Open Effective Field Theories

— PaulFest 2019 —

Peter Lepage

Paul's Thesis (1980-81)

From CLEO measurements (hadronic and leptonic branching fractions, R ...)

$$\frac{\Gamma(\Upsilon \to ggg\ldots)}{\Gamma(\Upsilon \to \mu^+\mu^-)} = \frac{10(\pi^2-9)}{9\pi}\frac{\alpha_s^3(\mu)}{\alpha_{\rm QED}^2}\left(1 + ?\,\frac{\alpha_s}{\pi} + \cdots\right)$$

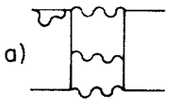
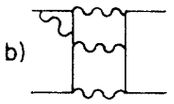
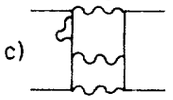
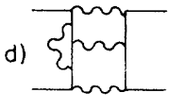
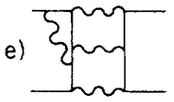
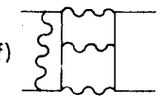
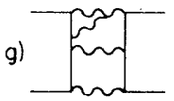
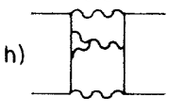
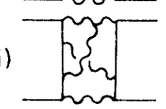
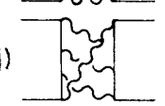
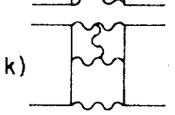
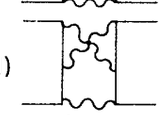
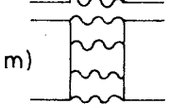
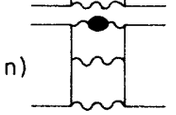
Calculate coefficient ⇒ new accurate determination of α_s.

A year later ...

P.B. Mackenzie + G.P. Lepage

Quantum Chromodynamic corrections to the gluonic width of the Υ meson

Phys. Rev. Lett. 47 (1981) 1244

	Sample Graph	Number of Graphs in Class	Coefficient of α_s/π: Finite Piece	Ultraviolet Divergence	Infrared Divergence
a)		24	-3.591	$-\frac{2}{3}P$	$-4/3 \ln \lambda^2/m_b^2$
b)		24	-0.726 ± .002	$-\frac{1}{6}P$	
c)		24	-0.802 ± .002	$-\frac{4}{3}P$	
d)		12	-0.143 ± .001	$-\frac{1}{12}P$	
e)		24	.594 ± .002		
f)		12	-10.424 ± .022		$4/3 \ln \lambda^2/m_b^2$
g)		24	3.73 ± .26	$\frac{9}{2}P$	
h)		12	9.76 ± .22	$\frac{9}{4}P$	
i)		24	0.0		
j)		12	-9.95 ± .30		
k)		18	10.59 ± .26		
l)		9	-3.02 ± .04		
m)		24	-0.19 ± .04		
n)		72	7.96 ± .02	$\frac{7}{4}P$	
Totals		315	3.79 ± .53	$\frac{25}{4}P$	0

Answer

Other activities at the time …

Problem #1: What scale μ? M_Υ, $M_\Upsilon/3$...? Hypersensitive!

Solution: BLM/LM scale setting

Brodsky, Lepage and Mackenzie

On the elimination of scale ambiguities in perturbative QCD

Phys. Rev. D28 (1983) 228

Lepage and Mackenzie

On the viability of lattice perturbation theory

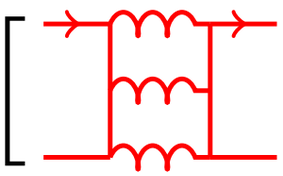
Phys. Rev. D48 (1993) 2250

$Q^* = 0.37\, M_\Upsilon$

$$\frac{\Gamma(\Upsilon \to ggg\ldots)}{\Gamma(\Upsilon \to \mu^+\mu^-)} = \frac{10(\pi^2-9)}{9\pi}\,\frac{\alpha_V^3(Q^*)}{\alpha_{\rm QED}^2}\left(1 - 8.0(5)\,\frac{\alpha_V}{\pi} + \cdots\right)$$

BIG!!

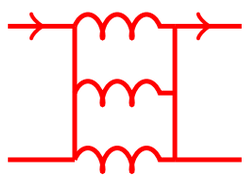
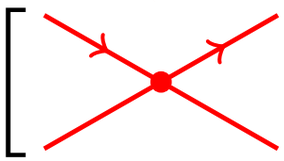
Problem #2: Is the QED-inspired formula

$$\Gamma(\Upsilon \to ggg\ldots) \propto \left|\psi_{\rm NR}(r=0)\right|^2 \,{\rm Im}\Bigg[$$  $$+\cdots\Bigg]$$

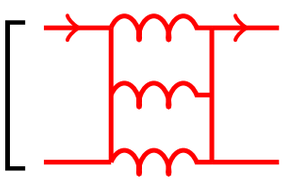
true in QCD?

Solution: NRQCD

local since far above *ggg* threshold

$$i\,{\rm Im}\Bigg[$$  $$+\cdots\Bigg] \quad\to\quad \delta\mathcal{L}_{\rm NRQCD} = i\,\frac{\gamma}{2}\Bigg[$$  $$+\,\mathcal{O}(v^2/c^2)\Bigg]$$

$\Rightarrow$

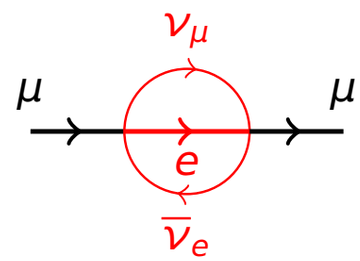
$$\Gamma(\Upsilon \to ggg\ldots) \propto \langle\Upsilon|$$ $$|\Upsilon\rangle_{\rm NRQCD} \quad {\rm Im}\Bigg[$$  $$+\cdots\Bigg]$$

cancels in $\dfrac{\Gamma(\Upsilon \to ggg\ldots)}{\Gamma(\Upsilon \to \mu^+\mu^-)}$

Problem #3 (Eg, for muon decay)

Muon NRQCD Hamiltonian has imaginary part —

$$\delta H_{\text{decay}} = -i \int d^3\mathbf{r}\ \mathrm{Im}\Bigg[$$



$$\Bigg]$$

$$= -i\,\frac{\Gamma_\mu}{2} \int d^3\mathbf{r}\ \psi_\mu^\dagger(\mathbf{r})\,\psi_\mu(\mathbf{r})$$

$$= -i\,\frac{\Gamma_\mu}{2}\,\hat{N}_\mu \quad \longleftarrow \quad \text{number operator}$$

— but the Hamiltonian conserves the number of muons.

How can it cause muons to disappear?

Muon NRQCD: Czarnecki, Lepage and Marciano, Phys. Rev. D61 (2000) 073001

Multi-muon states from the density matrix:

$$i\frac{d\hat{\rho}}{dt} = \left[H_{\mathrm{NRQCD}}\,,\,\hat{\rho}\right] - i\frac{\Gamma_\mu}{2}\left\{\hat{N}_\mu\,,\,\hat{\rho}\right\}$$

Define

$$P_n(t) = \sum_{X_n} \left\langle X_n \middle| \hat{\rho}(t) \middle| X_n \right\rangle$$

$= $ probability of finding n muons

implies

$$\frac{d}{dt}P_n(t) = -n\,\Gamma_\mu\,P_n(t)$$

Probability leaking away like $\exp(-n\Gamma_\mu t)$, but where is it going? Number of muons is unchanged.

Solution: Need effective density matrix, tracing out the decay products (*open effective theory*),

$$\hat{\rho}_{\rm eff}(t) \equiv {\rm Tr}_{decay}\big(\hat{\rho}(t)\big)$$

⇒ Lindblad Eq'n:

$$i\,\frac{d\hat{\rho}_{\rm eff}}{dt} = \Big[H_{\rm NRQCD}\,,\,\hat{\rho}_{\rm eff}\Big] - i\frac{\Gamma_\mu}{2}\Big\{\hat{N}_\mu\,,\,\hat{\rho}_{\rm eff}\Big\} + i\,\Gamma_\mu \int d^3\mathbf{r}\;\psi_\mu(\mathbf{r})\,\hat{\rho}_{\rm eff}\,\psi^\dagger_\mu(\mathbf{r})$$

⇒ for example,

$$\frac{d}{dt}P_n(t) = -n\,\Gamma_\mu\,P_n(t) + (n+1)\,\Gamma_\mu\,P_{n+1}(t)$$

prob. lost from decays of n-particle states

prob. gained from decays of $(n+1)$-particle states

Braaten, Hammer and Lepage, Phys. Rev. D94 (2016) 056006

Solution for Υs

$$\delta H_{\text{decay}} = -i\frac{\gamma}{2}\int d^3\mathbf{r}\ \psi_b^\dagger \boldsymbol{\sigma}\psi_{\bar{b}} \cdot \psi_{\bar{b}}^\dagger \boldsymbol{\sigma}\psi_b$$

$$\equiv -\frac{i}{2}\int d^3\mathbf{r}\ \mathbf{L}^\dagger(\mathbf{r})\cdot\mathbf{L}(\mathbf{r})$$

creates $b\bar{b}$'s

$\mathbf{L} \equiv \sqrt{\gamma}\,\psi_{\bar{b}}^\dagger \boldsymbol{\sigma}\,\psi_b$ annihilates $b\bar{b}$'s

implies

$$i\frac{d\hat{\rho}_{\text{eff}}}{dt} = \left[H_{\text{NRQCD}}\,,\,\hat{\rho}_{\text{eff}}\right] - \frac{i}{2}\left\{\int d^3\mathbf{r}\ \mathbf{L}^\dagger(\mathbf{r})\cdot\mathbf{L}(\mathbf{r})\,,\,\hat{\rho}_{\text{eff}}\right\}$$

$$+\ i\int d^3\mathbf{r}\ \mathbf{L}(\mathbf{r})\cdot\hat{\rho}_{\text{eff}}\,\mathbf{L}^\dagger(\mathbf{r})$$

Stochastic Wavefunction Evolution

Castin et al, 0805.4002

Complex Hamiltonian: $H = H_s - \frac{i}{2}\sum_m L_m^\dagger L_m$

Evolve $|\psi(t)\rangle$ in three steps:

1. Define $\left|\psi^{(1)}(t+dt)\right\rangle \equiv \left(1 - iH\,dt\right)\left|\psi(t)\right\rangle$ where

$$\left\langle\psi^{(1)}(t+dt)\middle|\psi^{(1)}(t+dt)\right\rangle = 1 - \sum_m \delta p_m$$

$$\delta p_m = dt\left\langle\psi(t)\middle|L_m^\dagger L_m\middle|\psi(t)\right\rangle$$

$$= \text{prob. of decay to state } L_m\left|\psi(t)\right\rangle$$

2. Random choice:

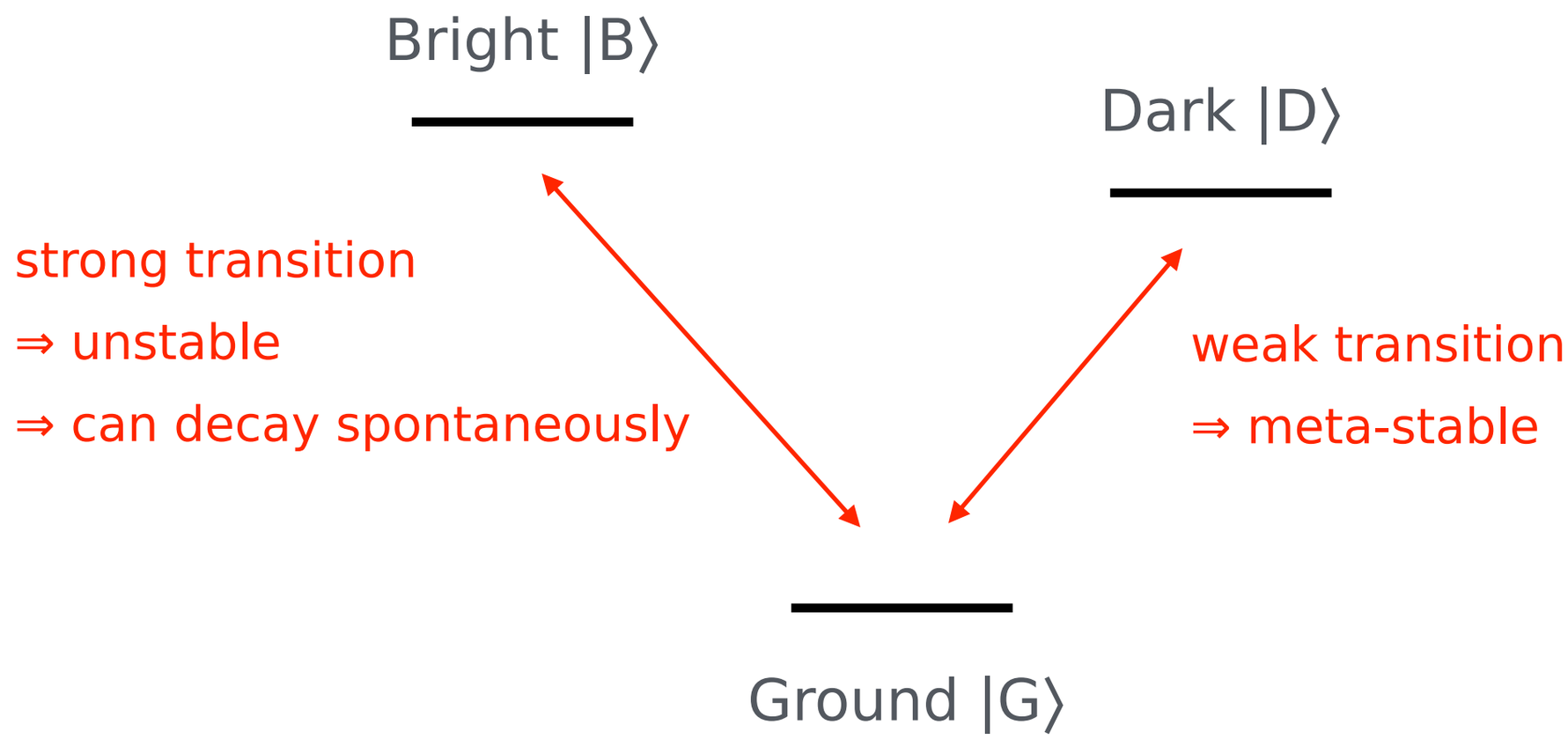
$$\left|\psi(t+dt)\right\rangle \propto \begin{cases} \left|\psi^{(1)}(t+dt)\right\rangle & \text{with prob. } 1 - \sum_m \delta p_m \\ L_m\left|\psi(t)\right\rangle & \text{with prob. } \delta p_m \end{cases}$$

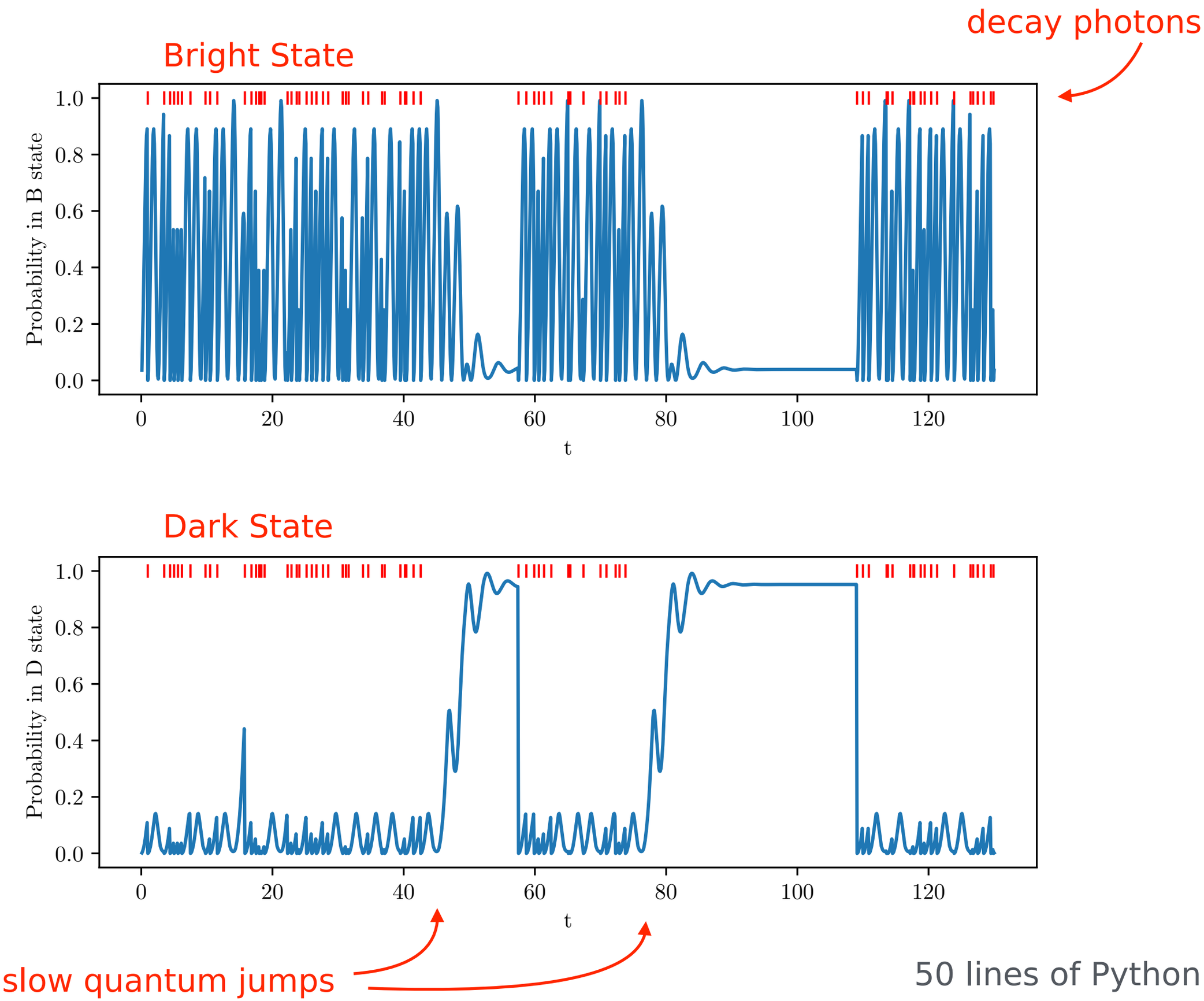
decay occurs, wavefunction collapses

3. Normalize wavefunction to 1.

Example: 3-level atom

Drive Ground → Bright and Ground → Dark transitions with tuned lasers:




decay photons
Bright State
Probability in B state
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100
120
t
Dark State
Probability in D state
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100
120
t
slow quantum jumps
50 lines of Python

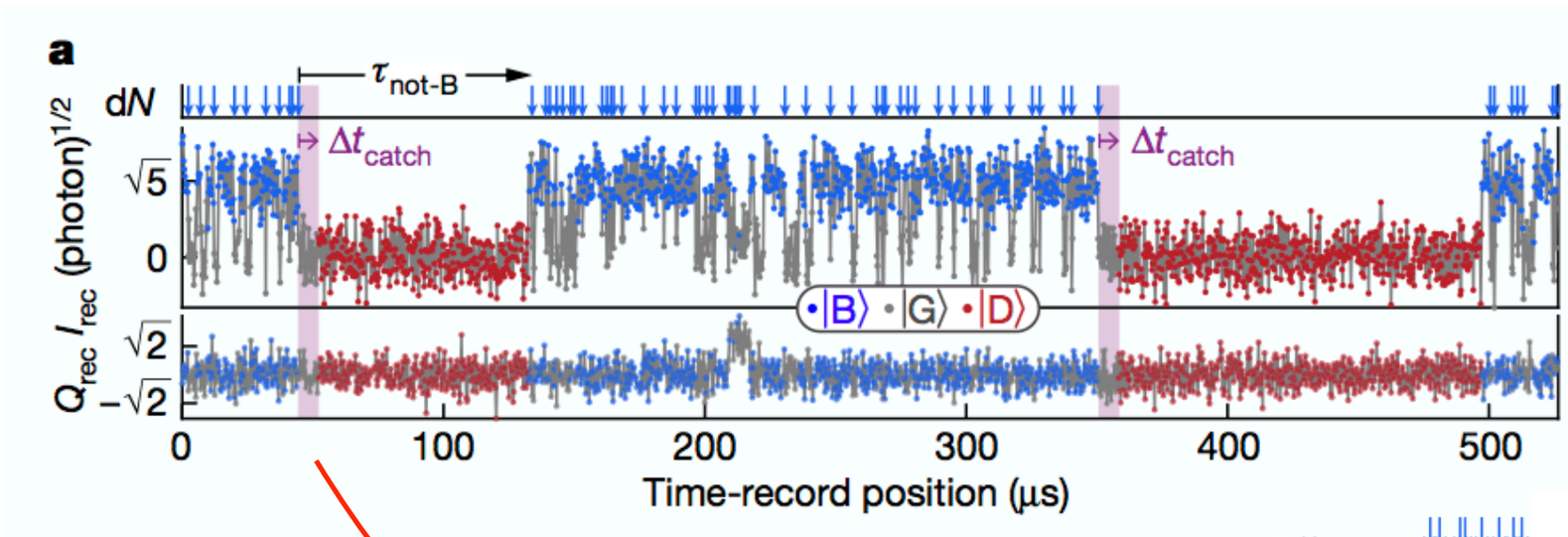
LETTER

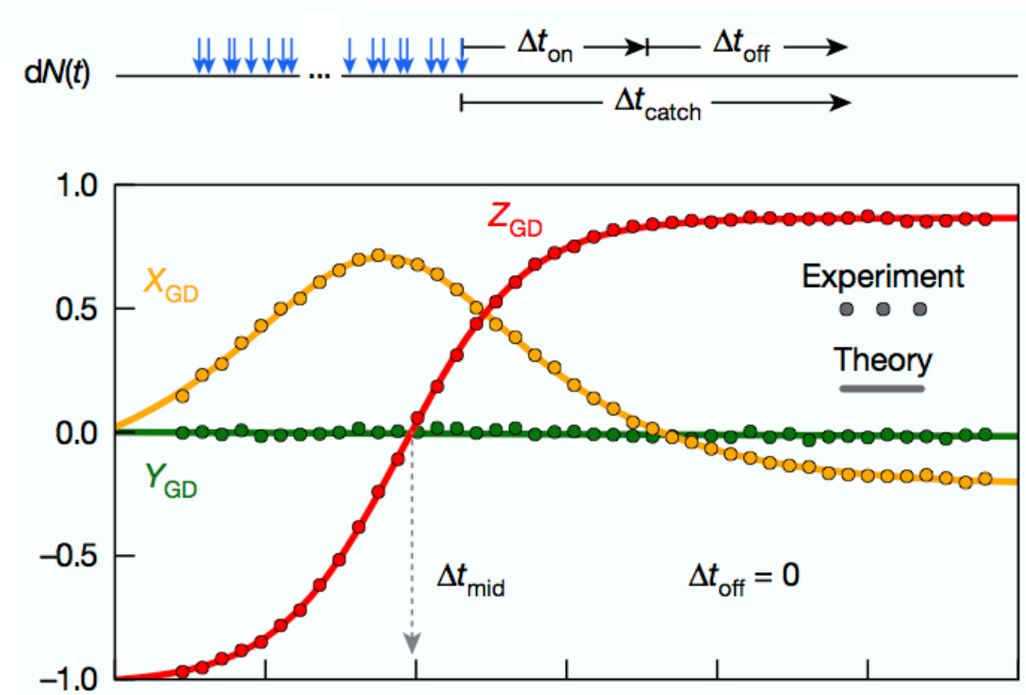
Nature 570 (2019) 200

https://doi.org/10.1038/s41586-019-1287-z

To catch and reverse a quantum jump mid-flight

Z. K. Minev[1,5]*, S. O. Mundhada[1], S. Shankar[1], P. Reinhold[1], R. Gutiérrez-Jáuregui[2], R. J. Schoelkopf[1], M. Mirrahimi[3,4], H. J. Carmichael[2] & M. H. Devoret[1]*





Happy Retirement

Paul!