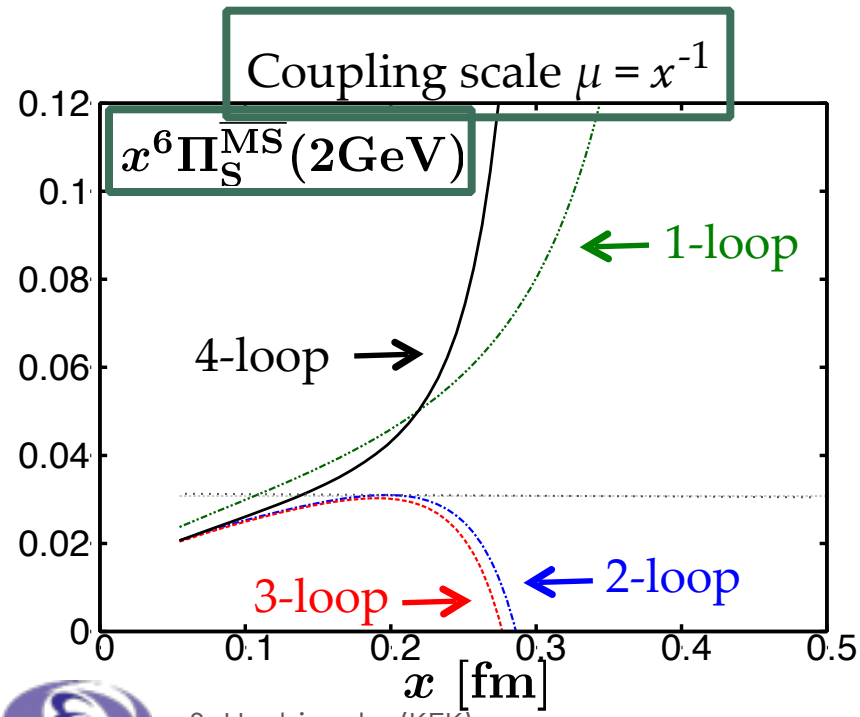
$$\Pi_{VV}^{\widetilde{MS}}(x) = \frac{6}{\pi^4 x^6}(1 + \tilde{a}_s^* + 0.083\tilde{a}_s^{*2} - 6\tilde{a}_s^{*3} + 18\tilde{a}_s^{*4}) \qquad \tilde{a}_s^* = a_s(\tilde{\mu}^*)$$

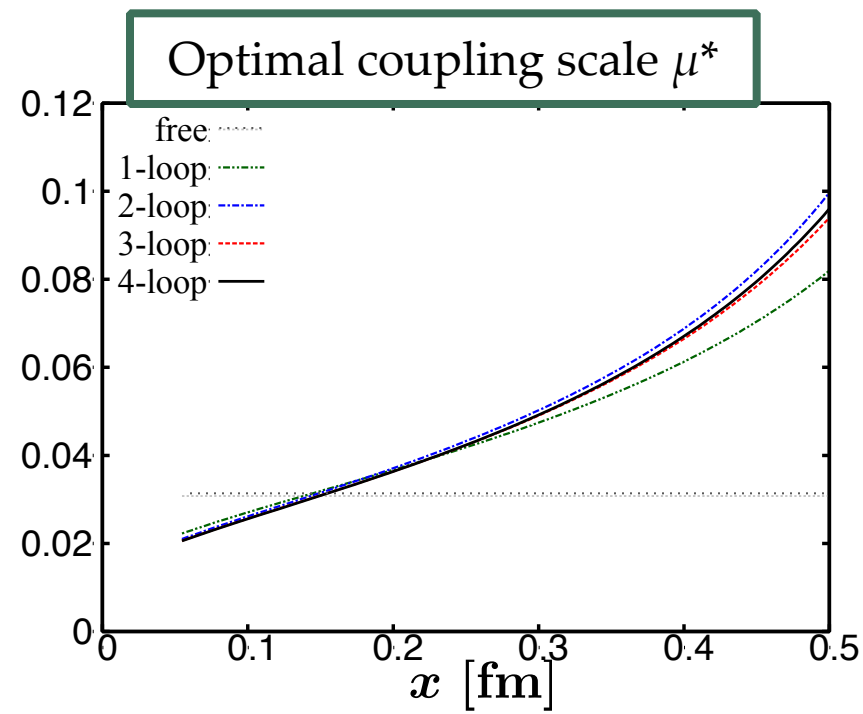
I am a big fan of BLM

$$\Pi = c_0 + c_1 a_s(\mu) + c_2 a_s(\mu)^2 + \cdots$$

$$a_s(\mu) = a_s(\mu^*) + O(a_s(\mu^*)^2)$$

$$= c_0 + c_1^* a_s(\mu^*) + c_2^* a_s(\mu^*)^2 + \cdots$$

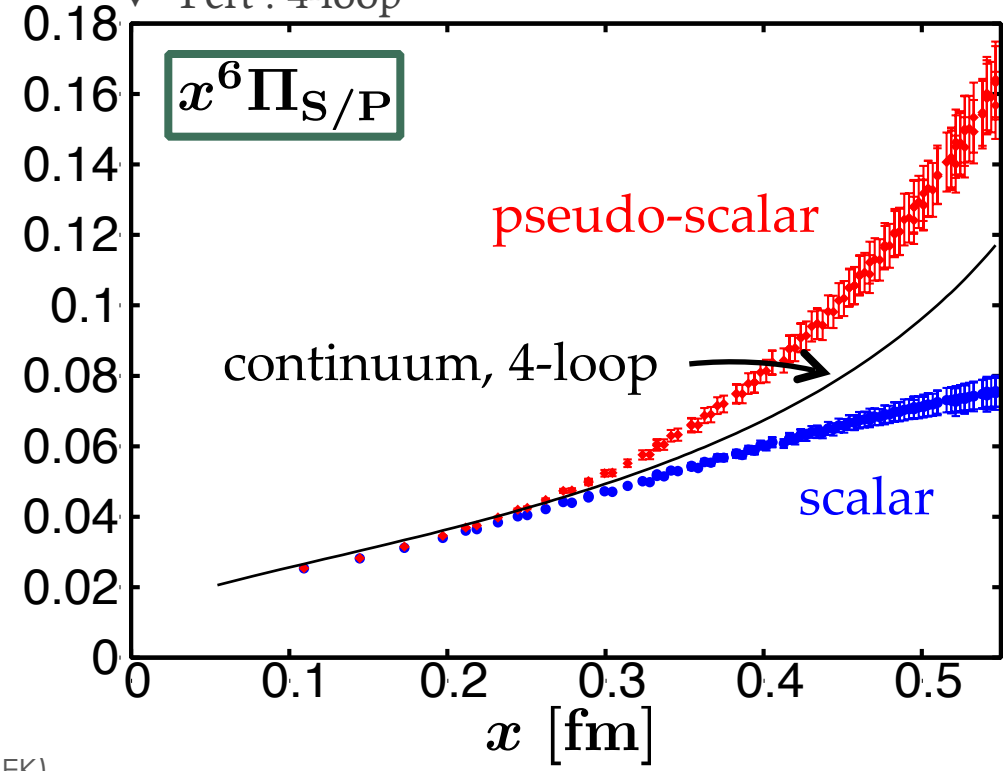




…, because we can

clearly see the transition from perturbative to non-perturbative region.

- Lattice : a^{-1} = 3.61 GeV, M_π = 300 MeV
- Pert : 4-loop



Tomii et al. (2017)

My days at Fermilab

- Oct 1997 ~ Sep 1998
  - as a postdoc, with Stephan Keller, Patricia Ball, Sinead Ryan, Hsin-Chia Cheng, Bogdan Dobrescu
  - age of 29-30, lived with my wife, Kumiko. No kids, yet.

  - a part time student of College of DuPage, on ESL classes
  - often listened to NPR (91.5 WBEZ Chicago): testimony, supreme-court, grand-jury, … impeachment (of president Clinton)
  - traveled a lot, to St. Louis, Urbana, Ohio/Kentucky, Cornell (+Niagara +Tronto), New York, Washington DC, Orlando, Madison, Boulder (+mountains+desert)

  - also worked on $B \rightarrow D^{(*)}$

Beginning: K→π

- Theory Group meeting in early October 1997
  - Everyone had to tell what he/she is interested in/going to study in a few minutes.
  - I don't remember what I talked about.
  - Paul's interest was about K→π form factor. He mentioned that double-ratio should be useful, as well as Ademollo-Gatto.

$$\frac{\langle K|V^0|\pi\rangle\langle\pi|V^0|K\rangle}{\langle K|V^0|K\rangle\langle\pi|V^0|\pi\rangle} \sim 1 + O((m_s - m_d)^2)$$

  - I knew I have only one year, and I needed a concrete project.
  - Sounds interesting, and the necessary semileptonic code is already there. So, … "Let me do it!"

$K \to \pi$

- Failed!
  - A ratio of very similar quantities … gave just a noise.
  - Quenched, β=5.7. A lot of "exceptional" configurations.
  - K and π are drastically different!
  - Indeed, the idea of MQA (modified quenched approximation) was there (Bardeen, Duncan, Eichten, Hockney, Thacker (1997)). I implemented and confirmed that the noise is gone. Nice, but none of us liked it. Anyway, my first encounter with the low-lying Dirac modes.
  - Got stuck.

$K \to \pi$

- Years later.
  - Becirevic et al. did it (2004). Still quenched, β=6.2. Much finer lattice helped.
  - We, JLQCD, did it (2005). Unquenched. No more exceptional configs.
  - many works followed…
  - And, of course, Fermilab Lattice and MILC collaborations achieved a very precise calculation ~0.2% (2018).

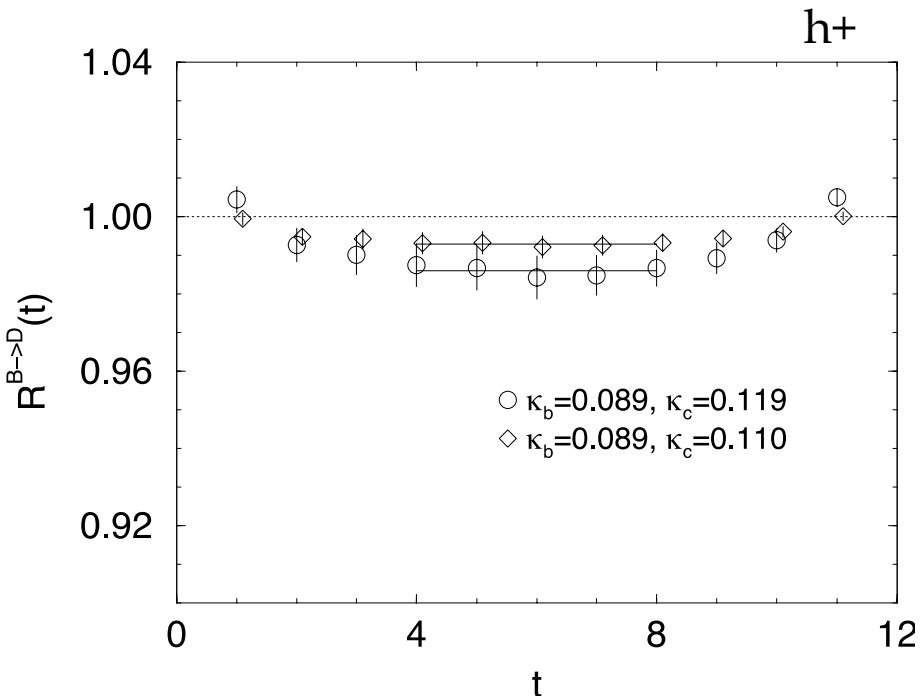
B→D

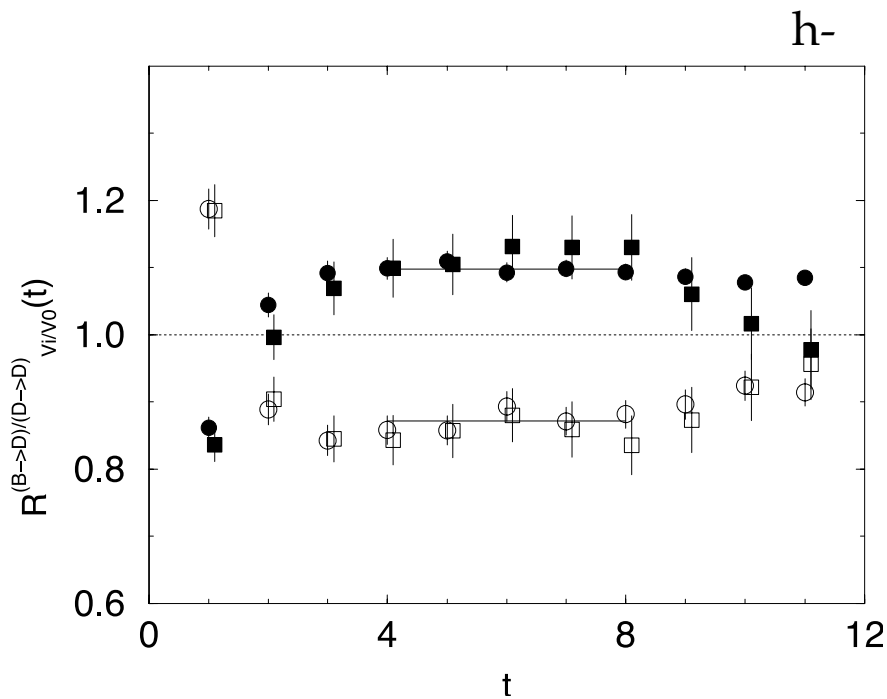
- Not a big jump to consider B→D instead
  - An item already in our first discussions
  - Decay between cousins, due to heavy quark symmetry
  - Luke's theorem (Boyd, Brahm (1991)), no O(1/m), is a version of Ademollo-Gatto for this case.

$$\frac{\langle B|V^0|D\rangle\langle D|V^0|B\rangle}{\langle B|V^0|B\rangle\langle D|V^0|D\rangle} \sim 1 + O\left(\left(\frac{1}{m_b} - \frac{1}{m_c}\right)^2\right)$$

  - HQS should work on the lattice, too… El-Khadra, Kronfeld, Mackenzie (1997).
  - Exceptional configuration is irrelevant for heavy quarks

Proof of concept: quenched β=5.7, $12^3\times24$





B→D*

- One step further
  - Something similar possible for B→D*? But, the initial and final states are different…
  - HQS helps again. No O(1/m) correction. The O(1/m^2) terms are

$$\langle D^*|A^k|B\rangle \sim 1 - \frac{\ell_V}{(2m_c)^2} + \frac{2\ell_A}{(2m_c)(2m_b)} - \frac{\ell_P}{(2m_b)^2}$$

  - The same coefficients appear in various channels

$$\frac{\langle D|V|B\rangle\langle B|V|D\rangle}{\langle D|V|D\rangle\langle B|V|B\rangle} \sim 1 - \ell_P\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2$$

$$\frac{\langle D^*|V|B^*\rangle\langle B^*|V|D^*\rangle}{\langle D^*|V|D^*\rangle\langle B^*|V|B^*\rangle} \sim 1 - \ell_V\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2$$

$$\frac{\langle D^*|A|B\rangle\langle B^*|A|D\rangle}{\langle D^*|A|D\rangle\langle B^*|A|B\rangle} \sim 1 - \ell_A\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2$$

Falk, Neubert (1992)

$B \to D^*$

- Lattice can do all!
  - With calculations at various m_c and m_b, we can determine the coefficients

$$\frac{\langle D|V|B\rangle\langle B|V|D\rangle}{\langle D|V|D\rangle\langle B|V|B\rangle} \sim 1 - \ell_P \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2$$

$$\frac{\langle D^*|V|B^*\rangle\langle B^*|V|D^*\rangle}{\langle D^*|V|D^*\rangle\langle B^*|V|B^*\rangle} \sim 1 - \ell_V \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2$$

$$\frac{\langle D^*|A|B\rangle\langle B^*|A|D\rangle}{\langle D^*|A|D\rangle\langle B^*|A|B\rangle} \sim 1 - \ell_A \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2$$

  - Going to higher orders, more combinations appear, for which we couldn't find an easy way to discriminate.

$B \to D^*$

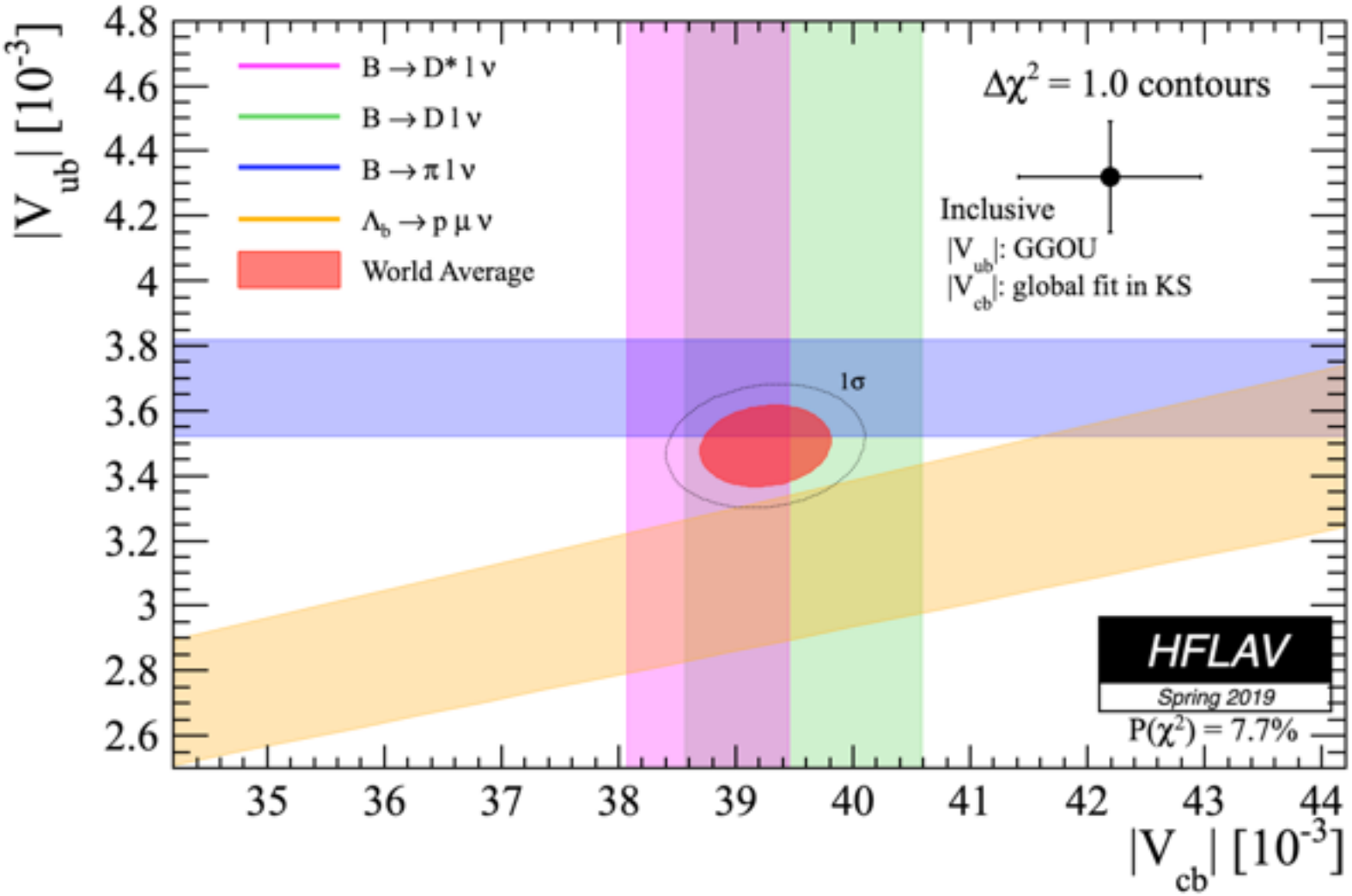
- Time is up. I had to leave.
  - Lots of things to do: product run at three β's. Various channels. (I cheated by modifying a few lines of the CANOPY code: $\gamma_5 \to \gamma_3$. HQS at work!): Jim took over the job and analysis.
  - Perturbative matching, analysis, writing…: Andreas took over.
  - And, of course, Paul supervised all the works.

  - So, I am proud of being one of the authors of
    - SH, El-Khadra, Kronfeld, Mackenzie, Ryan, Simone, $B \to D$ (1999).
    - SH, Kronfeld, Mackenzie, Ryan, Simone, $B \to D^*$ (2002).

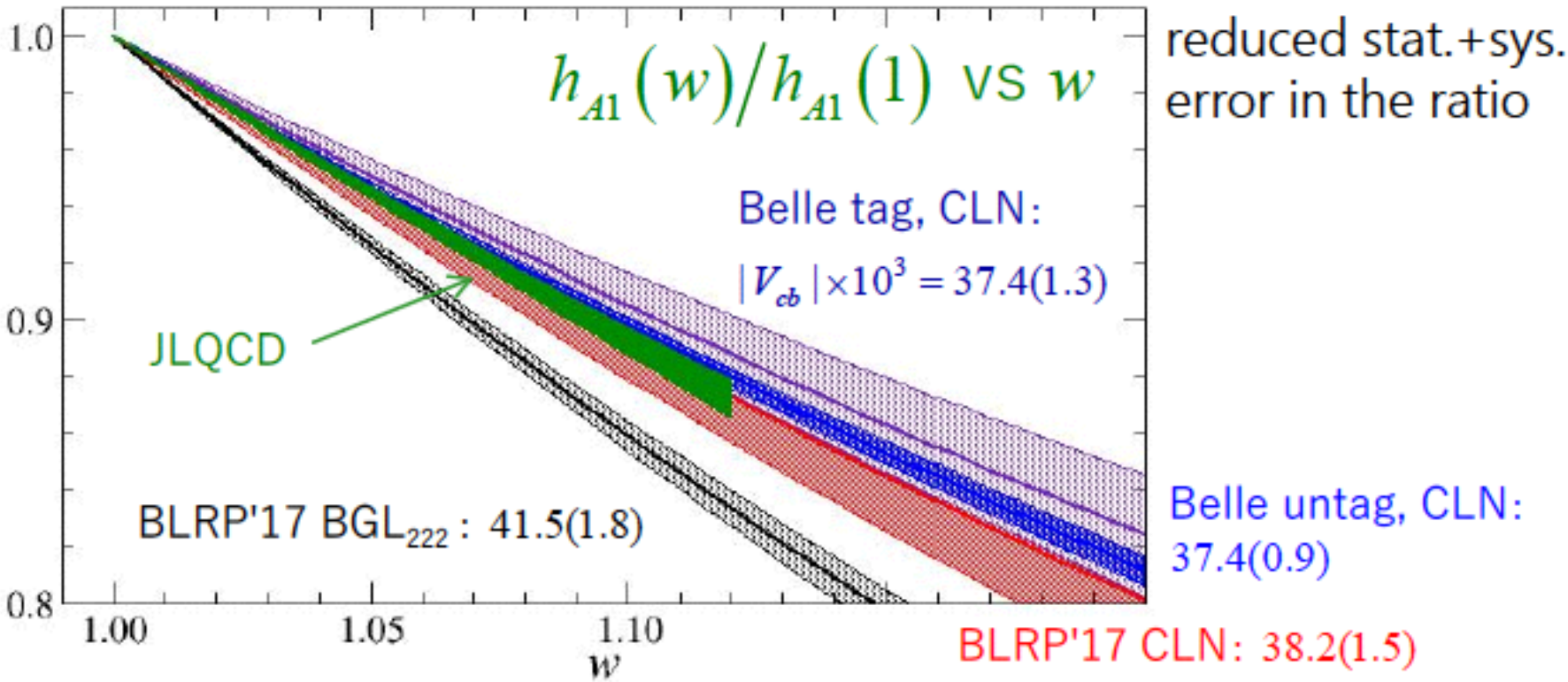
Years later

- Fermilab has been leading the field:
  - Laiho, Van de Water (2006~): Unquenched calculation, staggered χPT → Bernard et al. (2008)
  - Finer lattices: Bailey et al. (2014)
  - B→D non-zero recoil: Bailey et al. (2015)
  - Baquero, B→D* non-zero recoil (2017~)
  - more groups: HPQCD, LANL-SWME, JLQCD these days

Still, problems remain



Kaneko @ Lattice X intensity frontier WS (2019)



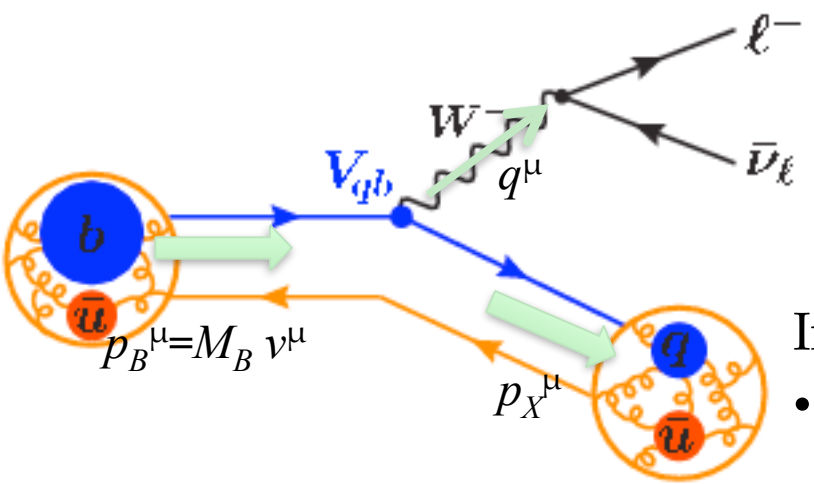
Experimental data alone cannot determine the slope well.
Lattice calculation will be of great help.

New directions?

- Everyone has their own. My favorite is the border between perturbative and non-perturbative regimes.
- Not just Euclidean current correlators. More interesting if connected to experiments.

Inclusive processes

- People say "inclusive modes are perturbative (plus OPE)."
  - Really True? Duality? I want to see.



2 kinematical variables:
- q^2 : lepton pair inv mass
- $v.q$: energy taken by leptons

Inclusive decays =
- $p_X{}^2 = m_X{}^2$ arbitrary

Summing all possible final states
- D, D^*, $D\pi$, $D\pi\pi$, …

Decay amplitude is an analytic function of q^2 and $v.q$.

Partial decay rate:

$$d\Gamma \sim |V_{cb}|^2 l^{\mu\nu} W_{\mu\nu}$$

Structure function:

$$W_{\mu\nu} = \sum_X (2\pi)^3 \delta^4(p_B - q - p_X) \frac{1}{2M_B} \langle B(p_B)|J_\mu^\dagger(0)|X\rangle\langle X|J_\nu(0)|B(p_B)\rangle$$

sum over all final states

Optical theorem: $-\frac{1}{\pi}\mathrm{Im}T_i = W_i$

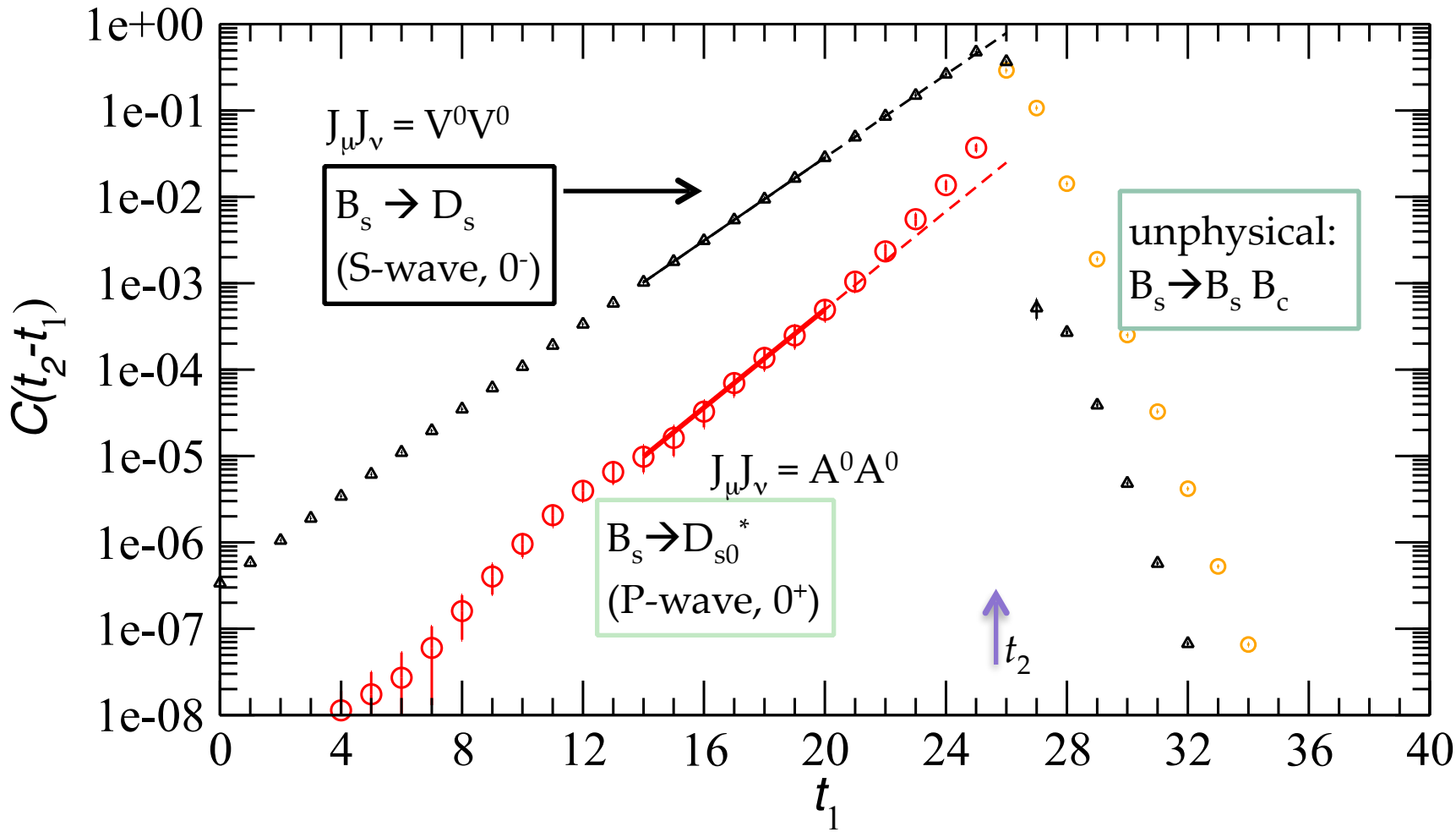
analytic function of q^2 and $v.q$

Forward scattering matrix element:

$$T_{\mu\nu} = i\int d^4x e^{-iqx} \frac{1}{2M_B} \langle B|T\{J_\mu^\dagger(x) J_\nu(0)\}|B\rangle$$

$$C^{JJ}_{\mu\nu}(t;\mathbf{q}) = \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \frac{1}{2M_B} \langle B(0)|J^\dagger_\mu(\mathbf{x},\overset{t_1}{t})\, J_\nu(\overset{t_2}{0})|B(0)\rangle$$

$m_b = 1.25^4 m_c$, zero recoil (**q=0**) 1/a = 3.6 GeV



$$T_{\mu\nu} = i\int d^4x e^{-iqx} \frac{1}{2M_B} \langle B|T\{J_\mu^\dagger(x) J_\nu(0)\}|B\rangle$$

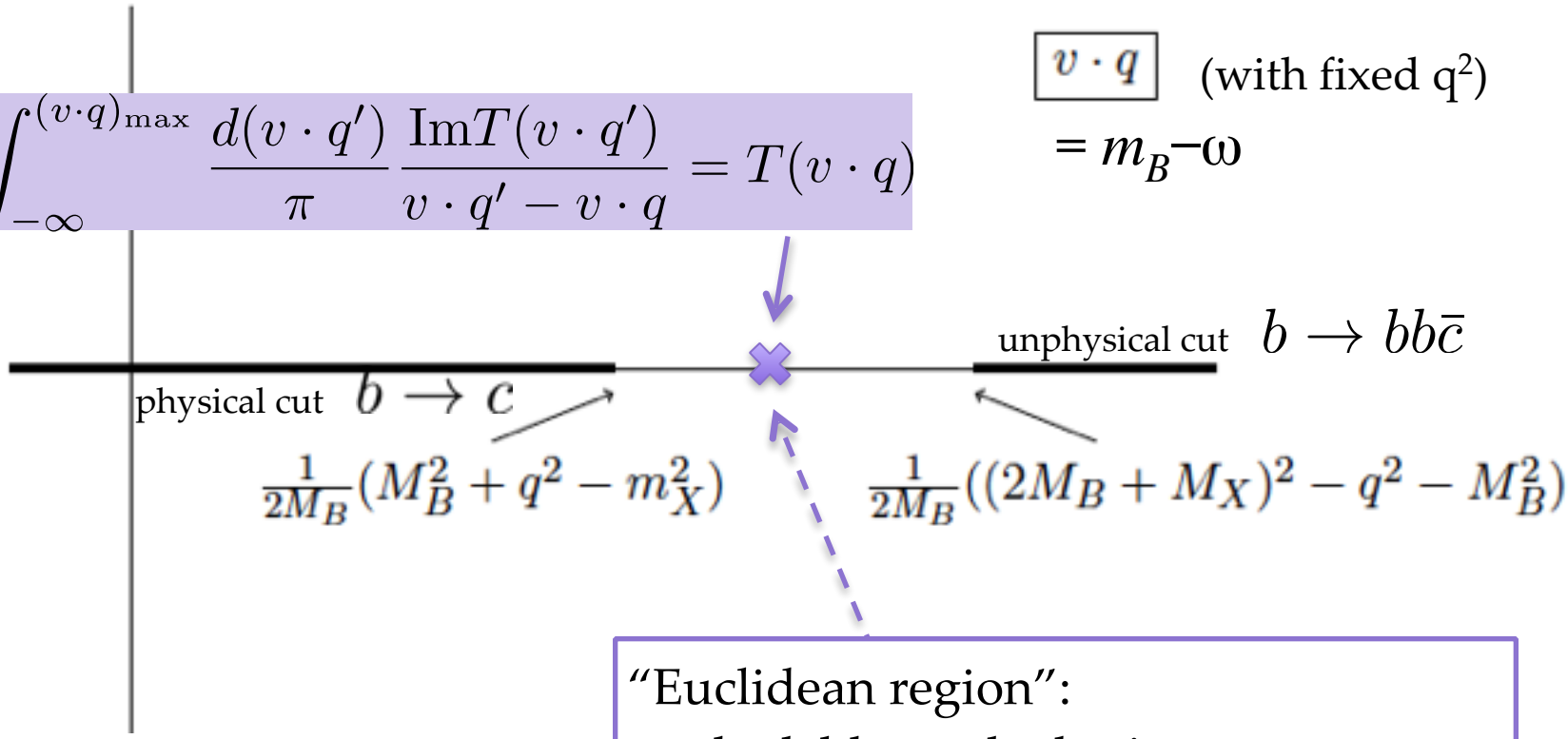
analytic function of q^2 and $v.q$

Analytic structure:

$$\int_{-\infty}^{(v\cdot q)_{\max}} \frac{d(v\cdot q')}{\pi} \frac{\mathrm{Im}T(v\cdot q')}{v\cdot q' - v\cdot q} = T(v\cdot q)$$

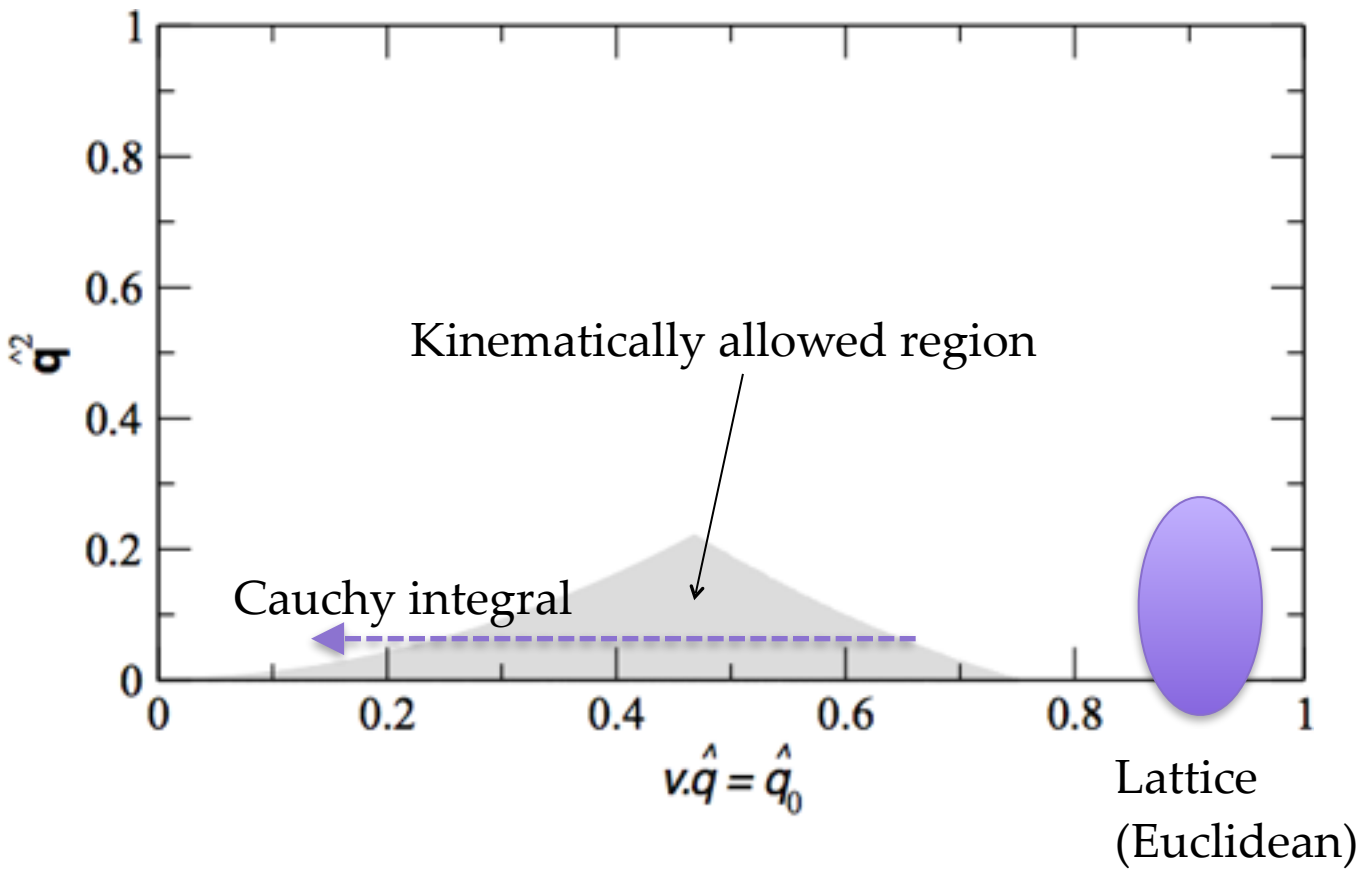
$v \cdot q$ (with fixed q^2)

$= m_B - \omega$



"Euclidean region":
calculable on the lattice
in the unphysical kinematical region

Rough sketch, yet…

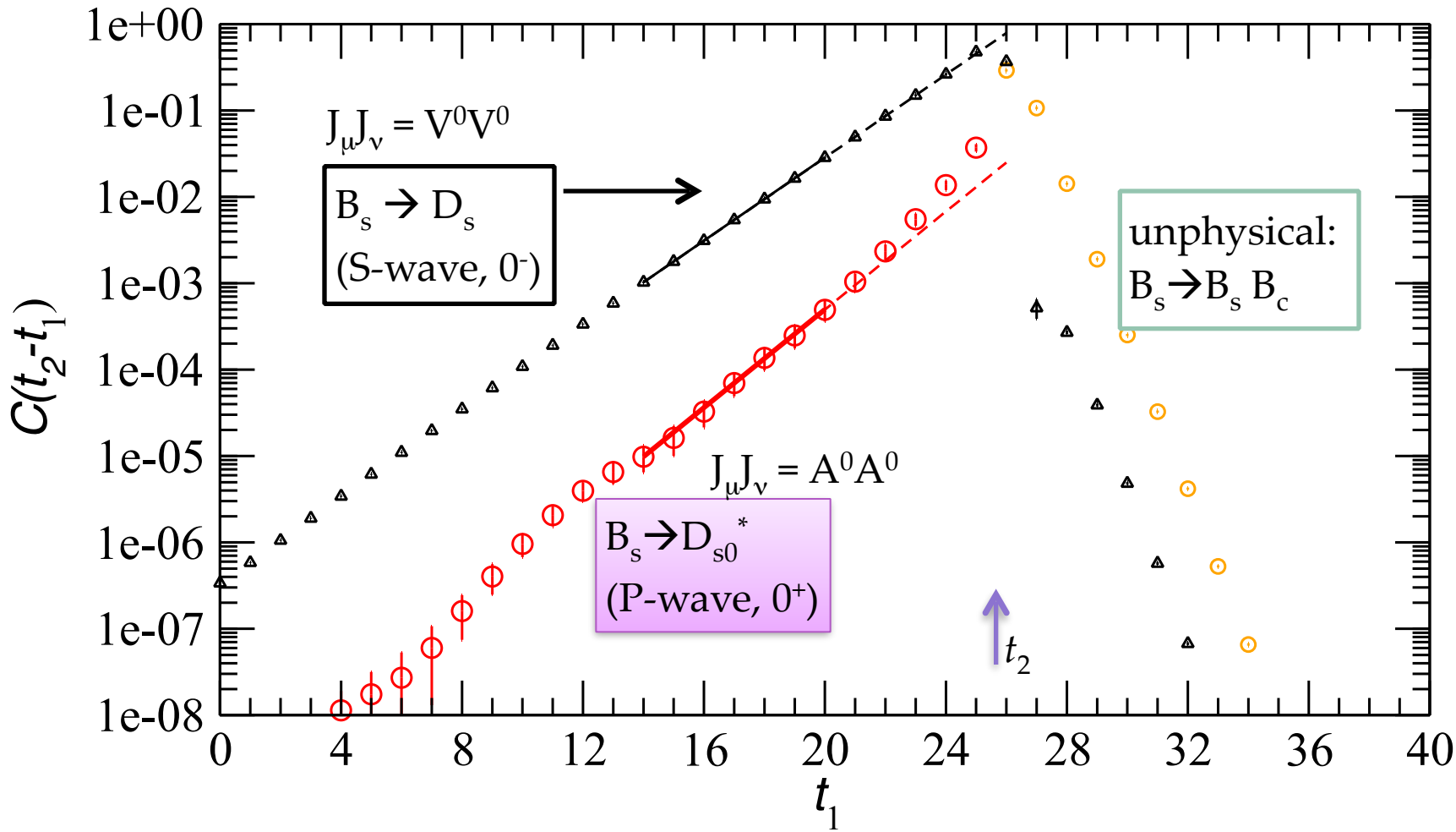


No conclusive results, yet.

But there are by-products.

$$C^{JJ}_{\mu\nu}(t;\mathbf{q}) = \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \frac{1}{2M_B} \langle B(0)|J^\dagger_\mu(\mathbf{x},\overset{t_1}{t})\, J_\nu(\overset{t_2}{0})|B(0)\rangle$$

$m_b = 1.25^4 m_c$, zero recoil (**q=0**) 1/a = 3.6 GeV

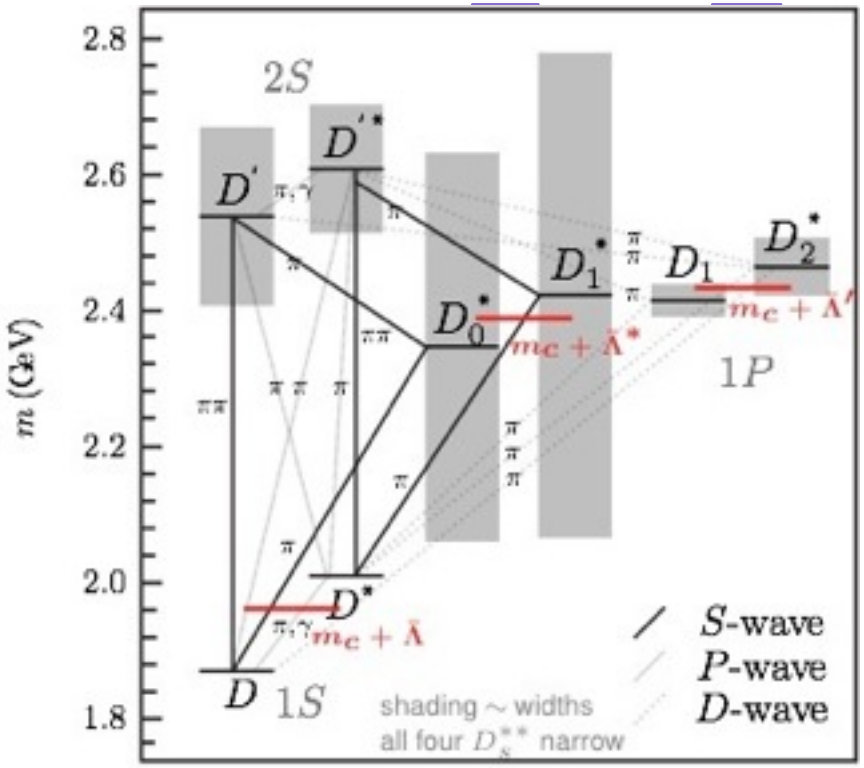


P-wave D mesons

Heavy quark symmetry: heavy quark spin decouples

- s_l = 1/2 or 3/2 for light degrees of freedom (½ + 1)

→ two doublets (D_0^*, D_1^*), (D_1, D_2^*)



Important to understand them as they appear in the event Monte Carlo used for the analysis of B→D$^{(*)}$.

"1/2 vs 3/2 puzzle" (Uraltsev et al.)

Bigi et al., EPJC 52, 975 (2007).

heavy quark limit, sum rule, quark model

theoretical estimates :

$$\Gamma(B \to D^{**}_{1/2}\ell\nu) \ll \Gamma(B \to D^{**}_{3/2}\ell\nu)$$

experimental data :

$$\Gamma(B \to D^{**}_{1/2}\ell\nu) \approx \Gamma(B \to D^{**}_{3/2}\ell\nu)$$

Are the broad states really P-wave?

Bernlochner, Ligeti, Turczyk, PRD85, 094033 (2012).

Bjorken and Uraltsev sum rules

Uraltsev, PLB 501, 86 (2001).
Le Yaouanc, Oliver, Raynal, PRD67, 114009 (2003).

- Heavy quark limit
- from B →($D^{(*)}$, D^{**}) →B

$$T^{\mu\nu}(q,v,u) = i\int d^4x\, e^{-iq\cdot x}\langle B(u)|T[J^\mu(x)J^{\nu\dagger}(0)]|B(v)\rangle$$

slope of the Isgur-Wise function (B→$D^{(*)}$)

BJ $$\rho^2 - \frac{1}{4} = 2\sum_m |\tau_{3/2}^{(m)}(1)|^2 + \sum_n |\tau_{1/2}^{(n)}(1)|^2$$

Uraltsev $$\frac{1}{4} = \sum_m |\tau_{3/2}^{(m)}(1)|^2 - \sum_n |\tau_{1/2}^{(n)}(1)|^2$$

zero-recoil (w=v.v′=1) form factors for B→D^{**}

Results

Bailas @ Lattice 2019

- assuming 1/M expansion
- "pole mass" for 1/m
- excitation energy from exp't

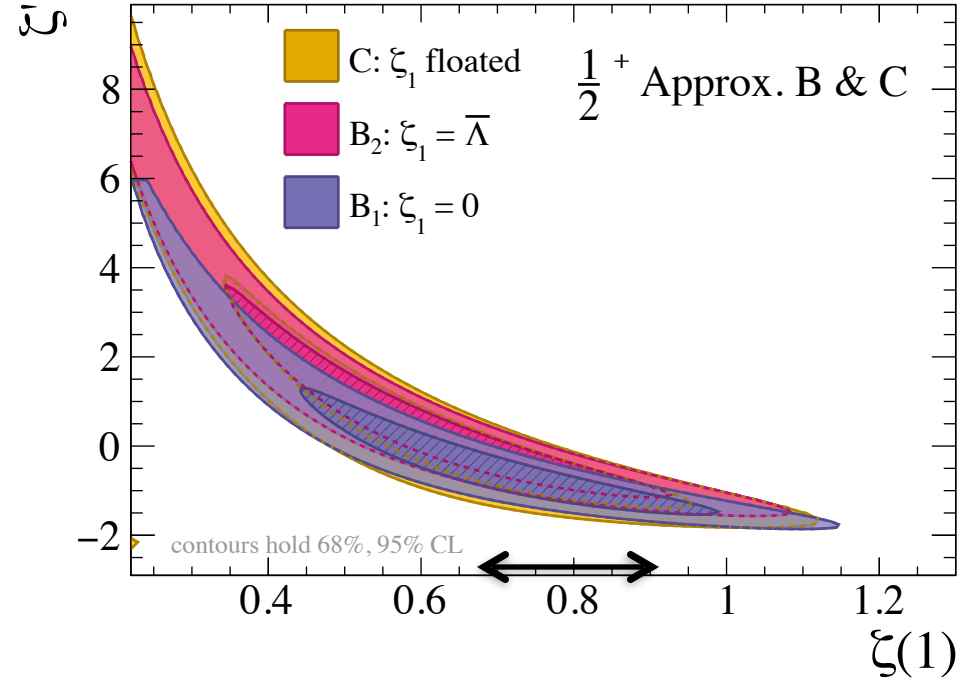
$$\zeta(1) = 0.77(13)$$
$$\tau(1) = 0.69(15)$$

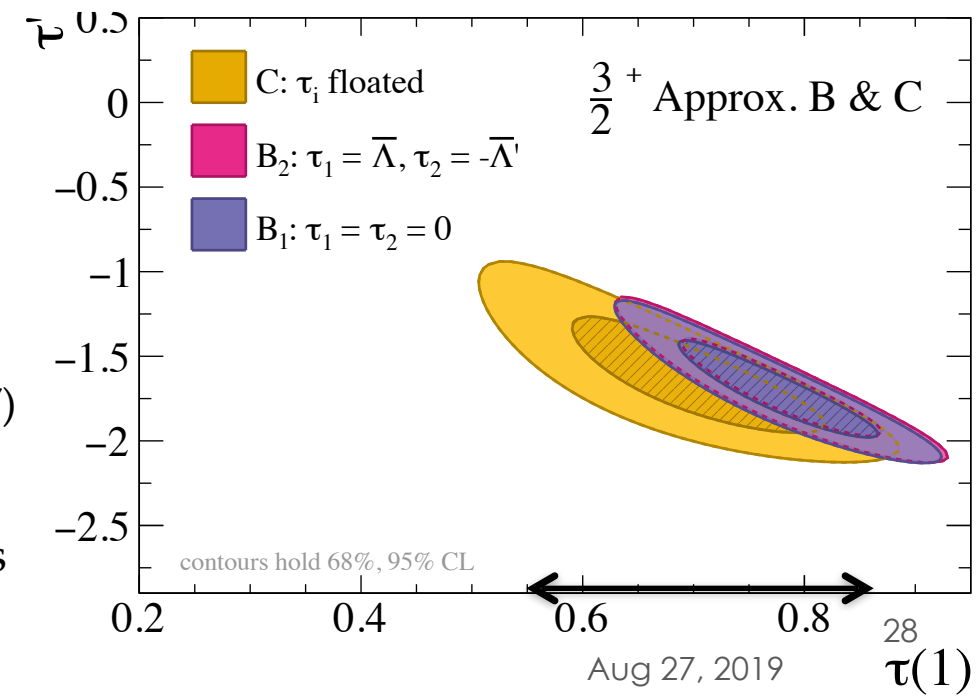
$$\tau^{(0)}_{1/2}(1) = \frac{1}{2}\zeta(1) \quad = 0.45(7)$$

$$\tau^{(0)}_{3/2}(1) = \frac{1}{\sqrt{3}}\tau(1) = 0.39(7)$$

Bernlochner, Ligeti, PRD95, 014022 (2017)

- fit of experimental values
- with some approximations





Paul said…

answering to a student's question.
(Maybe not very precise, my apology.)

Q: To me, theoretical physics is more interesting than computational physics. Why did you choose computational?

"I didn't choose. Good physicist should do **both**. There are many problems you can not solve purely theoretically. That's how physics has been developed and will."