Lattice Quantum Gravity

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Asymptotic Safety

Weinberg proposed idea that gravity might be Asymptotically Safe in 1976 [Erice Subnucl. Phys. 1976:1]. This scenario would entail:

- Gravity is effectively renormalizable when formulated non-perturbatively. Problem lies with perturbation theory, not general relativity.
- In a Euclidean lattice formulation the fixed point would show up as a continuous phase transition point, the approach to which would define a continuum limit.

Lattice gravity

- Euclidean dynamical triangulations (EDT) is a lattice formulation that was introduced in the ’90’s. [Ambjorn, Carfora, and Marzuoli, The geometry of dynamical triangulations, Springer, Berlin, 1997] Lattice geometries are approximated by triangles with fixed edge lengths. The dynamics is contained in the connectivity of the triangles, which can be added or deleted.

Einstein Hilbert Action

Continuum Euclidean path-integral:

$$Z = \int \mathscr{D}g \; e^{-S[g]}, \tag{1}$$

$$S[g_{\mu\nu}] = -\frac{k}{2} \int d^d x \sqrt{\det g}(R - 2\Lambda), \tag{2}$$

where $k = 1/(8\pi G_N)$.

Discrete action

Discrete Euclidean (Regge) action is

$$S_E = k \sum 2 V_2 \delta - \lambda \sum V_4, \tag{3}$$

where $\delta = 2\pi - \sum \theta$ is the deficit angle around a triangular face, V_i is the volume of an i-simplex, and $\lambda = k\Lambda$. Can show that

$$S_E = -\frac{\sqrt{3}}{2}\pi k N_2 + N_4 \left(\frac{5\sqrt{3}}{2} k \arccos \frac{1}{4} + \frac{\sqrt{5}}{96}\lambda \right) \tag{4}$$

where N_i is the total number of i-simplices in the lattice. Conveniently written as

$$S_E = -\kappa_2 N_2 + \kappa_4 N_4. \tag{5}$$

Measure term

Continuum calculations suggest a form for the measure

$$Z = \int \mathscr{D}g \prod_x \sqrt{\det g}^{\beta} \; e^{-S[g]}, \tag{6}$$

Going to the discretized theory, we have

$$\prod_x \sqrt{\det g}^{\beta} \to \prod_{j=1}^{N_2} \mathscr{O}(t_j)^{\beta}, \tag{7}$$

where $\mathscr{O}(t_j)$ is the order of triangle t_j, i.e. the number of 4-simplices to which a triangle belongs. Can incorporate this term in the action by taking exponential of the log. β is a free parameter in simulations. Can interpret as an ultra-local measure term, since it looks like a product over local 4-volumes.

New Idea

Revisiting the EDT approach because other formulations (renormalization group and other lattice approaches) suggest that gravity is asymptotically safe.

New(-ish) work done in collaboration with past students and postdoc: JL, S. Bassler, D. Coumbe, Daping Du, J. Neelakanta, (arXiv:1604.02745).

- Key new idea that inspired this study is that a fine-tuning of bare parameters in EDT is necessary to recover the correct continuum limit. This is in analogy to using Wilson fermions in lattice gauge theory to study quantum chromodynamics (QCD) with light or massless quarks. Striking similarities are seen.

- Previous work did not implement this fine-tuning, leading to negative results.

Main problems to overcome

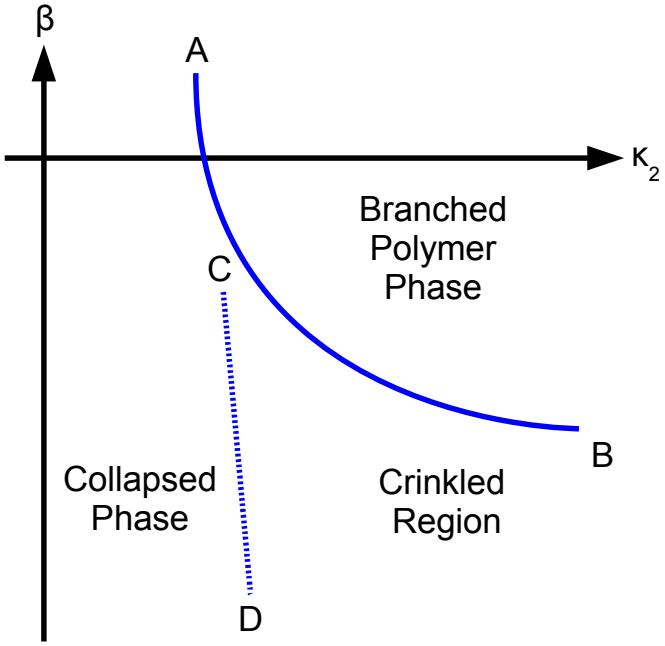
- Must show recovery of semiclassical physics in 4 dimensions.
- Must show existence of continuum limit at continuous phase transition.

Simulations

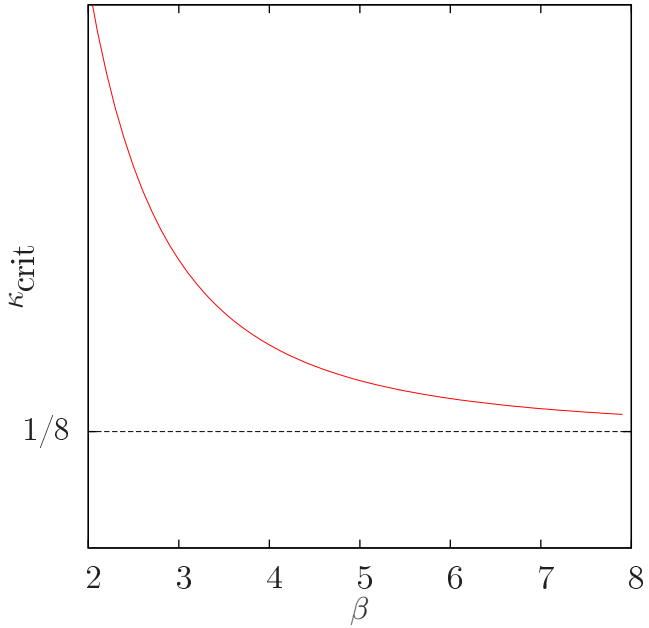
Methods for doing these simulations were introduced in the 90's. We wrote new code from scratch.

- The Metropolis Algorithm is implemented using a set of local update moves.

Phase diagram EDT vs. QCD with Wilson fermions

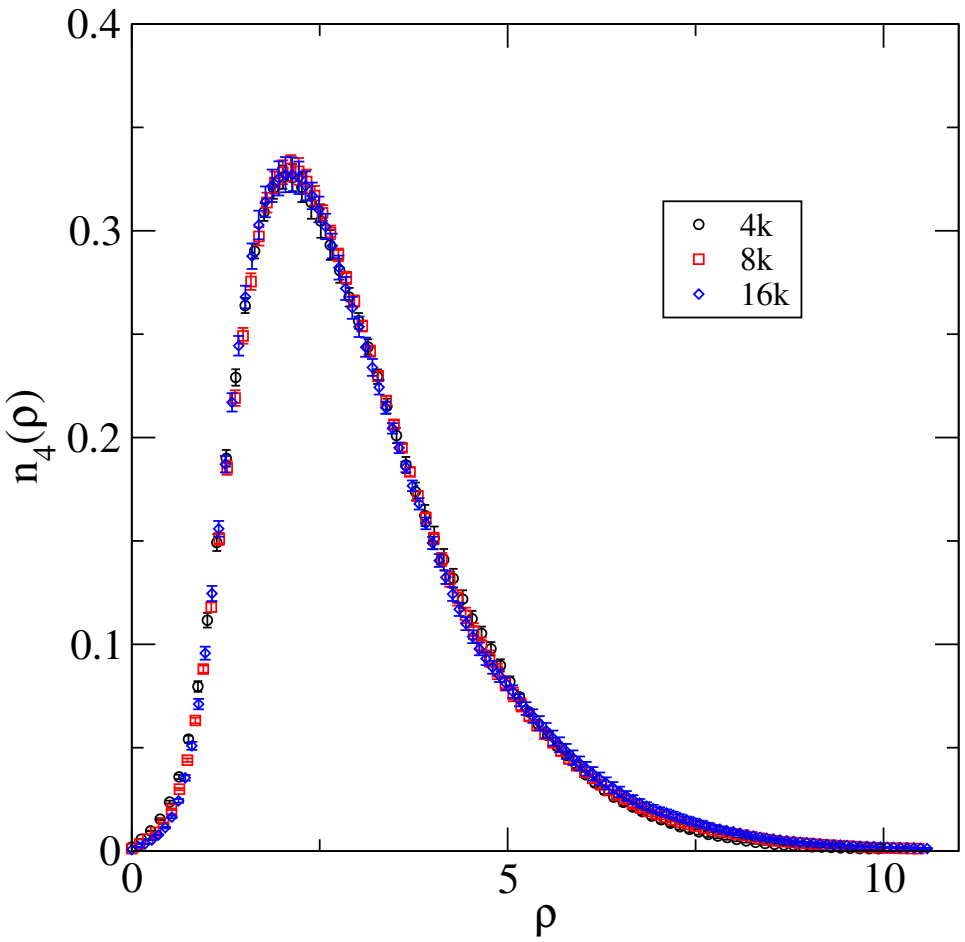


EDT

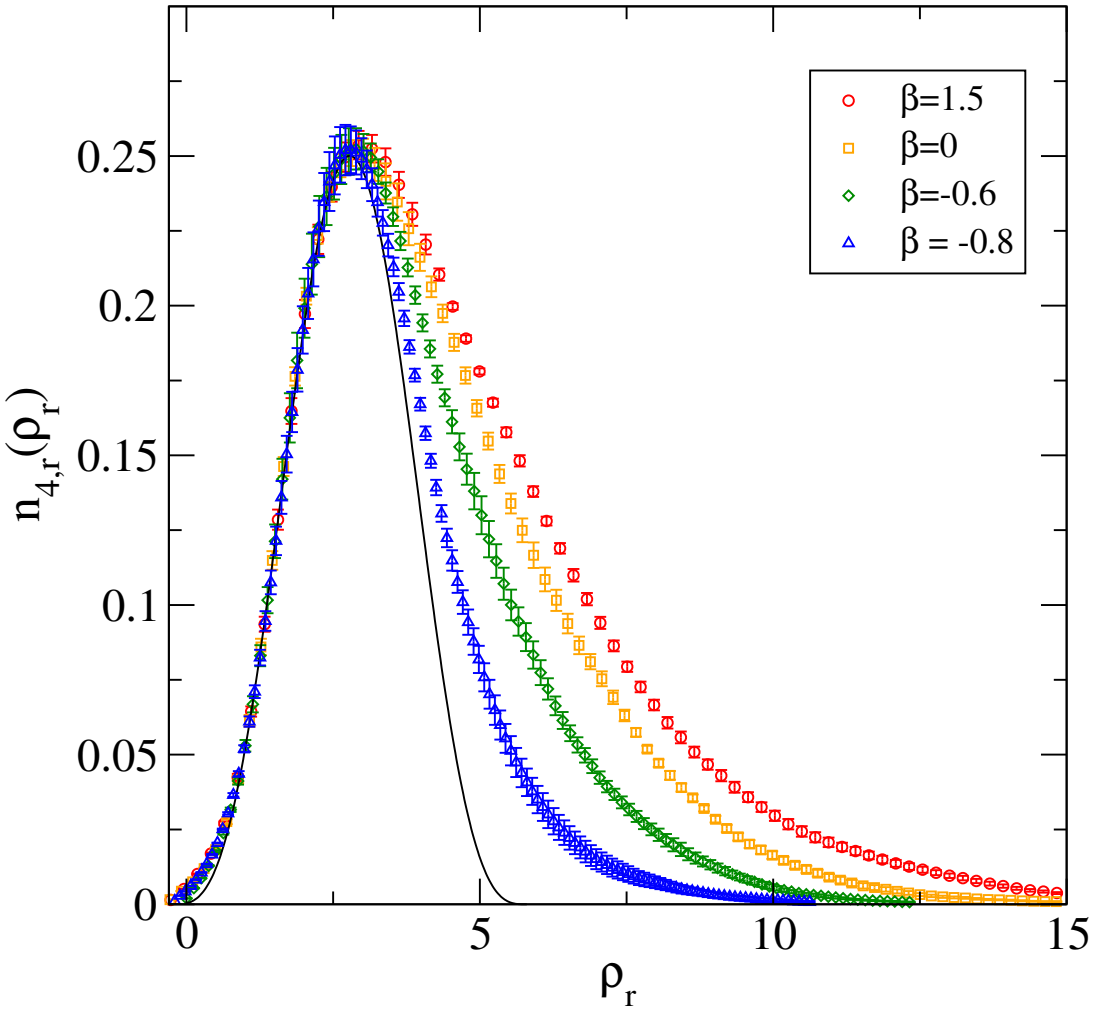


QCD

Three volume distribution



Three volume distribution



Diffusion process and the spectral dimension

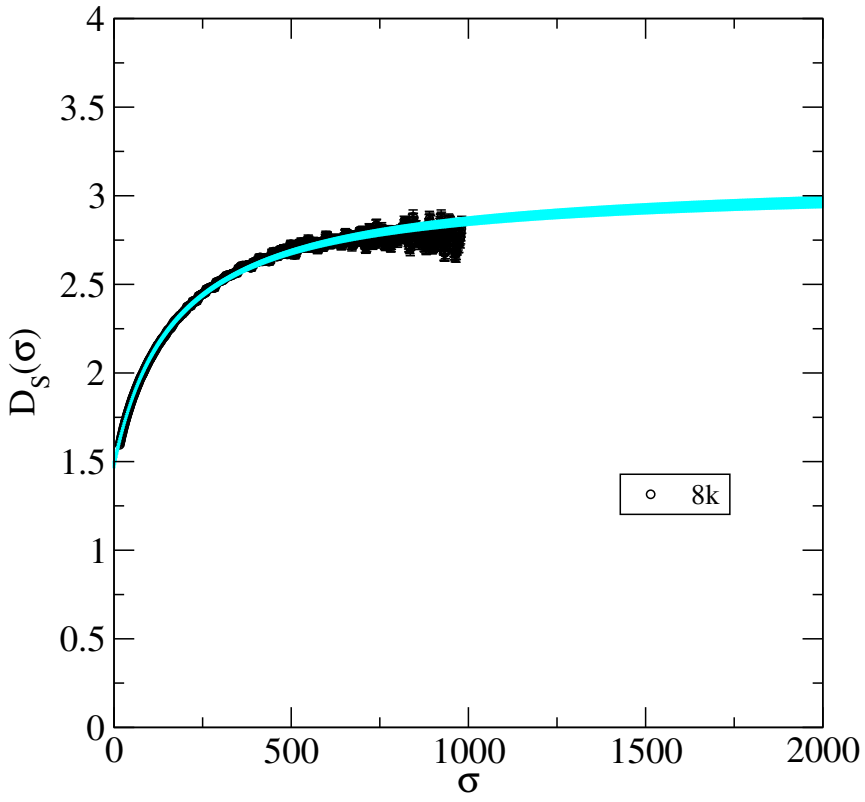
Spectral dimension is defined by a diffusion process

$$D_S(\sigma) = -2\frac{d\log P(\sigma)}{d\log\sigma}, \tag{8}$$

where σ is the diffusion time step on the lattice, and $P(\sigma)$ is the return probability, i.e. the probability of being back where you started in a random walk after σ steps.

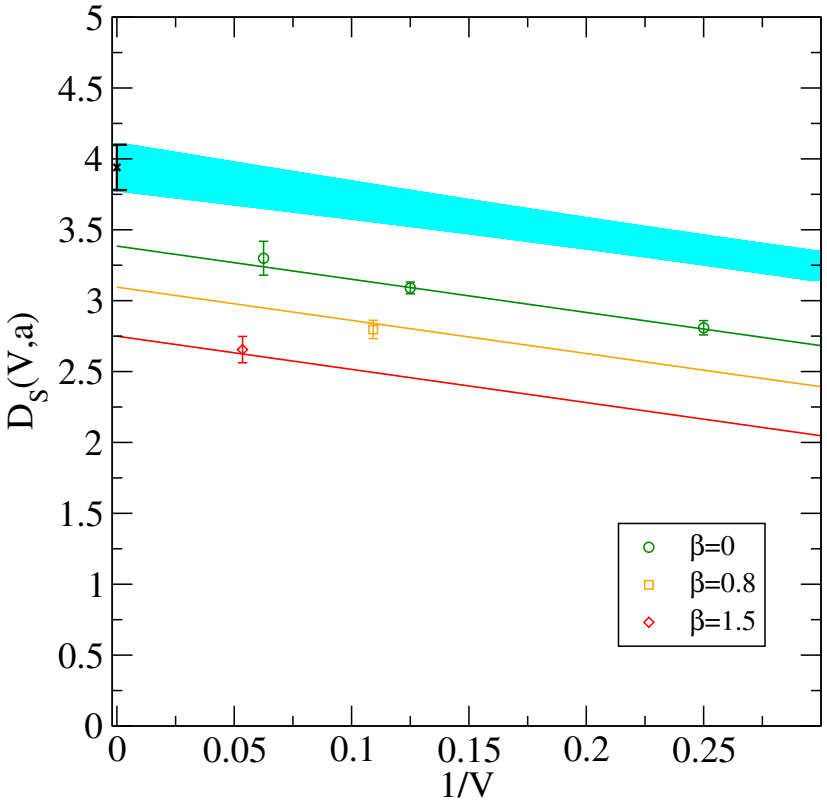
Spectral Dimension

χ^2/dof=1.25, p-value=17%
$D_S(\infty) = 3.090 \pm 0.041$, $D_S(0) = 1.484 \pm 0.021$



Infinite volume, continuum extrapolation

χ^2/dof=0.52, p-value=59%
$D_S(\infty) = 3.94 \pm 0.16$

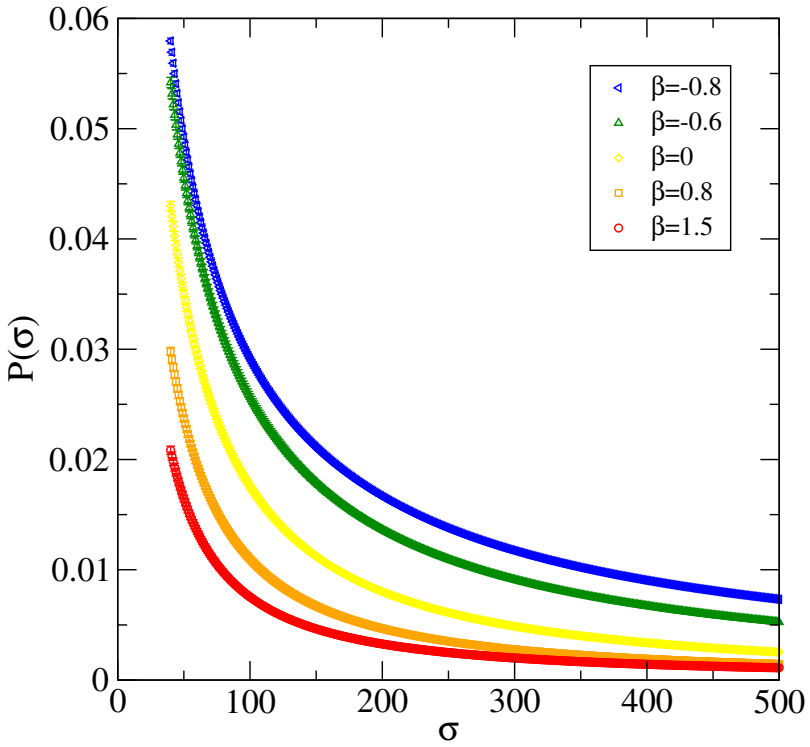


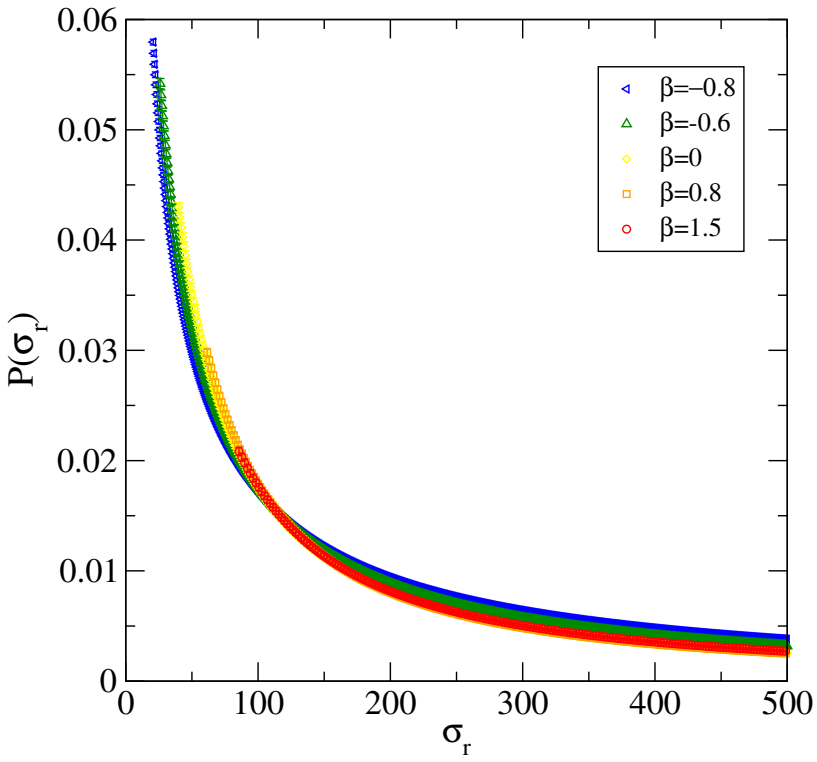
What does it mean?

Interesting results that suggest that the correct classical result might be restored in the continuum, large-volume limit. Analogy with Wilson fermions that inspired this study may tell us more.

We have to perform a fine-tuning, and long distance physics gets messed up by discretization effects. These things happen when the regulator breaks a symmetry of the quantum theory. In this case, natural to identify the symmetry as continuum diffeomorphism invariance.

Relative lattice spacing

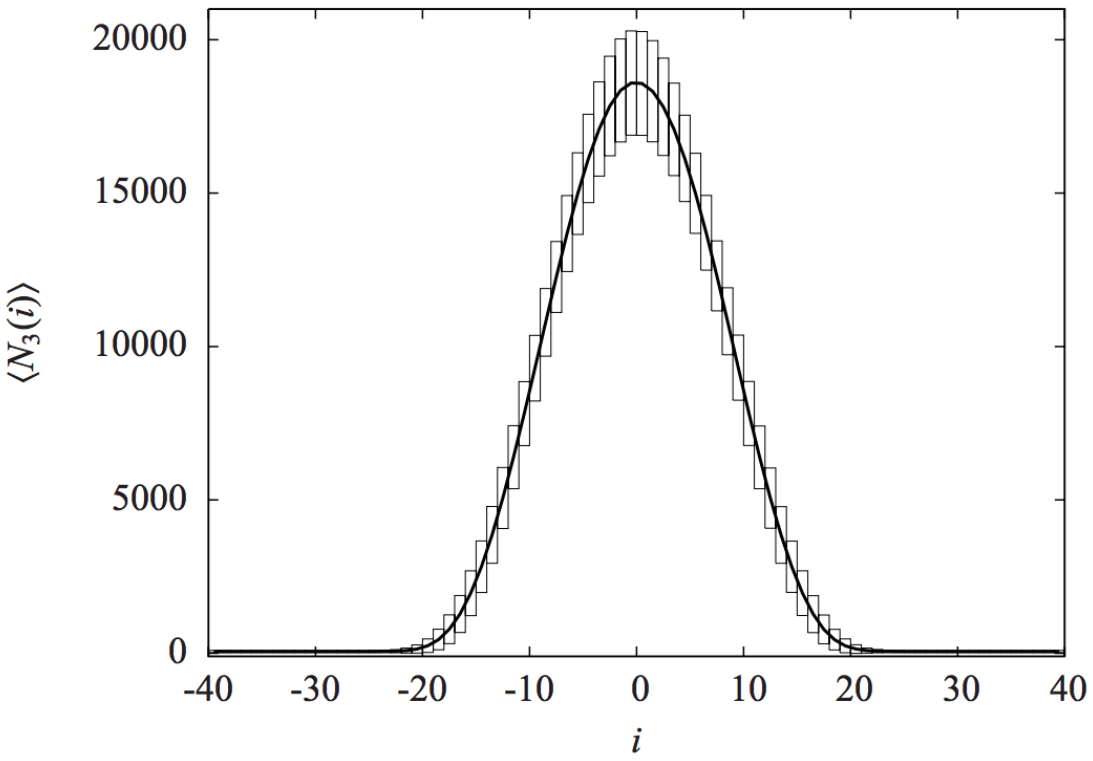




Return probability left and rescaled return probability right.

Causal dynamical triangulations

Euclidean de Sitter space solution from arXiv:1604.02745 (Ambjorn et al.)



Semiclassical fluctuations

Looking at quantum fluctuations about de Sitter space allow one to fix M_{Planck}. A simple minisuperspace model fits the CDT data well. Ambjorn et al. (arXiv:1604.02745) look at the correlator

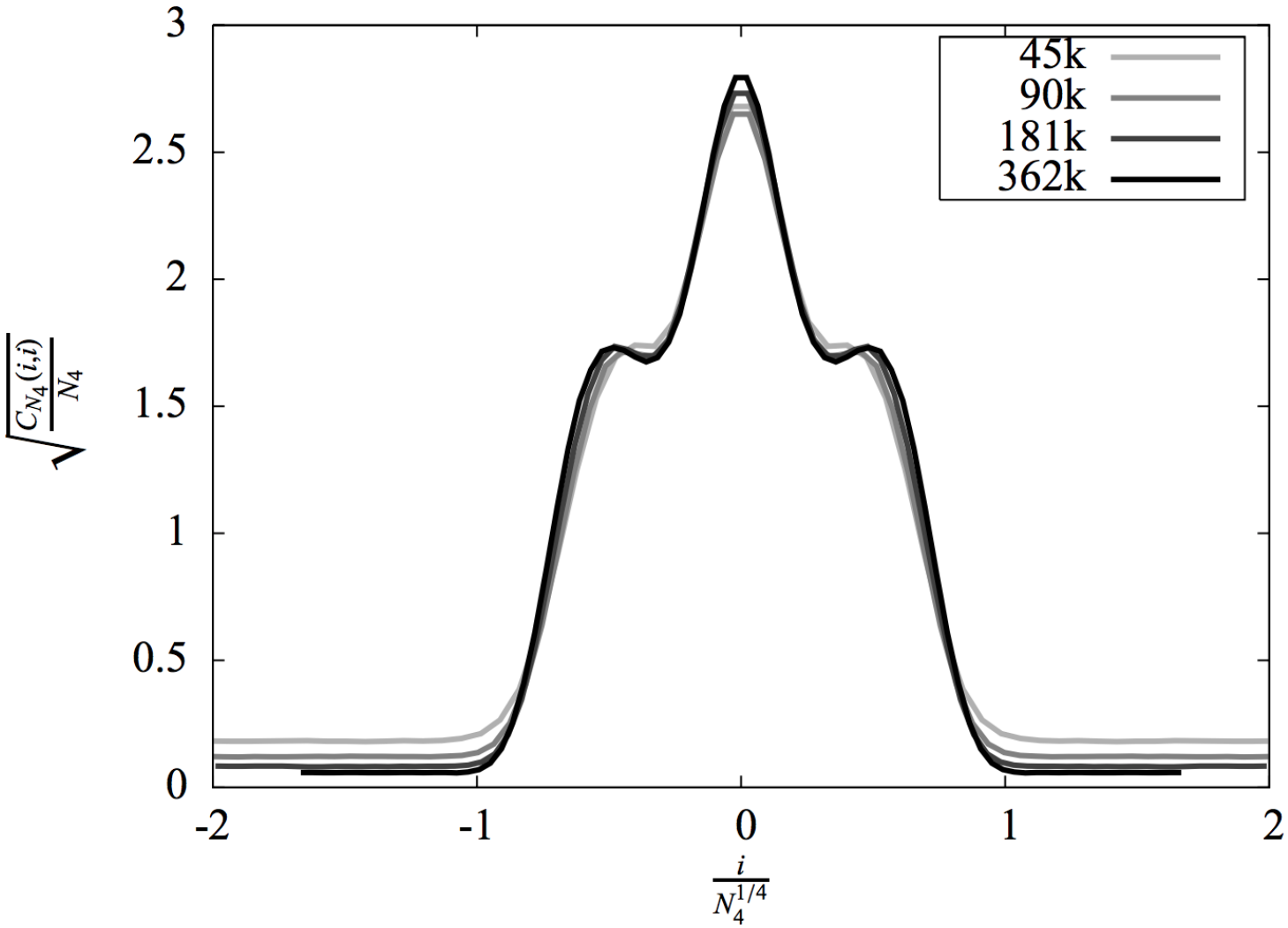
$$C(i,j) = \frac{1}{K}\sum_k (N_3^{(k)}(i) - \overline{N}_3(i))(N_3^{(k)}(j) - \overline{N}_3(j)), \tag{9}$$

where one can show that $C(i,j) \propto G_N$.

The size of these quantum fluctuations compared to the width of the de Sitter universe can be used to fix the lattice spacing.

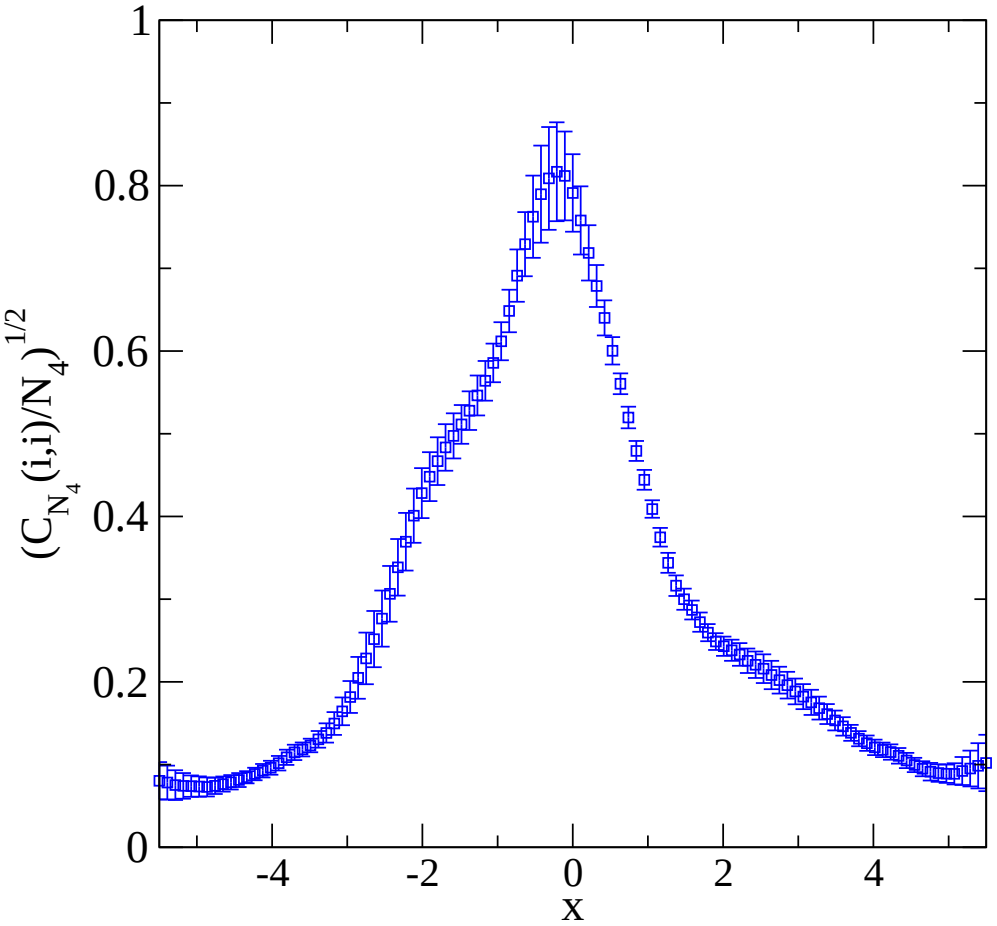
Causal dynamical triangulations

Semiclassical fluctuations about de Sitter, arXiv:1604.02745 (Ambjorn et al.)

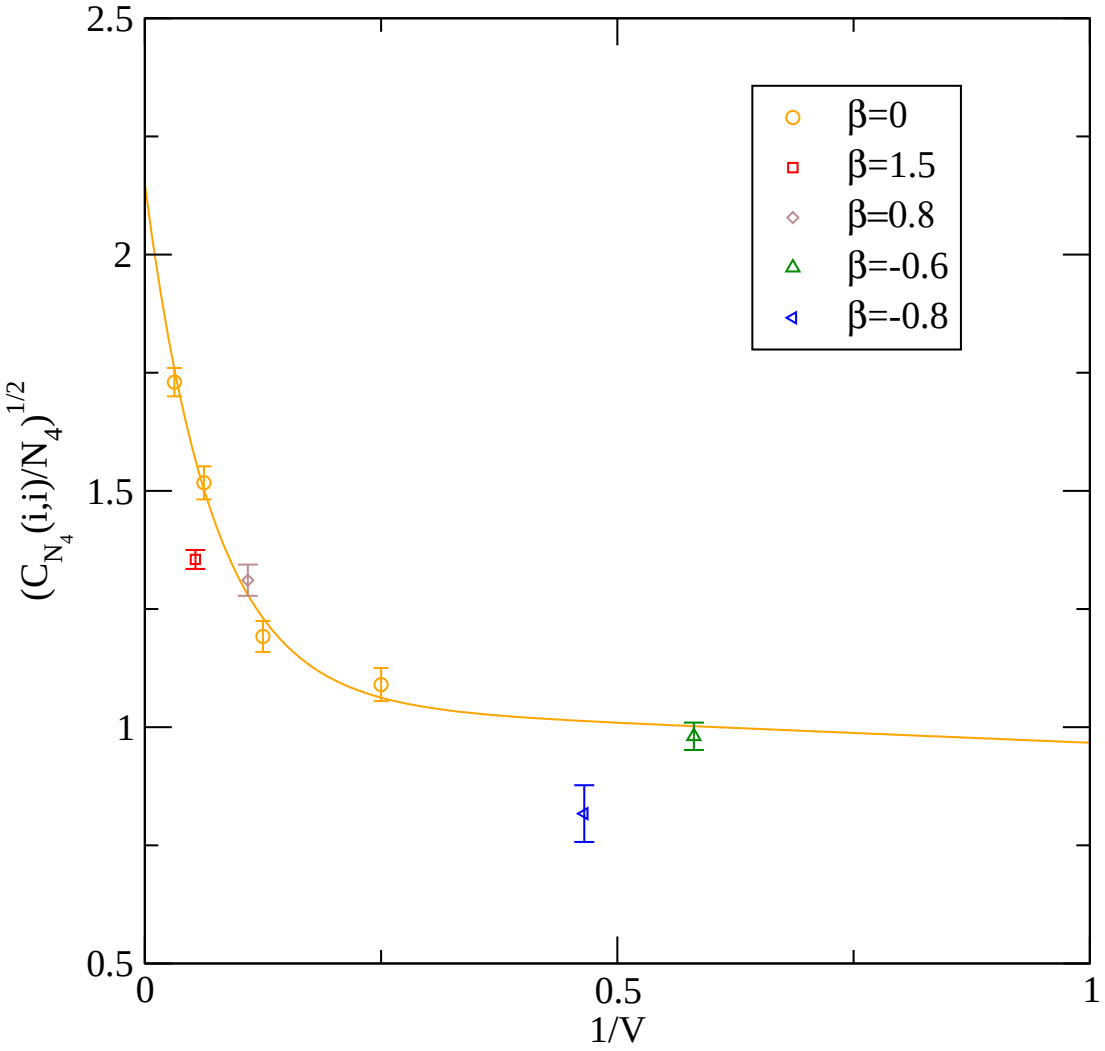


Semiclassical fluctuations from EDT

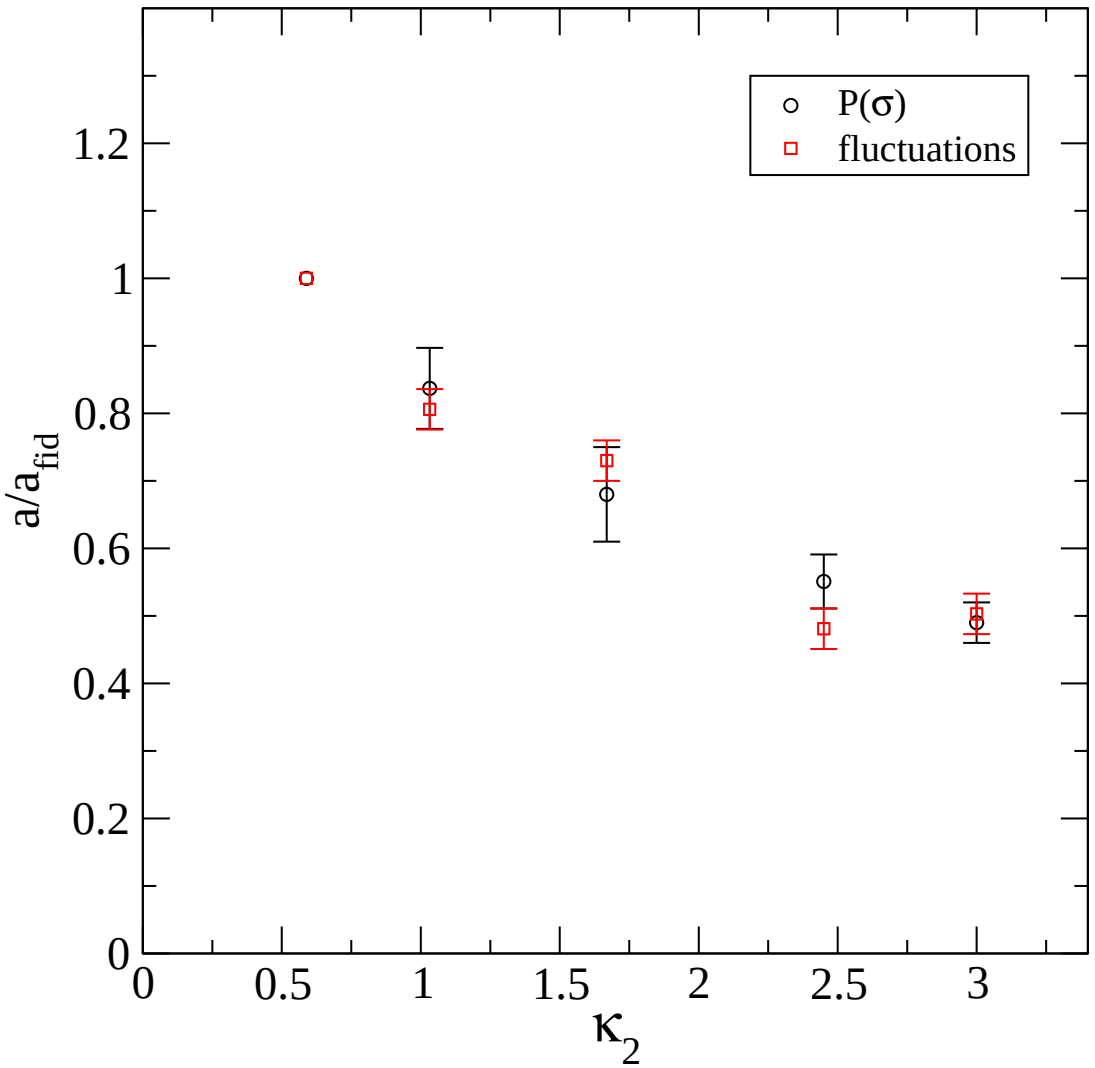
8k, $\beta = -0.8$



Finite volume effects



Relative lattice spacing



Adding matter

We have looked at adding both scalar and fermion matter fields.

Adding scalars and U(1) gauge fields dynamically (unquenched) was done already over twenty years ago.

For now we are revisiting things in the quenched approximation, where matter loops are neglected, since this allows us to reuse existing lattice ensembles.

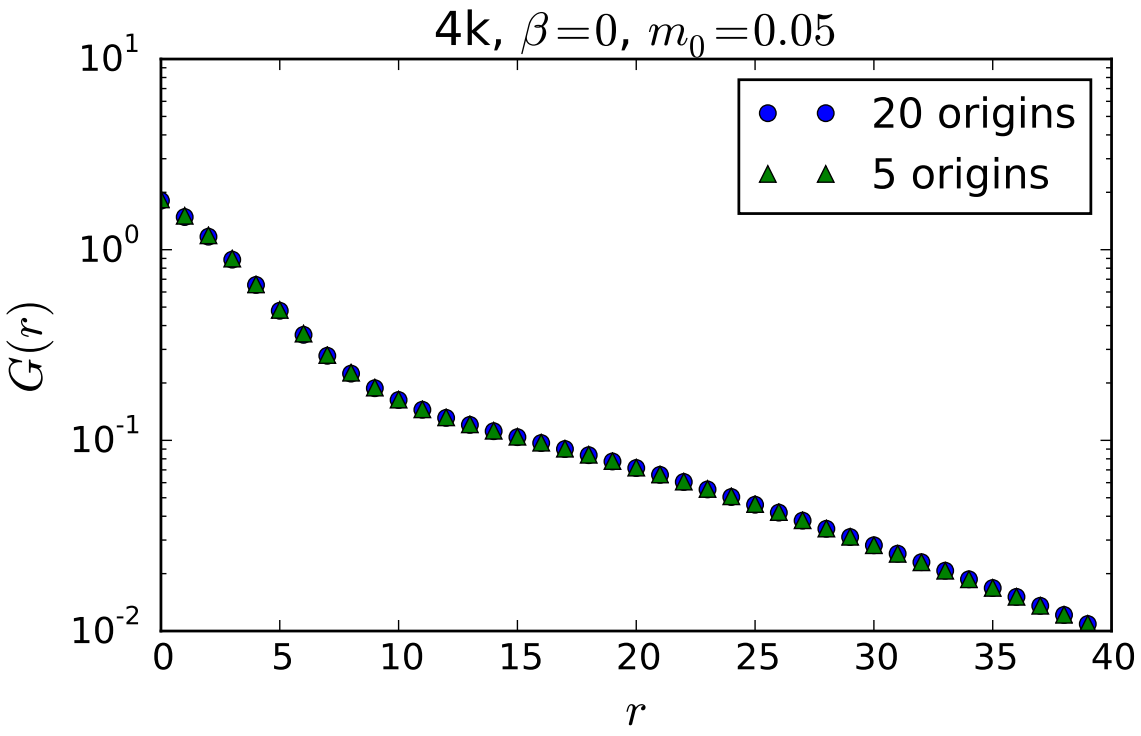
Scalar field

In the continuum we can add to the Einstein Hilbert action the action for the scalar field:

$$S[g,\phi] = \int d^4x \sqrt{g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} m_0^2 \phi^2 \right), \tag{10}$$

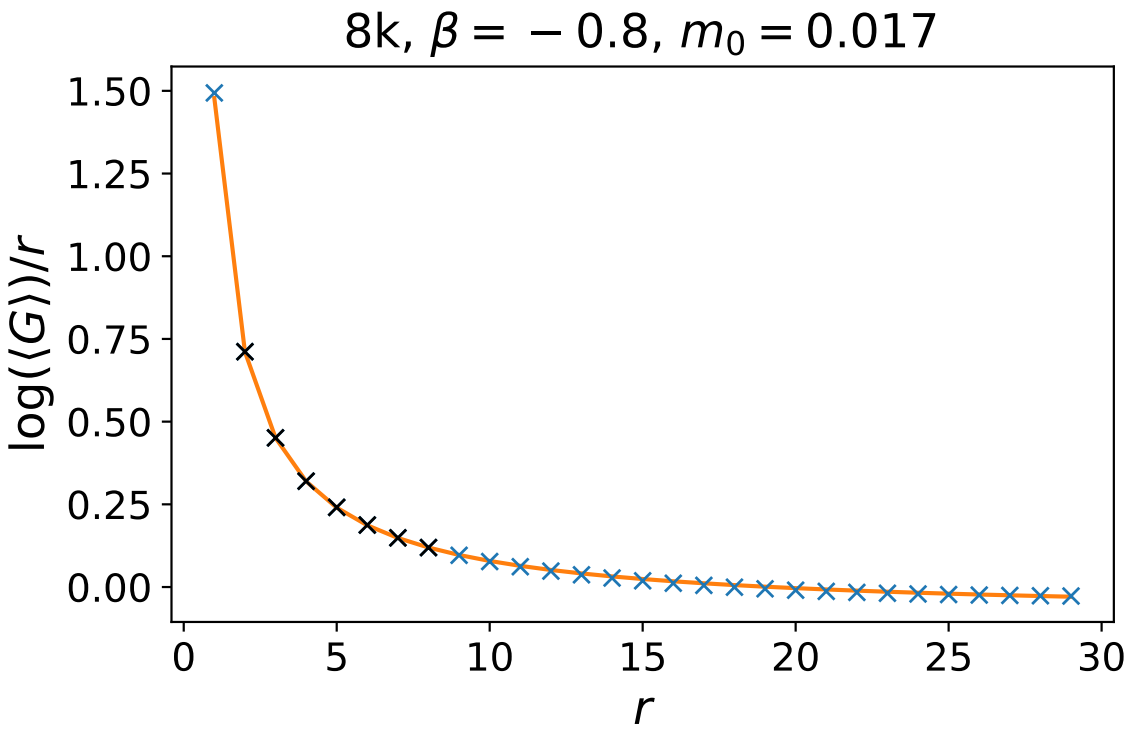
Lattice discretization is straightforward. Looking at scalar propagators on quenched configurations. This is being done by Judah Unmuth-Yockey.

Scalar propagator



$$G(r) = (-\Box + m_0^2)^{-1}_{0r} \tag{11}$$

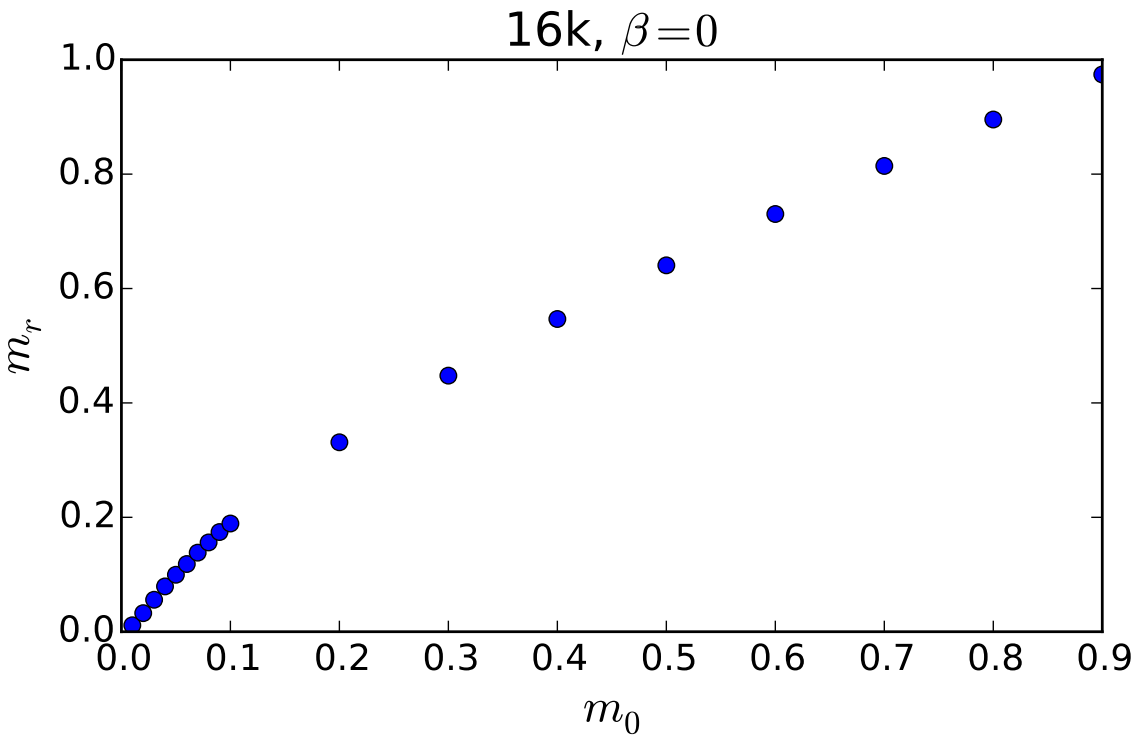
Correlator fit



Functional form of the correlator is $B' \exp(Ar)/r^C$.

We plot $f(r) = A + (B + C\log(r))/r$.

Mass dependence



The shift symmetry of the action ensures that $m_r \to 0$ as $m_0 \to 0$.

Binding energy

We follow the work of de Bakker and Smit, Nucl.Phys. B484 (1997) 476. They looked at gravitational binding on EDT near the transition, at what was effectively a single coarse lattice spacing. We revisit this work with our current understanding and ensembles.

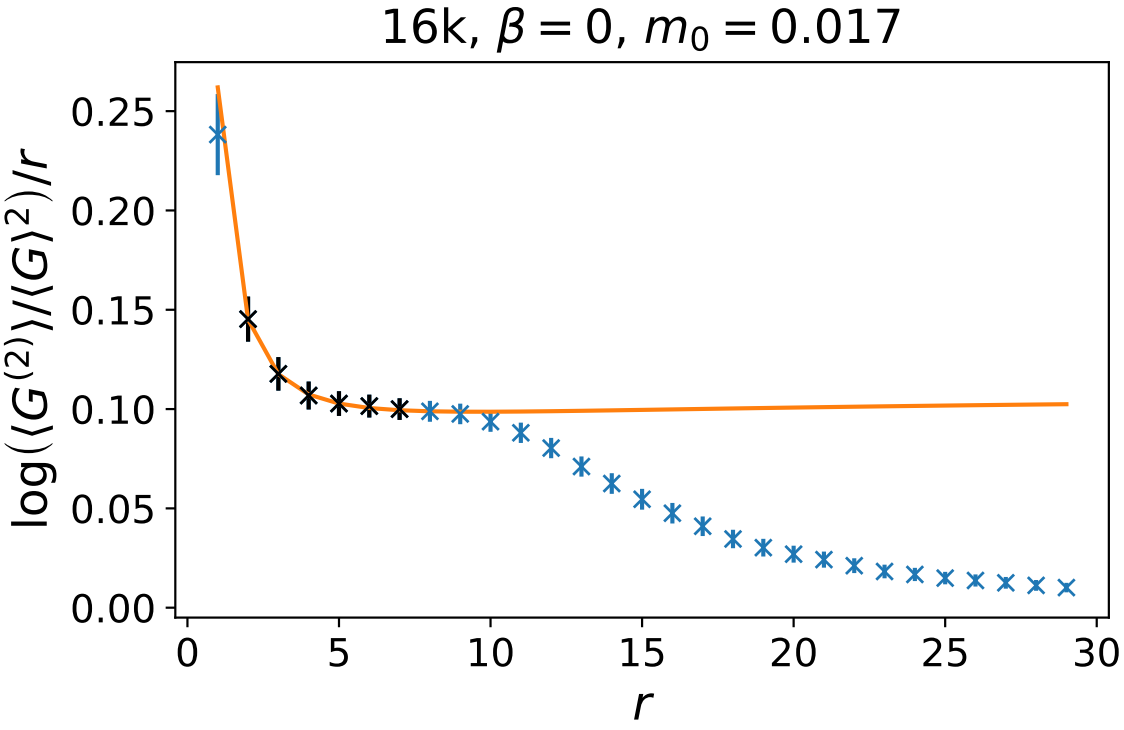
We are looking to calculate the binding energy:

$$E_b \equiv 2m - M = \frac{1}{4} G^2 m^5. \tag{12}$$

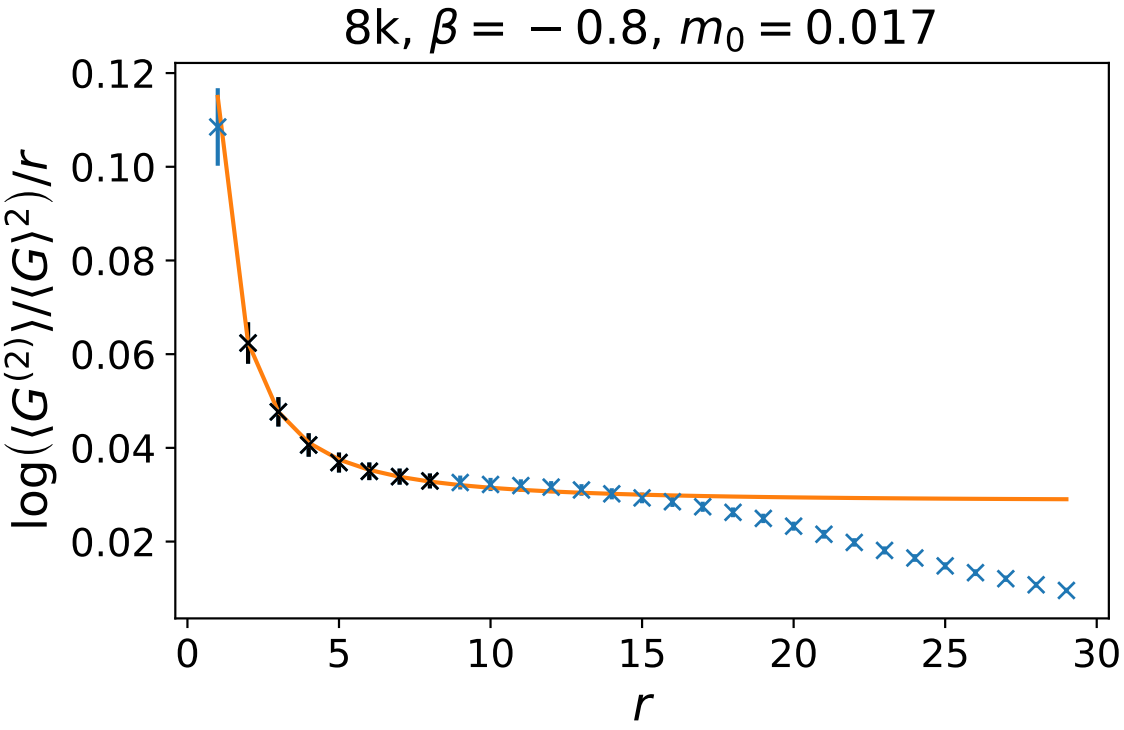
This is just the familiar energy of the hydrogen atom, $\alpha^2 m_{\mathrm{red}}/2$ but with $\alpha \to Gm^2$ and $m_{\mathrm{red}} \to m/2$. Presumably applies in the non-relativistic limit.

Also worth noting that we can calculate this for different dimensions. In three dimensions, $E_b \propto m^2$.

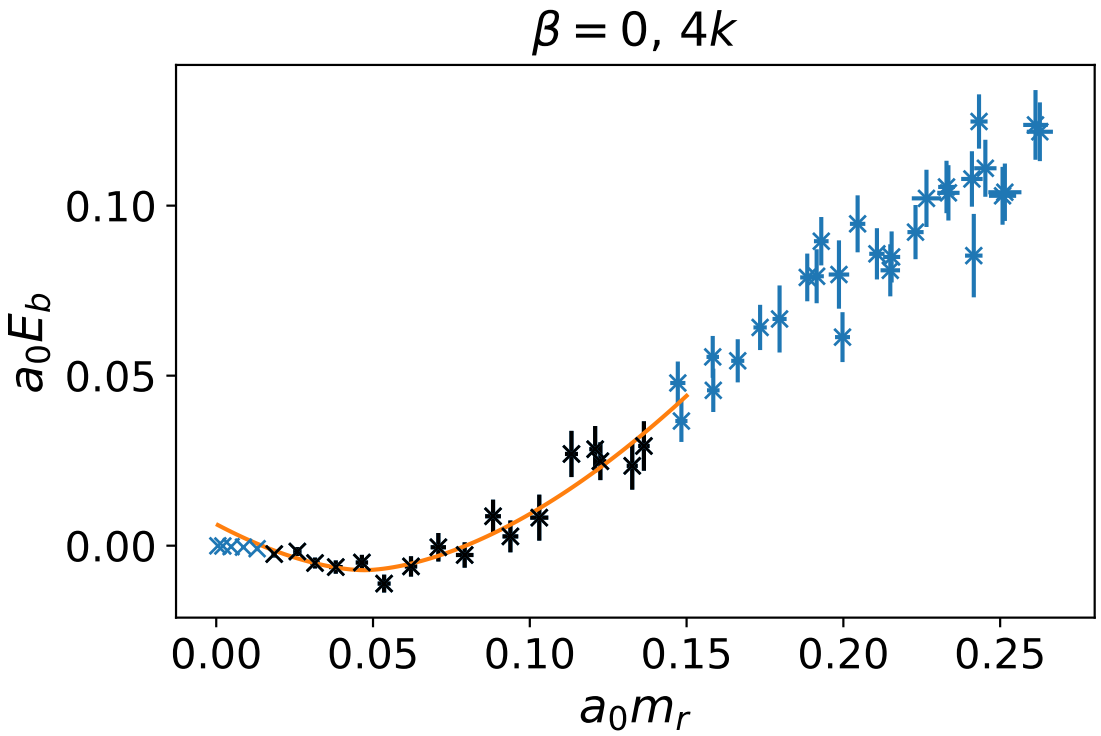
Binding Energy



Binding Energy (finer lattice)



Power law mass dependence

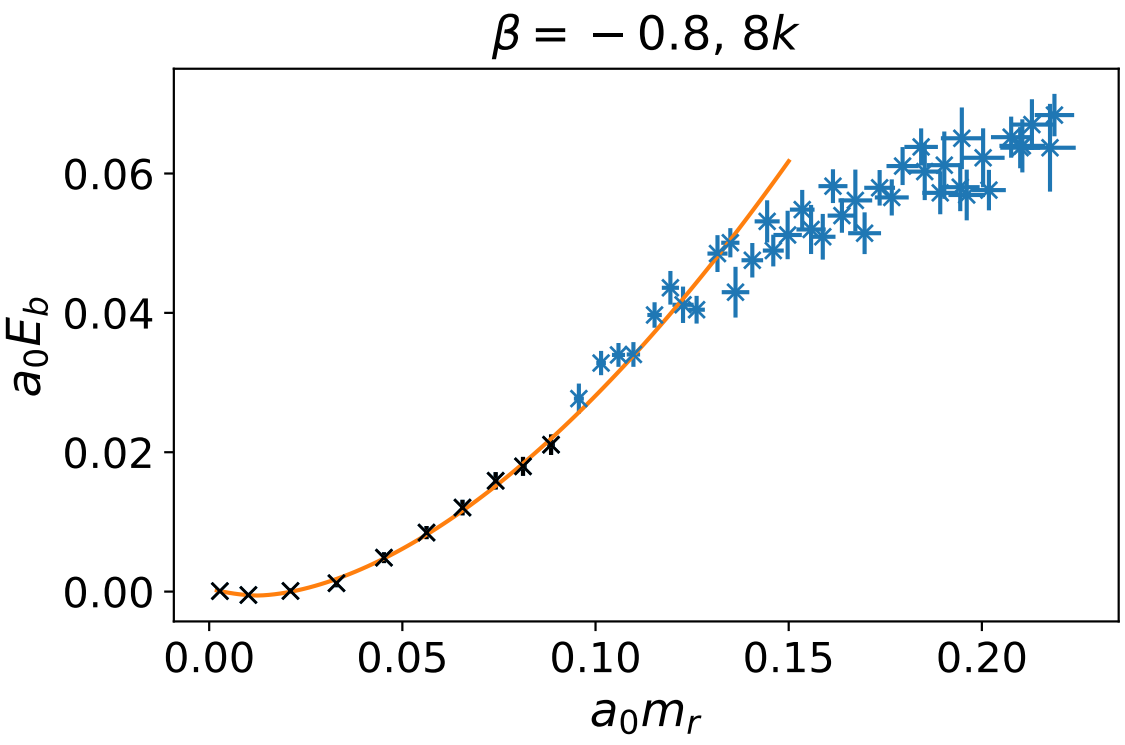


Fit to form

$$E_b = A(m - B)^C + D \tag{13}$$

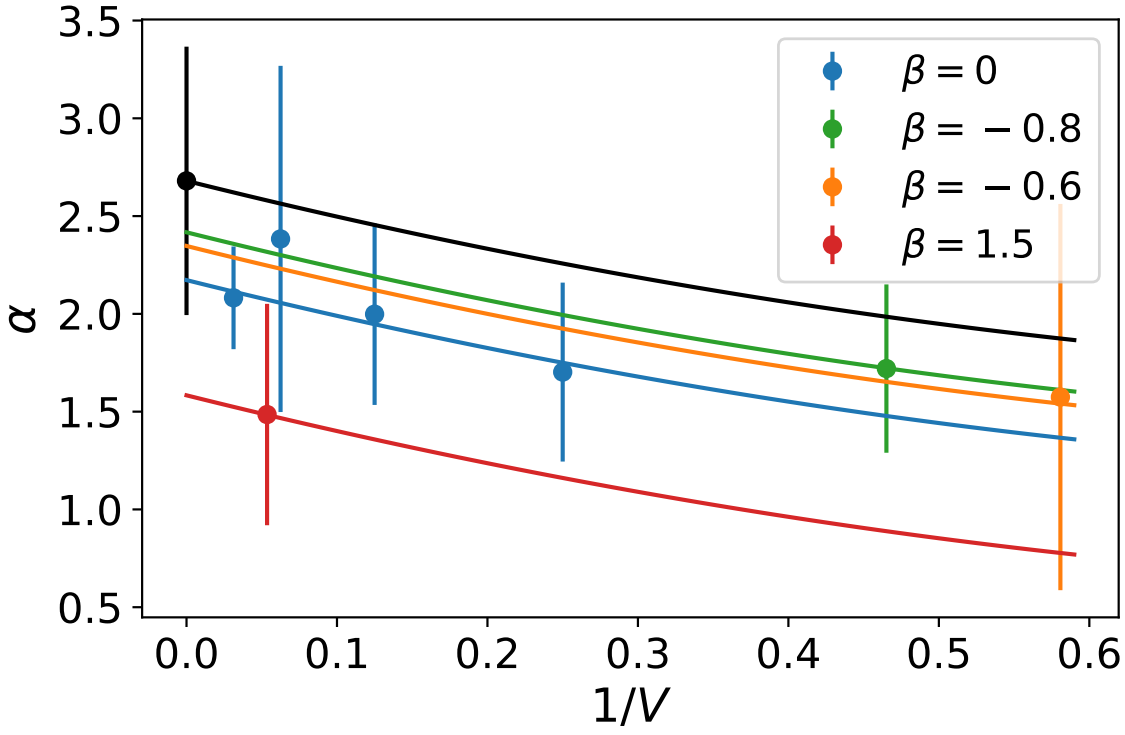
Note that B and D should be zero in the continuum, infinite volume limit. This appears to be the case. C should be 5, A is proportional to Newton's constant squared.

Power law mass dependence (finer lattice)



$$E_b = A(m - B)^C + D \tag{14}$$

Exponent result



Note: $d = 3$ corresponds to $\alpha = 2$,
$d = 3.5$ corresponds to $\alpha = 3$,
$d = 3.7$ corresponds to $\alpha \approx 3.5$.
Sensitive dependence of α on d makes it difficult to extrapolate to $d = 4$, $\alpha = 5$.

Conclusions

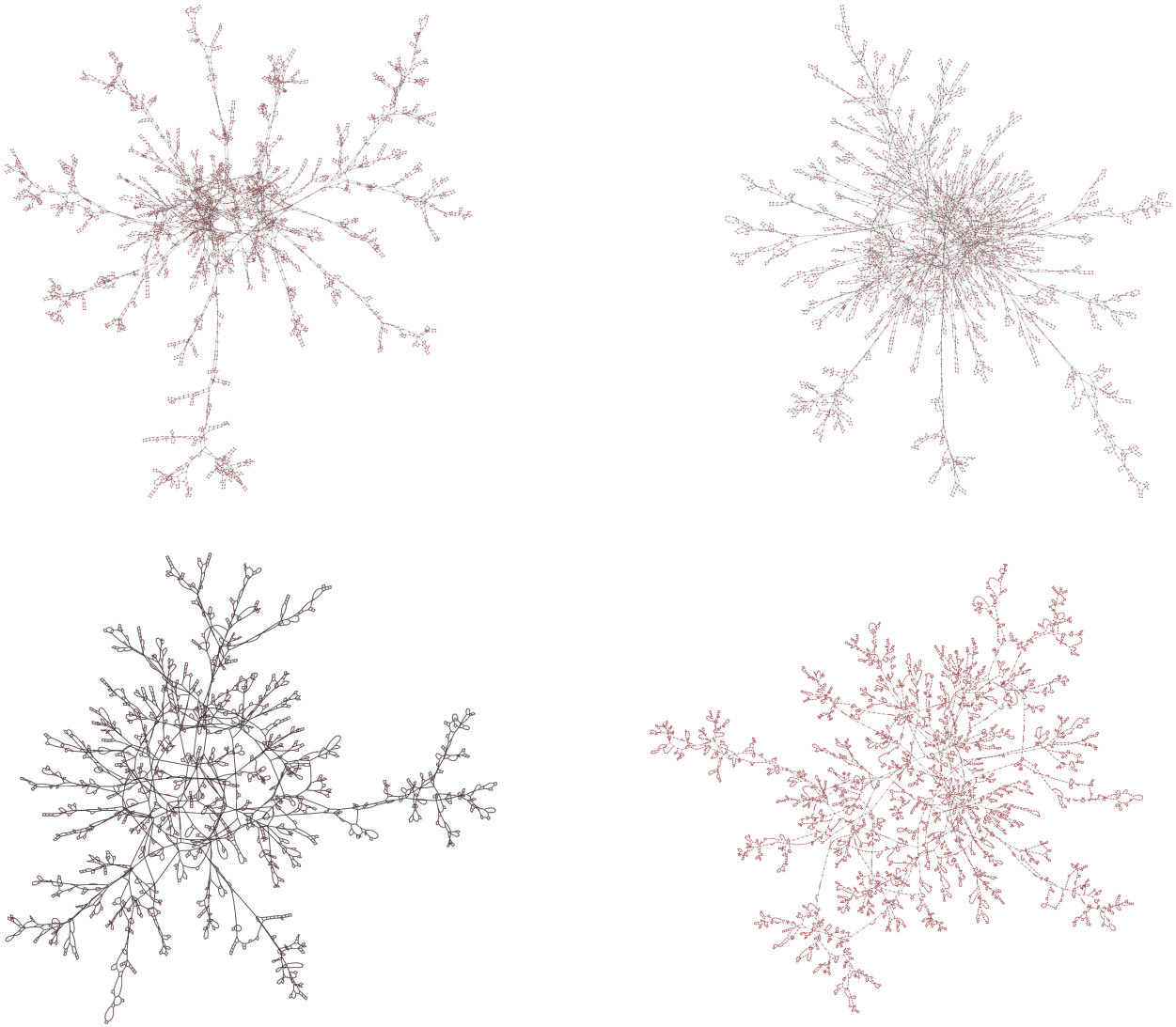
Two different ways to compute the relative lattice spacing. One makes contact with a minisuperspace model of cosmology and involves fluctuations around de Sitter space. The other involves nonperturbative short distance effects (from a diffusion process) that are not captured in the effective theory. Evidence that non-perturbative, strongly coupled physics is incorporated in lattice theory, not a lattice artifact.

Can compute matter interactions. Attractive force is found. Extrapolation to the infinite volume, continuum limit expected to have large finite-size/discretization effects. Bigger, finer lattices needed. Methodology for studying Newton's Law has been developed. Current result is between the $d = 3$ and $d = 4$ world.

Unphysical behavior gets smaller as the continuum limit is approached for a number of different types of observables (also true when we look at fermions propagating on the same ensembles). Evidence for a continuum limit.

Back-up Slides

Visualization of geometries



Coarser to finer, left to right, top to bottom.