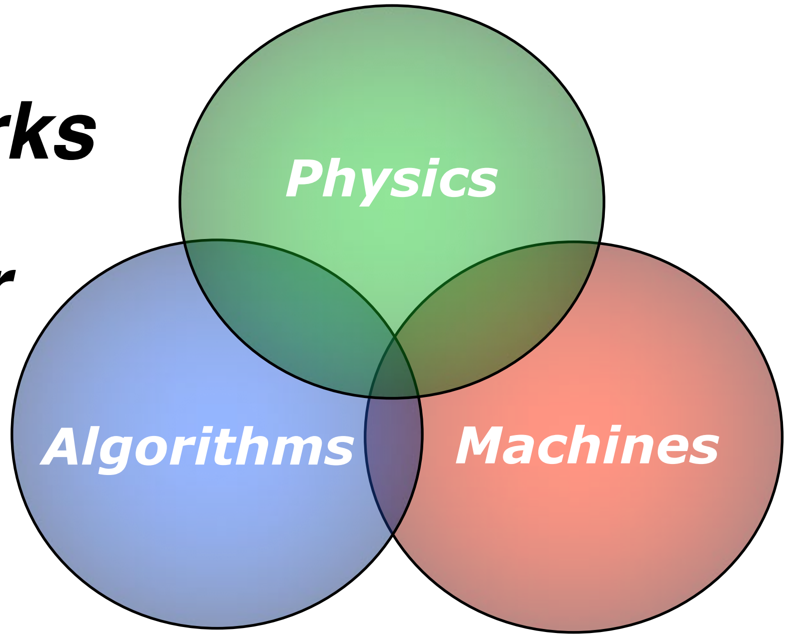
Lattice QCD at Fermilab
Celebrating the career of Paul Mackenzie
FNAL
7-8 November 2019

The History of Lattice QCD

Akira Ukawa
Professor Emeritus and Fellow
Center for Computational Sciences, University of Tsukuba
Director
Center for World Premier International Research Center Initiative
JSPS

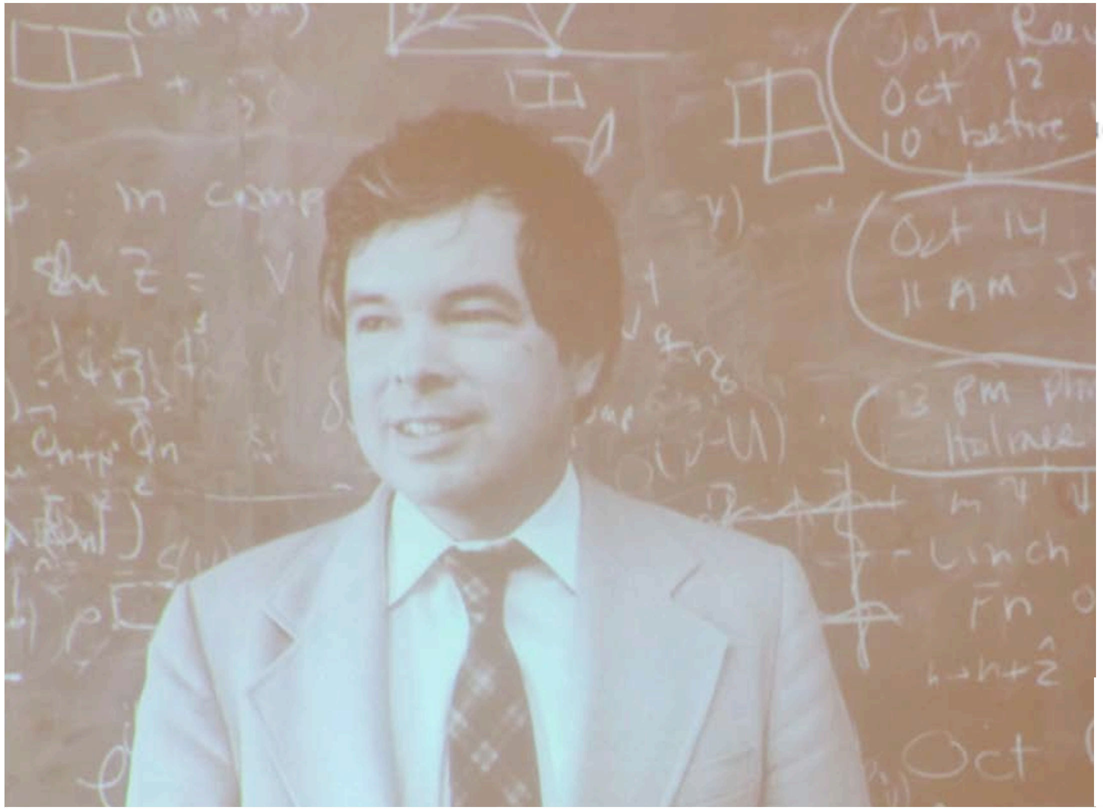
- The beginning
- Exploring the new frontier
- Building the instrument
- Getting nimble with quarks
- Putting them all together



The beginning

Wilson 1974

- Preprint CLNS-262 (February 1974)
- Gauge theory on a space-time lattice
- "Wilson loop" as order parameter
- Confinement for large values of coupling $g_0^2 \gg 1$



CAL REVIEW D VOLUME 10, NUMBER 8 15 OCTO

Confinement of quarks*

Kenneth G. Wilson
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850
(Received 12 June 1974)

A mechanism for total confinement of quarks, similar to that of Schwinger, is defined which requires the existence of Abelian or non-Abelian gauge fields. It is shown how to quantize a gauge field theory on a discrete lattice in Euclidean space-time, preserving exact gauge invariance and treating the gauge fields as angular variables (which makes a gauge-fixing term unnecessary). The lattice gauge theory has a computable strong-coupling limit; in this limit the binding mechanism applies and there are no free quarks. There is unfortunately no Lorentz (or Euclidean) invariance in the strong-coupling limit. The strong-coupling expansion involves sums over all quark paths and sums over all surfaces (on the lattice) joining quark paths. This structure is reminiscent of relativistic string models of hadrons.

I. INTRODUCTION

particles over short times and short d
prevent
al state
than 1/

See Wilson's Lattice 2004 talk for a personal historical account of the discovery

Cornell 1975

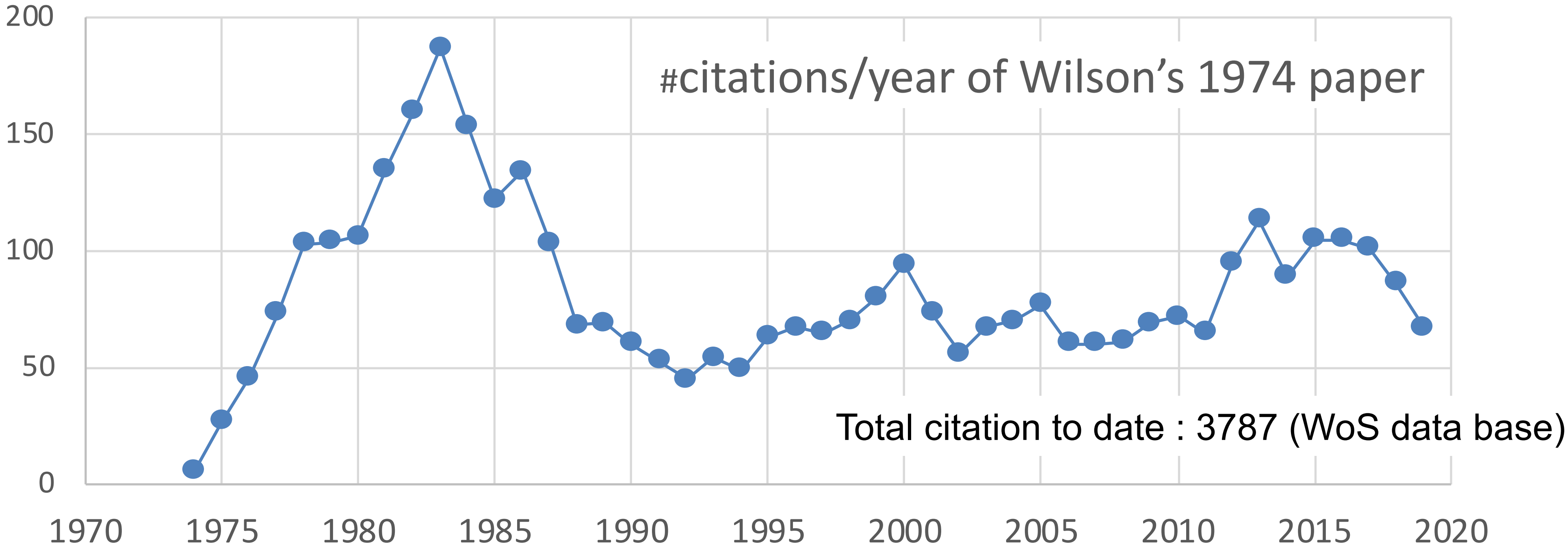
Grad students on 5th floor
- **Paul Mackenzie**
- Steve Shenker
- Serge Rudaz
- Junko Shigemitsu
- Belal Baaqui
- Michael Peskin

I was on 3rd floor.

Laboratory of Nuclear Studies
Cornell University

Faculty
- Ken Wilson
- John Kogut
- Tun-Mow Yan
- Tom Kinoshita
- Don Yennie
- Kurt Gottfried

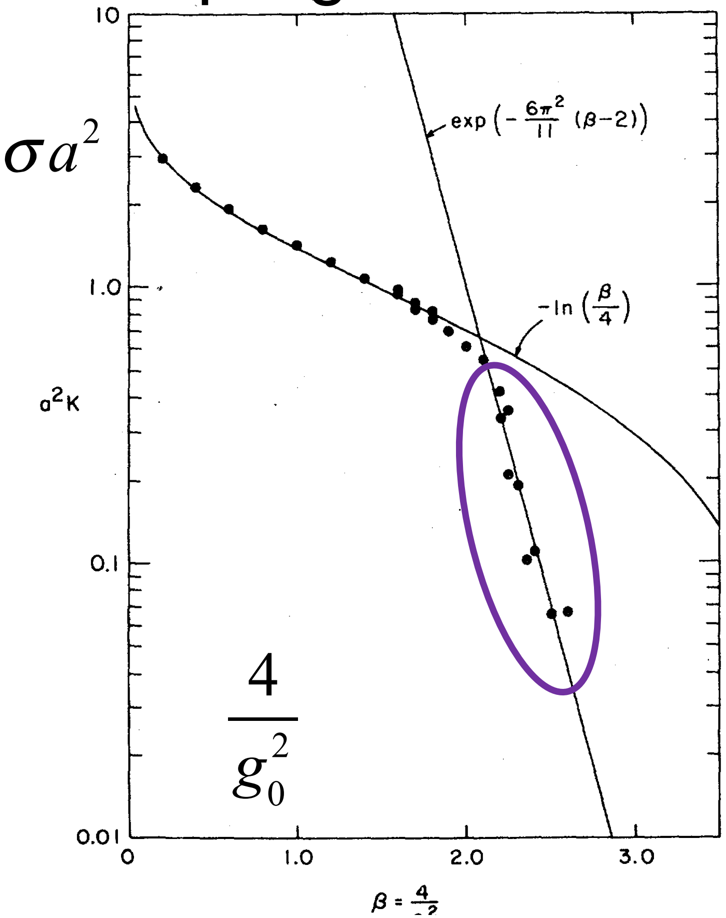
Early reception



- The paper attracted much interest, but in fact, there was little progress in the first 5 years…..
  - strong-coupling expansion of hadron masses (Kogut et al) did not lead to anywhere
  - "theoretical ideas" like "duality" , "monopole condensation" ('tHooft, Polyakov …) attracted more interest

Creutz 1979

- Preprint-79-0919(BNL)(September 1979)
- Computer evaluation of Wilson loop (VAX 11/780!)
- Non-zero string tension for smaller values of the coupling *consistent with asymptotic freedom*



Earlier for Z(2) :Creutz-Jacobs-Rebbi, PRL 42, 1390 (1979)
Also for S(2):Wilson, Cargese Summer Institute (1979)

' D VOLUME 21, NUMBER 8 15 APR

Monte Carlo study of quantized SU(2) gauge theory

Michael Creutz
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973

(Received 24 October 1979)

: Carlo techniques, we evaluate path integrals for pure SU(2) gauge fields. Wilson's procedure on a lattice of up to 10^4 sites controls ultraviolet divergences. Our renormalization ased on confinement, is to hold fixed the string tension, the coefficient of the asymptotic l between sources in the fundamental representation of the gauge group. Upon reducing the erve a logarithmic decrease of the bare coupling constant in a manner consistent with the normalization-group prediction. This supports the coexistence of confinement and asymptotic antized non-Abelian gauge fields.

Hamber-Parisi, Weingarten 1981

- Computer evaluation of hadron masses (MeV) (VAX11/780)

  It was a feat totally unthinkable in traditional field theory

Volume 47, Number 25 PHYSICAL REVIEW LETTERS 21 December

Numerical Estimates of Hadronic Masses in a Pure SU(3) Gauge Theory

H. Hamber
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973

and

G. Parisi
Istituto Nazionale di Fisica Nucleare, Frascati, Italy, and Istituto di Fisica della Facoltà di Ingegneria, Rome, Italy

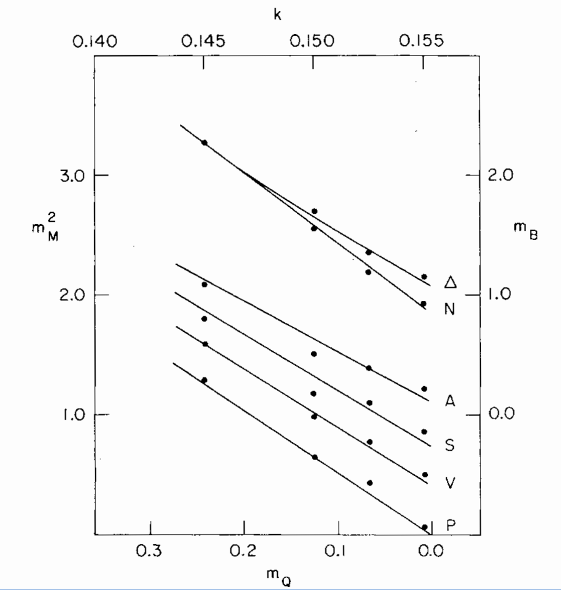
(Received 2 October 1981)

Volume 109B, number 1,2 PHYSICS LETTERS

MONTE CARLO EVALUATION OF HADRON MASSES IN LATTICE GAUGE THEORIES WITH FERMIONS

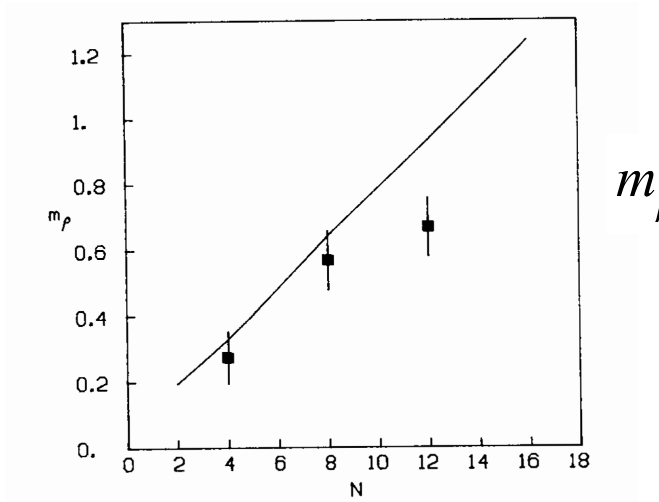
Don WEINGARTEN
Physics Department, Indiana University, Bloomington, IN 47405, USA

Received 13 October 1981



$$m_\rho = 800 \pm 100\text{MeV}$$

$$m_p = 950 \pm 100\text{MeV}$$



$$m_\rho = 670 \pm 100\text{MeV}$$

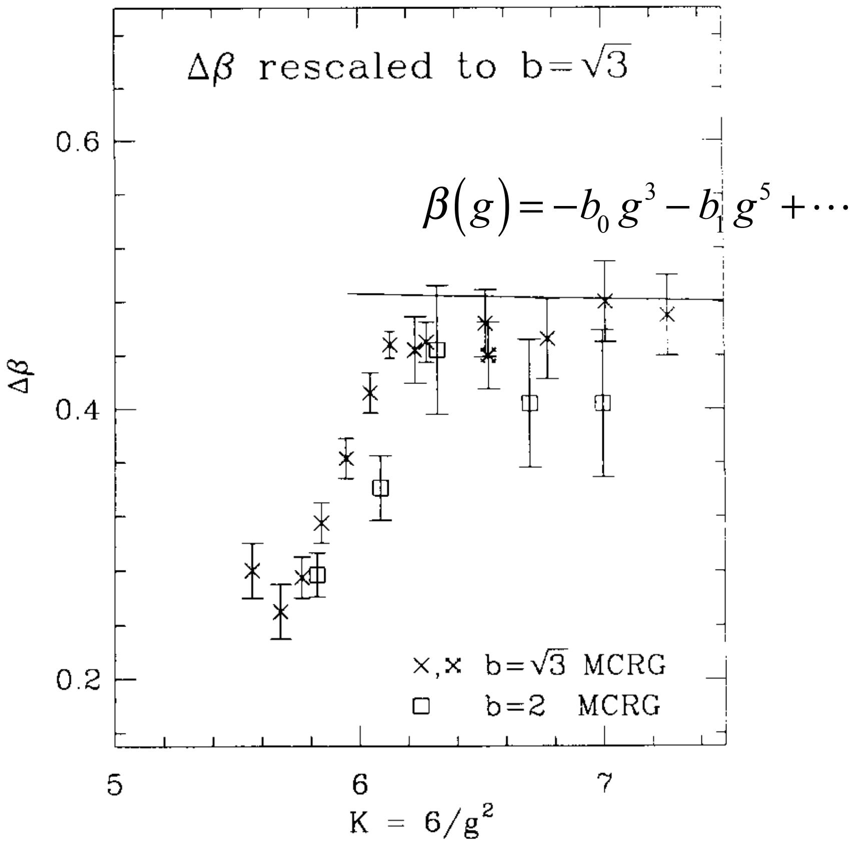
Exploring the new frontier

We can understand all those puzzles of strong interactions!

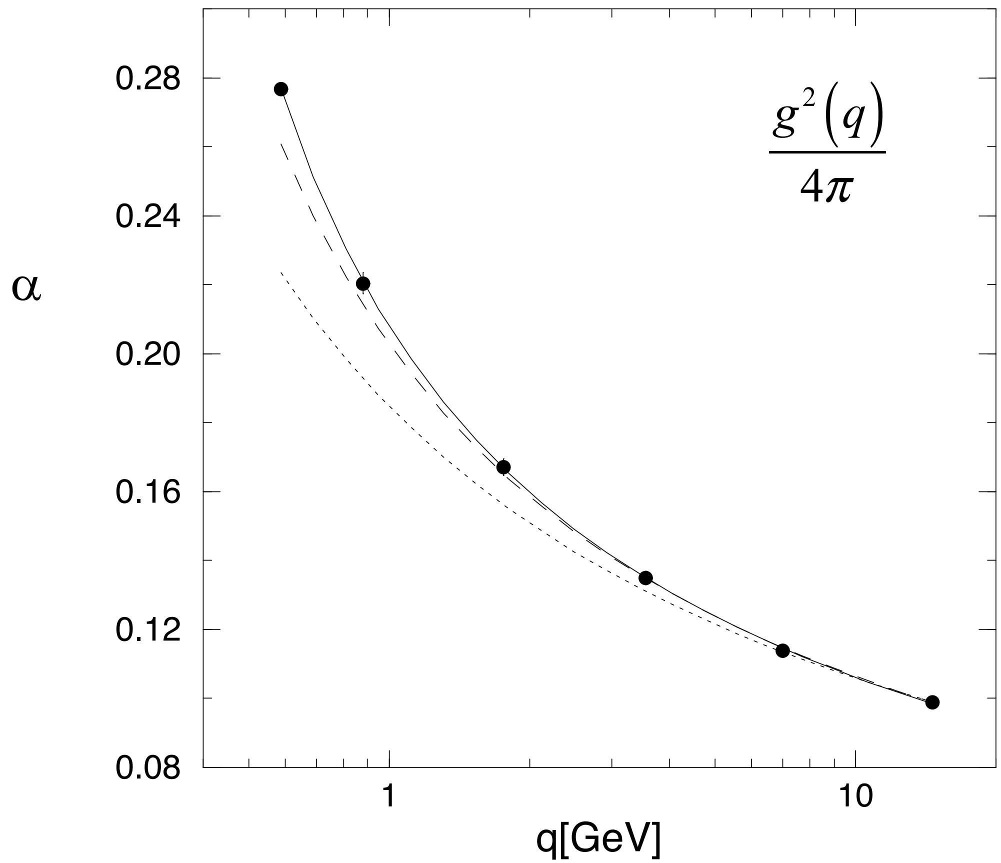
- *Confinement*
- *Spectroscopy of hadrons*
- *Strong interactions at high-temperature*
- *Weak interactions of hadrons*
- *Heavy quark physics*

Confinement for $\infty \geq g_0^2 \geq 0$

- RG running of the coupling $g^2(q)$
  - Monte Carlo studies following up on K. Wilson (1979)
  - Refined into the step-scaling method in the 90's



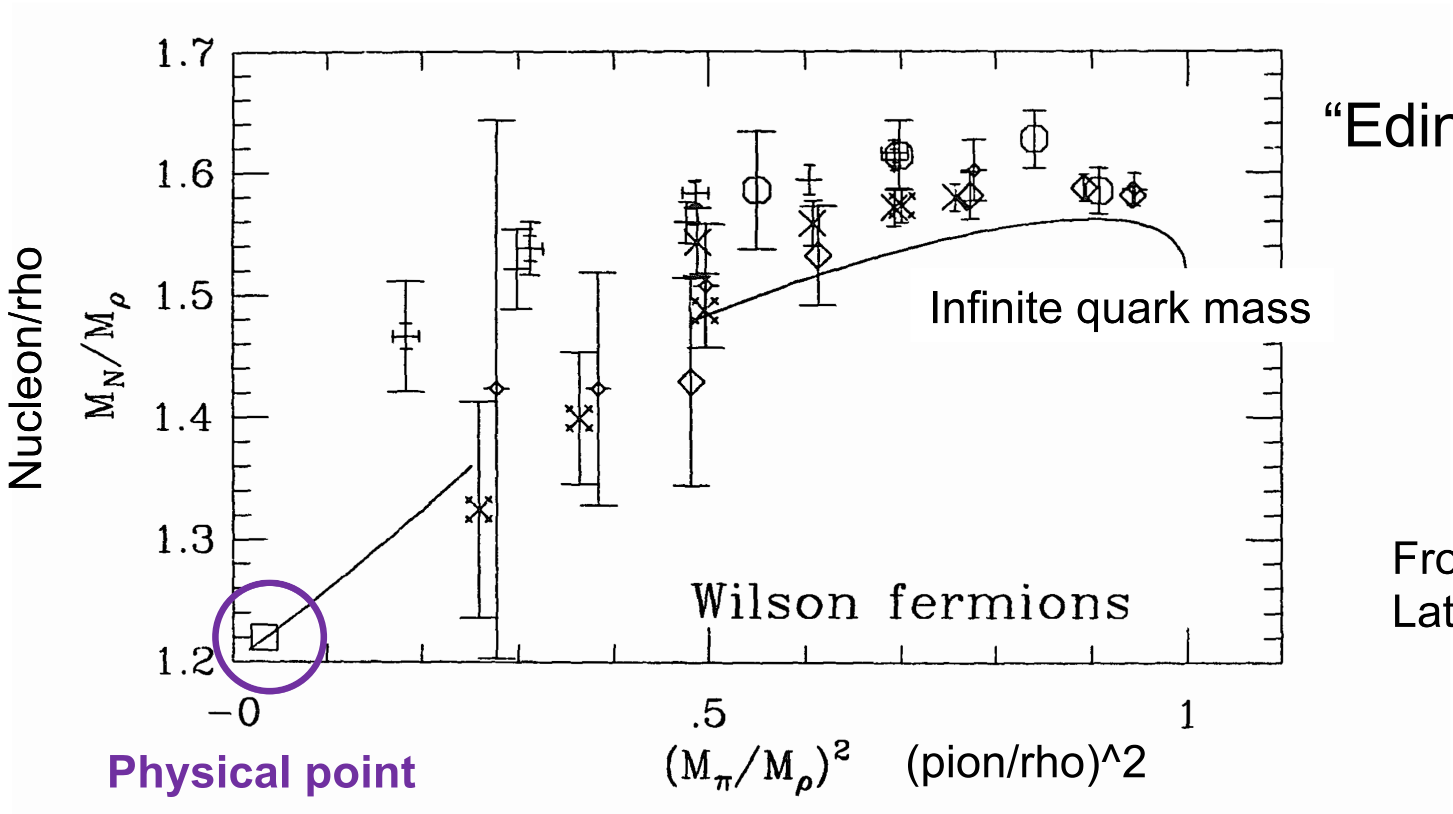
R. Gupta et al PLB211, 132 (1988)



M. Luescher et al, NPB413, 481 (1994)

Spectroscopy of hadrons

- Ground state hadron masses as a function of quark mass



"Edinburgh/APE plot"

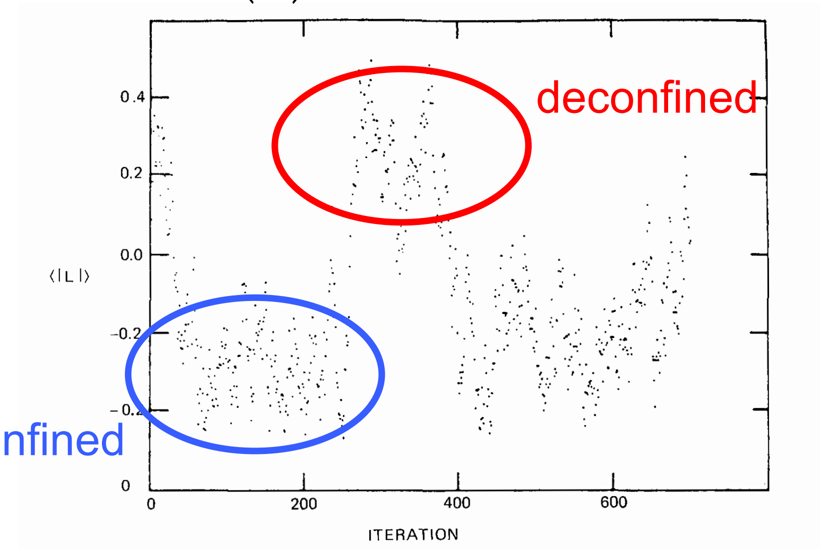
From R. Gupta, Latttice 89 review

- Experimentally unkown states, e.g.,
  - H dibaryon
    - Mackenzie-Thacker (no 1985)/Iwasaki et al (yes 1988) (still undecided today)
  - Pure glue spectrum, ……

High temperatures

- Deconfinement in pure gluon system?
  - Hagedorn's limiting temperature (1965) $T < T_c \approx m_\pi$
  - Theoretical prediction by Polyakov(1978)/Susskind(1979)
  - First Monte Carlo in 1980 and rapid development in '80s

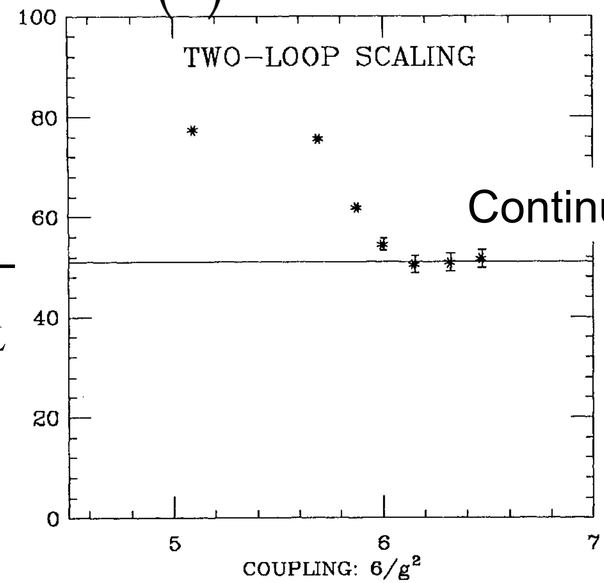
$SU(2)\ 7^3 \times 3$



MacLerran-Svetitsky, PL98B, 195 (1981)

Kuti et al PL98B, 199(1981)

$SU(3)\ 21^3 \times 14$



$T_c \approx 230$ MeV

Gottlieb et al PRL55, 1958 (1985)

Weak interaction of hadrons

- Puzzles from the 60's such as
  - $\Delta I = 1/2$ rule of $K \to 2\pi$ decays
  - CP violation and ε'/ε
- Possibilities of lattice elucidation pointed out in 1984

WEAK INTERACTIONS ON THE LATTICE

N. CABIBBO

i Fisica, II Università di Roma "Tor Vergata", INFN, Sezione c

G. MARTINELLI

INFN, Laboratori Nazionali di Frascati, Frascati, Italy

R. PETRONZIO[1]

CERN, Geneva, Switzerland

Received 5 December 1983
(Revised 16 April 1984)

ME 55, NUMBER 25 PHYSICAL REVIEW LETTERS

Lattice Calculation of Weak Matrix Elements

C. Bernard, T. Draper, G. Hockney, A. M. Rushton, and A. Soni

of California, Irvine, Cal.

1 Illinois 60439

IMBER 14 PHYSICAL REVIEW LETTERS

Calculation of Weak Transitions in Lattice QCD

Richard C. Brower and Guillermo Maturana

Institute for Particle Physics, University of California, Santa Cruz, California 95064

and

M. Belén Gavela[a]

Department of Physics, Brandeis University, Waltham, Massachusetts 02254

and

Rajan Gupta

Department of Physics, Northeastern University, Boston, Massachusetts 02115

(Received 23 February 1984)

Heavy quarks and CKM

- Kobayashi-Maskawa 1973
- Experimental discoveries: J/ψ 1974 Υ 1977
- Lattice studies began in the late 80's

HEAVY QUARKS ON THE LATTICE

E. EICHTEN *Static approximation*

Fermi National Accelerator Laboratory*, P.O. Box 500, Batavia, IL 60510

from Proceedings of Lattice 87 at Seillac

Abstract

ating heavy quarks is applied to lattice Q.C.D. for heavy
ıdition $m_H a > 1$. Explicit applications to the measurement c
neters are presented. Numerical results for B mesons are ol

EFFECTIVE LAGRANGIANS FOR SIMULATING OF HEAVY QUARK SYSTEMS

G.P. Lepage *NRQCD*

Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, NY 14853, U.S.A.; and D.A.M.T.P., University of Cambridge, Silver Street, Cambridge CB3 9EW, U.K.

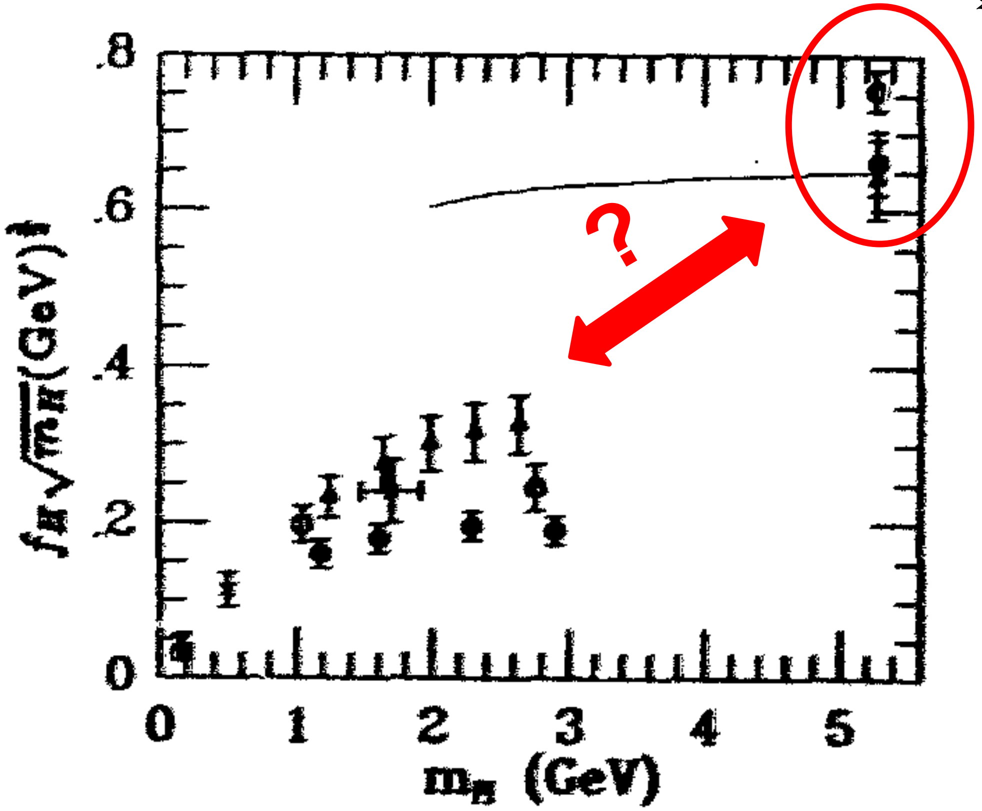
B.A. Thacker

Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, NY 14853, U.S.A.

How initial results looked in '89-'90

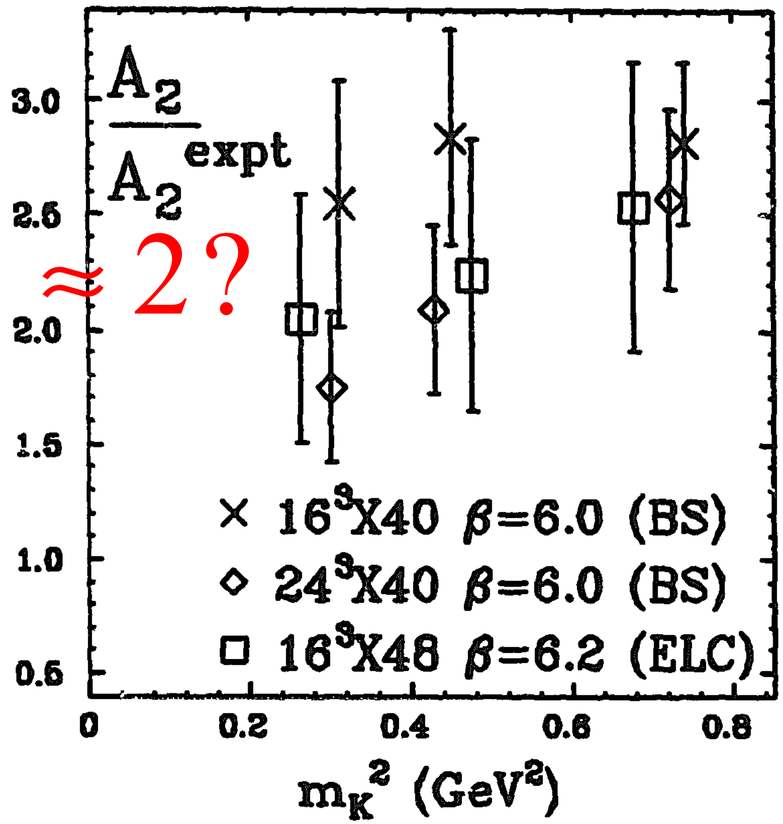
- Heavy-light decay constant
- $K \to \pi\pi$ decay amplitude

Static limit $m_Q \to \infty$



From Eichten's review at Lattice 1990

I=2 channel



From Sharpe's review at Lattice 1989

From S. Sharpe's review at Lattice 89

What?	Why?	Who? [6]	Level
Nucleon matrix elements			
f_π/m_N, f_K/f_π	check	MANY	2
Axial vector matrix elements: g_A ...	check	Sömmer [7]	2
EM form factors: $G_M(q^2)$, ...	check	Wilcox, Draper/Liu [8]	2
Structure functions	check	Rossi [9]	1
Neutron Electric Dipole Moment	measure θ_{QCD}	Goksch [10]	1
Heavy-light mesons			
f_D, f_B, B_D, B_B	$\overline{D}D$ and $\overline{B}B$ mixing	Eichten, Martinelli [11]	1-2
$D \to Ke\nu$, $(B \to \pi e\nu)$, ...	measure V_{cs}, V_{ub}	El Khadra, [12] Sachrajda [13]	1-2
$D \to K\pi$	check	Sachrajda, Simone	1
K decay and mixing amplitudes			
B_K	extract δ from ϵ	Bernard, Kilcup, [14] Martinelli	3
$K \to \pi\pi$ ($\Delta I = 1/2$ rule)	check	Bernard, Kilcup, Martinelli	2
ϵ'	over-determine δ	Kilcup, Bernard	2

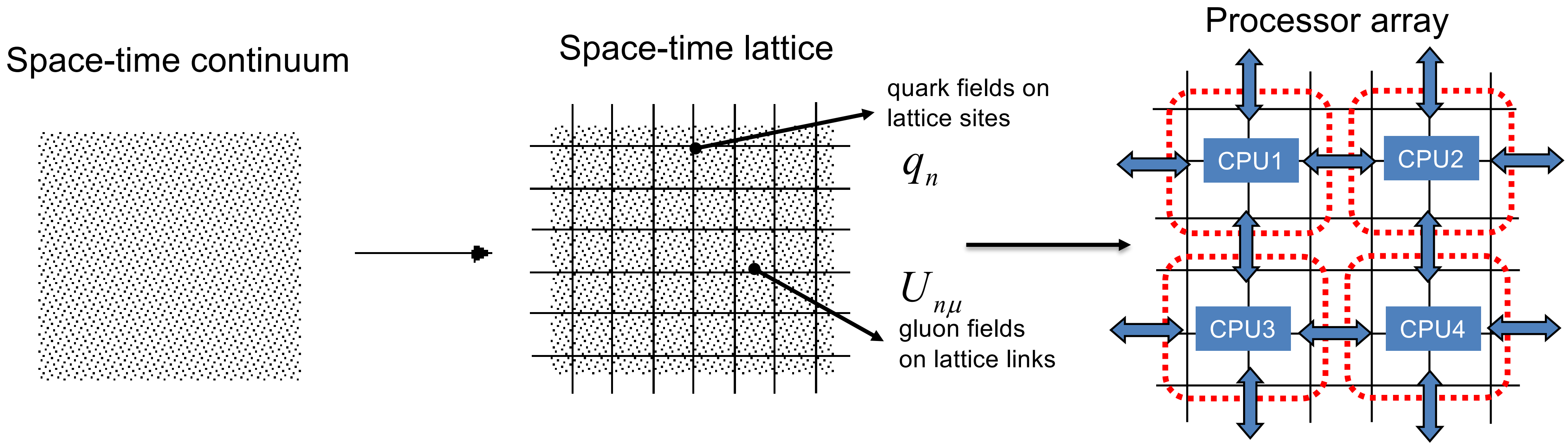
Table 1: Work done on weak matrix elements in the year preceding September 1989

- Level 1: exploratory, setting up methods, resolve problems of principle
- Level 2: larger scale calculation, reasonably small errors (10-20%)
- Level 3: reliable quenched calculation with small errors

Building the instrument

Clearly people needed (much) more computing power…..

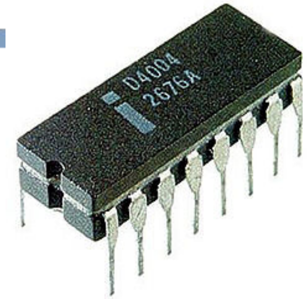
QCD and parallel computing

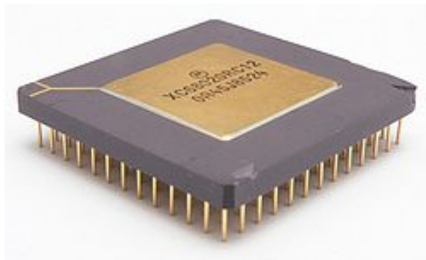


- QCD is a local field theory; only nearest neighbor interactions
- Computation on each CPU and communication between nearest neighbor CPUs, scalable to “infinite” lattice size
- *An ideal case of massively parallel computation!*

Microprocessors in the 70's

- 1971 Intel 4004 4 bit $60/chip
- 1972 Intel 8008 8 bit $120
- 1974 Intel 8080 8 bit
- 1974 Motorola 6800 8 bit $360
- 1978 Intel 8086 16 bit $320
- 1979 Motorola 68000 32 bit



pictures from Wikipedia

In the 70's, the key element for a parallel system became affordable for a reasonable price even for academics!

Parallel QCD machines in the 80's

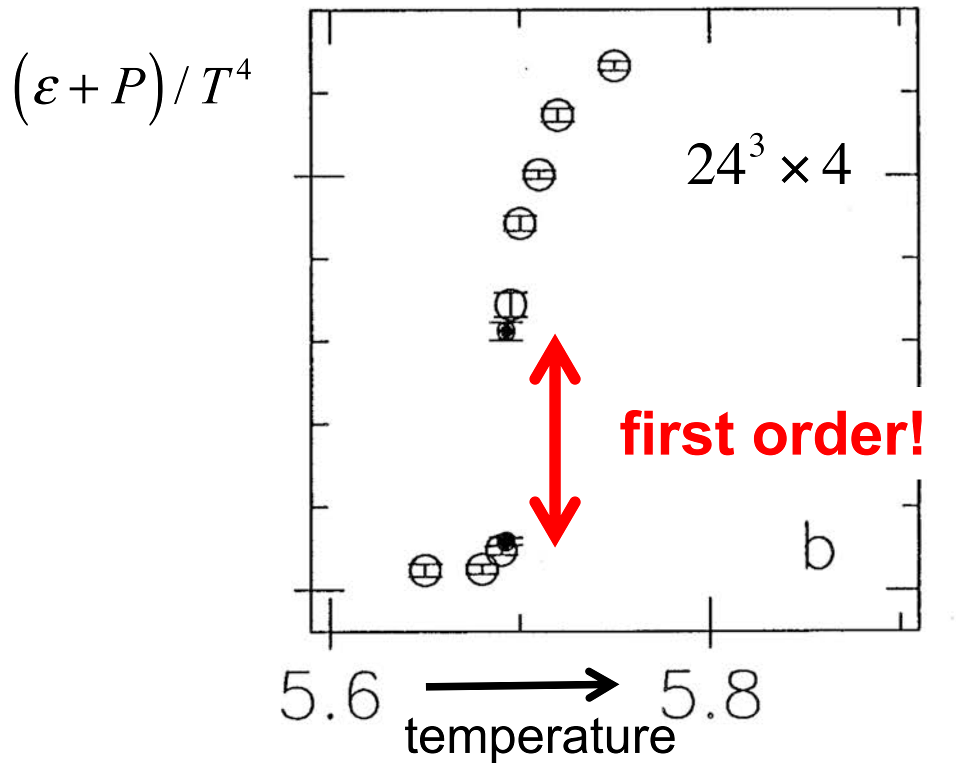
			CPU	FPU	peak
Columbia	1984	Christ-Terrano	PDP11	TRW 16MPY	
Columbia-16	1985	Christ et al	Intel 80286	TRW	0.25GF
APE1	1987	Cabibbo-Parisi	3081/E	Weitek	1GF
Columbia-64	1987	Christ et al	Intel 80286	Weitek	1GF
Columbia-256	1988	Christ et al	M68020	Weitek 3332	16GF
ACPMAPS	1991	Mackenzie et al	micro VAX	Weitek 8032	5GF
QCDPAX	1989	Iwasaki-Hoshino	M68020	LSI-logic 64133	14GF
GF11	1992	Weingarten	PC/AT	Weitek 1032/33	11GF

- Parallel array of (commodity CPU + custom made FPU)
- More or less hand-made by academics
- *Overtook vector supercomputers in speed in the late 80's*

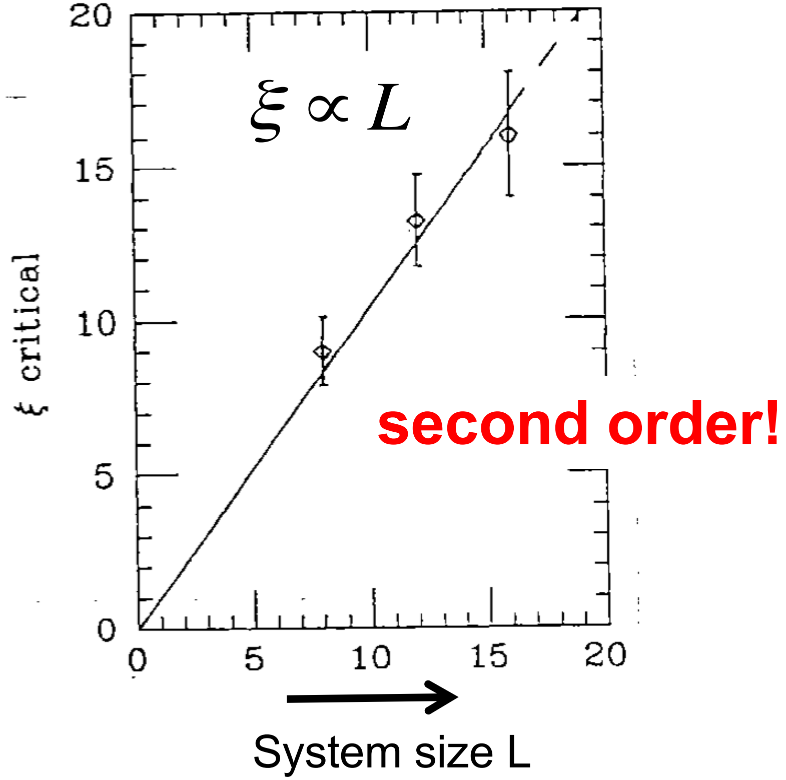
Order of the deconfining phase transition

- Columbia Group (N. Christ et al 1988): physical quantities has a jump, so **first-order** F. R. Brown et al, PRL61, 2058 (1988)
- APE Group (G. Parisi et al 1988): correlation length increases with system size, so **second order** P. Bacilieri et al, PRL61, 1545 (1988)

Columbia-64 computer



APE-1 computer

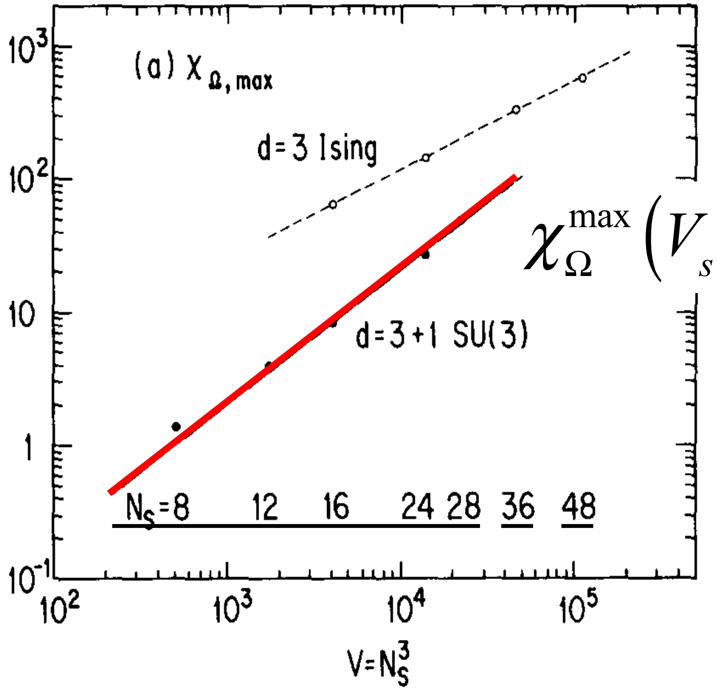


Which one is right?

- ***First-order*** making full use of finite-size scaling theory:
  - Susceptibility peak grows linearly with spatial volume
  - Correlation length, though large, stays finite

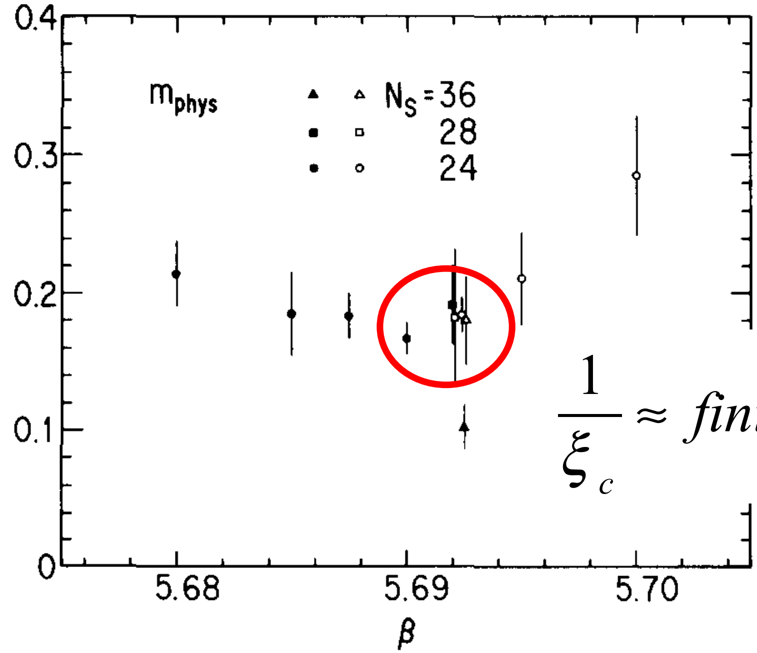
Fukugita et al, PRL63, 1768 (1989)

Susceptibility peak scaling



$$\chi_\Omega^{\max}\left(V_s\right) \propto V_s$$

Correlation length

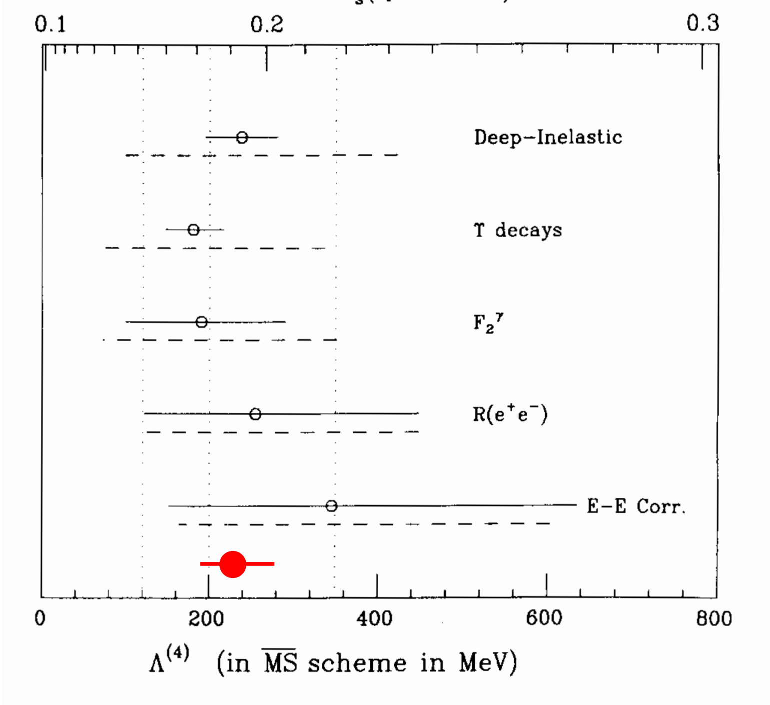


$$\frac{1}{\xi_c} \approx finite$$

Fundamental constants of QCD

- QCD coupling

PDG 1990



$$\alpha_s^{MS}(5\text{GeV}) = 0.174 \pm 0.012$$

El-Kahdra et al, PRL69, 729 (1992)

Energy scale from charmonium spectrum in quenched lattice QCD

- Quark masses

RPP 1996

u

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Mass m = 2 to 8 MeV [a]

Charge = $\frac{2}{3}$ e $I_z = +\frac{1}{2}$

m_u/m_d = 0.25 to 0.70

d

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Mass m = 5 to 15 MeV [a]

Charge = $-\frac{1}{3}$ e $I_z = -\frac{1}{2}$

m_s/m_d = 17 to 25

s

$I(J^P) = 0(\frac{1}{2}^+)$

Mass m = 100 to 300 MeV [a]

Charge = $-\frac{1}{3}$ e Strangeness = −1

$(m_s - (m_u + m_d)/2)/(m_d - m_u)$ = 34 to 51

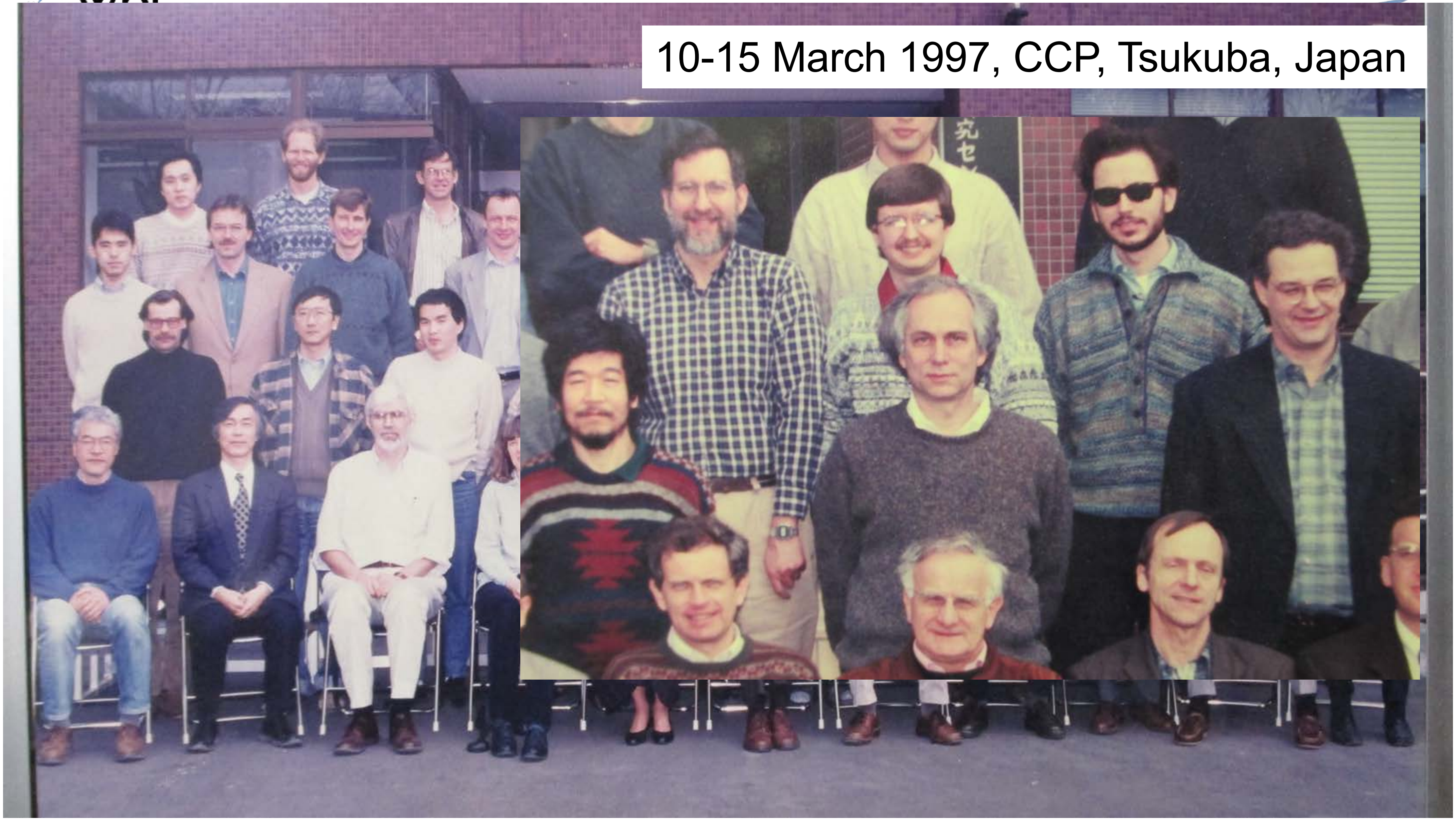
$$\bar{m}_{ud}(2\text{GeV})^{N_f=0} = 3.6(6)\ \text{MeV}$$

$$\bar{m}_s(2\text{GeV})^{N_f=0} = 95(16)\,\text{MeV}$$

Gough et al, PRL79, 1622 (1997)

Both simulations done on ACPMAPS

“Lattice QCD on Parallel Computers”



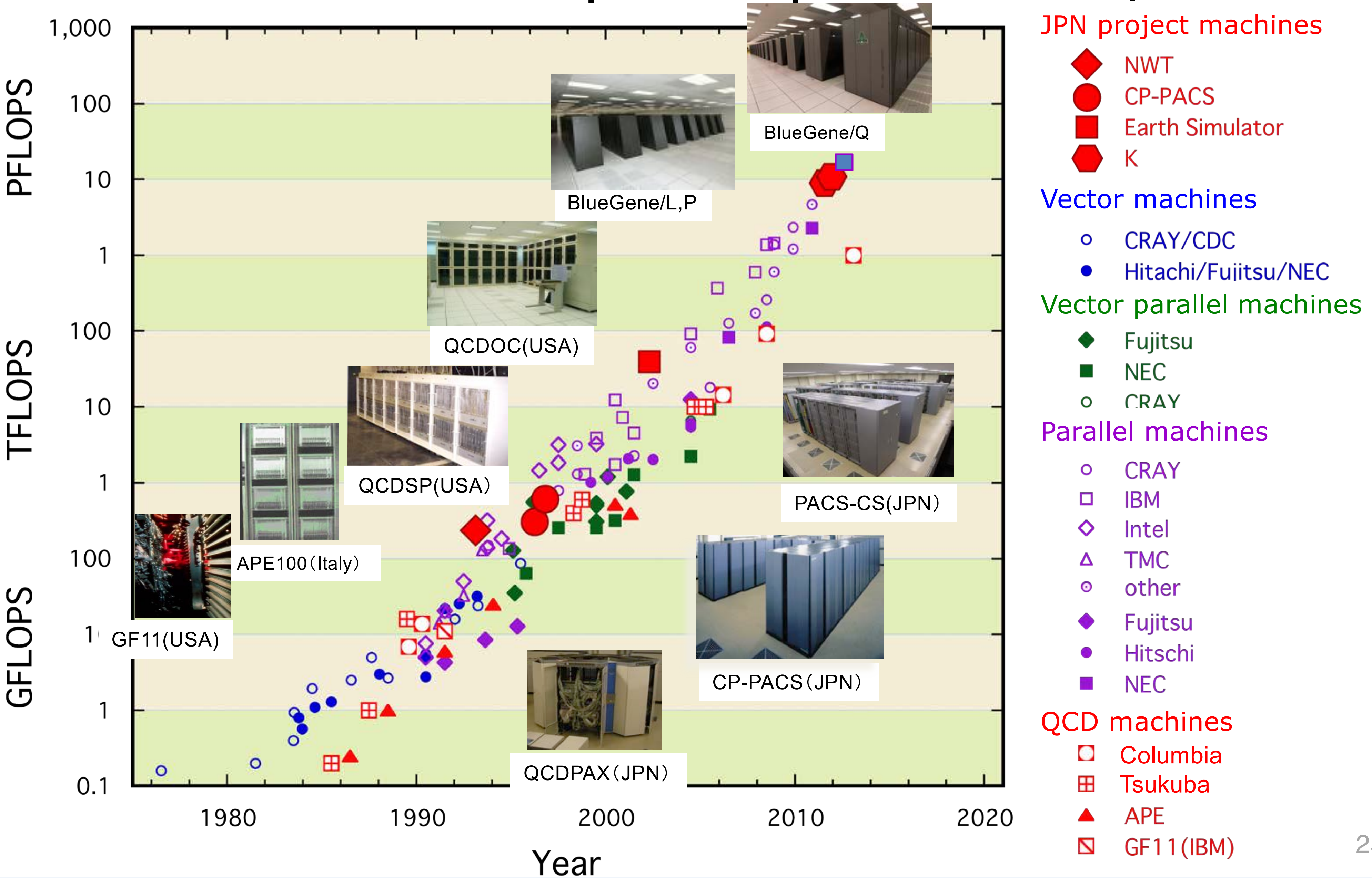
10-15 March 1997, CCP, Tsukuba, Japan

QCD machines in the 90's and later

			CPU	vendor	peak
CP-PACS	1996	Iwasaki et al	PA-RISC	Hitachi	0.6TF
QCDSP	1998	Christ et al	TI DSP	--	0.6TF
APEmille	2000	APE Collab.	custom	--	0.8TF
QCDOC	2005	Christ et al	PPC-based	IBM(BG/L)	10TF
PACS-CS	2006	Ukawa et al	Xeon	Hitachi	14TF
QCDCQ	2011	Christ et al	PPC-based	IBM(BG/Q)	0.5PF
QPACE	2012	Wettig et al	PowerXCell	--	0.2PF

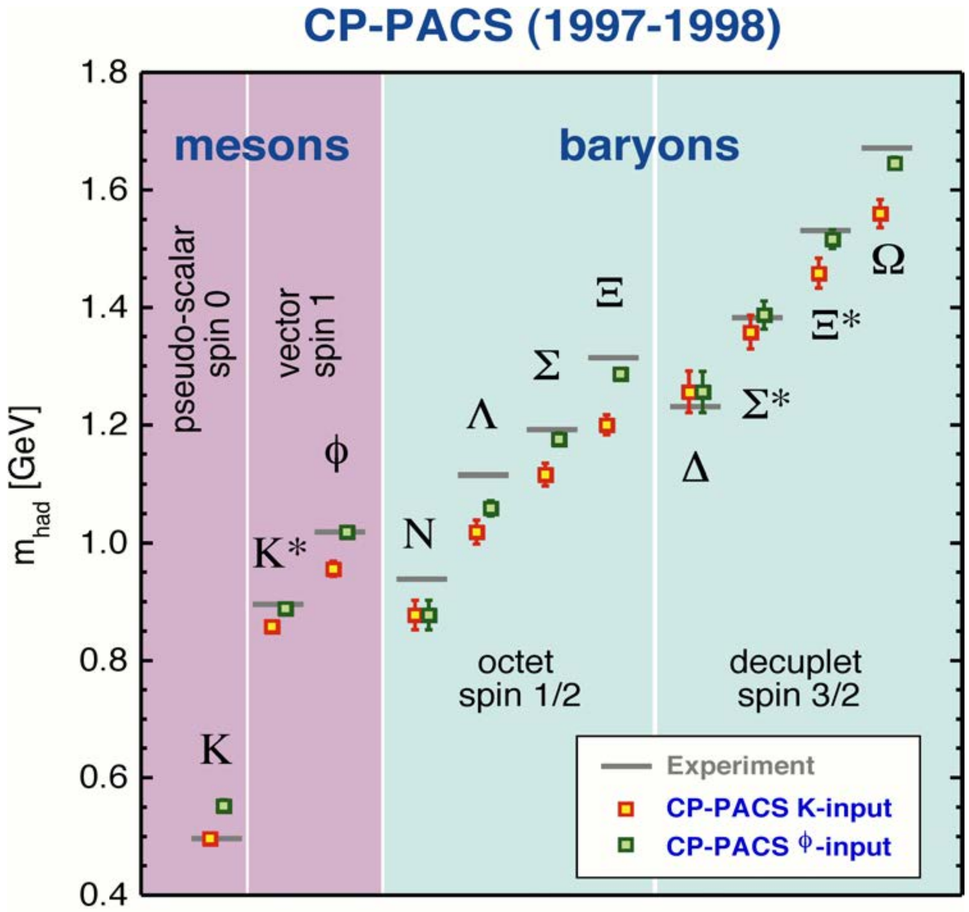
- Big success continued, with increasing involvement of big vendors (Hitachi, IBM) to secure necessary technology
- Gradual loss of control of physicists as the HPC vendors embraced the parallel paradigm

Lattice QCD machines and supercomputer development



Advances in the 90's

- Quenched hadron spectrum (1997)
  - Determination with a few % error revealed a definite discrepancy with experiment
  - ***Signaled the end of "quenched" era***



- Big shift to full QCD (late 90's to early 00's)
  - MILC (staggered)
  - CP-PACS (Wilson-clover)
  - RBRC (domain-wall)
  - …..

An interlude

Building the community (I)

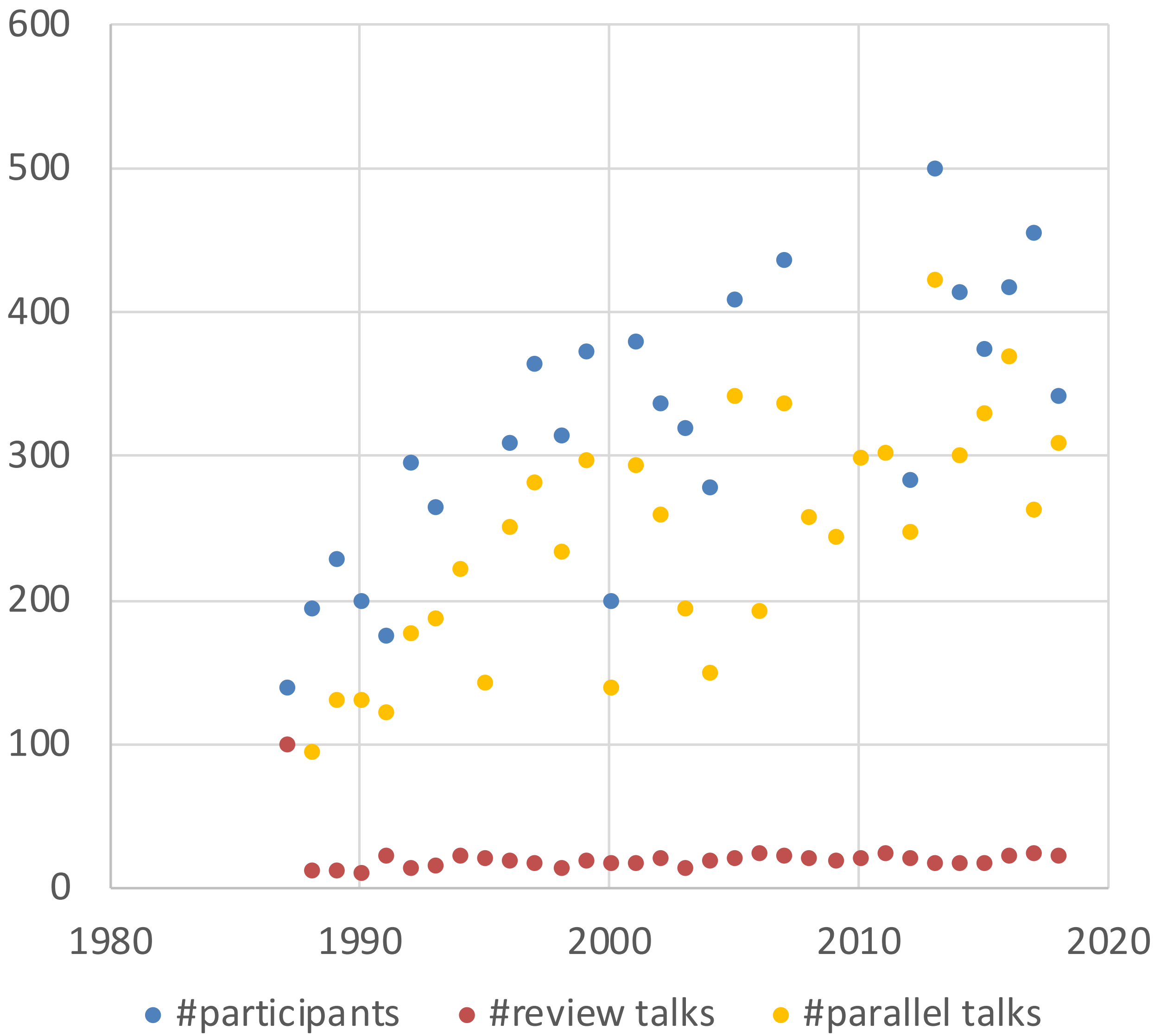
- Lattice conference (LATTICE XX) : the meeting place
  - Started in 1984 and being held every year
  - A few dozen of review talks + hundreds of parallel talks (everyone talks!) style quickly established
  - Rotates among USA-Europe-Asia

- ***LatticeNews*** mailing list: the communication tool
  - ***latticejobs*** started in 1994
  - ***latticeannounce*** started in 2001/renamed as ***LatticeNews*** in 2004
  - Created and maintained by S. Gottlieb
  - Current registrations: *latticejobs* 636 *LatticeNews* 839

- arXiv (hep-lat)
  - Started in 1992

Lattice XX participants and talks

1984	Argonne	USA
1985	Tallahassee	USA
1986	BNL	USA
1987	Seillac	France
1988	FNAL	USA
1989	Capri	Italy
1990	Tallahassee	USA
1991	Tsukuba	Japan
1992	Amsterdam	Netherland
1993	Dallas	USA
1994	Bielefeld	Germany
1995	Melbourne	Australia
1996	St Louis	USA
1997	Edinburgh	Scotland
1998	Boulder	USA
1999	Pisa	Italy
2000	Bangalore	India
2001	Berlin	Germany



Building the

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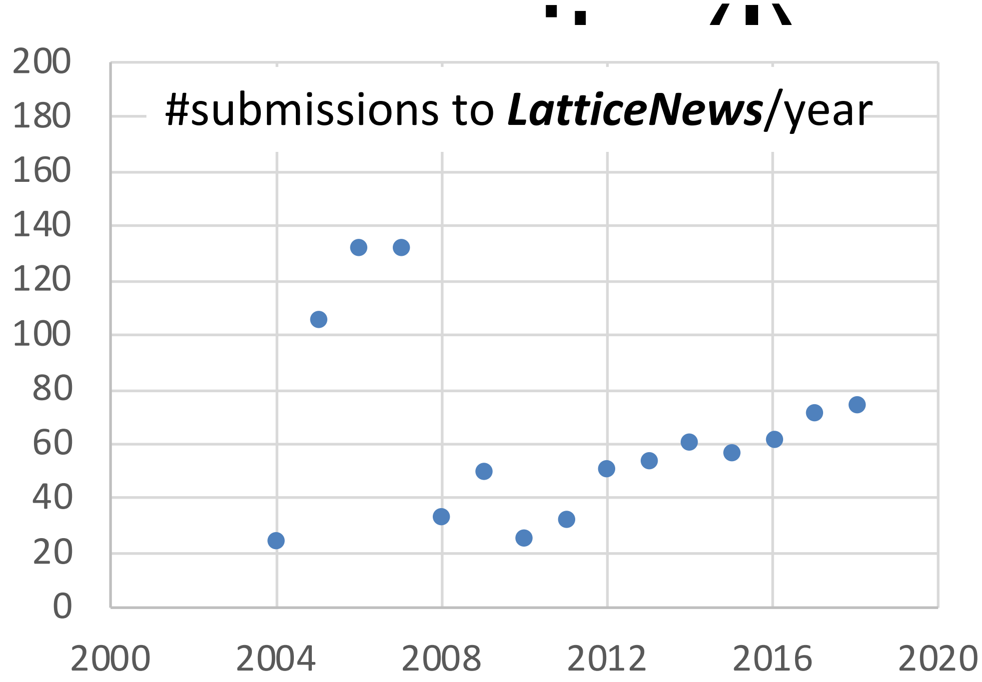
  - A few dozen of review talks + talks!) style quickly establishe
  - Rotates among USA-Europe-



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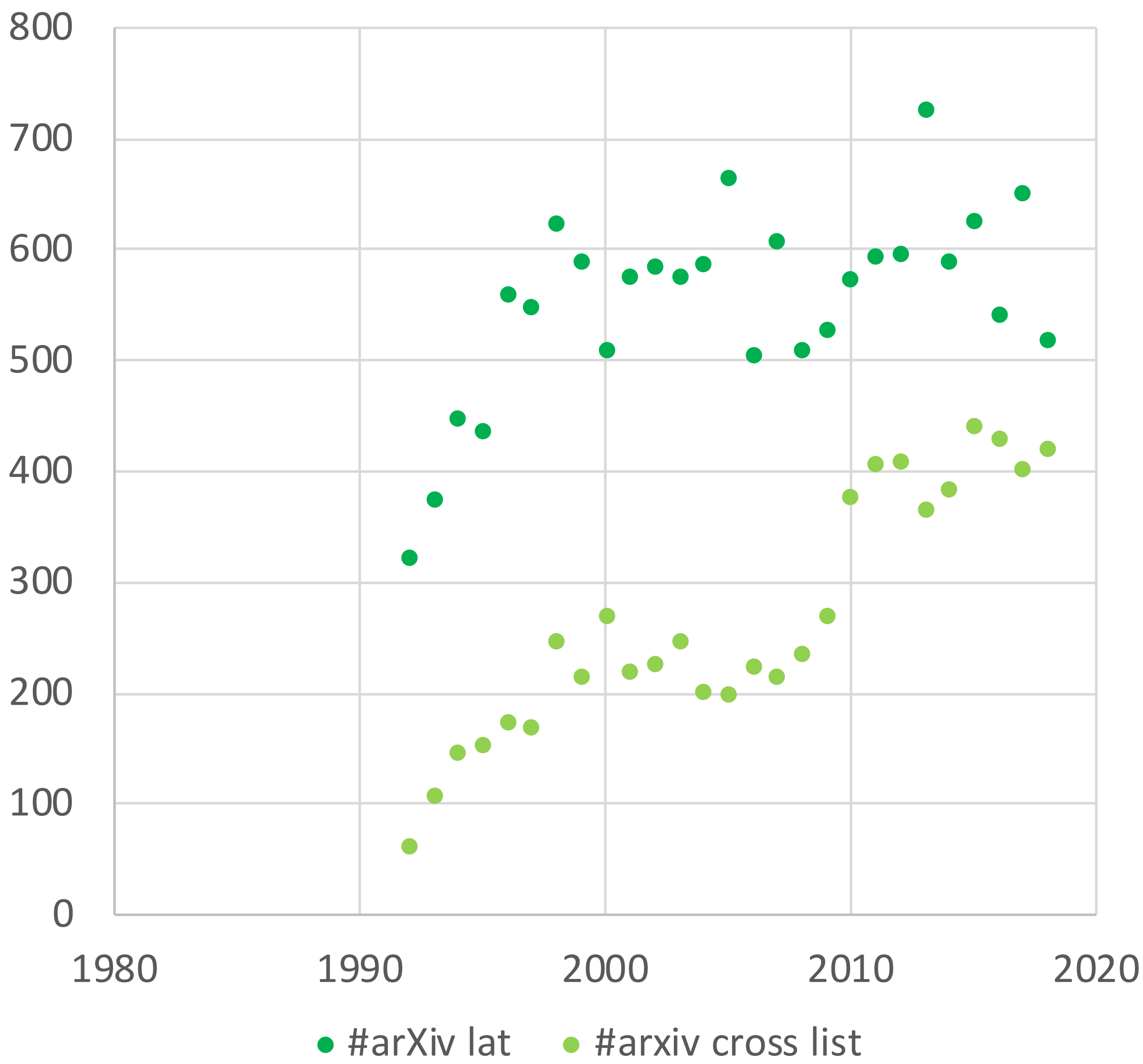
  - Current registrations: *latticejobs* 636 *LatticeNews* 839

- arXiv (hep-lat)
  - Started in 1992

Lattice XX and hep-lat

2002	Boston	USA
2003	Tsukuba	Japan
2004	FNAL	USA
2005	Dublin	Ireland
2006	Tucson	USA
2007	Regensburg	Germany
2008	Williamsburg	USA
2009	Beijin	China
2010	Villasimius	Italy
2011	Squaw Valley	USA
2012	Cairns	Australia
2013	Mainz	Germany
2014	New York	USA
2015	Kobe	Japan
2016	Southampton	England
2017	Granada	Spain
2018	East Lansing	USA
2019	Wuhan	China
2020	Bonn	Germany

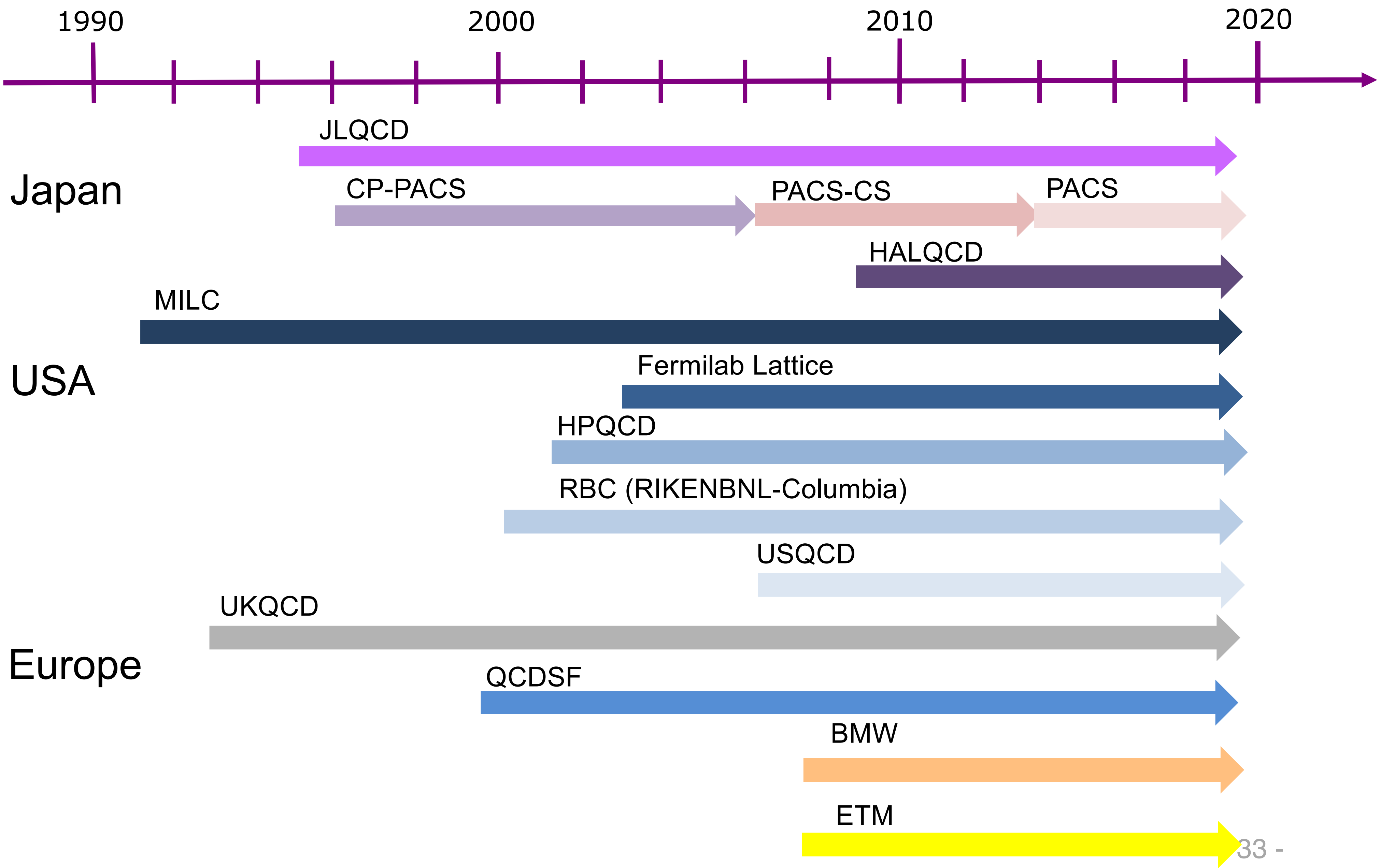


Building the community (II)

- International Lattice Data Grid (ILDG):sharing the resources
  - Started in 2002
  - Federation of regional configuration repositories
  - NERSC(USA)/LGT(Europe)/JLDG(Japan)

- FLAG (Flavor Lattice Averaging Group):outreaching the results
  - Critical review and summary of lattice QCD results relevant for experiments
  - Started in 2007 in Europe, now a global effort across Europe, America, Asia
  - Publication every few years: 2011, 2014, 2017, 2019; well cited: 363, 570, 528, 94 (inspire Nov. 2019)

Lattice QCD collaborations



Kenneth G. Wilson Award

for Excellence in Lattice Field Theory

2011 ***Xu Feng, Marcus Petschlies, Karl Jansen, Dru B. Renner***

2012 ***T. Blum, P.A. Boyle, N.H. Christ, N. Garron, E. Goode,***
T. Izubuchi, C. Jung, C. Kelly, C. Lehner, M. Lightman, Q. Liu,
A.T. Lytle, R.D. Mawhinney, C.T. Sachrajda, A. Soni, C. Sturm

2013 ***André Walker-Loud***

2014 ***Gergely Endrödi***

2015 ***Stefan Meinel***

2016 ***Antonin Portelli***

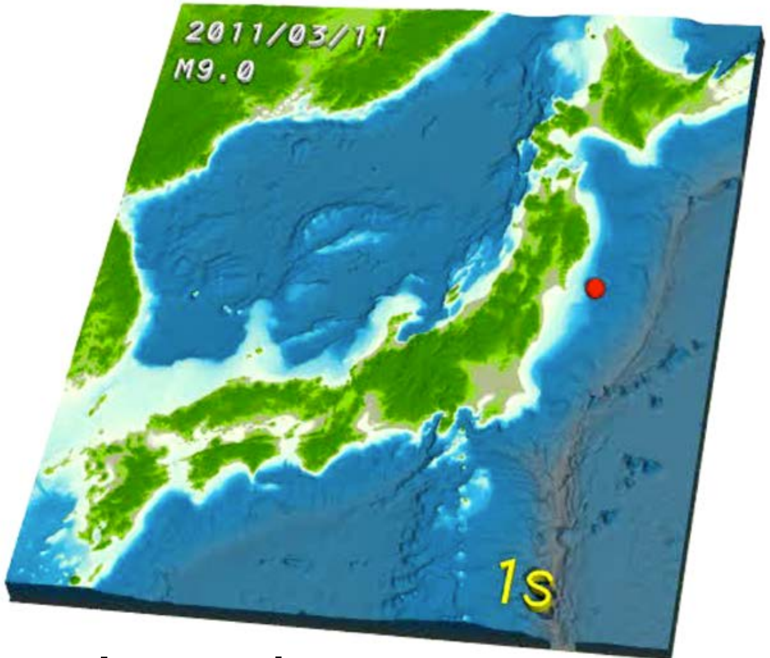
2017 ***Raul Briceno***

2018 ***Zohreh Davoudi***

2019 ***Luchang Jin***

A time of crisis

- Eastern Japan Great Earth Quake 2011.3.11
  - The earthquake 14:46 JST on 11 March
  - The tsunamis 15:00-16:00 on 11 March
  - The Nuclear Power Plant 11 – 15 March



- No serious damage to supercomputers for lattice, but they were likely to be down for a half year or more due to power shortage, stopping research in serious ways

A case of international support

- Generous offer of international support
  - 3 April mail from Paul Mackenzie for USQCD
  - 4 April mail from Nick Samios for RBRC
  - May 2011 to March 2012 6 projects in Japan benefited from the US resources
- March 2012
  - end of the projects and report to USQCD and RBRC

Date: Sun, 3 Apr 2011 17:36:23 -0500
From: paul mackenzie <mackenzie@fnal.gov>
To: Akira Ukawa <ukawa@ccs.tsukuba.ac.jp>
Subject: lattice supercomputing in Japan in the

Dear Akira,

Members of the USQCD Executive Commit
we would like to find a way to be of help to you
electricity has been restored, but facilities which
are still off, and will remain so for some time. If
of our cluster resources would be helpful for Jap

Date: Thu, 7 Apr 2011 10:36:01 -0400
From: "Samios, Nicholas P" <samios@bnl.gov>
To: <ukawa@ccs.tsukuba.ac.jp>
Cc: "Taku Izubuchi" <izubuchi@quark.phy.bnl.gov>
"Norman Christ" <nhc@phys.columbia.edu>
Subject: computing

Dear Akira,

If useful RBRC is setting aside 4 racks of the Q
complex for use by any interested Japanese physicis

Getting nimble with quarks

Quarks posed many headaches!

- Light quarks $q(x) \to e^{i\theta\gamma_5} q(x)$
  - Chiral symmetry, while playing a fundamental role in the strong interactions, is *incompatible with lattice* under rather general conditions (Nielsen-Ninomiya 1981)

- Heavy quarks $m_h a > 1$
  - Large quarks masses of charm and bottom leads to *serious distortions* if simulated on lattices with coarse lattice spacings

- Computation $\int \prod_n d\bar{q}_n dq_n \; e^{-\sum_{n,m} \bar{q}_n D_{nm}(U) q_m} = \det D(U) = \int \prod_n d\bar{\phi}_n d\phi_n \; e^{-\sum_{n,m} \bar{\phi}_n \left(\frac{1}{D(U)}\right)_{nm} \phi_m}$
  - Sea quark effects requires *computation of the determinant or inverse of lattice Dirac operator*, whose cost blows up for light quarks

$$\text{computational cost}\left(D^{-1}\right) \propto \frac{1}{m_q a} \times \left(\frac{L}{a}\right)^4$$

Chiral symmetry

- Condition for exact (but modified) chiral symmetry

  $$\gamma_5 D + D\gamma_5 = a D\gamma_5 D$$ Ginsparg-Wilson '82

  - It took a full decade before the explicit formalisms were found:
    - Domain-wall formalism — Kaplan 1992
    - Overlap formalism — Narayanan-Neuberger 1995
    - Perfect action — Hasenfratz-Niedermeyer 1998
  - Simulations started in late '90s and early '00s
    - Christ et al, RBC collaboration — domain-wall
    - KEK group, JLQCD collaboration — overlap

- Fine-tuning of conventional fermions

  Wilson formalism (Wilson 1975) / Staggered formalism (Susskind 1977)

  - Add terms reducing O(a^n) effects (Wilson '79, Symanzik '83)
  - Smearing of gauge links (APE '85, HYP '99, STOUT '02)
  - Complicated but highly improved actions e.g., HISQ '07 used today

Heavy quarks

- Static approximation — Eichten, Lattice 87 NPBP 4, 170 ('88)
  - Systematic expansion in $1/m_h a$
- NRQCD — Lepage-Thacker, Lattice 87 NPBP 4, 199 ('88)
  - Non-relativistic reformulation of QCD by integrating out all momenta higher than the heavy quark mass M
  - Systematic expansion in powers of heavy quark velocity

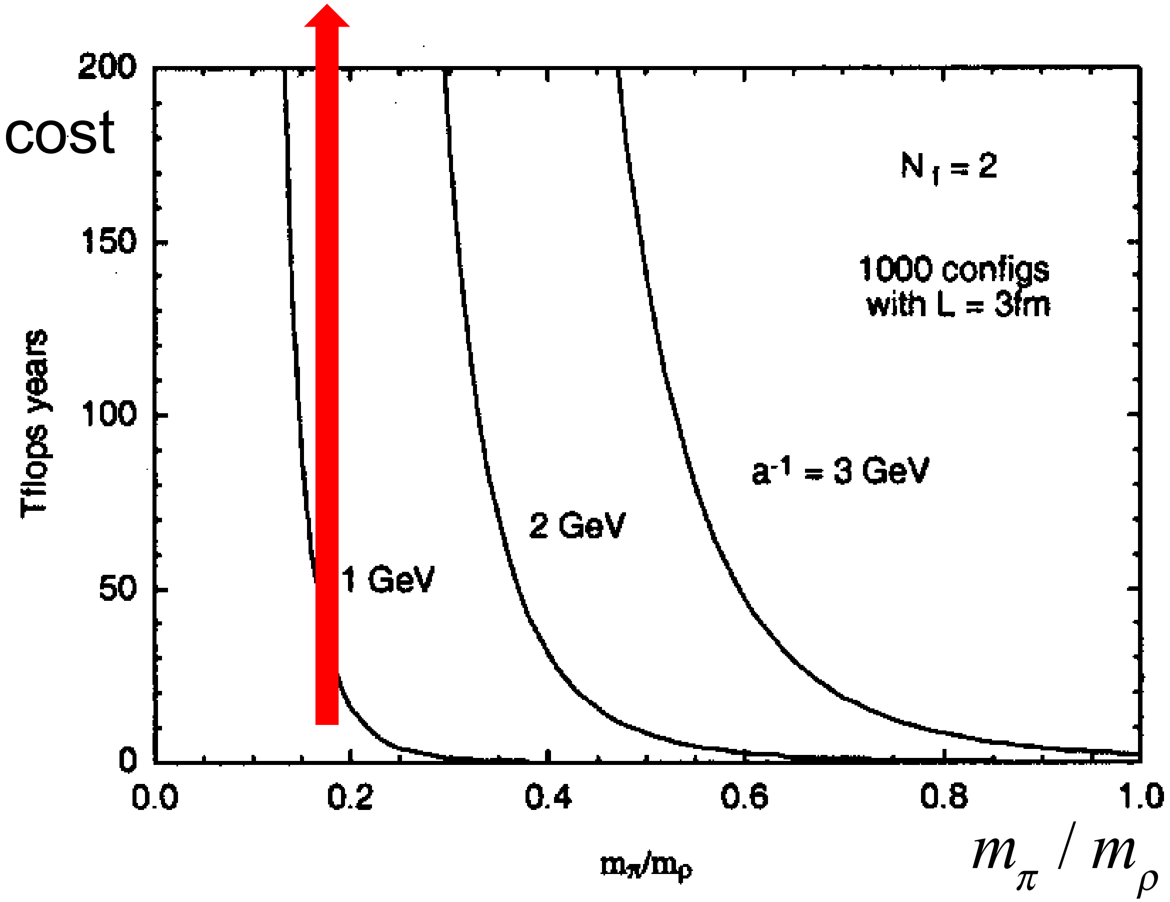
$$L_{NRQCD} = \psi^{\dagger}\left(iD_t - \frac{D^2}{2M}\right)\psi + c_1 \frac{g}{2M}\psi^{\dagger}\vec{\sigma}\bullet\vec{B}\psi + \cdots$$

- Relativistic heavy quark formalism — El-Kahdra et al, PRD55, 3933 (1997) also, Aoki et al, PTP 109, 383 (2003) Christ et al, PRD 76, 074505 (2007)
  - Systematic reformulation of action so as to reduce effects of $m_h a > 1$

$$S = \sum_{n,m} \bar{q}_n D_{Wilson}(U)_{n,m} q_m + \frac{i}{2} c_B \kappa_s \sum_n \bar{q}_n \varepsilon_{ijk} \sigma_{ij} B_{n,k} q_n + i c_E \kappa_s \sum_n \bar{q}_n \sigma_{0i} E_{n,i} q_n + \cdots$$

Cost of dynamical quark simulations

- Panel discussion at Lattice 2001 Berlin
  - Panelists: C. Bernard, N. Christ, S. Gottlieb, K. Jansen, R.Kenway(chair), T. Lippert, M. Luescher, P. Mackenzie, F. Niedermeyer, S. Sharpe, F. Tripicione, A. Ukawa, H. Wittig



Computational cost blows up toward the physical point:

$$\text{cost} \propto \frac{L^5}{m_q^3 a^8}$$

CP-PACS/JLQCD 2001

Removing the critical slowing down

- Cost of dynamical full QCD calculation

$$\text{cost} \propto N_{inv} \times \frac{1}{\delta\tau} \times \left(\frac{L}{a}\right)^4 \times \tau_{aut}$$

- With HMC

Duane et al, PLB195, 216 (1987)

$$\text{cost} \propto \frac{1}{m_q a} \times \frac{L/a}{m_q a} \times \left(\frac{L}{a}\right)^4 \times \frac{1}{m_q a} = \frac{L^5}{m_q^3 a^8}$$

Sexton-Weingarten, NPB380, 665 (1992)

- With UV/IR separation

Hasenbusch, PLB519, 177 (2001)

$$\text{cost} \propto \frac{1}{m_q a} \times \frac{L}{a} \times \left(\frac{L}{a}\right)^4 \times 1 = \frac{L^5}{m_q a^6}$$

Luescher, CPC165, 199 (2005)

- With deflation/multi-grid

Babich et al, PRL 105, 201602 (2010)

$$\text{cost} \propto 1 \times \frac{L}{a} \times \left(\frac{L}{a}\right)^4 \times 1 = \frac{L^5}{a^5}$$

Frommer et al, SIAM J 36, A1581 (2014)

Note: topology has to be still treated, e.g., by open boundary condition

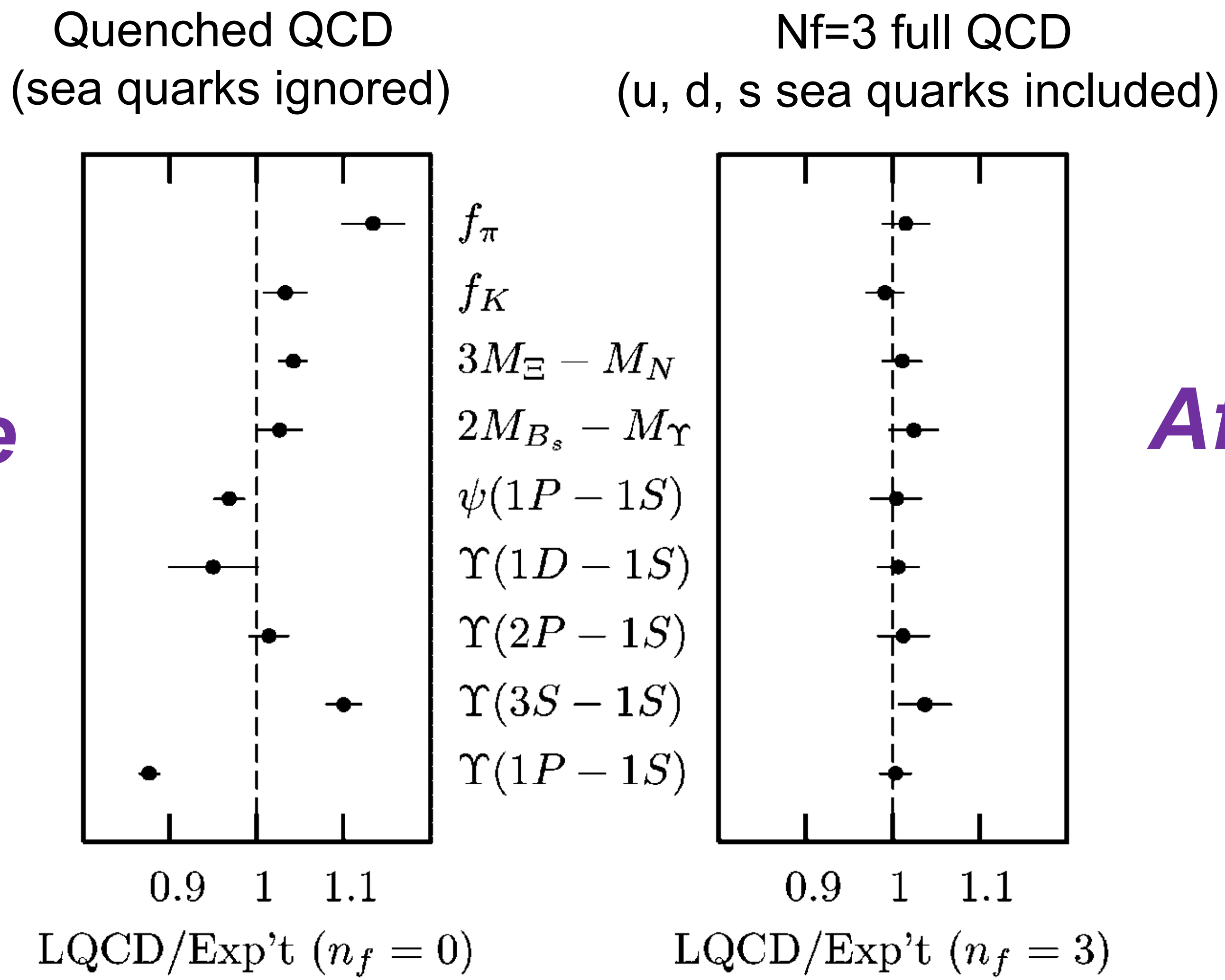
Putting them all together

Begininng of maturity

"High Precision Lattice QCD Confronts Experiment"
HOQCD/UKQCD/MILC/Fermilab Lattice Collaborations
PRL 92, 022001 (2004)

Before

After



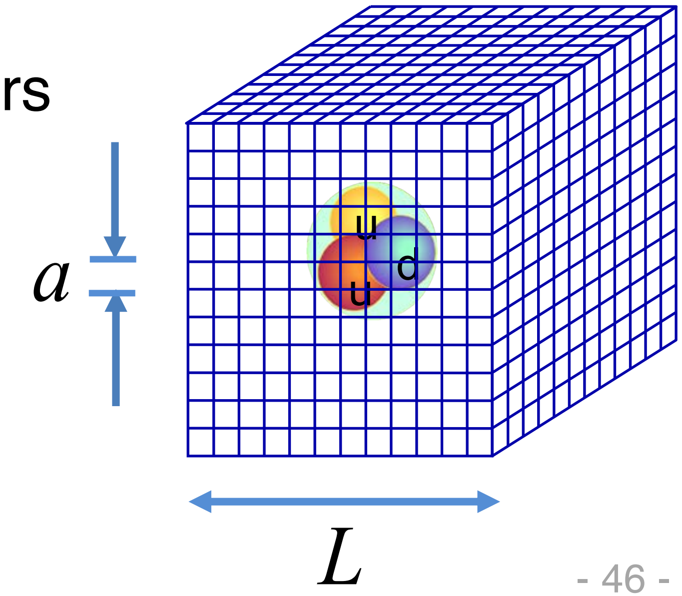
“Lattice QCD Simulations via International Research Network”

Lattice calculations after 2000s

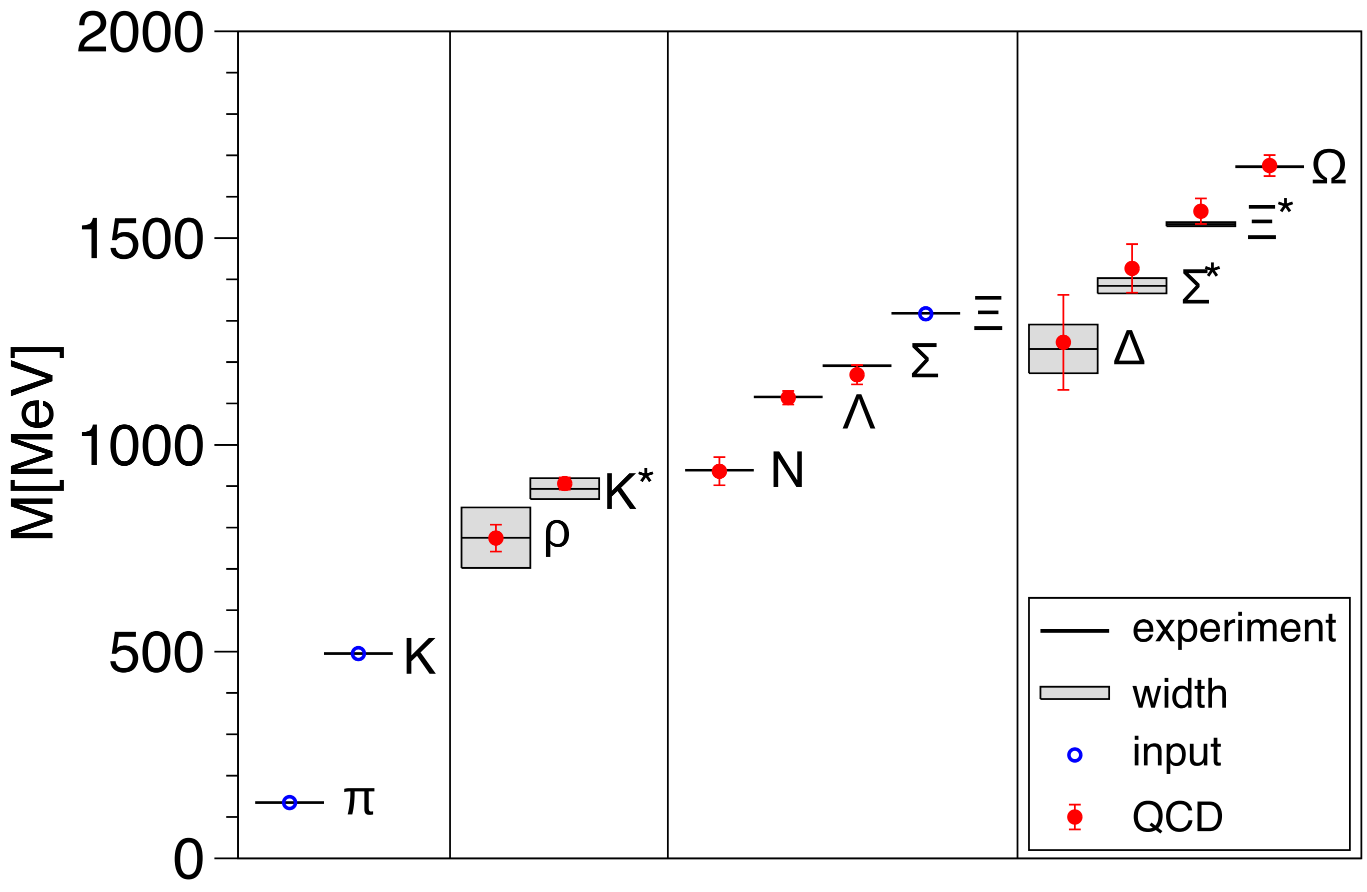
- Lattice size $L \geq 3-5\text{fm}$ $m_\pi L \geq 4-5$

  to control finite size effects $L \to \infty$
- Lattice spacings $a \leq 0.1-0.04\text{fm}$ $a^{-1} \geq 2-5\text{GeV}$

  to control continuum extrapolation $a \to 0$
- Sea quark effects fully incorporated for u, d, s, c $N_f = 2+1+1$ with their physical masses
- Heavy quarks treated by effective theory or directly simulated for small enough a
- Use of non-perturbative renormalization factors
- Multiple set of parameter points, each with O(1000) independent configurations
- Sophisticated fitting and error estimation to extract physical results



Light hadron spectrum



BMW Collaboration, Durr et al, Science 322, 1224 (2008)

Isospin breaking in hadron masses

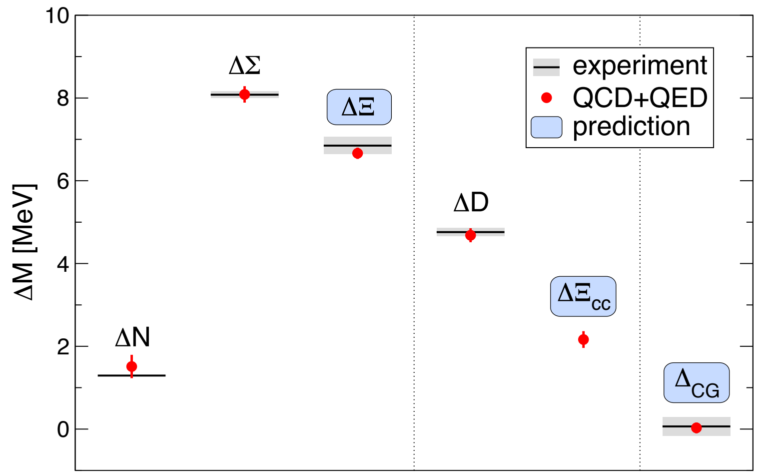
- important for understanding Nature

  e.g., neutron-proton mass difference is crucial for Big Bang nucleo-synthesis / stability of nuclei

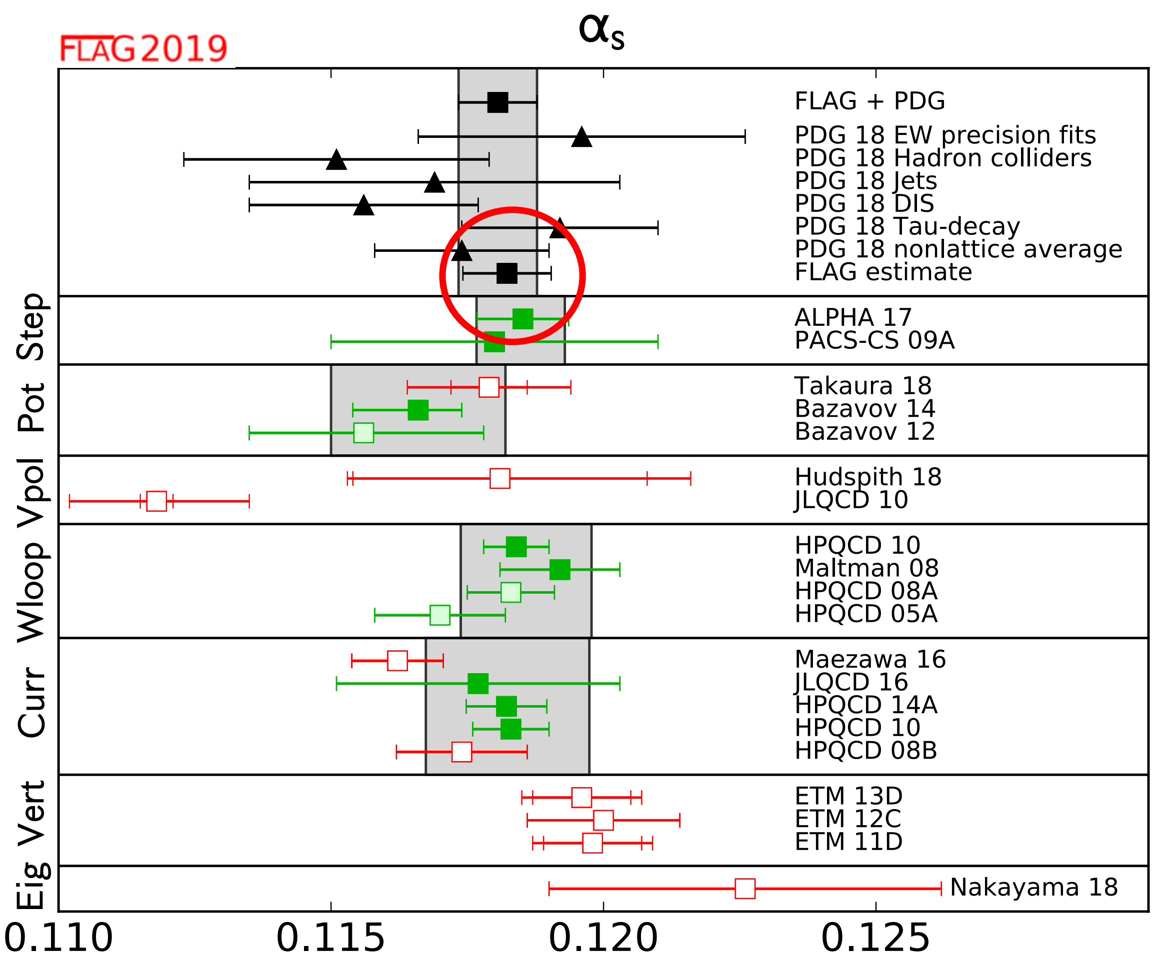
- History
  - Duncan-Eichten-Thacker, PRL76, 3894(1996)
  - RBC/UKQCD, Blum et al, PRD82, 094508 (2010)
  - BMW, Borsanyi et al, Science 347, 1452 (2015)

$$m_n - m_p = +1.51(28)\,\text{MeV}$$
$$= +2.52(30)_{QCD} - 1.00(16)_{QED}\ \text{MeV}$$

$cf. +1.2933321(5)$ MeV RPP 2018



QCD coupling constant



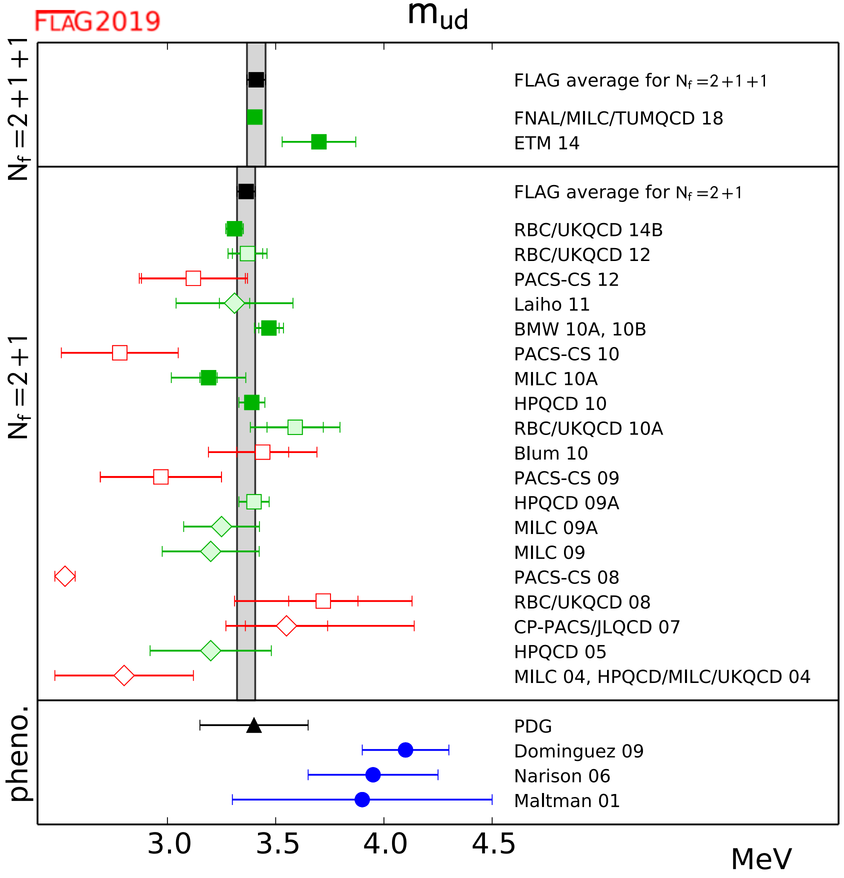
experiment

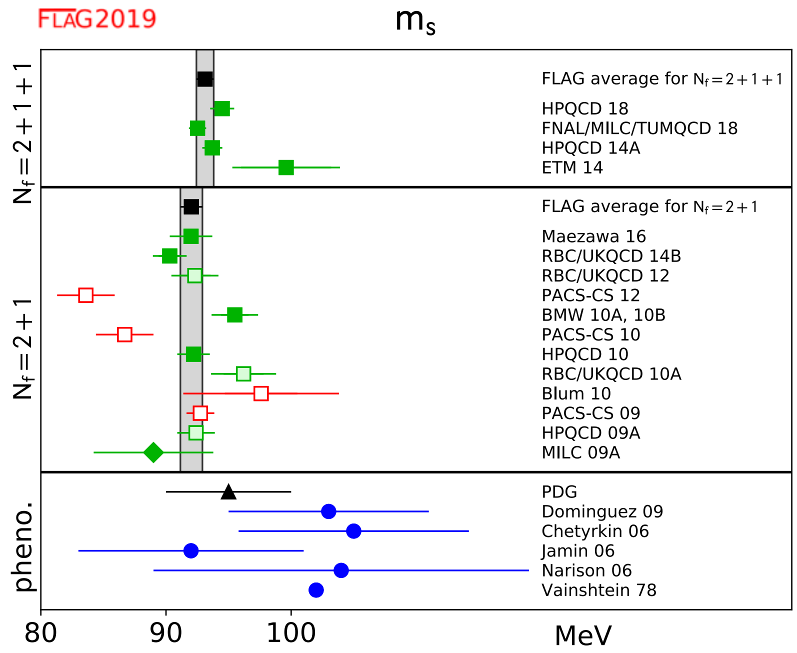
Lattice QCD

filled green results:
systematics under control, so included in the lattice estimate

$$\alpha^{(5)}_{\overline{MS}}(M_Z) = \begin{cases} 0.1174(16) & RPP2019(\text{nonlattice}) \\ 0.1182(8) & FLAG2019 \end{cases}$$

Quark masses

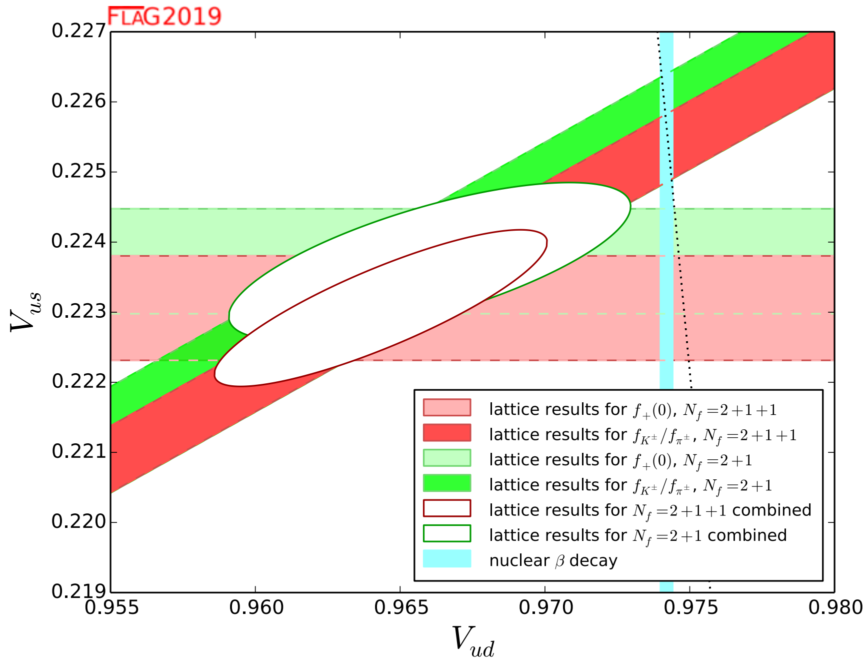




N_f	$m_u^{MS}(2\mathrm{GeV})$	$m_d^{MS}(2\mathrm{GeV})$	$m_s^{MS}(2\mathrm{GeV})$	$m_c^{MS}(m_c)$	$m_b^{MS}(m_b)$
$2+1$	2.27(9) MeV	4.67(9) MeV	92.03(88) MeV	1.275(5) GeV	4.164(23) GeV
$2+1+1$	2.50(17) MeV	4.88(20) MeV	93.44(68) MeV	1.280(13) GeV	4.198(12) GeV

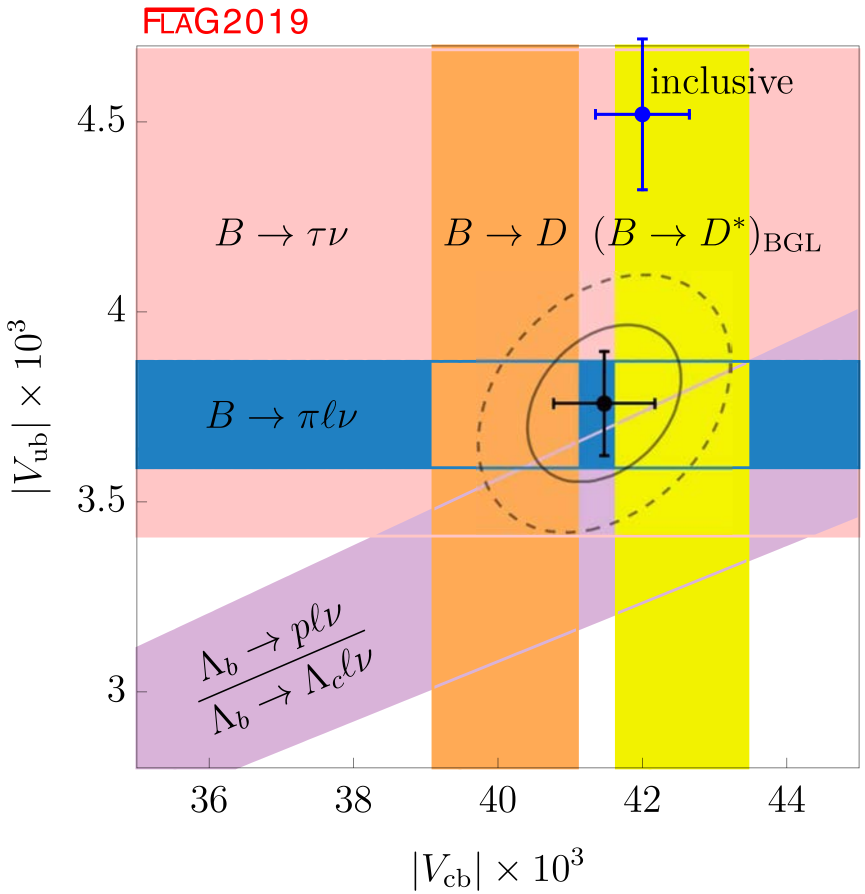
CKM matrix elements

V_{us}, V_{ud}



Tension with nuclear beta decay?

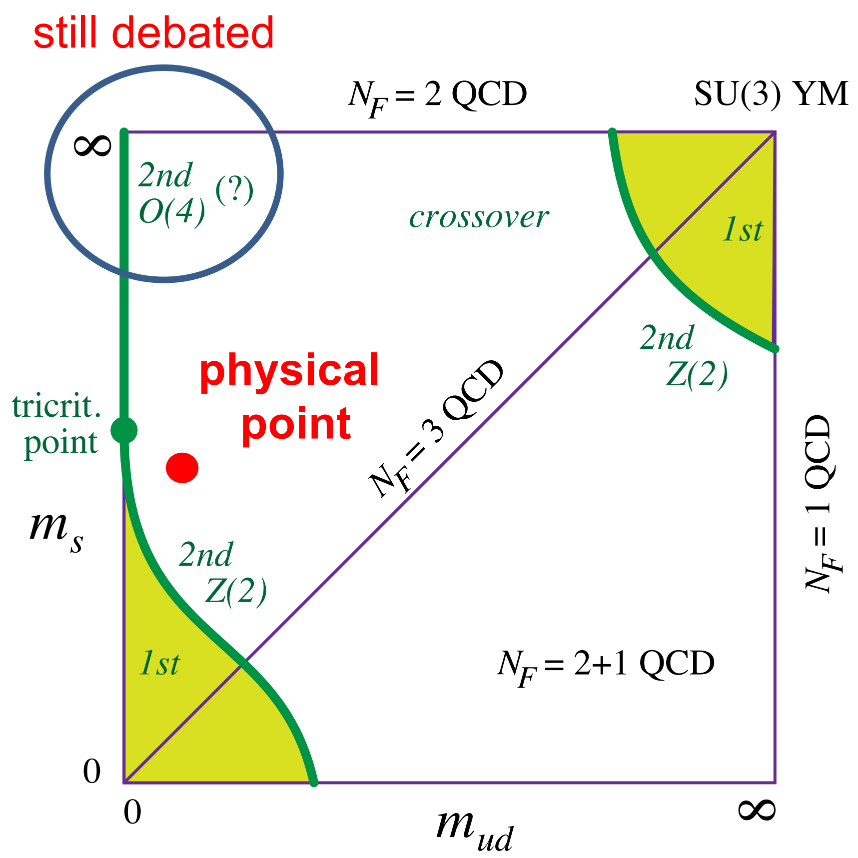
V_{ub}, V_{cb}



Tension between exclusive and inclusive determinations?

High temperatures

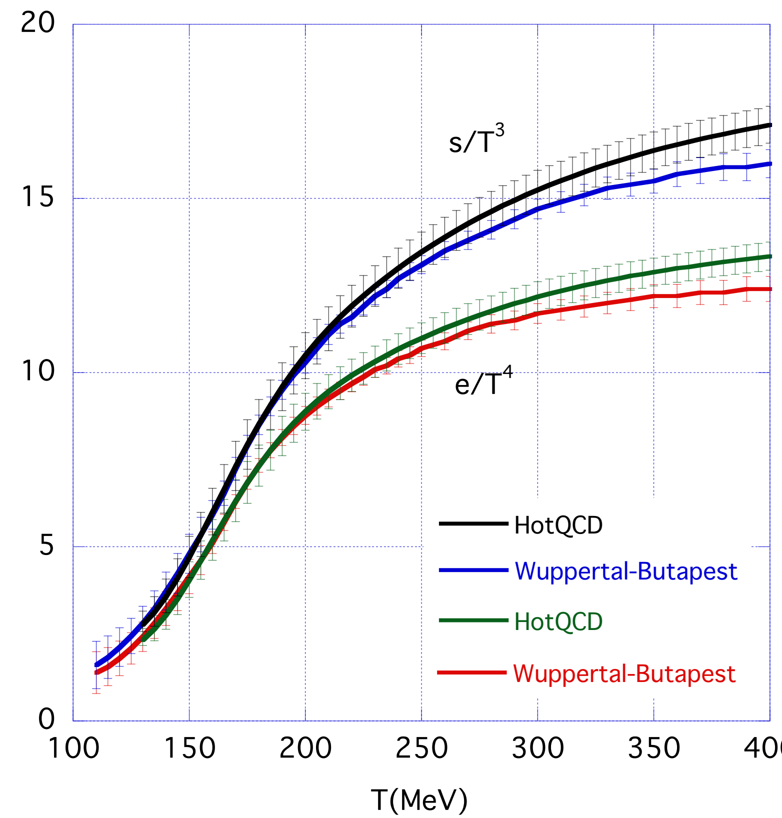
- Phase diagram



Physical point is a crossover.

Aoki et al, Nature 443, 675 (2006)

- Equation of state



HotQCD Bazavov et al, PRD90, 094503 (2014)

WB Borsanyi et al, PLB730, 99 (2014)

Muon g-2

$$a_\mu^{\exp} - a_\mu^{SM} = 27.9(6.3)^{\exp}(3.6)^{SM} \times 10^{-10}$$

- Huge ongoing effort by lattice community to calculate hadronic contributions

  Fermilab/HPQCD/MILC, RBC/UKQCD, BMW, ETMC, Mainz, PACS-CS, …

- Sub % accuracy in the near future?

Lattice QCD today

- *45 years since Wilson's seminal paper (February 1974)*
- *Amazing progress in physics, algorithms, machines*
- *Has matured as particle theory*
  - *Direct calculation at the physical quark masses on large lattices (L～4-5fm) and small lattice spacings (a～0.05fm or less) with physical sea quarks*
  - *Many important hadron properties now calculated (masses, decay constants, form factors etc), verifying the validity of QCD at % level or better, and providing valuable constraints on the CKM matrix and the Standard Model.*

What now?

What will be the next chapter of the History of Lattice QCD,

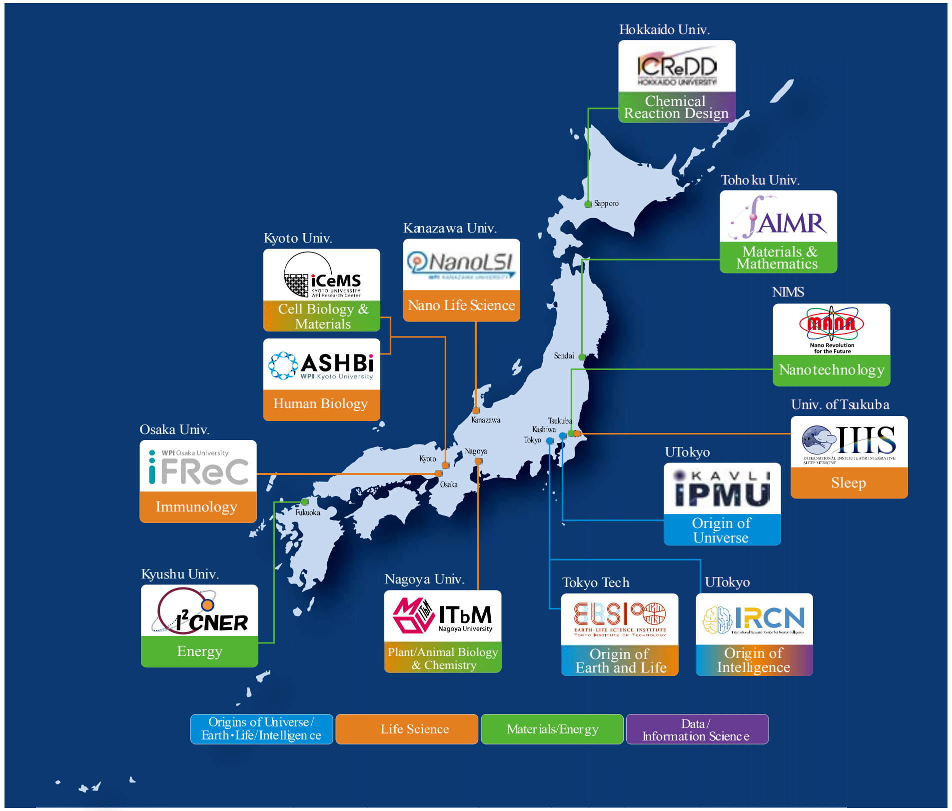
and who will be the Maker(s) of it?

Appendix

WPI Program

(World Premier International Research Center Initiative)

Shaping the future science in Japan beyond borders and barriers



13 centers as of 2019
(Planned up to 20 centers)

Four basic features:

1. Critical mass of outstanding PIs
2. International environment
3. Long term financial support
4. Robust follow up

Four missions:

1. Top-notch Science
2. Fusion Research
3. Internationalization
4. System reform