Paper LLRF2019/ 207

Ponderomotive Instability of RF Cavities with Vector Sum, and Cure By Difference Control

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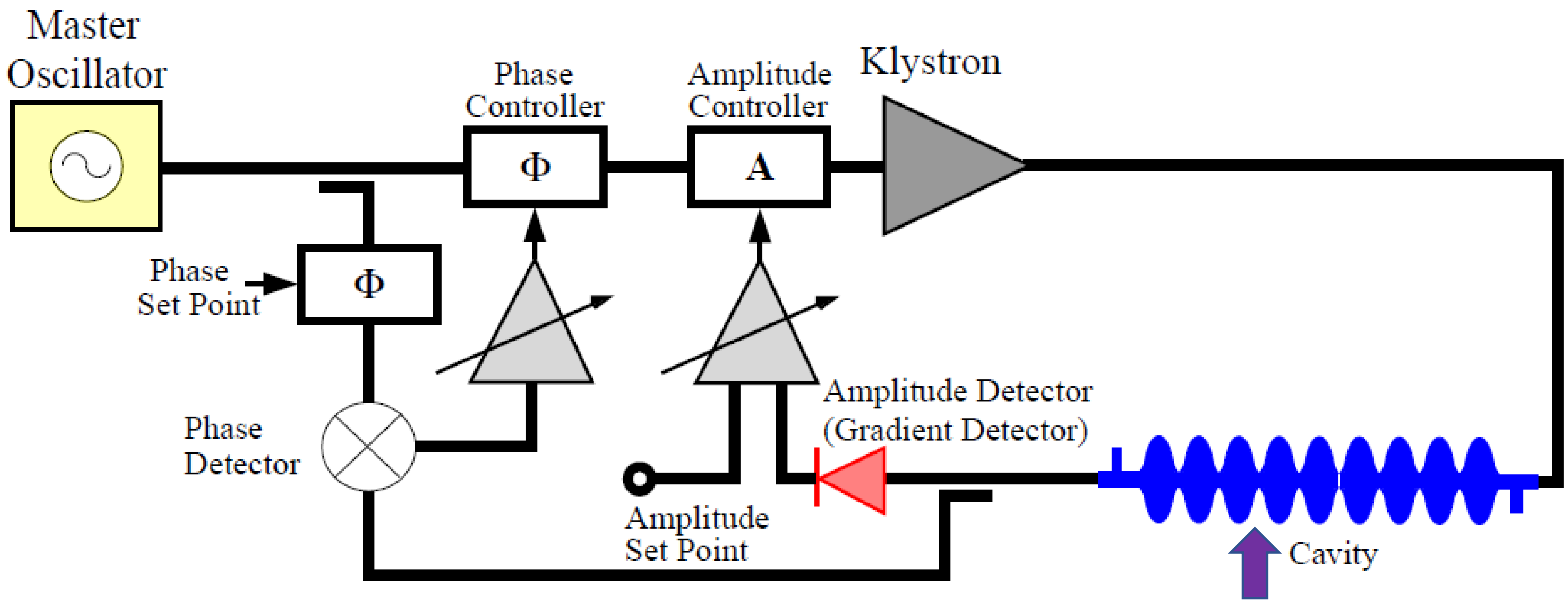
Generator Driven (GD) presented here

Most GD results have SE analogs. Any GD result that relies on a “symmetry argument” carries over to SE case.

Self Excited (SE) Loop: see references

All results have been derived analytically (Routh-Hurwitz analysis in *Mathematica*)
Many results presented here are numerical examples – to avoid writing equations.

Generator Driven (GD)



External tuning control, and internal detuning from Lorentz Force and microphonics

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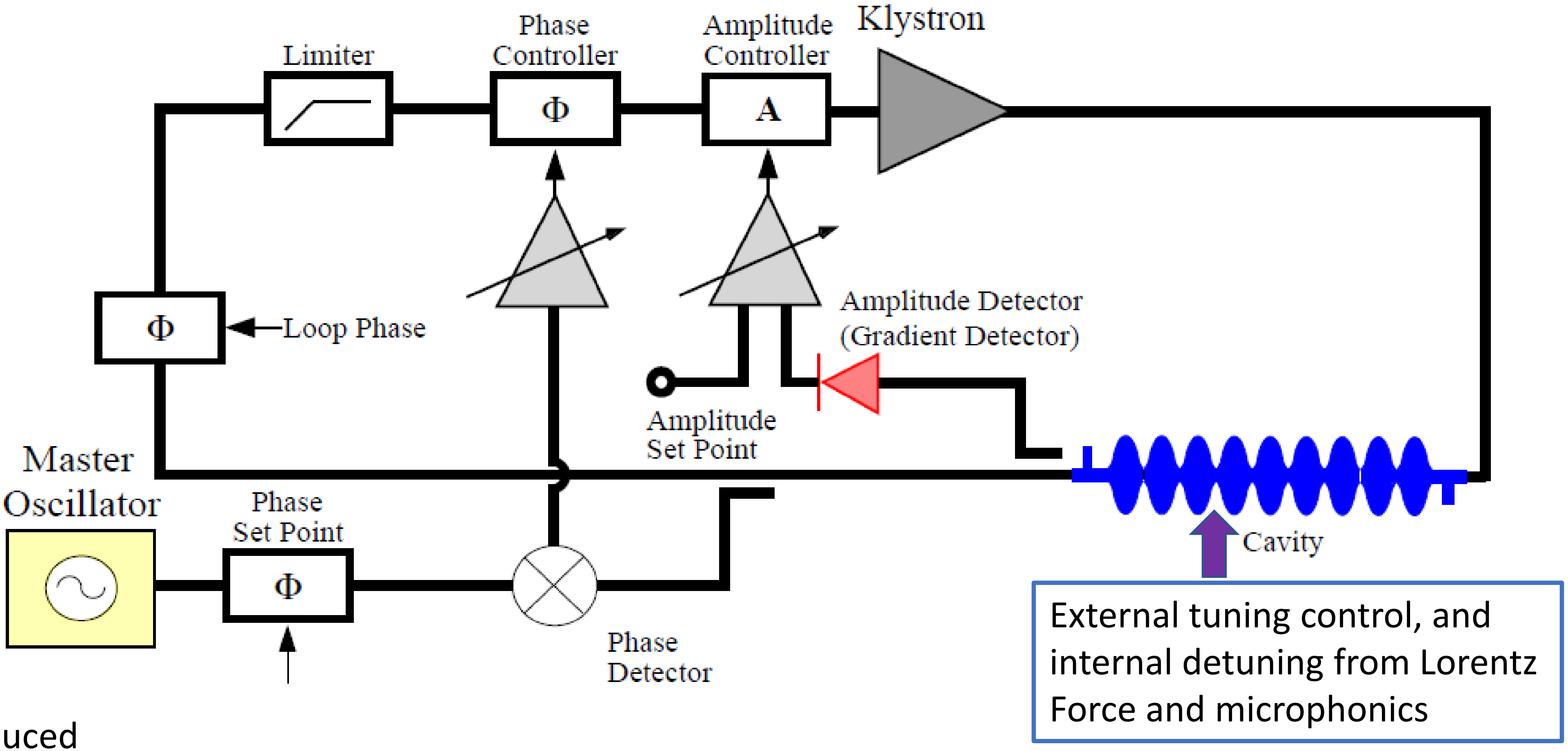
Self-Excited (SE) Loop



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Static Lorentz Force Detuning

$$\Delta\Omega == \Omega - \Omega 0$$

$$\frac{\Delta\Omega}{\Omega 0} == \frac{-1}{2}\left(\frac{\Delta C}{C} + \frac{\Delta L}{L}\right)$$

For pill-box cavity

$$\frac{\Delta C}{C} == -E^2 \Delta Ve$$

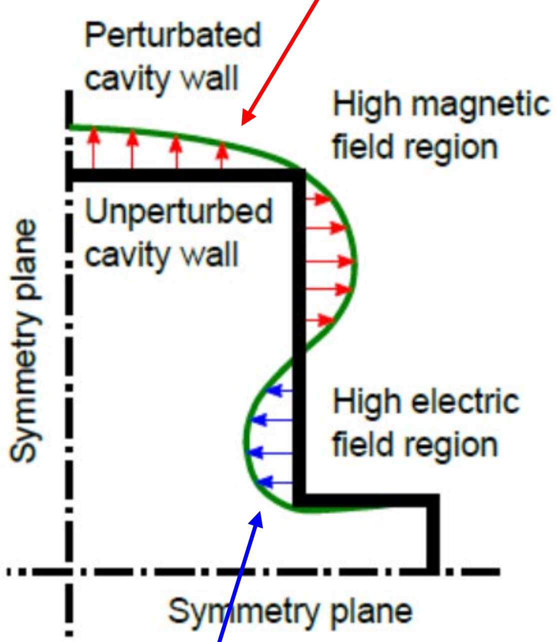
$$\frac{\Delta L}{L} == H^2 \Delta Vh$$

$$\frac{\Delta\Omega}{\Omega 0} == \frac{E^2 \Delta Ve}{2} - \frac{H^2 \Delta Vh}{2}$$

$\Delta\Omega < 0$ $\quad\quad$ $\Omega < \Omega 0$ always

Calculation adapted from Y. Yamazaki, Proc. Frontiers of Accelerator Technology, World Scientific, 1996

$\Delta Vh > 0$



$\Delta Ve < 0$

Internal radiation pressure:

$$P(\mathbf{x}) = (\mu_0|H_0|^2 - \epsilon_0|E_0|^2)/4\ ,$$

- Cavity responds to forces by changing shape, resulting in volume changes ΔV.
- Slater's Theorem gives the change in resonance frequency Ω.

Qualitative Lorentz pressure for TM010 mode in a pillbox cavity.

Diagram from
M. Parise 2018 *JINST* **13** T05010

If the electric & magnetic field amplitudes (E &H) are time varying, then the Lorentz force detuning is time dependent; and includes dynamics of cavity inertia and elasticity.

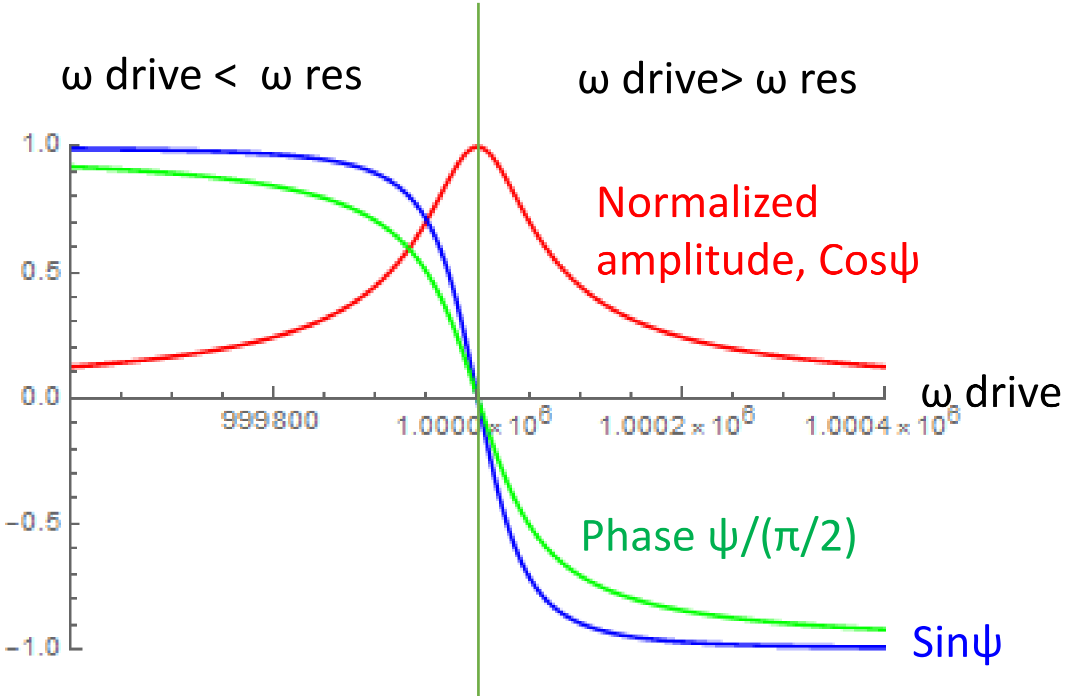
Cavity Mechanical and Electrical Response

Cavity Mechanical Response

- Coupling coefficients between frequency shift and cavity voltage are defined at DC.
- $\Delta\omega_c = -k_{DC} V_0^2$ where $k_{DC} = \Sigma_m k_m$ is sum of mechanical modes
- Mode m response to (DC) voltage modulations (a_c):
- $\delta\omega_c = -2k_m V_0^2 a_v$
- Let $-\delta\omega_c \tau_c \equiv K_m a_v$ and $K_L = \Sigma_m K_m$ be normalized values
- Mechanical resonator response (slide #9) extends dynamical behavior to AC.

Cavity Electromagnetic (EM) Response

- Cavity EM mode modelled by parallel resonance LCR circuit.
- Pure sinusoidal oscillations Exp[+jωt]
- Drive current source I
- Response is voltage V = Z×I
- Impedance Z=R Cos[ψ]Exp[+jψ]
- Detuning angle (ψ) quantifies the difference between drive frequency ω and resonance frequency ω_c of the cavity.



$\Psi > 0$
Voltage leads current (inductive reactance dominates)

$\Psi < 0$
Voltage lags current (capacitive reactance dominates)

$\mathrm{Tan}[\Psi] = (\omega_c^2 - \omega^2)/(2\alpha\omega)$ and $\alpha = \omega_c/(2Q_c)$

$V_0 = V_g \,\mathrm{Cos}[\Psi]$ and $\tau_c = 1/\alpha$

“Static Lorentz Force Detuning” instability (a.k.a montonic instability)

Have enough information to find a fundamental GD stability condition: $\Psi < 0 \rightarrow \omega$ drive> ω res

Changes of frequency and voltage depend on local derivative of the resonance curve

MM Resonator Response: $(\partial\omega/\partial a_V) \approx -2K_L\mathrm{Cos}[\psi]$

EM Resonator Response: $(\partial a_V/\partial\omega) \approx -\mathrm{Sin}[\psi]$

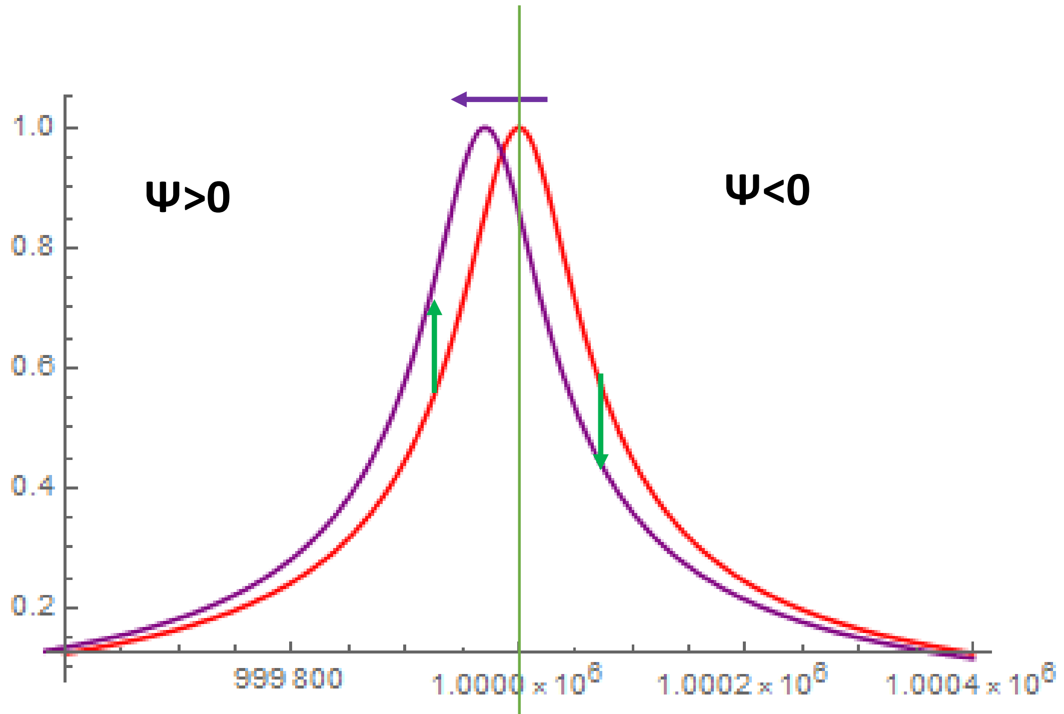
At threshold, product of amplification factors is unity.

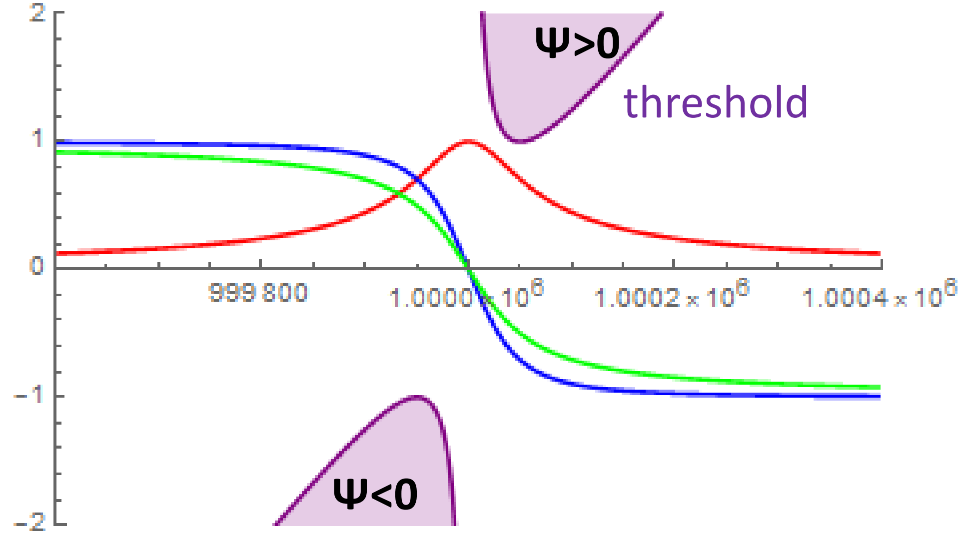
$(\partial\omega/\partial aV)^{MM}(\partial aV/\partial\omega)^{EM} \approx 2K_L\mathrm{Cos}[\psi]\mathrm{Sin}[\psi] = 1$

Hence threshold for monotonic instability

$K_L < 1/(2\mathrm{Cos}\psi\ \mathrm{Sin}\psi) = 1/\mathrm{Sin}[2\psi]$

$K_L \propto V_0^2$ so eventually the instability is always reached.





Excitation pulls resonance curve to lower frequency, which increases amplitude of response

⇒ Positive feedback within the cavity

⇒ Instability

Excitation pulls resonance curve to lower frequency, which reduces amplitude of response

⇒ Negative feedback within the cavity

⇒ stability

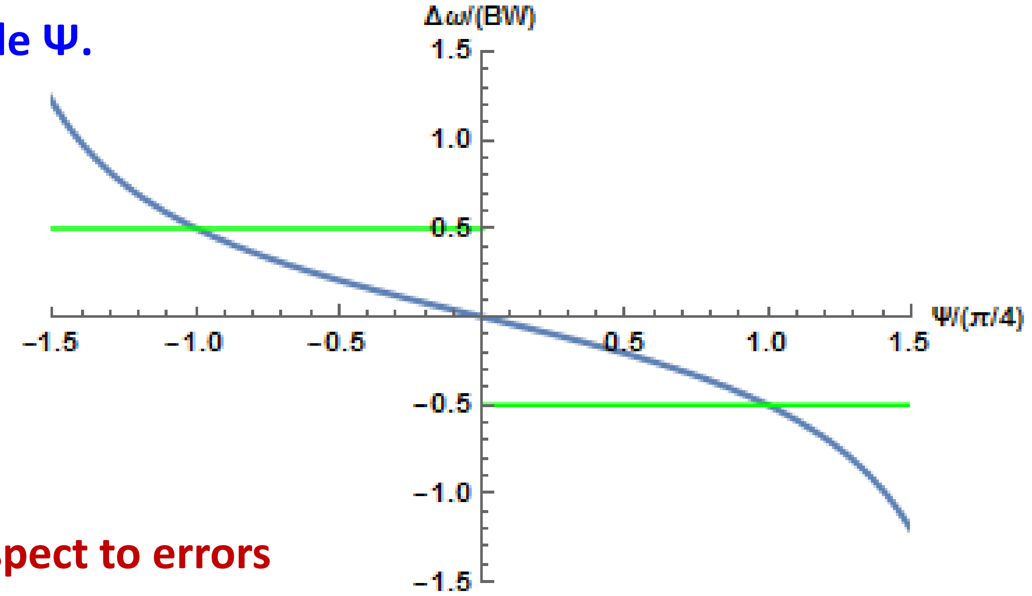
Instability is near DC, because this is the only frequency at which mechanical modes can all cooperate

Coordinate ω versus coordinate Ψ

- **For mathematical analysis, the natural and simplest variable is angle Ψ.**
- **But in the real world we deal with angular frequency ω**
- Therefore we are interested how the threshold varies with ω

$$\partial K/\partial\omega = (\partial K/\partial\Psi)\,(\partial\Psi/\partial\omega) \approx (\partial K/\partial\Psi).(-2Q_c/\omega_c)\,\mathrm{Cos}^2\Psi$$

Hence the sensitivity is greater for small cavity bandwidth ω_c/Q_c (i.e. high Q_c).



- **To get a real feeling for how very sensitive is the threshold with respect to errors in cavity tuning, consider the following:**
- In the range Ψ =[0, π/4] which maps to the tuning range Δω = [0, ½ bandwidth], the threshold varies from infinite to K_L=1.
- The range Δω = one bandwidth corresponds to Ψ = 1.1 radian.
- FYI, π/4 ≈ 0.785

A “microphonic” is an excitation of a mechanical mode. A subset of the modes couple to the EM resonance frequency. Such modes will eat into the stable detuning region and stability margin.

Oscillatory instability

- EM resonator pumps the mechanical mode (MM) displacement amplitude
- MM pumps the voltage modulation index $a_v = \delta V/V_0$ – but not V_0.

τ_c = EM filling time
= 2/(cavity EM bandwidth)

MM oscillates at (or near) its resonance frequency Ω_m.
Response is boosted by MM quality factor Q_m.
Hence $\delta\omega_c\, \tau_c \approx -2Q_mK_m\, a_v$

During mechanical oscillation, the cavity EM resonance moves up and down in frequency

The effect is to drive the cavity at upper and lower sidebands ($\omega \pm \Omega_m$) with FM depth $\delta\omega_c$, leading to changes in the amplitude response.

Differencing of sidebands leads to a net excitation $\propto \mathrm{Cos}(\psi+\delta\psi)\ -\mathrm{Cos}(\psi-\delta\psi)$ with $\delta\psi \approx \Omega_m\tau_c$, leading to

$a_v \approx 2\ \mathrm{Sin}\Psi\mathrm{Cos}\Psi\ (\delta\omega_c\tau_c)(\Omega_m\tau_c)$

At threshold, the net amplification factor is unity $[\delta\omega_c/ a_v]^{MM}[a_v/\delta\omega_c]^{EM} \approx -4Q_mK_m(\Omega_m\tau_c)\mathrm{Sin}\Psi\mathrm{Cos}\Psi = 1$

Hence threshold $K_m \approx -(1+\rho^2)/[2\rho Q_mK_m\ \mathrm{Sin}(2\Psi)]$ where $\rho= \Omega_m\tau_c$.

[expression is valid for $\rho \approx 1$]

Oscillatory instability occurs above resonance (i.e. $\Psi<0$)

Monotonic threshold is insensitive to dynamics

Oscillatory threshold

- Depends on dynamics
- parameter $\rho = \tau_c\,\Omega_m$ = (EM time constant)×(MM frequency)
- ρ answers question: is Ω_m inside or outside cavity EM bandwidth?
- ρ has large dynamic range:
  - $\rho \gg 1$ light loading of cavity Q_c (see references); typically very stable
  - $\rho \approx 1$ heavy loaded Q_c regime (example here); typically prone to instability

Mechanical Mode (MM) dynamics:

$$MM[s] \rightarrow \frac{Q_m\,\Omega_m^2}{s\,\Omega_m + Q_m\left(s^2 + \Omega_m^2\right)}$$

Instability Analysis

- Find steady state solution of nonlinear dynamical equation for RF cavity coupled to linear mechanical resonator
- Make small perturbations & discard products of small quantities
- Laplace transform (convert ODE to algebraic equation)
- Obtain characteristic equation
- Apply Rough-Hurwitz criteria, or find roots numerically.

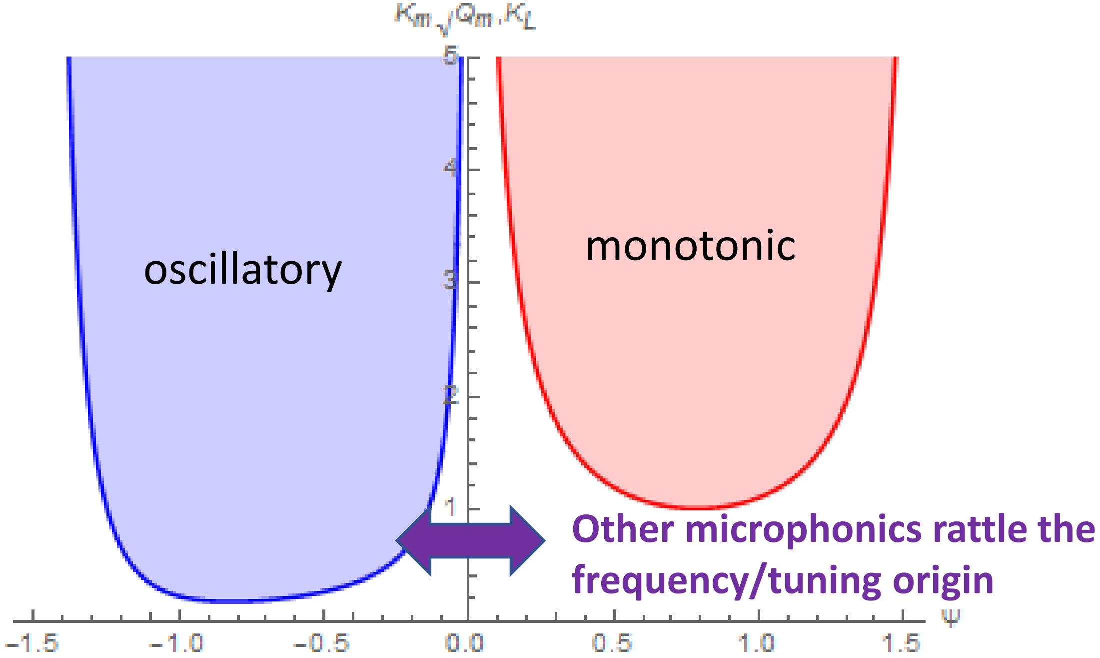
Instability Classification

- Complex frequency $s = \sigma + i\omega$
- Monotonic instability: $\sigma \geq 0$ and $\omega = 0$
  - threshold depends on K_L & ψ
- Oscillatory instability: $\sigma \geq 0$ and $\omega \neq 0$
  - threshold depends on K_m, Q_m, ρ & ψ

Note Bene: these ponderomotive instabilities have some similarity to the Robinson instability in rings.
MM takes role of charged particle beam.

Single Cavity – no loops

- monotonic threshold: $K_L < 1/\mathrm{Sin}[2\psi]$
- oscillatory threshold $\propto \rho/Q_m$ which may be less than K_m



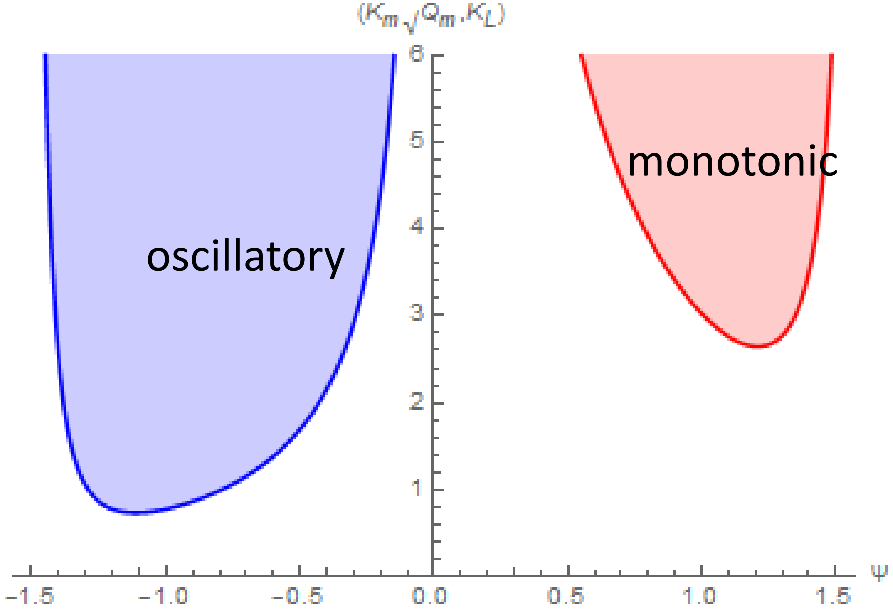
$$\begin{pmatrix} a_v\,(1 + s\,\tau_c) + p_v\,\mathrm{Tan}[\Psi] \\ -\delta\omega_m\,\tau_c + p_v\,(1 + s\,\tau_c) - a_v\,\mathrm{Tan}[\Psi] \\ 2\,\mathrm{MM}[s]\,a_v\,K_L + \delta\omega_m\,\tau_c \end{pmatrix} == 0$$

Characteristic polynomial:

$$(1 + s\,\tau_c)^2 - 2\,\mathrm{MM}[s]\,K_L\,\mathrm{Tan}[\Psi] + \mathrm{Tan}[\Psi]^2$$

Single Cavity – with only fast* tuning control

- monotonic threshold $K_L < 1/2\,(K_t\,\mathrm{Cot}[\psi] + \mathrm{Tan}[\psi])$
- oscillatory threshold $K_m \propto K_t/Q_m$ when $\rho \approx 1$;
- need minimum of $K_t > \sqrt{Q_m}$ when $\rho \approx 1$



$$\begin{pmatrix} a_v\,(1 + s\,\tau_c) + p_v\,\mathrm{Tan}[\Psi] \\ -\delta\omega_m\,\tau_c - \delta\omega_T\,\tau_c + p_v\,(1 + s\,\tau_c) - a_v\,\mathrm{Tan}[\Psi] \\ K_t\,p_v + \delta\omega_T\,\tau_c \\ 2\,\mathrm{MM}[s]\,a_v\,K_L + \delta\omega_m\,\tau_c \end{pmatrix} == 0$$

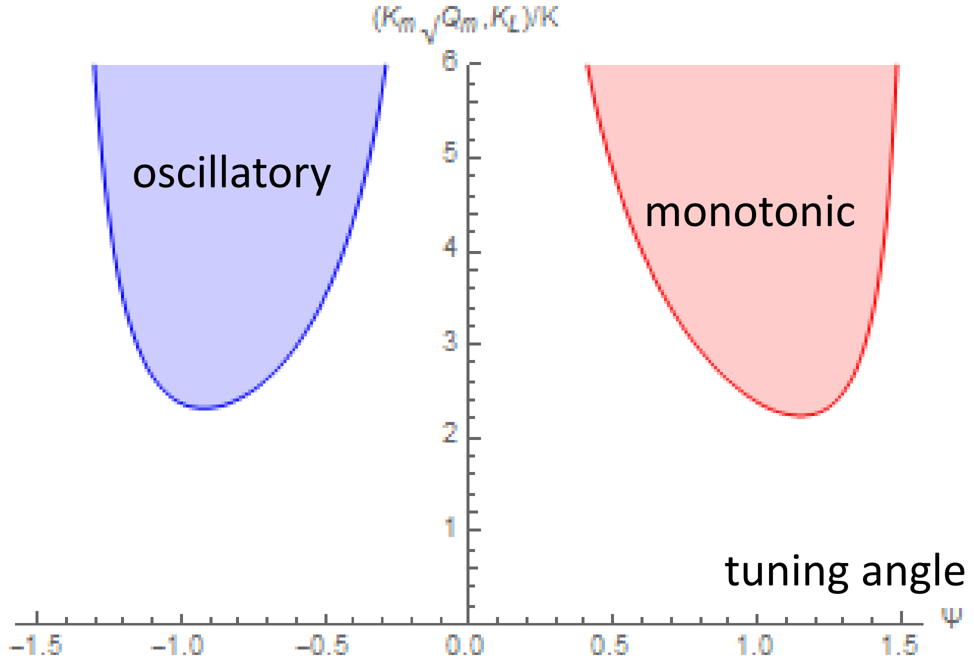
$$(1 + s\,\tau_c)\,(K_t + s\,\tau_c) - 2\,\mathrm{MM}[s]\,K_L\,\mathrm{Tan}[\Psi] + \mathrm{Tan}[\Psi]^2$$

* If tuner time constant $> \tau_c$ then get additional instability

Single Cavity – all loops

- proportional control gains Ka, Kp, Kt (amplitude, phase, tuning)
- monotonic threshold boosted by Ka, Kt (not Kp): $K_L <$ 1/2 Ka (Kt Cot[ψ] + Tan[ψ])
- oscillatory threshold also raised: when ρ≈1 need gain Ka×Kp×Kt > Q_m

$$\begin{pmatrix} -a_g + a_v\,(1 + s\,\tau_c) - p_g\,\mathrm{Tan}[\Psi] + p_v\,\mathrm{Tan}[\Psi] \\ -p_g - \delta\omega_m\,\tau_c - \delta\omega_T\,\tau_c + p_v\,(1 + s\,\tau_c) + a_g\,\mathrm{Tan}[\Psi] - a_v\,\mathrm{Tan}[\Psi] \\ a_g + a_v\,K_a \\ p_g + K_p\,p_v \\ -K_t\,p_g + K_t\,p_v + \delta\omega_T\,\tau_c \\ 2\,\mathrm{MM}[s]\,a_v\,K_L + \delta\omega_m\,\tau_c \end{pmatrix} == 0$$



Matrix determinant ≡
characteristic polynomial:

$$(K_a + s\,\tau_c)\,(K_p\,K_t + s\,\tau_c) + K_p\,\mathrm{Tan}[\Psi]\,\left(-2\,\mathrm{MM}[s]\,K_L + K_a\,\mathrm{Tan}[\Psi]\right)$$

Vector sum control

- Sum cavity voltages together and apply to the input of a single AM/PM feedback loop to the shared RF source

Motivation: Shared RF source

- High power RF sources are expensive
- More cost effective to drive several cavities from one source

Cavity strings

- We attempt to build cavities with identical EM modes.
- So the mechanical modes will also be very similar.
- Leads to equations that are equal/symmetrical in MM[s]

Two cavities in vector sum (see slide #13)

From control view point, behaves exactly like **two virtual cavities:**

Characteristic polynomials

a) Cavity with all loops present $(K_a + s\,\tau_c)\,(K_p K_t + s\,\tau_c) - 2\,MM[s]\,K_m K_p \,Tan[\Psi] + K_a K_p \,Tan[\Psi]^2$

b) Cavity with no loops except tuner $(1 + s\,\tau_c)\,(K_t + s\,\tau_c) - 2\,MM[s]\,K_m \,Tan[\Psi] + Tan[\Psi]^2$

Virtual cavity “b” goes unstable before cavity “a”.

Follows from symmetry; therefore also true for SEL.

Consequence: no matter how hard we push the gains K_a, K_p there is no improvement of ponderomotive stability - because virtual cavity “b” is the problem.

Notation

- ω_c (instantaneous) cavity electromagnetic (EM) resonance (angular) frequency
- Q_c cavity (loaded) EM quality factor
- $\Delta\omega_c = \omega_c/Q_c = 2/\tau_c$ cavity (angular frequency) bandwidth
- $\tau_c = 2Q_c/\omega_c$ cavity (loaded) electrical time constant
- $\delta\omega$ or $\delta\omega_i$ deviation of frequency, cavity index i
- $\Delta\omega_\mu$ steady state Lorentz force detuning
- $\delta\omega_\mu$ dynamical Lorentz force detuning
- a_v or a_{vi} amplitude modulation index $(\delta V/V_0)$ for cavity voltage, cavity index i
- p_v or p_{vi} phase modulation index $(\delta\phi)$ for cavity voltage, cavity index i
- a_g and p_g amplitude $(\delta V_g/V_{g0})$ and phase $(\delta\phi_g)$ modulation idices for generator (equivalent) voltage
- Ψ or Ψ_i cavity detuning angle (radian), cavity index i
- Θ or Θ_i loop phase (radian), cavity index i; Delayen denotes this Θ_L.
- Ω or Ω_μ cavity mechanical mode (MM) resonance angular frequency, mode index m
- Q or Q_m cavity mechanical mode quality factor, mode index m
- $\rho = \tau_c \times \Omega_m$ product of cavity EM time constant and cavity MM angular frequency.
- s Laplace frequency having real and imaginary parts.
- H and E electric and magnetic fields, respectively.

Case A: Two cavities in vector sum with simple tuning control of each cavity

Cavity and control is treated symmetrically

$$\begin{pmatrix}
-a_g + (1 + s\,\tau_c)\,a_{v,1} - p_g \operatorname{Tan}[\Psi] + p_{v,1} \operatorname{Tan}[\Psi] \\
-p_g + (1 + s\,\tau_c)\,p_{v,1} - \tau_c\,\delta\omega_{m,1} - \tau_c\,\delta\omega_{T,1} + a_g \operatorname{Tan}[\Psi] - a_{v,1} \operatorname{Tan}[\Psi] \\
-\frac{1}{2} K_t\,p_g + \frac{1}{2} K_t\,p_{v,1} + \frac{1}{2}\tau_c\,\delta\omega_{T,1} \\
2\,\mathrm{MM}[s]\,K_L\,a_{v,1} + \tau_c\,\delta\omega_{m,1} \\
a_g + \frac{1}{2} K_a\,a_{v,1} + \frac{1}{2} K_a\,a_{v,2} \\
p_g + \frac{1}{2} K_p\,p_{v,1} + \frac{1}{2} K_p\,p_{v,2} \\
-a_g + (1 + s\,\tau_c)\,a_{v,2} - p_g \operatorname{Tan}[\Psi] + p_{v,2} \operatorname{Tan}[\Psi] \\
-p_g + (1 + s\,\tau_c)\,p_{v,2} - \tau_c\,\delta\omega_{m,2} - \tau_c\,\delta\omega_{T,2} + a_g \operatorname{Tan}[\Psi] - a_{v,2} \operatorname{Tan}[\Psi] \\
-\frac{1}{2} K_t\,p_g + \frac{1}{2} K_t\,p_{v,2} + \frac{1}{2}\tau_c\,\delta\omega_{T,2} \\
2\,\mathrm{MM}[s]\,K_L\,a_{v,2} + \tau_c\,\delta\omega_{m,2}
\end{pmatrix} == 0$$

Characteristic equation factors into 2 polynomials; see previous slide.

Introduce vector sum and difference* variables

V_{sum} = (V1+V2)/2 and V_{diff} =(V1-V2).

And likewise for all other variables in the state vector

V_{sum} = {a_1+a_2, ϕ_1+ ϕ_2}
V_{diff} = {a1-a2, ϕ_1- ϕ_2}
Likewise for $\delta\omega_c$, etc

Find that system equations divide into 2 separate sets.

V_{sum} is governed by virtual cavity "a" (all loops present)

V_{diff} is governed by virtual cavity "b" (tuning loop only)

=> The difference mode will be the first to go unstable.

1. **Vector sum control ineffective for the V1 - V2 state variable**
2. **→ Lowered threshold for ponderomotive Instability**

1) Restoring control over individual V_i, implies:

- Introducing control that is different between the cavities
- Only place to do this is at the (fast) cavity tuners

2) Raising or removing ponderomotive threshold implies:

- Targeted DC-coupled feedback for the monotonic instability
- Targeted AC-coupled feedback for the oscillatory instability

Recall damped harmonic oscillator: x"+kx'+Ω^2
For damping, need a quadrature term like kx'.
So take:
$\delta\omega_1$ = +K_t s $\Delta\phi$
$\delta\omega$2 = -K_t s $\Delta\phi$
$\Delta\phi$ = (ϕ_1 – ϕ_2)

Step 1) Adopt difference variable control for the tuners:

Step 2) Add derivative control to tuning loop

* Introduced March 2019, see references.

Case B: Two cavities

Retain sum-variable control for amplitude and phase loops

Introduce difference-variable control for tuning loops

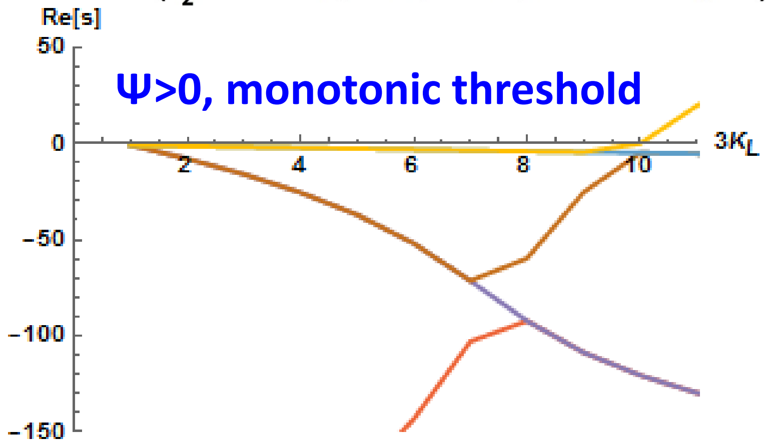
$$\begin{pmatrix} \frac{1}{2}\left(-K_t\left(p_g - p_{v,1} + p_{v,2}\right) + \tau_c\, \delta\omega_{T,1}\right) \\ \frac{1}{2}\left(-K_t\left(p_g + p_{v,1} - p_{v,2}\right) + \tau_c\, \delta\omega_{T,2}\right) \end{pmatrix}$$

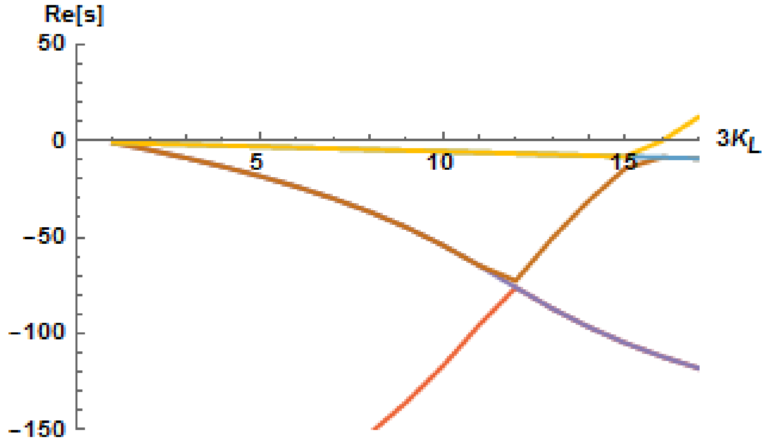
Case C: Two cavities

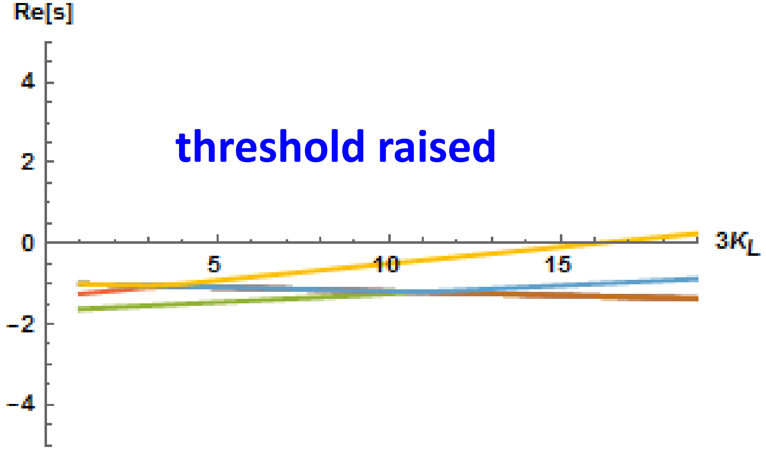
Retain controls as for case B

Add derivative control to tuning loops

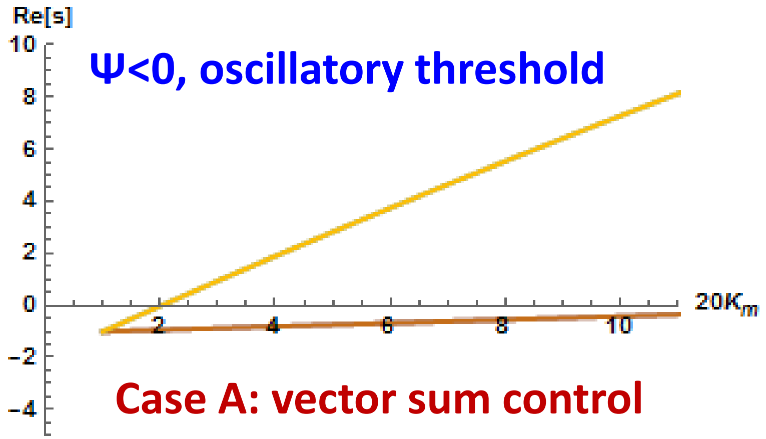
$$\begin{pmatrix} \frac{1}{2}\left(-\left(1 + s\,T\right) K_t\left(p_g - p_{v,1} + p_{v,2}\right) + \tau_c\, \delta\omega_{T,1}\right) \\ \frac{1}{2}\left(-\left(1 + s\,T\right) K_t\left(p_g + p_{v,1} - p_{v,2}\right) + \tau_c\, \delta\omega_{T,2}\right) \end{pmatrix}$$



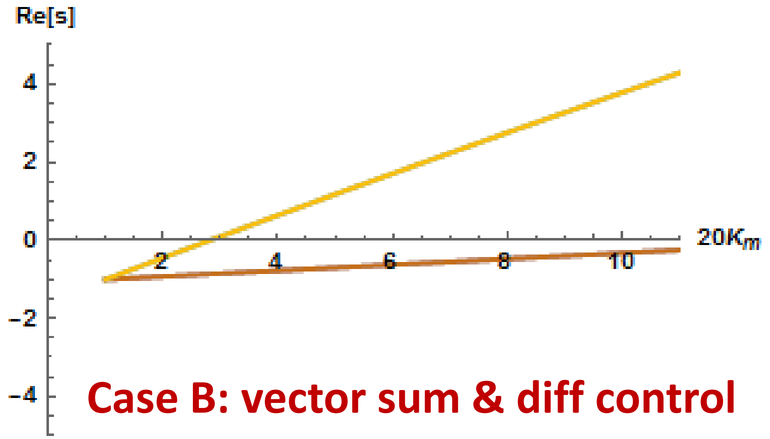




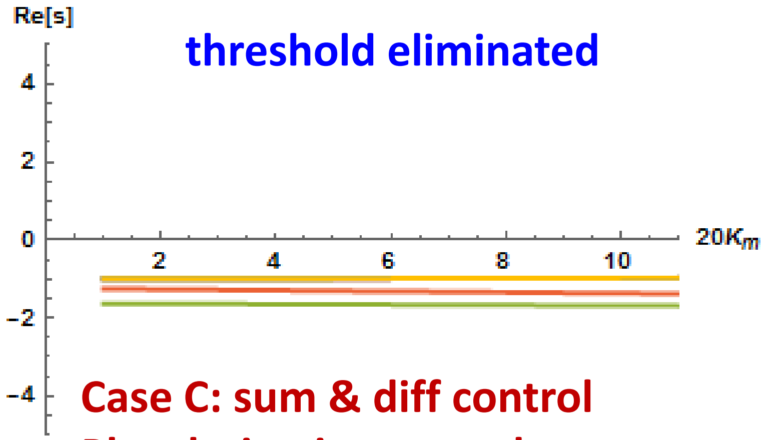
both thresholds raised



Case A: vector sum control



Case B: vector sum & diff control



Case C: sum & diff control
Plus derivative control

N-cavity Vector Sum (& Difference) Control

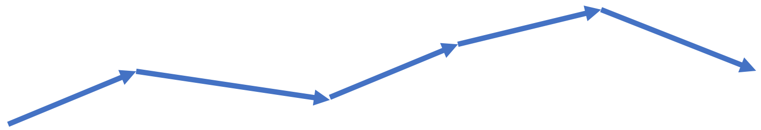
N cavities with shared single RF source and vector sum control

- Characteristic equation factors into N polynomials
- Behaves exactly as:
  - 1 virtual cavity with all loops present
  - N-1 virtual cavities with no control loops except a tuner for each
  - This means we can employ the stability criteria for single cavities
  - All the ponderomotive difference modes have lower threshold than the sum mode
- In short pulse operation, the instability is no concern.
  - The initial values are periodically reset before the instability has time to take off.
- For long pulse, near c.w. or c.w. operation the instabilities can become manifest.

Hence we need to add Difference Control

[Note, the sum and difference variables do not change when extra microphonic modes are added, because the variables are a property of the vector sum configuration.]

Why Use Vector Sum-and-Difference Control?



- Above threshold, simple tuning control cannot straighten the vector sum*
- Without changing the cavity Q_c or access to the individual a_i,p_i
  - Sum & diff is the only way to raise the threshold/damp the ponderomotive difference modes
- System matrix becomes simpler, fewer coupling elements
- Simpler matrix is easier to comprehend: reveals underlying structure/causation/nature of system
- System matrix takes block-diagonal form
  - Hence coordinates are orthogonal between virtual cavities
- Can add feedback that does not cause un-intended couplings
  - Additional feedbacks based on differences does not compromise the pre-existing dominant/defining feedback based on the sum

*However, sum & diff will not make the maximum length vector (for which you need all the individual a_i,p_i)

Vector Sum-and-difference control: change of basis

- From linear algebra theory, we know the characteristic equation is independent of basis vector
- But the appearance of the system matrix **P** depends on the choice of basis vector
- So, underlying symmetry can be made manifest by suitable choice of basis
- Any linear superposition of the old bases, is also a basis
- The transform to vector sum and difference co-ordinates is generated by a matrix **T**
- [If **T** can be inverted ($\mathbf{T}^{-1}$ exists) then old and new basis vectors are independent]
- Old basis vector **v**; new basis vector **v'=Tv**
- New system matrix $\mathbf{P'}=\mathbf{T}\,\mathbf{P}\,\mathbf{T}^{-1}$.

N-Cavity System

- We have to generalize the concept of "sum" and "difference" to N variables.
- The differences are formed pairwise and cyclically permuted.
- The 3 cavity system will demonstrate the principle.
  - For brevity, we omit the microphonic mode coupling – but it is easy to restore.

Old system matrix, P Old base vector, v

$$
\begin{pmatrix}
1+s\,\tau_c & \mathrm{Tan}[\Psi] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -\mathrm{Tan}[\Psi] \\
-\mathrm{Tan}[\Psi] & 1+s\,\tau_c & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{Tan}[\Psi] & -1 \\
0 & \frac{K_t[s]}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{3}K_t[s] \\
0 & 0 & 0 & 1+s\,\tau_c & \mathrm{Tan}[\Psi] & 0 & 0 & 0 & 0 & -1 & -\mathrm{Tan}[\Psi] \\
0 & 0 & 0 & -\mathrm{Tan}[\Psi] & 1+s\,\tau_c & -1 & 0 & 0 & 0 & \mathrm{Tan}[\Psi] & -1 \\
0 & 0 & 0 & 0 & \frac{K_t[s]}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & -\frac{1}{3}K_t[s] \\
0 & 0 & 0 & 0 & 0 & 0 & 1+s\,\tau_c & \mathrm{Tan}[\Psi] & 0 & -1 & -\mathrm{Tan}[\Psi] \\
0 & 0 & 0 & 0 & 0 & 0 & -\mathrm{Tan}[\Psi] & 1+s\,\tau_c & -1 & \mathrm{Tan}[\Psi] & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{K_t[s]}{3} & \frac{1}{3} & 0 & -\frac{1}{3}K_t[s] \\
\frac{K_a[s]}{3} & 0 & 0 & \frac{K_a[s]}{3} & 0 & 0 & \frac{K_a[s]}{3} & 0 & 0 & 1 & 0 \\
0 & \frac{K_p[s]}{3} & 0 & 0 & \frac{K_p[s]}{3} & 0 & 0 & \frac{K_p[s]}{3} & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
av_1 \\ pv_1 \\ \tau\,\delta\omega_1 \\ av_2 \\ pv_2 \\ \tau\,\delta\omega_2 \\ av_3 \\ pv_3 \\ \tau\,\delta\omega_3 \\ a_g \\ p_g
\end{pmatrix}
= 0
$$

New base vector, v’

[Moving a_g,p_g from bottom to top has the effect of diagonalizing the matrix; it is nice but not essential]

$$
\begin{pmatrix}
a_g \\ p_g \\ A_1 \\ P_1 \\ \tau\,O_1 \\ A_2 \\ P_2 \\ \tau\,O_2 \\ A_3 \\ P_3 \\ \tau\,O_3
\end{pmatrix}
\equiv
\begin{pmatrix}
a_g \\ p_g \\ \frac{1}{3}\,(av_1+av_2+av_3) \\ \frac{1}{3}\,(pv_1+pv_2+pv_3) \\ \frac{1}{3}\,(\tau\,\delta\omega_1+\tau\,\delta\omega_2+\tau\,\delta\omega_3) \\ av_1-av_3 \\ pv_1-pv_3 \\ \tau\,\delta\omega_1-\tau\,\delta\omega_3 \\ av_1-av_2 \\ pv_1-pv_2 \\ \tau\,\delta\omega_1-\tau\,\delta\omega_2
\end{pmatrix}
$$

Subscript = virtual cavity index Subscript = real cavity index

Transformation matrix T

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\
0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

New system matrix, P’ **New base vector, v’**

$$\begin{pmatrix}
1 & 0 & K_a[s] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & K_p[s] & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & -\mathrm{Tan}[\Psi] & 1+s\,\tau_c & \mathrm{Tan}[\Psi] & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathrm{Tan}[\Psi] & -1 & -\mathrm{Tan}[\Psi] & 1+s\,\tau_c & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{3}K_t[s] & 0 & \frac{K_t[s]}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1+s\,\tau_c & \mathrm{Tan}[\Psi] & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\mathrm{Tan}[\Psi] & 1+s\,\tau_c & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{K_t[s]}{3} & \frac{1}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1+s\,\tau_c & \mathrm{Tan}[\Psi] & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathrm{Tan}[\Psi] & 1+s\,\tau_c & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{K_t[s]}{3} & \frac{1}{3}
\end{pmatrix}
\begin{pmatrix} a_g \\ p_g \\ A_1 \\ P_1 \\ \tau\, O_1 \\ A_2 \\ P_2 \\ \tau\, O_2 \\ A_3 \\ P_3 \\ \tau\, O_3 \end{pmatrix} = 0$$

Virtual cavity with all loops

Virtual cavity with tuner only

Virtual cavity with tuner only

$$\begin{pmatrix}
a_g + A_1 K_a[s] \\
p_g + P_1 K_p[s] \\
-a_g + (1+s\,\tau_c)\,A_1 + (-p_g + P_1)\,\mathrm{Tan}[\Psi] \\
-\tau\,O_1 - p_g + P_1 + s\,\tau_c\,P_1 + (a_g - A_1)\,\mathrm{Tan}[\Psi] \\
\frac{1}{3}\,(\tau\,O_1 + (-p_g + P_1)\,K_t[s]) \\
(1+s\,\tau_c)\,A_2 + P_2\,\mathrm{Tan}[\Psi] \\
-\tau\,O_2 + (1+s\,\tau_c)\,P_2 - A_2\,\mathrm{Tan}[\Psi] \\
\frac{1}{3}\,(\tau\,O_2 + P_2\,K_t[s]) \\
(1+s\,\tau_c)\,A_3 + P_3\,\mathrm{Tan}[\Psi] \\
-\tau\,O_3 + (1+s\,\tau_c)\,P_3 - A_3\,\mathrm{Tan}[\Psi] \\
\frac{1}{3}\,(\tau\,O_3 + P_3\,K_t[s])
\end{pmatrix} = 0$$

Opposite Cavity Detuning

Take a hint from alternating gradient focusing.

Cav#1: Ψ<0, oscillatory instability is reduced by Cav#2

Vector sum control

Cav#2 Ψ>0, monotonic instability is reduced by Cav#1

System equations same as slide #13, except one cavity has $+\psi \rightarrow -\psi$

- Opposite detunings => characteristic does not factor => octic equation
- Polynomial coefficients contain only powers of $(\mathrm{Tan}\psi)^2$; so characteristic has identical roots at $\pm\psi$
- 1 virtual cavity that behaves the same at $\pm\psi$.
- Equal monotonic thresholds at $\pm\psi$; and equal oscillatory thresholds at $\pm\psi$
- Both thresholds are higher than for single cavity with tuning loop alone
- Both thresholds are lower than for single cavity with all loops present
- Hence "opposite detuning" wins because its more stable than virtual cavity with tuning loop alone.
- E.g. compare the monotonic thresholds:

$$2\,K_L < \mathrm{Cot}[\Psi]\,K_t + \mathrm{Tan}[\Psi]$$

Tuner only

$$2\,K_L < \sqrt{K_a}\,\left(\mathrm{Cot}[\Psi]\,K_t + \mathrm{Tan}[\Psi]\right)$$

Opposite detuning

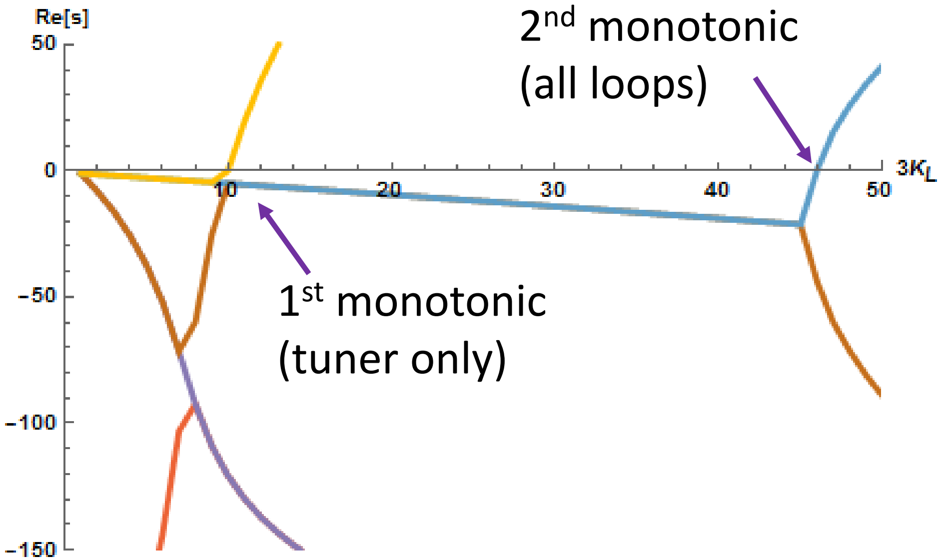
$$2\,K_L < K_a\,\left(\mathrm{Cot}[\Psi]\,K_t + \mathrm{Tan}[\Psi]\right)$$

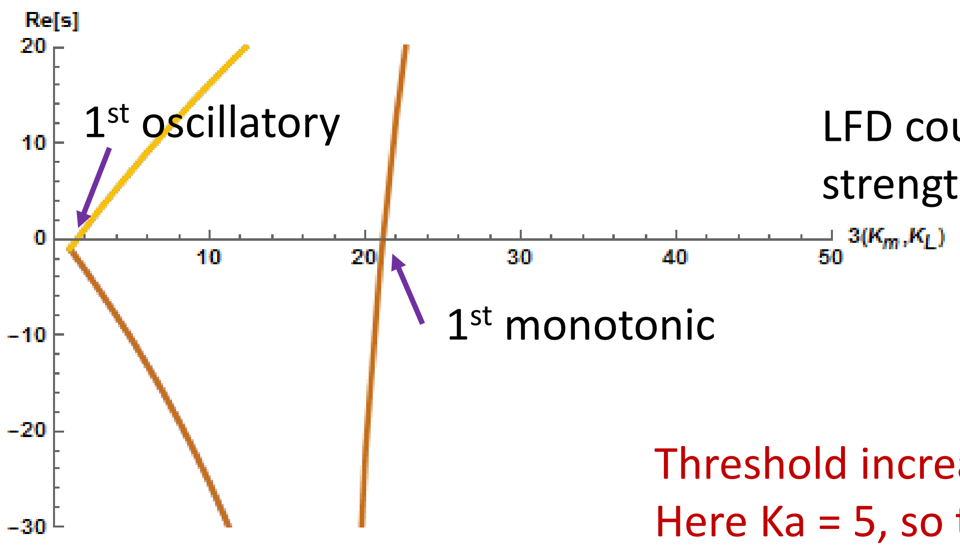
All loops

Equal versus Opposite Cavity Detuning

σ=Re[root]

Ψ>0

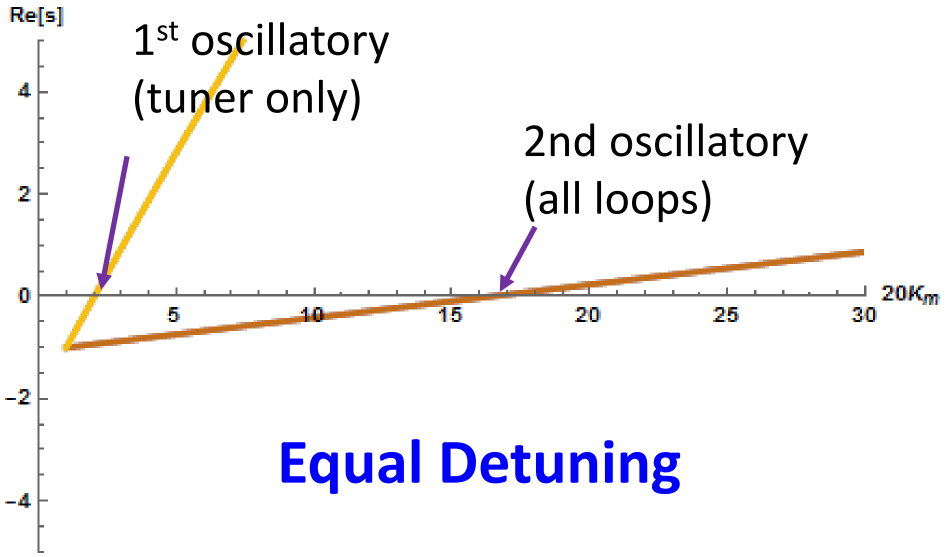


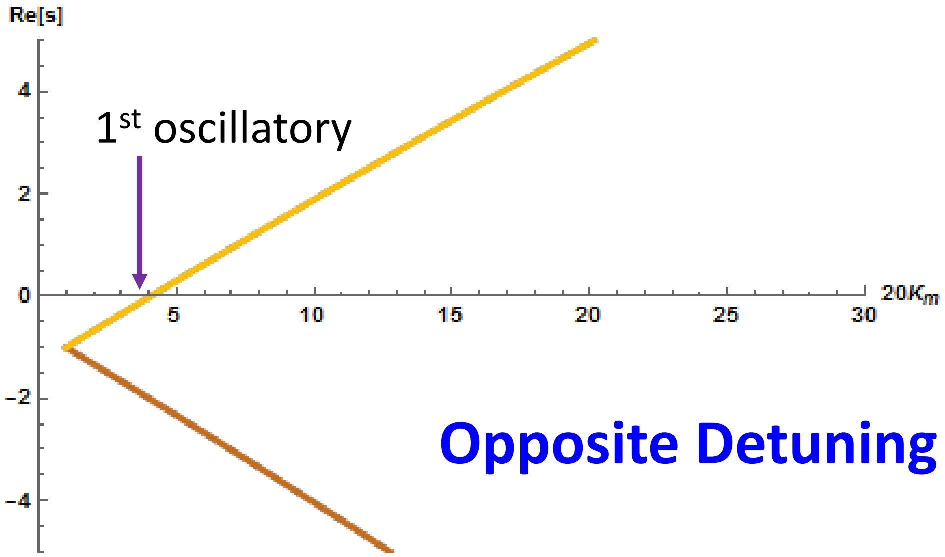


LFD coupling strength K_m ,K_L

Threshold increased by ≈√Ka
Here Ka = 5, so threshold is doubled

Ψ<0



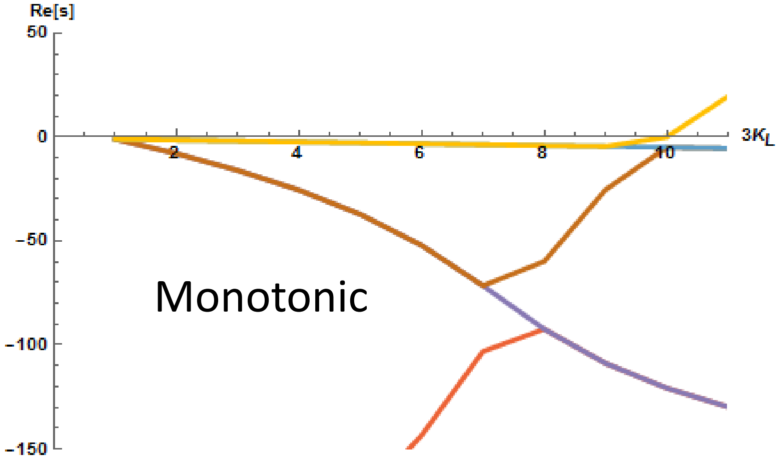


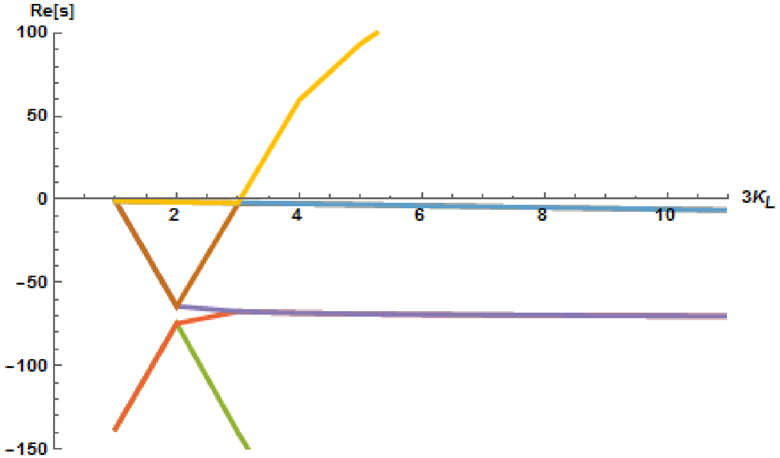
Other Tuning Controls

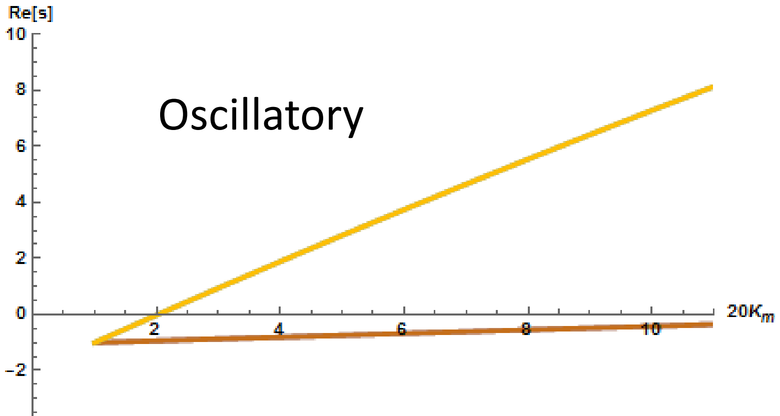
Opposite Phase

$$\begin{pmatrix} \frac{1}{2}\left(-K_t\,(p_g+p_{v,1})+\tau_c\,\delta\omega_{T,1}\right) \\ \frac{1}{2}\left(K_t\,(-p_g+p_{v,2})+\tau_c\,\delta\omega_{T,2}\right) \end{pmatrix} == 0$$

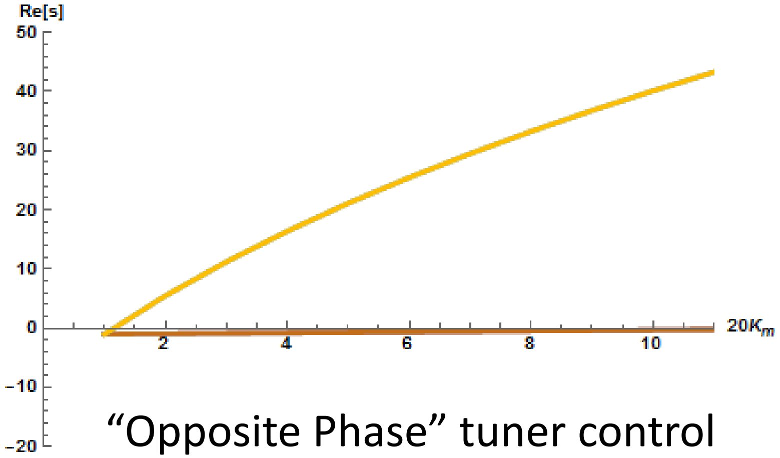
Thresholds are reduced, by unintended coupling







Regular tuning control – slide # 13



"Opposite Phase" tuner control

Derivative of Amplitude Difference

$$\begin{pmatrix} \frac{1}{2}\left(-K_t\,(p_g+s\,T\,a_{v,1}-s\,T\,a_{v,2}-p_{v,1}+p_{v,2})+\tau_c\,\delta\omega_{T,1}\right) \\ \frac{1}{2}\left(-K_t\,(p_g-s\,T\,a_{v,1}+s\,T\,a_{v,2}+p_{v,1}-p_{v,2})+\tau_c\,\delta\omega_{T,2}\right) \end{pmatrix} == 0$$

Although this can be "made to work", it is finicky w.r.t. the derivative gain parameter T. Moreover, not possible to increase the threshold at both signs of the detuning angle. The idea is best avoided!

CONCLUSIONS

$\rho = \tau_c \Omega_m$ is important parameter for oscillatory instability
Heavy loaded regime $\rho \approx 1$ is prone to instability

Multiple cavities driven in vector sum have an instability threshold equal that of a single cavity with only a tuning loop. Does not matter how large are made the amplitude and phase loop gains.

Introducing sum-and-difference control at the individual cavity tuners allows to straighten the vector sum; And to raise the monotonic threshold; And to eliminate the oscillatory instability.

Generalizing the concept of sum-and-difference, enables the technique to be applied to N cavities driven from one source. Opens the way for use of vector-sum in c.w. and near-c.w. applications.

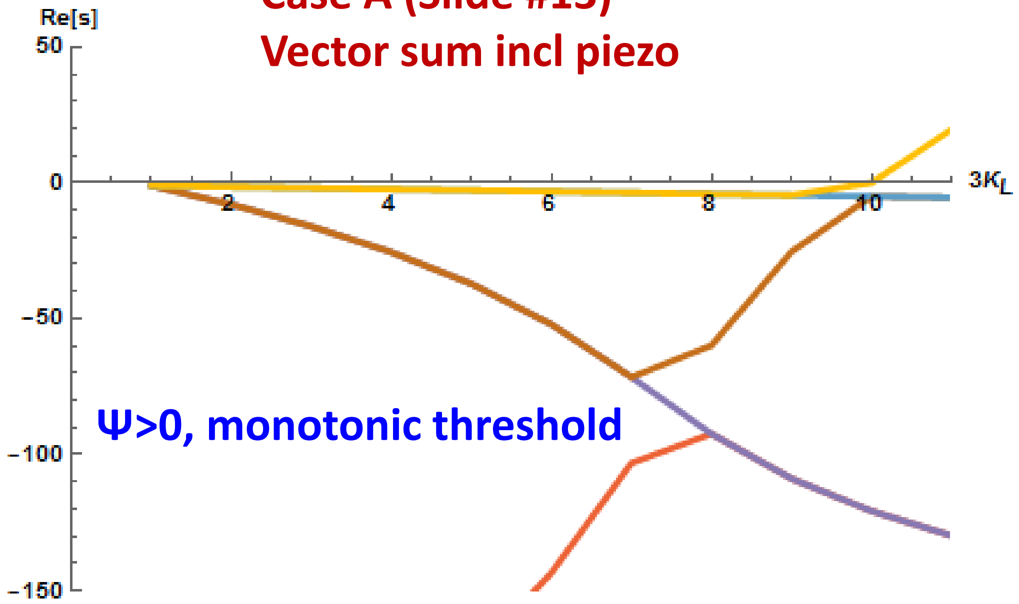
Depends only on "symmetry arguments"; so applies equally well to SE loop.

We note "counter detuning" of alternating cavities as a means to raise thresholds without applying additional feedback.

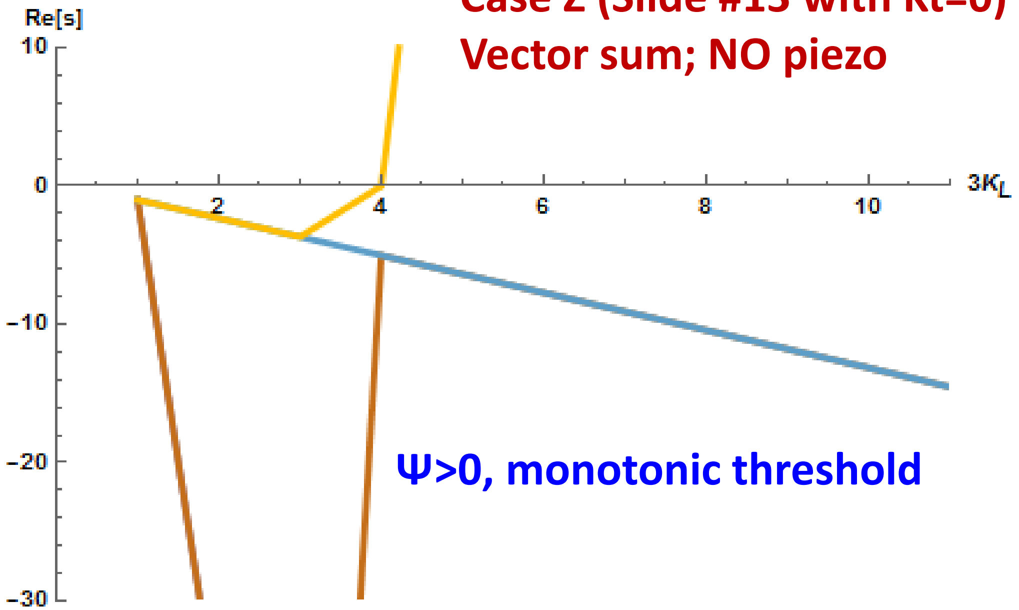
CAVEAT

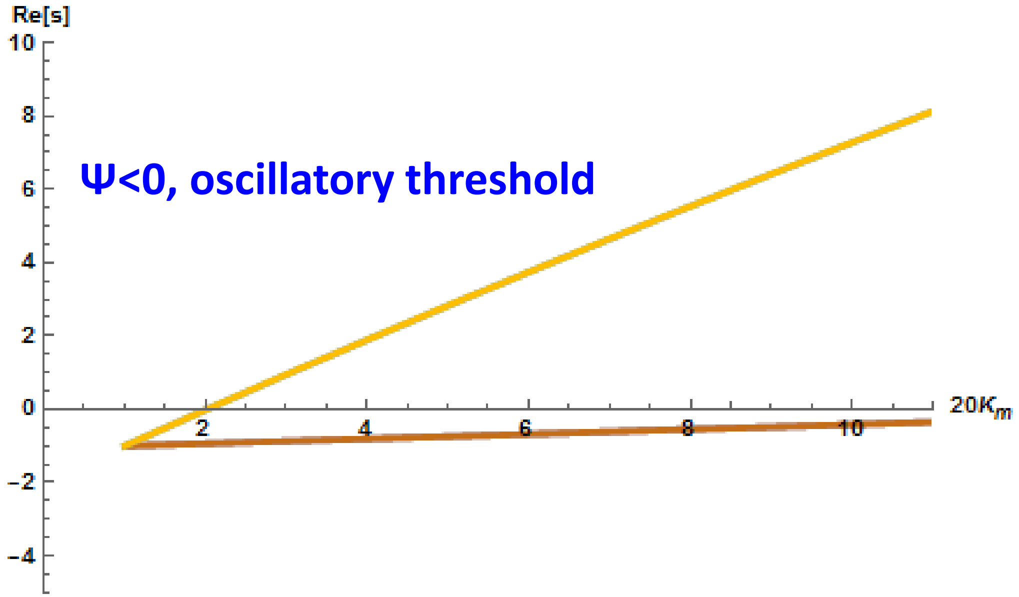
All preceding slides assume that each cavity has the same microphonic mode.
If one cavity is missing the mechanical mode, the situation becomes both more complicated and often more stable than the limits described here. We have performed analysis for such a case with two-cavities; but not reported here.

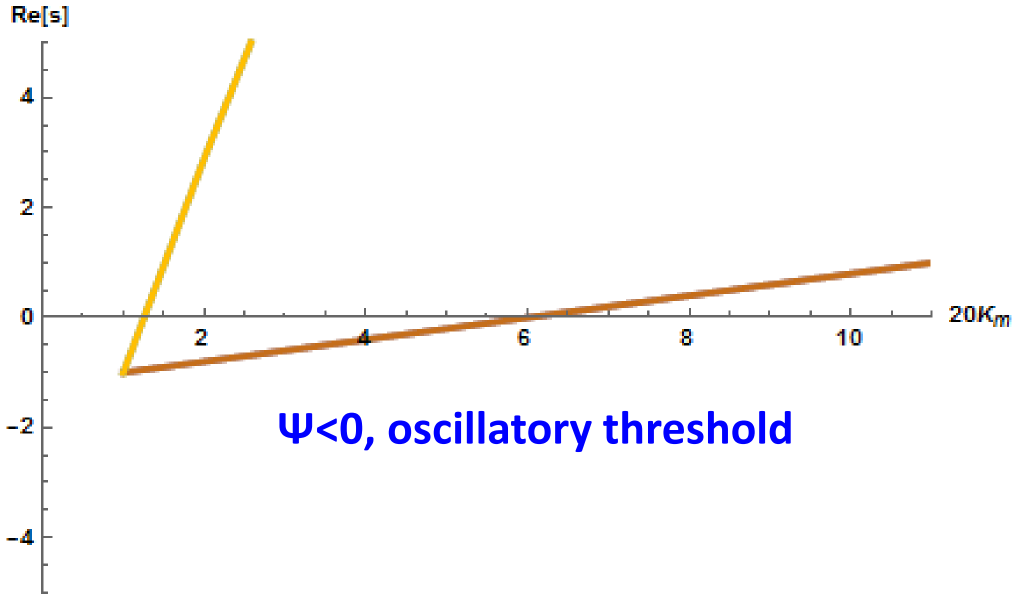
Case A (Slide #13)
Vector sum incl piezo



Case Z (Slide #13 with Kt=0)
Vector sum; NO piezo







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