BSM Physics from Kaon Decays

Lattice QCD at Fermilab:
Celebrating the Career of
Paul Mackenzie

November 7-8, 2019

N.H. Christ

RBC/UKQCD Collaboration

Outline

- Thoughts about Paul
- Physics
  - $K_L \to \mu^+ \mu^-$
  - $K \to \pi\pi$ decay and ε'
- Conclusions

Thoughts about Paul

- Many places where Paul’s work has had a large impact on me:
  - Tadpole improvement of lattice perturbation theory.
  - QCD machines: ACPMAPS
  - Fermilab heavy quark action
- Founding member and then spokesperson for USQCD.
- Common physics goal of searching for phenomena not predicted by the standard model.

The RBC & UKQCD collaborations

BNL and BNL/RBRC

Yasumichi Aoki (KEK)
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Meifeng Lin
Aaron Meyer
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

UC Boulder

Oliver Witzel

CERN

Mattia Bruno

Columbia University

Ryan Abbot
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
Masaaki Tomii
Jiqun Tu
Bigeng Wang
Tianle Wang
Yidi Zhao

University of Connecticut

Tom Blum
Dan Hoying (BNL)
Luchang Jin (RBRC)
Cheng Tu

Edinburgh University

Peter Boyle
Luigi Del Debbio
Felix Erben
Vera Gülpers
Tadeusz Janowski
Julia Kettle
Michael Marshall
Fionn Ó hÓgáin
Antonin Portelli
Tobias Tsang
Andrew Yong
Azusa Yamaguchi

University of Liverpool

Nicolas Garron

MIT

David Murphy

Peking University

Xu Feng

University of Regensburg

Christoph Lehner (BNL)

University of Southampton

Nils Asmussen
Jonathan Flynn
Ryan Hill
Andreas Jüttner
James Richings
Chris Sachrajda

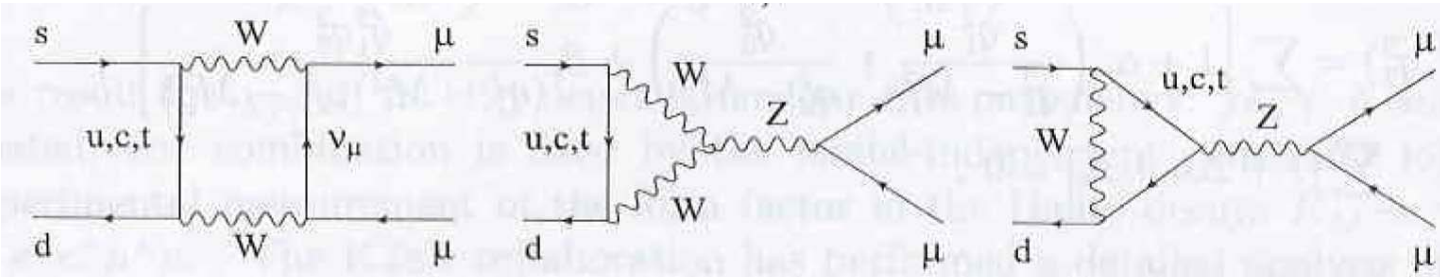
Stony Brook University

Jun-Sik Yoo
Sergey Syritsyn (RBRC)

$$K \rightarrow \mu^+ \mu^-$$

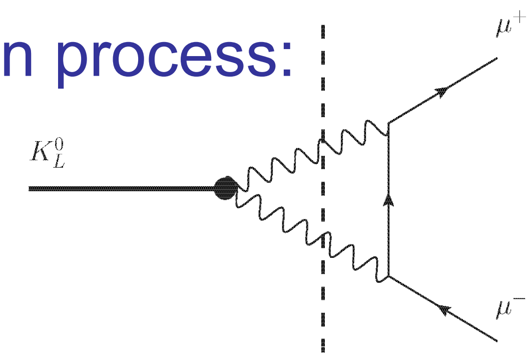
Physics of $K_L \rightarrow \mu^+ \mu^-$

- A second order weak, ``strangeness changing neutral current''



(Cirigliano, *et al.* , Rev. Mod. Phys., **84**, 2012)

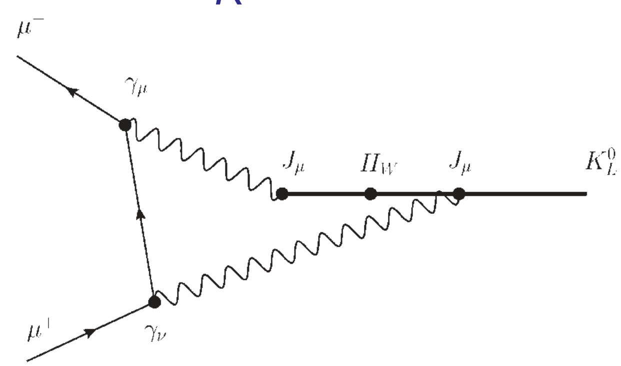
- $K_L \rightarrow \mu^+ \mu^-$ decay rate is known:
  - BR($K_L \rightarrow \mu^+ \mu^-$) = (6.84 ± 0.11) x 10^{-9}
- Large ``background'' from two-photon process:
  - Third-order electroweak amplitude
  - Optical theorem gives imaginary part.
  - $K_L \rightarrow \gamma\gamma$ decay rate is known



Physics of $K_L \rightarrow \mu^+ \mu^-$ (con't)

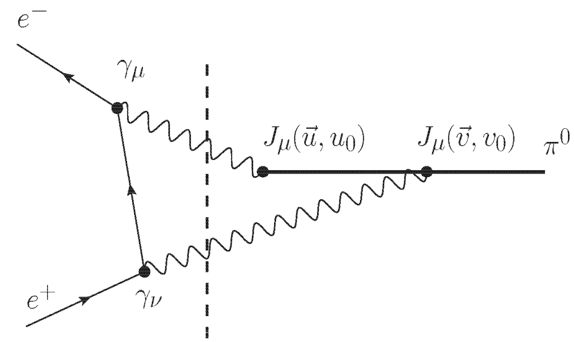
- Define: $$\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K_L \to \gamma\gamma)} = 2\beta_\mu \left(\frac{\alpha}{\pi}\frac{m_\mu}{M_K}\right)^2 \left(|F_{\rm imag}|^2 + |F_{\rm real}|^2\right)$$
- Optical theorem + experiment determine:

  $$|F_{\rm real}| = |(F_{\rm real})_{\rm E\&M} + (F_{\rm real})_{\rm Weak}| = 1.167 \pm 0.094$$
- Standard model: $(F_{\rm real})_{\rm Weak} = -1.82 \pm 0.04$
- A 10% lattice calculation of $(F_{\rm real})_{\rm E\&M}$ would allow a test of $(F_{\rm real})_{\rm Weak}$ with 6 – 17% accuracy
- Lattice calculation more difficult than ΔM_K
  - 5 vertices, 60 time orders
  - many states $|n\rangle$ with $E_n < M_K$
- First try simpler $\pi^0 \rightarrow e^+ e^-$

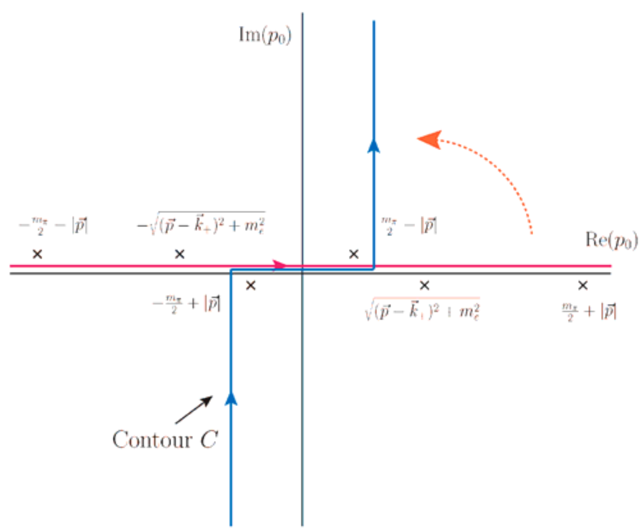


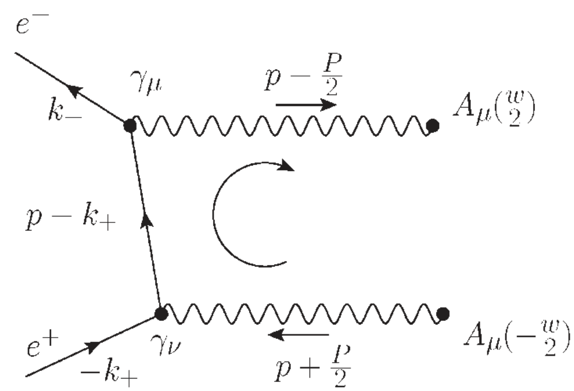
Consider simpler $\pi^0 \rightarrow e^+ e^-$

- Euclidean non-covariant P.T. difficult:
  - 12 time orders,
  - $E_{\gamma\gamma} < M_{\pi^0}$
- Try something different:
  - Evaluate in Minkowski space
  - Wick rotate internal time integral:



$$\mathcal{A}_{\pi^0 \to e^+ e^-} \to \int d^4 w \, \widetilde{L}(k_-, k_+, w)_{\mu\nu} \langle 0 | T \left\{ J_\mu(\frac{w}{2}) J_\nu(-\frac{w}{2}) \right\} | \pi^0(\vec{P} = 0) \rangle$$



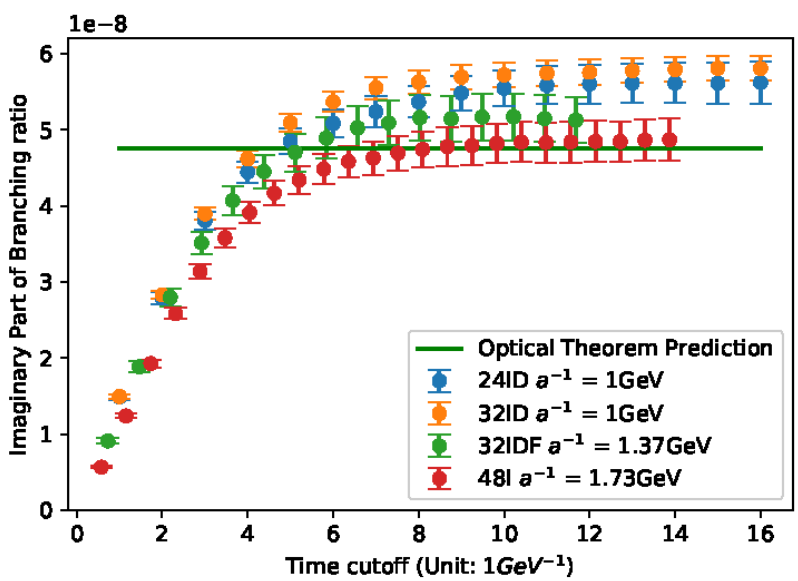


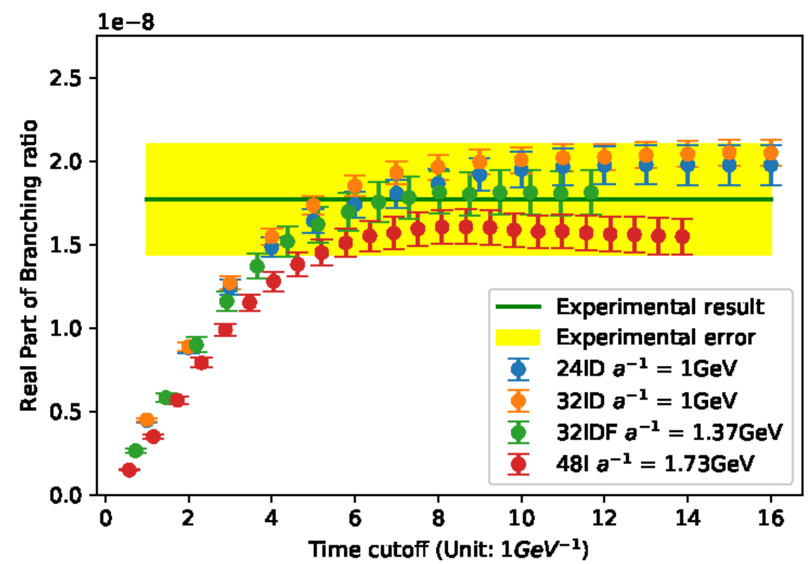
Lattice Results

(Yidi Zhao)

$$\mathcal{A}_{\pi^0\to e^+e^-} \to \int d^4w\, \widetilde{L}(k_-,k_+,w)_{\mu\nu}\langle 0|T\left\{J_\mu(\frac{w}{2})J_\nu(-\frac{w}{2})\right\}|\pi^0(\vec{P}=0)\rangle$$

- Lattice result is literally complex:
  - Exponentially small FV corrections
  - Physical kinematics, $1/a \leq 1.73$ GeV :
    - Im(A) = 35.94(1.01)(1.09) [Expt: 35.07(37)]
    - Re(A) = 20.39(72)(70). [Expt: 21.51(2.02)]





$K \rightarrow \pi\pi$ decay and ε'

Cabibbo-Kobayashi-Maskawa mixing

- $W^{\pm}$ emission scrambles the quark flavors

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix} \overset{W^{\pm}}{\longleftrightarrow} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CP violation!

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\overline{\rho}-i\overline{\eta}) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\overline{\rho}-i\overline{\eta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\lambda = 0.22535 \pm 0.00065\,, \qquad A = 0.811^{+0.022}_{-0.012}\,,$$

$$\bar{\rho} = 0.131^{+0.026}_{-0.013}\,, \qquad \bar{\eta} = 0.345^{+0.013}_{-0.014}\,.$$

CP violation

- CP violating, experimental amplitudes:

$$\eta_{+-} \equiv \frac{\langle \pi^+\pi^- | H_w | K_L \rangle}{\langle \pi^+\pi^- | H_w | K_S \rangle} = \epsilon + \epsilon'$$

$$\eta_{00} \equiv \frac{\langle \pi^0\pi^0 | H_w | K_L \rangle}{\langle \pi^0\pi^0 | H_w | K_S \rangle} = \epsilon - 2\epsilon'$$

- Where:

Indirect: $|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3}$

Direct: $\mathrm{Re}(\varepsilon'/\varepsilon) = (1.66 \pm 0.23) \times 10^{-3}$

Direct CP violation in $K \to \pi\,\pi$

- Final $\pi\pi$ states can have I = 0 or 2.

$$\langle \pi\,\pi(I=2)|H_w|K^0\rangle = A_2 e^{i\delta_2} \qquad \Delta I = 3/2$$

$$\langle \pi\,\pi(I=0)|H_w|K^0\rangle = A_0 e^{i\delta_0} \qquad \Delta I = 1/2$$

- CP symmetry requires A_0 and A_2 be real.
- Direct CP violation in this decay is characterized by:

$$\epsilon' = \frac{i\,e^{\delta_2-\delta_0}}{\sqrt{2}}\left|\frac{A_2}{A_0}\right|\left(\frac{\mathrm{Im}\,A_2}{\mathrm{Re}A_2}-\frac{\mathrm{Im}\,A_0}{\mathrm{Re}A_0}\right)$$

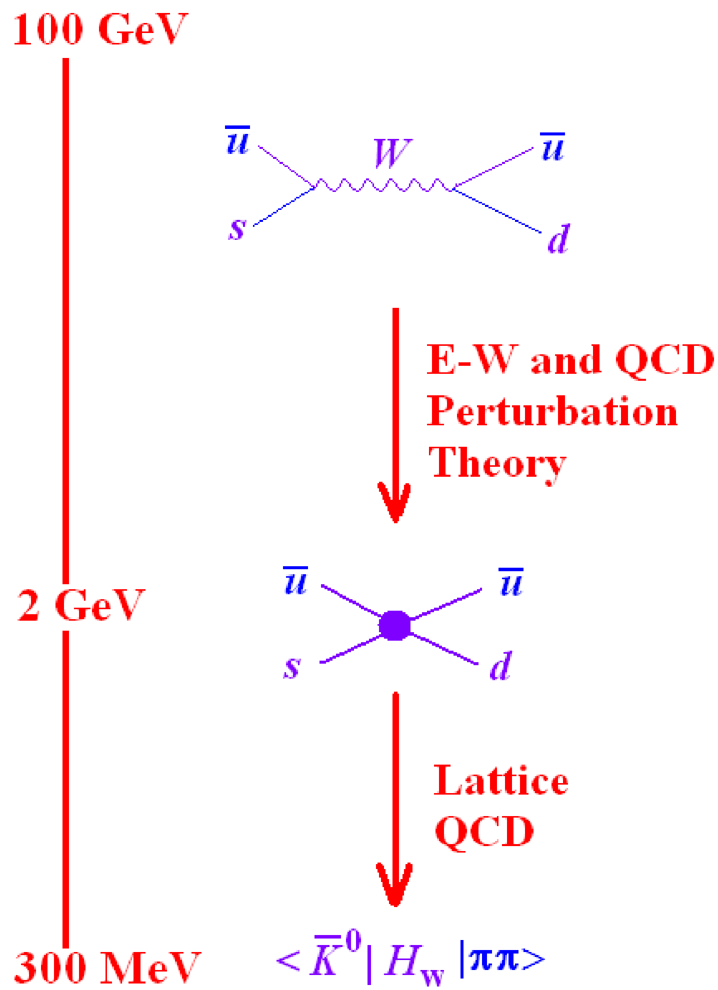
Direct CP violation

Low Energy Effective Theory

- Represent weak interactions by local four-quark Lagrangian

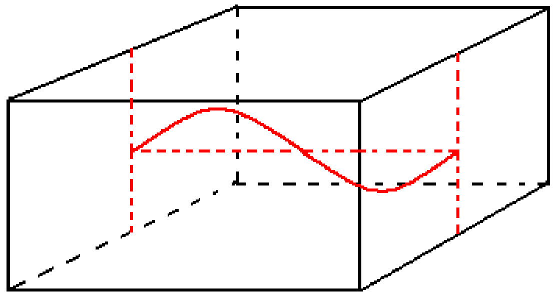
$$\mathcal{H}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) + \tau y_i(\mu) \right] Q_i \right\}$$

- $\tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} = (1.543 + 0.635i) \times 10^{-3}$
- $V_{qq'}$ – CKM matrix elements
- z_i and y_i – Wilson Coefficients
- Q_i – four-quark operators



Lattice calculation of $<\pi\pi|H_W|K>$

- The operator product $\overline{d}(x)s(x)$ easily creates a kaon.
- Use finite-volume energy quantization (Lellouch-Luscher) and adjust L so that n^{th} excited state obeys: $E_{\pi\pi}^{(n)} = M_K$



$p = 2\pi/L$

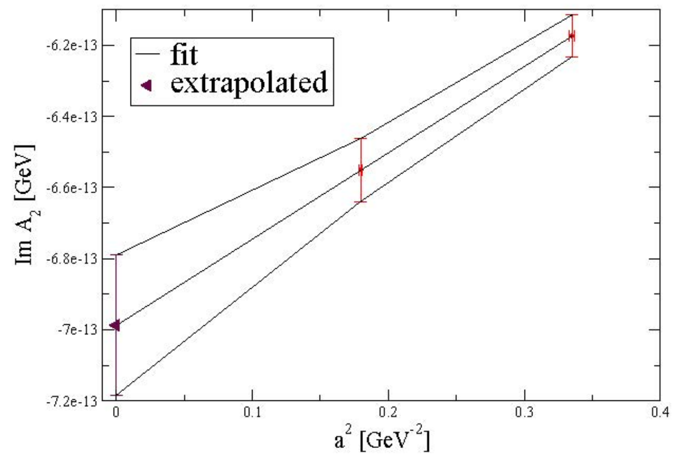
$$\langle \pi^+\pi^- | H_W | K^0 \rangle \quad \propto \quad \langle \overline{d}u(t_{\pi_1})\overline{u}d(t_{\pi_2})\ H_W(t_{\text{op}})\ \overline{d}u(t_K) \rangle$$

- Use boundary conditions on the quarks: $E_{\pi\pi}^{(\text{gnd})} = M_K$
- For $(\pi\pi)_{I=2}$ make d anti-periodic
- For $(\pi\pi)_{I=0}$ use G-parity boundary conditions: **arXiv:1908.08**

Calculation of A_2

$\Delta I = 3/2$ – Continuum Results

(M. Lightman, E. Goode, T. Janowski)



- Use two large ensembles to remove a^2 error (m_π=135 MeV, L=5.4 fm)
  - 48^3 x 96, 1/*a*=1.73 GeV
  - 64^3 x 128, 1/*a*=2.28 GeV
- Continuum results:
  - $\mathrm{Re}(A_2) = 1.50(0.04_{stat})\,(0.14)_{syst} \times 10^{-8}$ GeV
  - $\mathrm{Im}(A_2) = -6.99(0.20)_{stat}\,(0.84)_{syst} \times 10^{-13}$ GeV
- Experiment: $\mathrm{Re}(A_2) = 1.479(4)\ 10^{-8}$ GeV
- $E_{\pi\pi} \rightarrow \delta_2 = -11.6(2.5)(1.2)^o$
- [Phys. Rev. **D91**, 074502 (2015)]

Calculation of A_0 and ε'

Overview of 2015 calculation
(Chris Kelly and Daiqian Zhang)

- Use 32^3 x 64 ensemble
  - *1/a* = 1.3784(68) GeV, *L* = 4.53 fm.
  - G-parity boundary condition in 3 directions
  - 216 configurations separated by 4 time units
- Essentially physical kinematics:
  - M_π = 143.1(2.0)
  - M_K = 490.6(2.2) MeV
  - $E_{\pi\pi}$ = 498(11) MeV

2015 Results

[Phys. Rev. Lett. 115 (2015) 212001]

- $E_{\pi\pi}$(499 MeV) determines δ_0 :
  - I = 0 $\pi\pi$ phase shift: $\delta_0 = 23.8(4.9)(2.2)^o$
  - Dispersion theory result: $\delta_0 = 34^o$ [G. Colangelo, *et al.*]
- Re(ε'/ε) = $(1.38 \pm 5.15_{stat} \pm 4.59_{sys}) \times 10^{-4}$
  - Expt.: $(16.6 \pm 2.3) \times 10^{-4}$
  - 2.1 σ difference
- Unanswered questions:
  - Is this 2.1 σ difference real? → Reduce errors
  - Why is δ_0 so different from the dispersive result? → Introduce more $\pi\pi$ operators to distinguish excited states

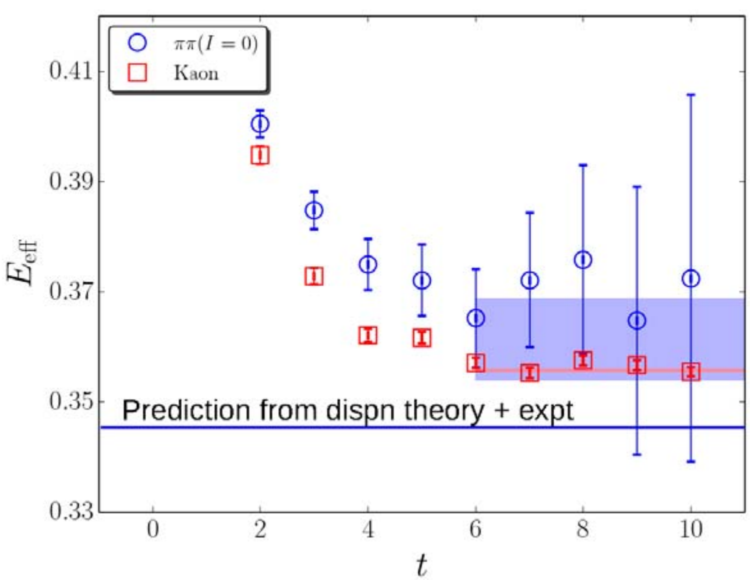
Extend and improve calculation

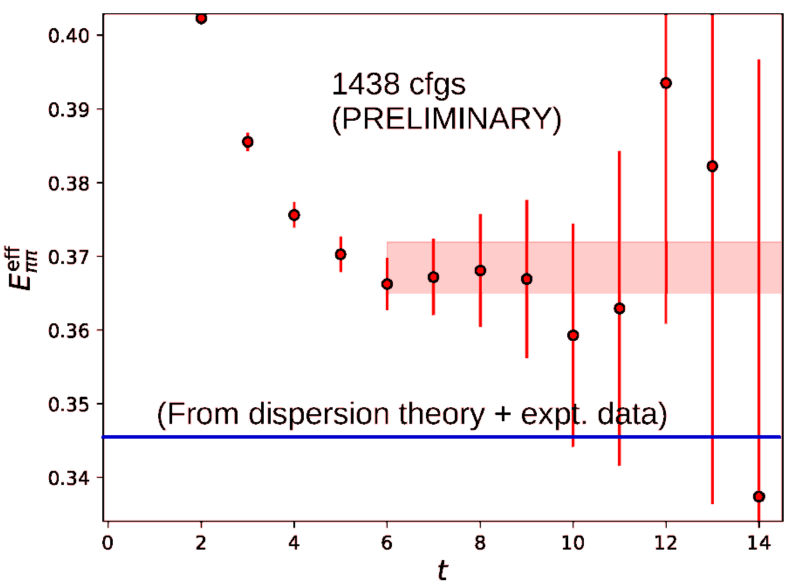
(Chris Kelly and Tianle Wang)

- ✓- Increase statistics: 216 → 1438 configs.
  - – Reduce statistical errors
  - – Allow deeper study of systematic errors
- ✓- Study operators neglected in our NPR implementation
- ✓- Use step-scaling to allow perturbative matching at a higher energy
- ✓- Use an expanded set of $\pi\pi$ operators
- • Use X-space NPR to cross charm threshold (Masaaki Tomii).

Adding more statistics

- Increasing statistics: 216 → 1438 configs.
  - $\pi\pi - \pi\pi$ correlator well-described by a single $\pi\pi$ state
  - $\delta_0 = 23.8(4.9)(2.2)^o \rightarrow 19.1(2.5)(1.2)^o$
    $\chi^2 / \mathrm{DoF} = 1.6$





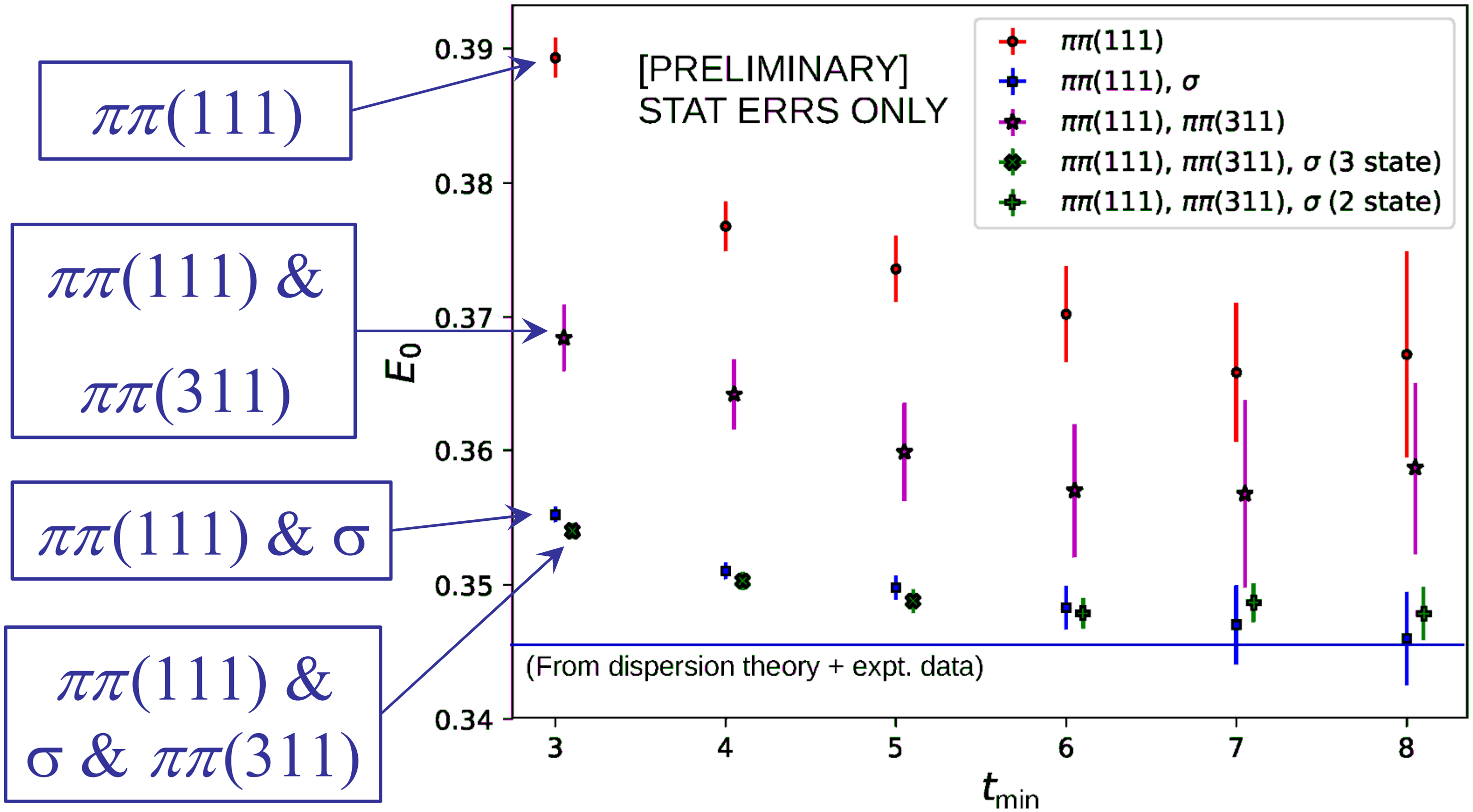
Adding more $\pi\pi$ operators

- Adding a second σ-like ($\bar{u}u+\bar{d}d$) operator reveals a second state!
- If only one state, 2 x 2 correlator matrix will have determinant = 0. For $t_f - t_i = 5$:

$$\det\begin{pmatrix} \langle \pi\pi(t_f)\pi\pi(t_i)\rangle & \langle \pi\pi(t_f)\sigma(t_i)\rangle \\ \langle \sigma(t_f)\pi\pi(t_i)\rangle & \langle \sigma(t_f)\sigma(t_i)\rangle \end{pmatrix} = 0.439(50)$$

- Add a third operator giving each pion a larger momentum: $p = \pm\,(3,1,1)\,\pi/L$
- Label operators as $\pi\pi(111)$, σ, $\pi\pi(311)$
- Only 741 configurations with new operators

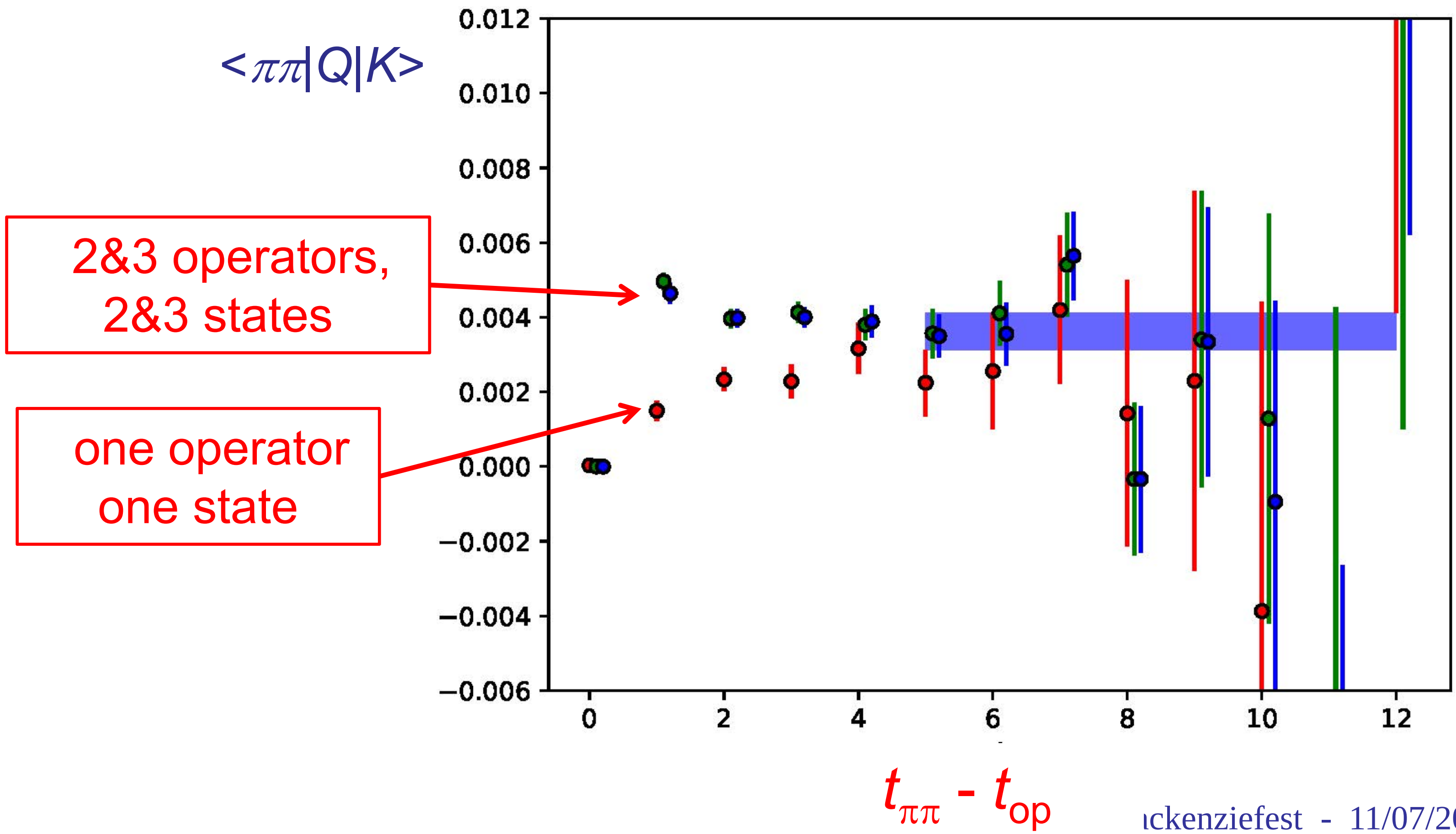
$I = 0$ $\pi\pi$ scattering with three operators



- Third $\pi\pi(311)$ operator not important.
- δ_0 = 31.7(6)° vs 34° prediction (5-15 fit, statistical errors only).

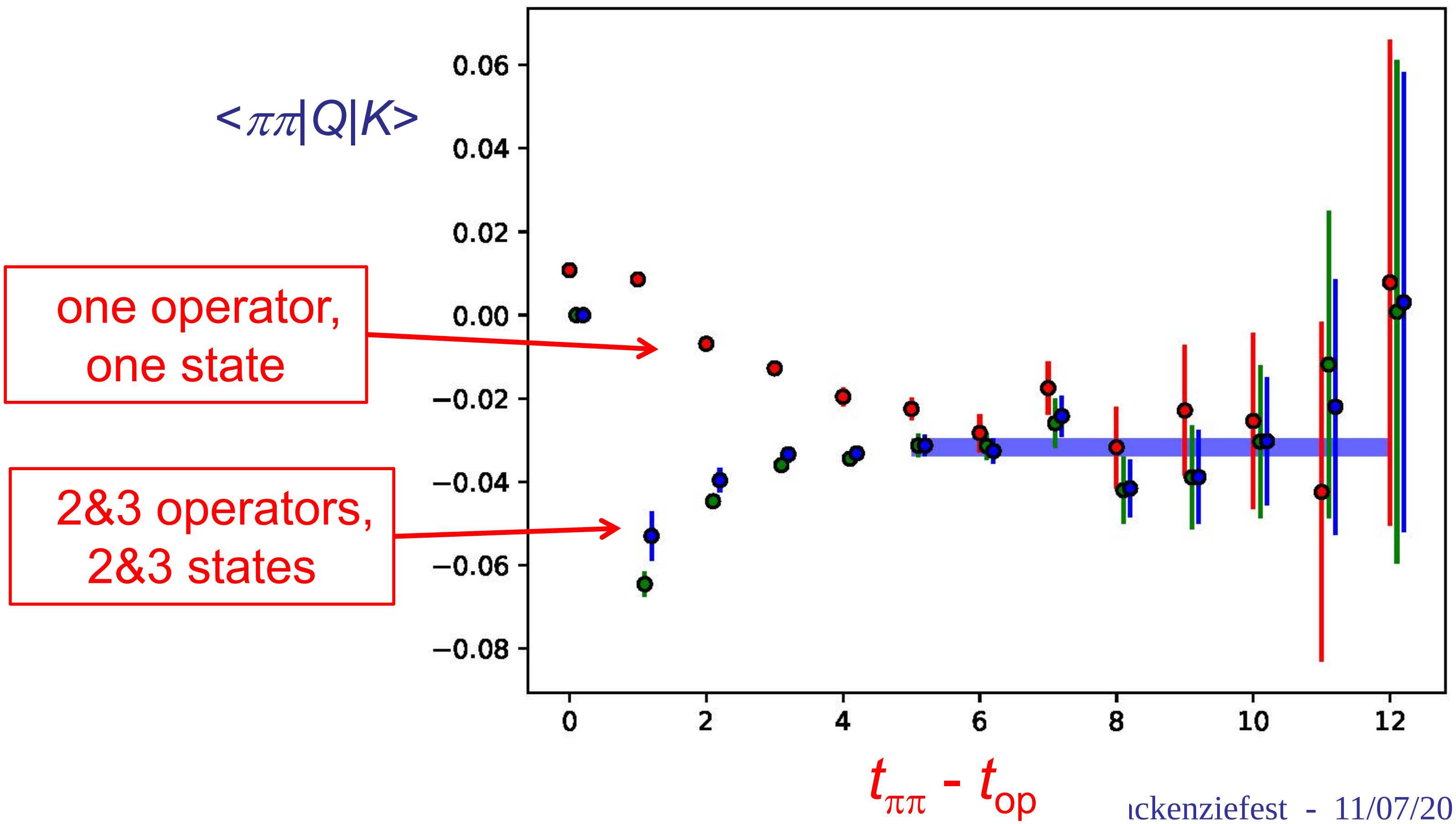
K → $\pi\pi$ from 3-operator fits (case I)

- Fit using up to 3 operators and 3 states with energies and amplitudes from $\pi\pi$ scattering:



K → $\pi\pi$ from 3-operator fits (case II)

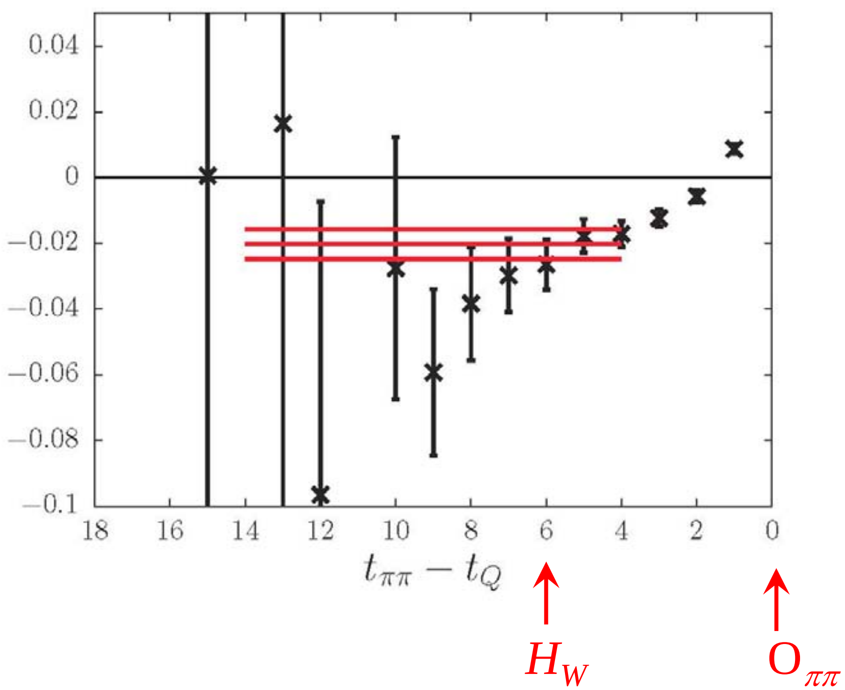
- Fit using up to 3 operators and 3 states with energies and amplitudes from $\pi\pi$ scattering:



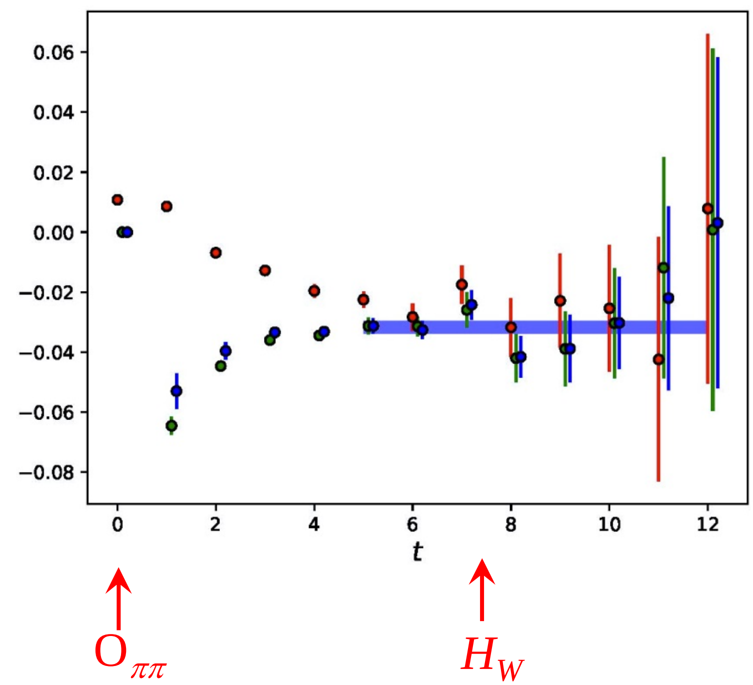
$K \rightarrow \pi\pi$ matrix elements

- Compare old and new results (case I)

215 samples [2015]



741 samples [2019]



Two data analysis challenges

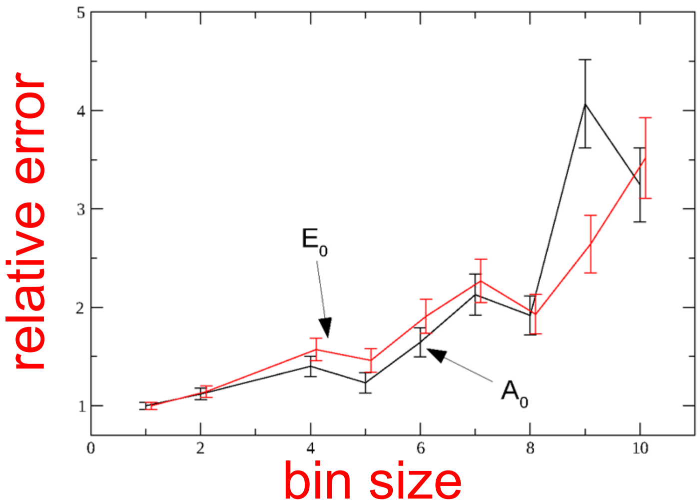
- Auto-correlations – we must be careful that our errors are accurate
- We need estimates of goodness of fit (*p*-values)
  - Demonstrate that our fits describe the data.
  - Decide if alternative fits used to estimate systematic errors are plausible.
  - However, our lattice QCD *p*-values are traditionally unreasonably small!

Auto-correlations

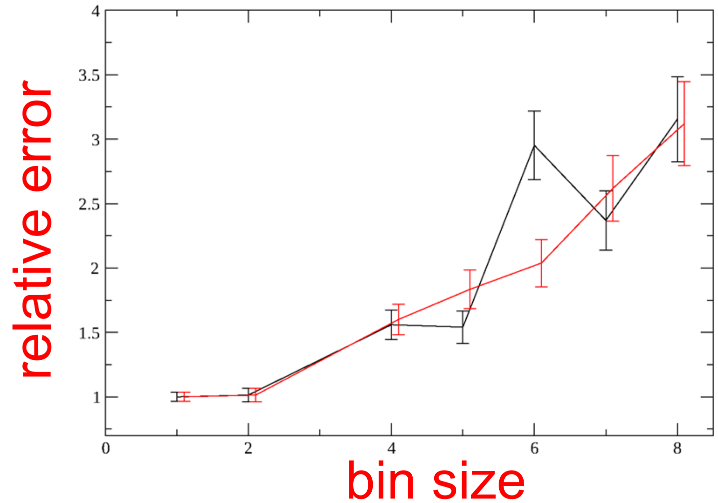
- Our measurements are made every 4 MD time units and are mildly correlated.
- While we have N=741 configurations, the covariance matrix for three operators and t = 5-15 time slices is 66 x 66!
- Noise grows as we bin the data and have fewer samples to measure the fluctuations.
- Solved by the *blocked jackknife* method:
  - Identify *N*/*B* blocks of size *B.*
  - Sequentially remove each block and analyze the remaining N-B (not N/B-1) samples

I=0 $\pi\pi$ two-point function fit errors

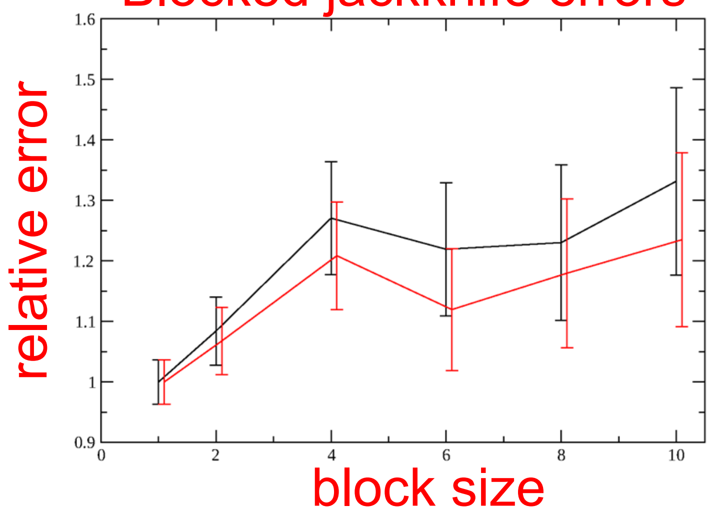
Binned data errors



Binned scrambled data errors



Blocked jackknife errors



Poor *p*-values

- We obtain *p*-values of 0.1– 0.2 for most “best fits”!
- This is often caused by ignoring fluctuations in the covariance matrix (Tanmoy Bhattacharya).
- Including covariance matrix fluctuations broadens the χ^2 distribution into the Hotelling T^2 distribution (related to *F* distrib.).

Hotelling T^2 is *insufficient*

- Hotelling assumes that the data (not their averages) are Gaussian and uncorrelated.
- Both are not true for our case.
- Use a bootstrap analysis to determine the correct generalized χ^2 distribution from the data. (C. Kelly)

q^2 distribution

- Define $$q^2 = \sum_{t,t'=t_{min}}^{t_{max}} \left[\bar{v}_t - f(t,\vec{p})\right] \left[C^{-1}\right]_{tt'} \left[\bar{v}_{t'} - f(t',\vec{p})\right]$$

  where $$C_{tt'} = \frac{1}{N(N-1)} \sum_{i=1}^{N} \left[v_{i,t} - \bar{v}_t\right] \left[v_{i,t'} - \bar{v}_{t'}\right]$$

- Find $P(q^2)$ where

  $$\int_0^\infty P(q^2)dq^2 = 1$$ and $$p_{int}(q^2) = \int_{q^2}^\infty P(q^2)dq^2$$

- Here $p_{int}(q^2)$ gives the ``p-value”

Find q^2 distribution from the data (Chris Kelly)

- Start with the original ensemble $\{v_{it}\}_{1 \le i \le N}$
- Draw N values from this set (allowing the same value to be drawn multiple times).
- Create N_{boot} such ensembles of N values: $\{b_{it}^{\alpha}\}_{1 \le i \le N}$ where $1 \le \alpha \le N_{\mathrm{boot}}$
- *Recenter* these ensembles so $f(t,\vec{p})$ will fit the average over boot strap ensembles perfectly:

$$b_{i,t}^{\alpha} \to \tilde{b}_{i,t}^{\alpha} = b_{i,t}^{\alpha} - \bar{v}_t + f(t,\vec{p})$$

- Here the parameters $\vec{p}$ fit the average data $\bar{v}_t$

q^2 distribution

- $\tilde{b}^{\alpha}_{i,t}$ has the fluctuation in the population but is fit perfectly by $f(t,\vec{p})$

$$b^{\alpha}_{i,t} \rightarrow \tilde{b}^{\alpha}_{i,t} = b^{\alpha}_{i,t} - \bar{v}_t + f(t,\vec{p})$$

Fit to ensemble α

- Thus

$$(q^2)^{\alpha} = \sum_{t,t'=t_{\min}}^{t_{\max}} \left[\bar{\tilde{b}}^{\alpha}_t - f(t,\vec{p}^{\alpha})\right] \left[(C^{\alpha})^{-1}\right]_{tt'} \left[\bar{\tilde{b}}^{\alpha}_{t'} - f(t',\vec{p}^{\alpha})\right]$$

will obey (and give) the correct q^2 distribution.

- $p(q^2) = N(q^2)/N_{\text{boot}}$ where $N(q^2)$ is the number of bootstrap ensembles with $(q^2)^{\alpha} > q^2$.
- Now p-values can be computed for any definition of q^2 including for uncorrelated fits!

$K \rightarrow \pi\pi$ calculation (2019)

- Calculation of $K \rightarrow \pi\pi$ decay substantially improved over 2015 result.
- 216 → 741 configurations.
- Three $\pi\pi$ interpolating operators: discriminate between ground and excited states → $\delta_0\,(E\!=\!M_K) = 31.7(6)°$
- Errors reduced by using correlated fits.
- Bootstrap-determined q^2 distribution gives correct p-values. [p=0.261(BS) vs 0.037(χ^2)]
- Results available soon.

Thanks !

- Precision measurement + lattice QCD is a long-term direction of great promise.
  - Paul is a leading contributor
  - Important new results lie ahead.
- Importance of Paul's national leadership is hard to overstate:
  - USQCD hardware has enabled frontier calculations
  - Many careers advanced by USQCD projects.
  - Collaborative good will and combined strengths of USQCD attract new talent and enhanced funding!