Perspectives on cavity field control

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Low-Level RF Workshop, 2019-10-03

Outline

Thoughts from an automatic-control (and linac) perspective

1. Complex-coefficient LTI systems
2. Energy-based parametrization of cavity dynamics
3. Normalized cavity dynamics, sensitivity to disturbances
4. Parasitic modes

Baseband cavity models

Complex SISO representation

$$P_a(s) = \frac{\omega_{1/2}}{s + \omega_{1/2} - i\Delta\omega}$$

Real TITO representation

$$\boldsymbol{P}_a(s) = \frac{\omega_{1/2}}{(\Delta\omega)^2 + (s + \omega_{1/2})^2} \begin{bmatrix} \omega_{1/2} & -\Delta\omega \\ \Delta\omega & \omega_{1/2} \end{bmatrix}$$

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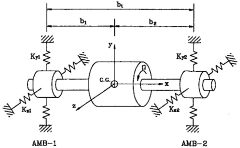
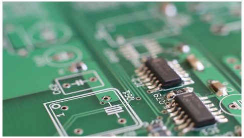
Other control applications:



Vibration damping of rotating machinery



FB linearization of RF amps.

Relation between real and complex representations

Complex SISO representation

$$G(s) = G_{\mathrm{Re}}(s) + iG_{\mathrm{Im}}(s)$$

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$$G_{\mathsf{equiv}}(s) = \begin{bmatrix} G_{\mathrm{Re}}(s) & -G_{\mathrm{Im}}(s) \\ G_{\mathrm{Im}}(s) & G_{\mathrm{Re}}(s) \end{bmatrix}$$

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Consider eigendecomposition

$$G_{\mathsf{equiv}}(s) = U \begin{bmatrix} {\color{blue}G(s)} & 0 \\ 0 & {\color{green}G^*(s)} \end{bmatrix} U^{\mathsf{H}}, \qquad U = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -i & i \end{bmatrix}.$$

Note: ${\color{green}G^*(i\omega) = \overline{G(-i\omega)}}$. Positive and negative frequencies are intertwined in $G_{\mathsf{equiv}}(s)$

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Advantages of the complex-coefficient representation

- Simplifies understanding, calculations, life in general, etc
- Structure is implicit, good for system identification
- More efficient computations

Control theory for complex-coefficient systems

Standard tools and results apply but

- Change A^{T} to A^{H}
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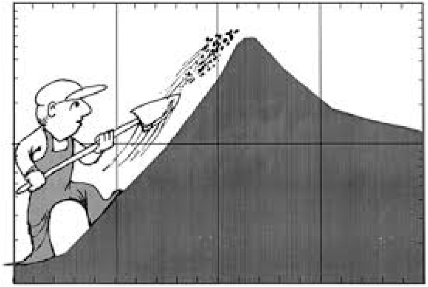

Illustration of Bode's sensitivity integral (the water-bed effect)

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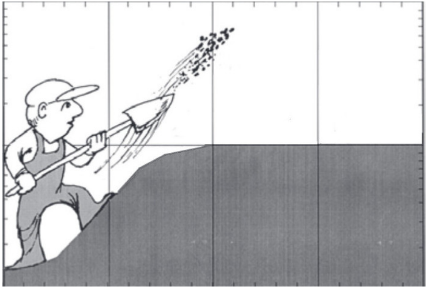

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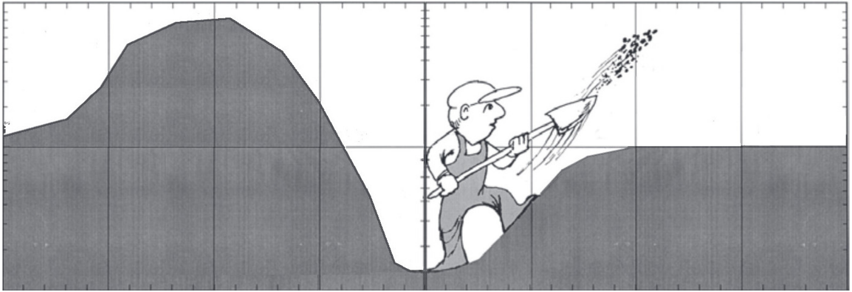

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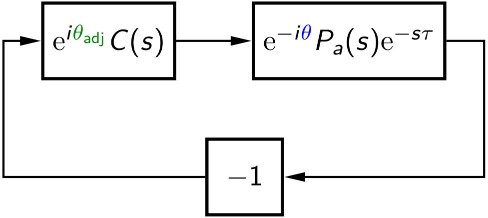
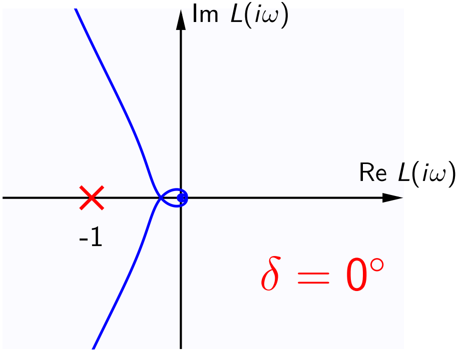
Intuitive understanding of loop-phase adjustment

Open-loop system: $e^{i\theta_{adj}}C(s)e^{-i\theta}P_a(s)e^{-s\tau} = e^{\delta}L_0(s)$ where $\delta := \theta_{adj} - \theta$.





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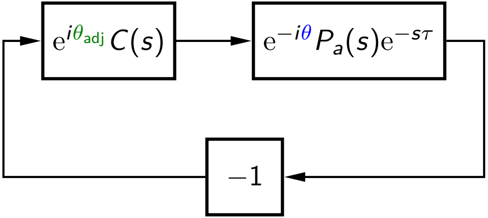

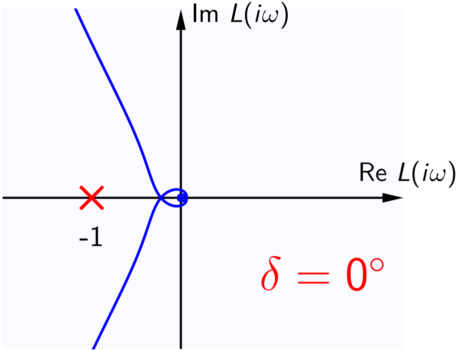

Loop-phase-adjustment error δ gives corresponding phase-margin reduction!

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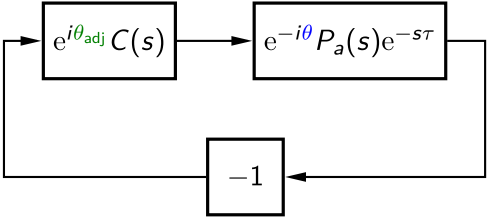

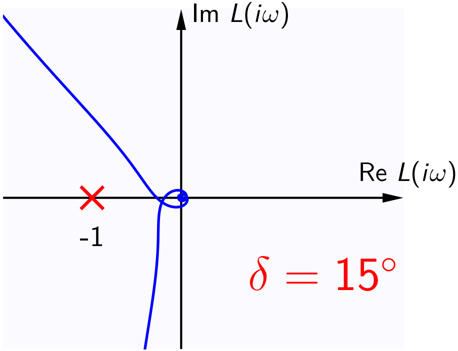

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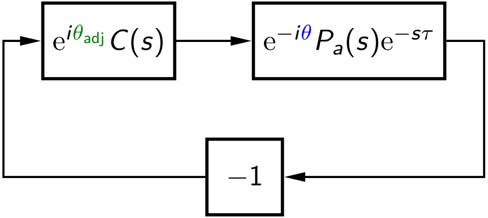

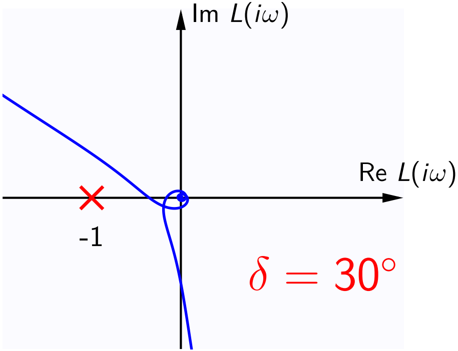

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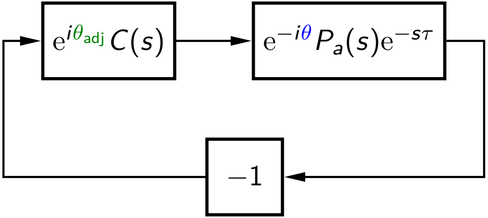

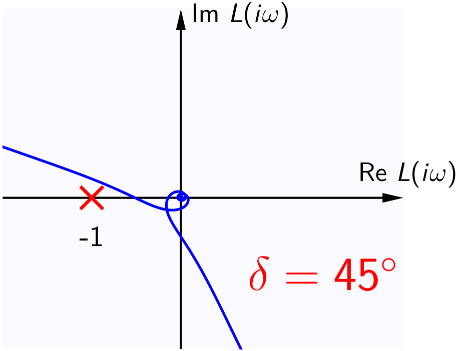

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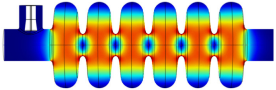
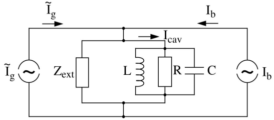
Energy-based parametrization of cavity dynamics

Accelerator Cavity Modeling (1/2)





Equivalent-circuit parametrization:

$$\frac{d\mathbf{V}}{dt} = (-\omega_{1/2} + i\Delta\omega)\mathbf{V} + R_L\omega_{1/2}\left(2\mathbf{I_g} + \mathbf{I_b}\right)$$

$$P_g = \frac{1}{4}\frac{r}{Q}Q_{\text{ext}}\left|\mathbf{I_g}\right|^2$$

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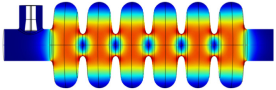

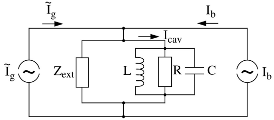

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RF drive is modeled as fictitious generator current $\mathbf{I_g}$. Problematic.

"*One word of caution is required here:* /.../ for considerations where Q_{ext} varies /.../ or where (R/Q) varies /.../ the model currents cannot be considered constant; they have to be re-adapted" [Tückmantel (2011)]

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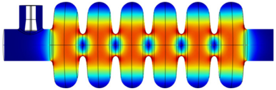

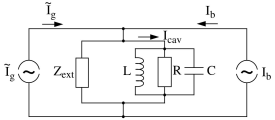

Energy-based parametrization:

$$\frac{d\mathbf{A}}{dt} = (-\gamma + i\Delta\omega)\mathbf{A} + \sqrt{2\gamma_{\text{ext}}}\mathbf{F_g} + \frac{\alpha}{2}\mathbf{I_b}$$

$\mathbf{A}$ – Mode amplitude $[\sqrt{\text{J}}]$

$\mathbf{F_g}$ – Forward wave $[\sqrt{\text{W}}]$

$\mathbf{V} = \alpha\mathbf{A} \qquad \left(\alpha = \sqrt{\omega_a(r/Q)}\right)$

$P_g = |\mathbf{F_g}|^2$

Haus (1984) *Waves and fields in optoelectronics*
plus beam loading

Equivalent-circuit parametrization:

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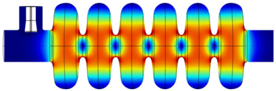

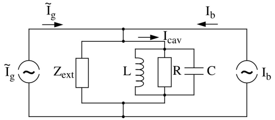

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Advantages of energy-based parameterization:

- Cleaner expressions, e.g., $P_g = |\mathbf{F_g}|^2$
- States and parameters are well defined
- Direct connection to physical quantities of interest

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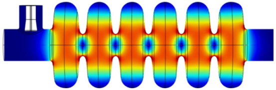

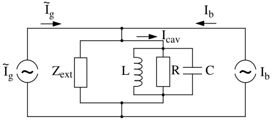

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Helpful to think of γ as both decay rate and bandwidth.

The total decay rate $\gamma = \gamma_0 + \gamma_{\text{ext}}$, not so intuitive if considered as bandwidths

Common for laser cavities

Optimal coupling and detuning

$$\text{Minimize} \quad |\mathbf{F_{g0}}|^2 = \frac{1}{2\gamma_{\mathrm{ext}}} \left| (-\gamma_0 - \gamma_{\mathrm{ext}} + i\Delta\omega)\mathbf{A_0} + \frac{\alpha}{2}\mathbf{I_{b0}} \right|$$

with repsect to $\Delta\omega$ and γ_{ext}

Solution: $\Delta\omega = -\frac{1}{\mathrm{A}_0}\mathrm{Im}\ \frac{\alpha}{2}\mathbf{I_{b0}}, \quad \gamma_{\mathrm{ext}} = \gamma_0 - \frac{1}{\mathrm{A}_0}\mathrm{Re}\ \frac{\alpha}{2}\mathbf{I_{b0}}$

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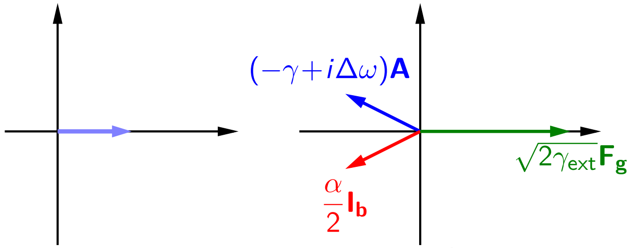

Mode amplitude $\mathbf{A}$ $[\sqrt{\text{J}}]$ Terms of $\frac{d}{dt}\mathbf{A}$ $[\sqrt{\text{J}}/\text{s}]$

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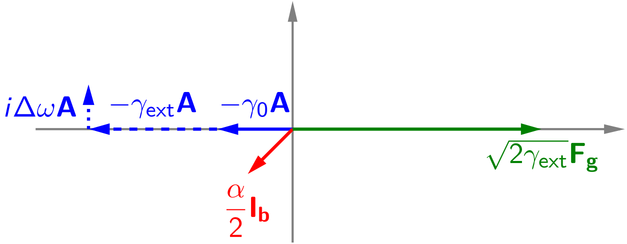

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$$\text{with repsect to}\quad \Delta\omega \text{ and } \gamma_{\text{ext}}$$

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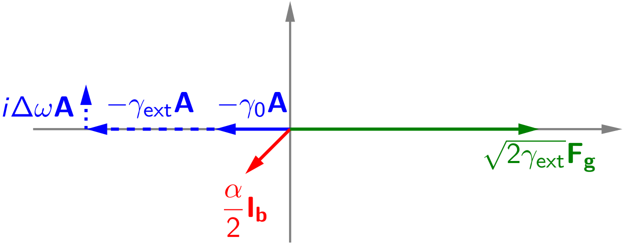


Terms of $\frac{d}{dt}\mathbf{A}$ $[\sqrt{\text{J}}/\text{s}]$

Normalized cavity dynamics

Normalization

$$\mathbf{a} := \frac{1}{A_0}\mathbf{A}, \qquad \mathbf{f_g} := \frac{1}{\gamma A_0}\sqrt{2\gamma_{\mathrm{ext}}}\mathbf{F_g}, \qquad \mathbf{i_b} := \frac{1}{\gamma A_0}\frac{\alpha \mathbf{I_b}}{2}.$$

Normalized cavity dynamics

$$\dot{\mathbf{a}} = (-\gamma + i\Delta\omega)\mathbf{a} + \gamma(\mathbf{f_g} + \mathbf{i_b}).$$

At nominal operating point, with optimal coupling and tuning,
$1 \leq \mathbf{f_{g0}} \leq 2, \quad 0 \leq \mathrm{Re}\,\mathbf{i_{b0}} \leq 1.$
Relative disturbances $\boldsymbol{d}_\mathrm{b}$ and $\boldsymbol{d}_\mathrm{b}$ give rise to

$$\begin{aligned}\mathbf{f_g} &\approx (1 + \boldsymbol{d}_\mathrm{b})(\mathbf{f_{g0}} + \tilde{\mathbf{f}}_\mathbf{g}) \approx \mathbf{f_{g0}} + \tilde{\mathbf{f}}_\mathbf{g} + \mathbf{f_{g0}}\boldsymbol{d}_\mathrm{b}\\ \mathbf{i_b} &= (1 + \boldsymbol{d}_\mathrm{b})\mathbf{i_{b0}}\end{aligned}$$

Introducing the relative field error $\boldsymbol{z} = 1 - \mathbf{a}$, we have

$$\dot{\boldsymbol{z}} = (-\gamma + i\Delta\omega)\boldsymbol{z} + \gamma\tilde{\mathbf{f}}_\mathbf{g} + \gamma\mathbf{f_{g0}}\boldsymbol{d}_\mathrm{b} + \gamma\mathbf{i_{b0}}\boldsymbol{d}_\mathrm{b}$$

Transfer functions around operating point

At nominal operating point

$$\dot{\boldsymbol{z}} = (-\gamma + i\Delta\omega)\boldsymbol{z} + \gamma\tilde{\mathbf{f}}_{\mathbf{g}} + \gamma\mathbf{f}_{\mathbf{g0}}\boldsymbol{d}_{\mathbf{g}} + \gamma\mathbf{i}_{\mathbf{b0}}\boldsymbol{d}_{\mathbf{b}}$$

The transfer function from control action to the cavity field is given by

$$P_a(s) :- \frac{\gamma}{s + \gamma - i\Delta\omega}$$

Transfer functions from relative disturbances to relative field errors are given by

$$P_{d_\mathrm{g}\to z}(s) = \mathbf{f}_{\mathbf{g0}} P_a(s) \tag{1}$$
$$P_{d_\mathrm{b}\to z}(s) = \mathbf{i}_{\mathbf{b0}} P_a(s) \tag{2}$$

For optimally tuned and coupled superconducting cavities $\mathbf{f}_{\mathbf{g0}} = 2$.

Additional factor 2 in disturbance sensitivity to relative amplifier variations!

Impact of disturbances

For clarity, assume that $\Delta\omega = \phi_b = 0$, so $\gamma \mathbf{i_{b0}} = \gamma_{\text{beam}}$ and $\gamma \mathbf{f_{g0}} = \gamma_0 + \gamma_{\text{ext}} + \gamma_{\text{beam}}$

Transfer functions from relative disturbances to relative field errors are given by

$$P_{d_g \to z}(s) = \frac{\gamma_0 + \gamma_{\text{ext}} + \gamma_{\text{beam}}}{s + \gamma_0 + \gamma_{\text{ext}}}, \tag{3}$$

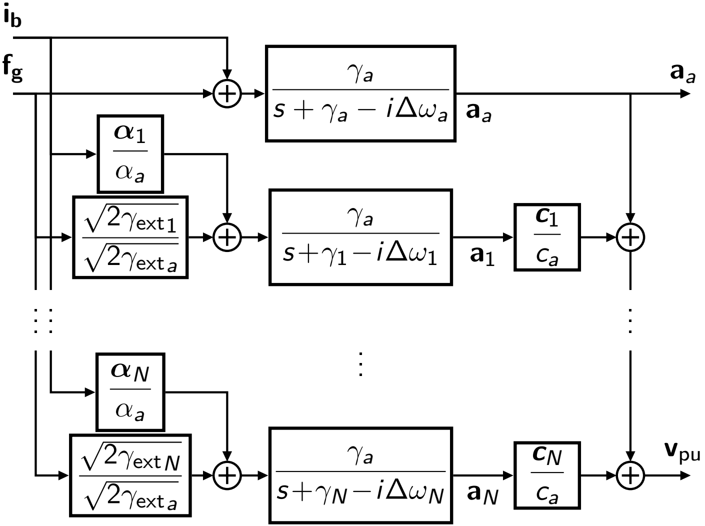
$$P_{d_b \to z}(s) = \frac{\gamma_{\text{beam}}}{s + \gamma_0 + \gamma_{\text{ext}}}. \tag{4}$$

Sensitivity to amplifier ripple, equation (4), cannot be made smaller than $\dfrac{\gamma_0 + \gamma_{\text{beam}}}{s + \gamma_0}$

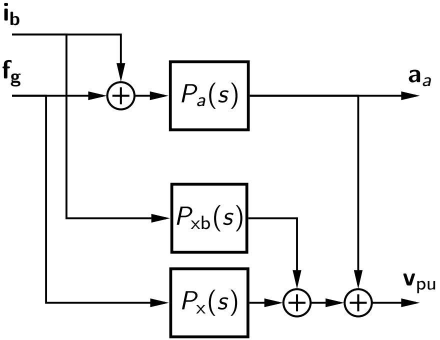
Difficulty of field control is determined by γ_0 and γ_{beam}, but typically $\gamma = 2\gamma_0 + \gamma_{\text{beam}}$

Parasitic modes

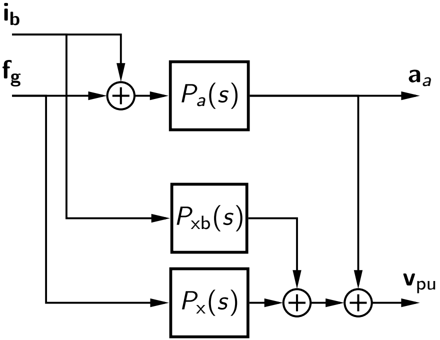
Parasitic modes



Parasitic modes



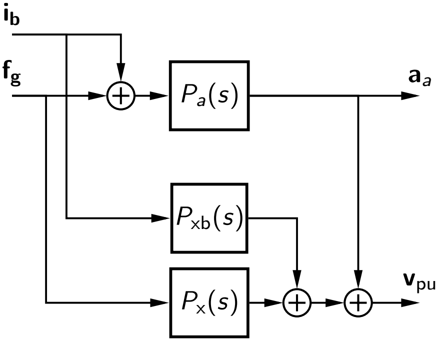
Parasitic modes



A. Calibrate setpoint $\mathbf{v}_{\mathrm{pu}}$ for $\mathbf{a}_a = \mathbf{a}_a^{\star}$ with short/low-current beam pulses ($\mathbf{i_{b0}} = 0$)

B. Operation with nominal beam current and regulation to the set point

Parasitic modes



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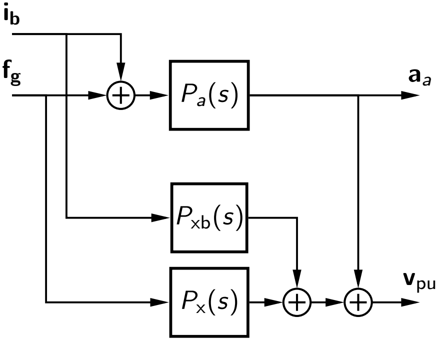
B. Operation with nominal beam current and regulation to the set point

Gives error steady-state error:

$$\delta = \mathbf{a}_a^B - \mathbf{a}_a^\star = \frac{P_x(0) - P_{xb}(0)}{P_a(0) + P_x(0)} P_a(0)\mathbf{i_{b0}}$$
$$\approx (P_x(0) - P_{xb}(0))\mathbf{i_{b0}}$$

Parasitic modes



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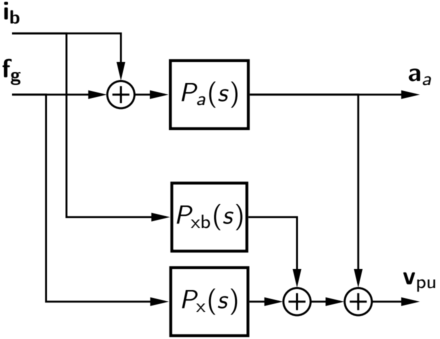
$$\approx (P_{\mathrm{x}}(0) - P_{\mathrm{xb}}(0))\mathbf{i_{b0}}$$

ESS medium-β cavity: $\boldsymbol{\delta} = P_{\mathrm{x}}(0)\mathbf{i_{b0}} \approx \dfrac{\gamma_{5\pi/6}}{i\Delta\omega_{5\pi/6}} = \dfrac{R_5^2\gamma_\pi}{i\Delta\omega_{5\pi/6}} \approx 0.00187i \leftrightarrow 0.11^\circ$

Parasitic modes



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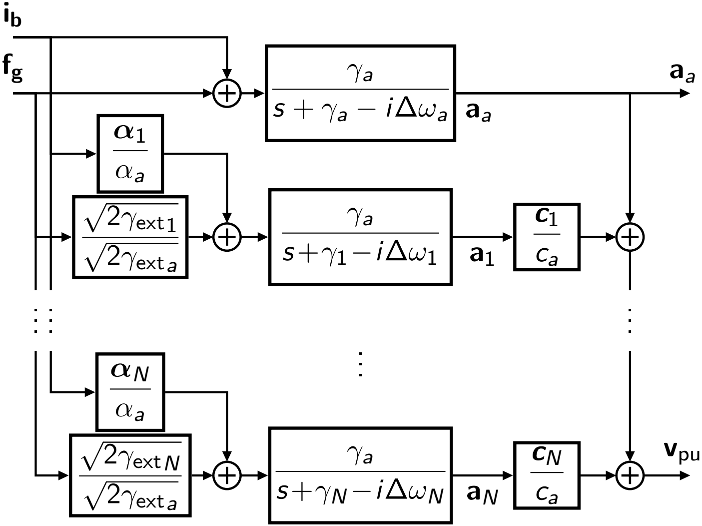
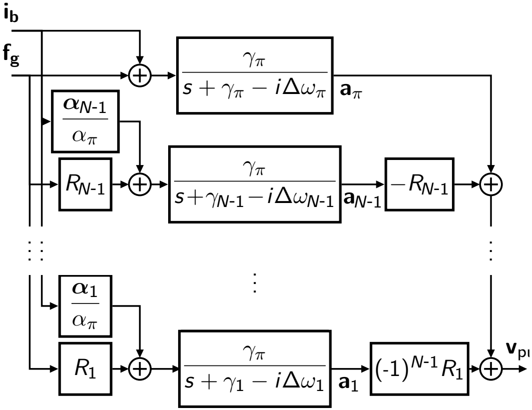
How to handle this? Do nothing, Kalman filter, re-calibrate?

Parasitic modes



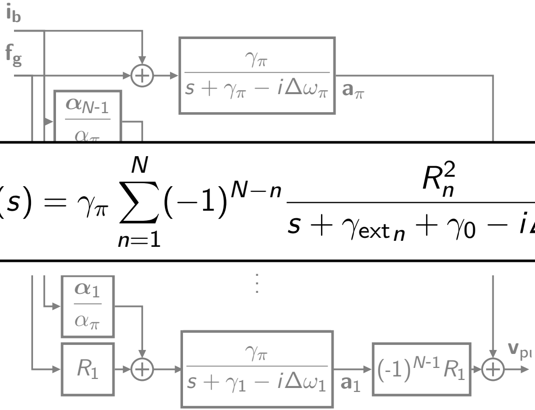
Relations between same-order modes



$$\gamma_{\text{ext}\,n} = R_n^2 \gamma_{\text{ext}\,\pi}$$
$$\Delta\omega_n \approx (R_n^2 - 2) k_{cc} \omega_{\text{cell}}$$

where $R_n := \sqrt{2}\sin(n\pi/(2N))$

Relations between same-order modes



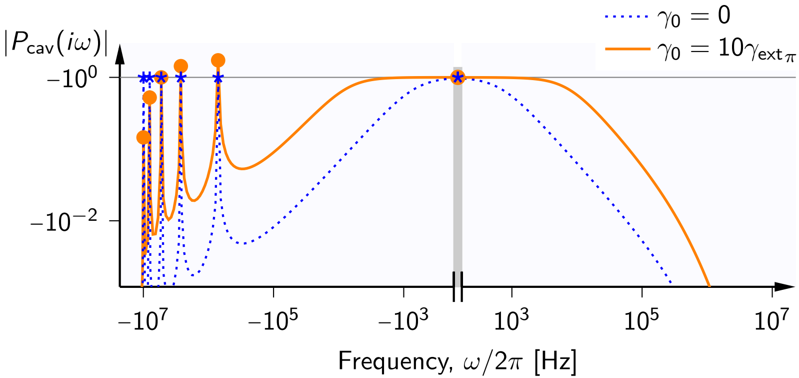
$$\boxed{P_{\text{cav}}(s) = \gamma_\pi \sum_{n=1}^{N} (-1)^{N-n} \frac{R_n^2}{s + \gamma_{\text{ext}\,n} + \gamma_0 - i\Delta\omega_n}}$$

$$\gamma_{\text{ext}\,n} = R_n^2 \gamma_{\text{ext}\,\pi}$$

$$\Delta\omega_n \approx (R_n^2 - 2) k_{cc} \omega_{\text{cell}}$$

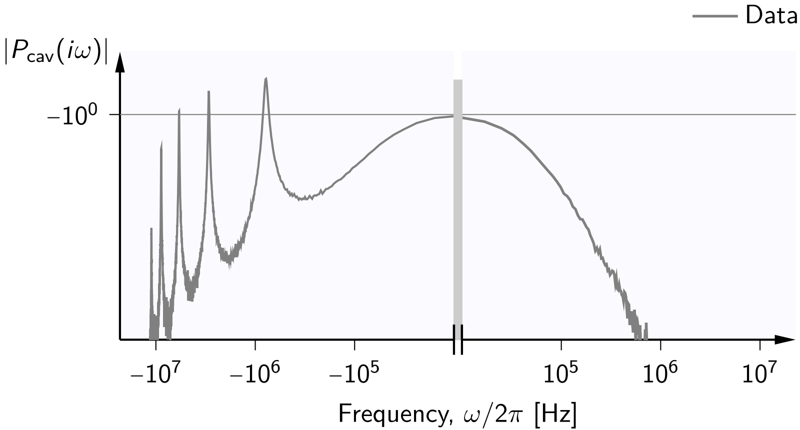
where $R_n := \sqrt{2}\sin(n\pi/(2N))$

Bode magnitude plot for 6-cell cavity



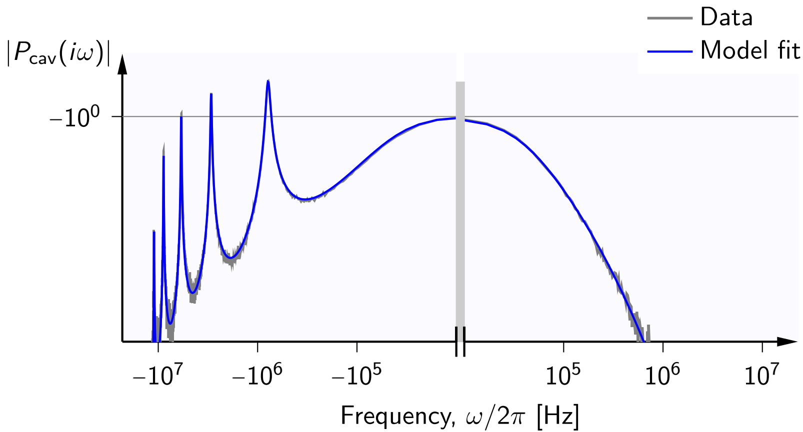
Similar to ESS medium-β cavity, $\gamma_{\mathrm{ext}\,\pi} = 700\,\mathrm{Hz}$

Fit to measured data



Measurements by P. Pierini on *warm* 6-cell ESS medium-β cavity

Fit to measured data



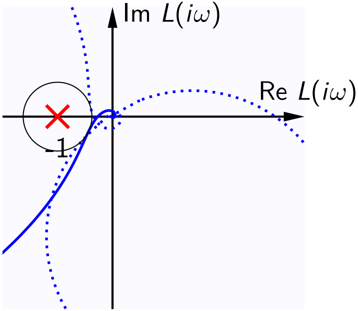
Measurements by P. Pierini on *warm* 6-cell ESS medium-β cavity

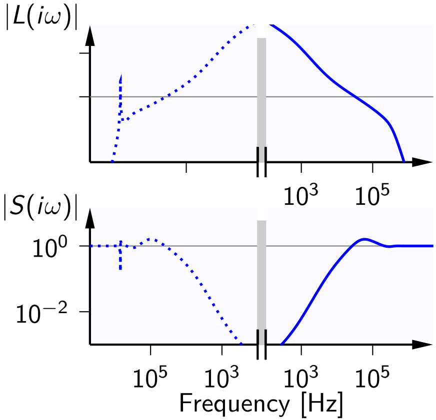
Four parameters were fitted. Estimated resistive decay rate, $\gamma_0/2/\pi = 35\,\text{kHz}$.

Example Control Strategies for Parasitic Modes (1/3)

PI controller + 3rd order filter

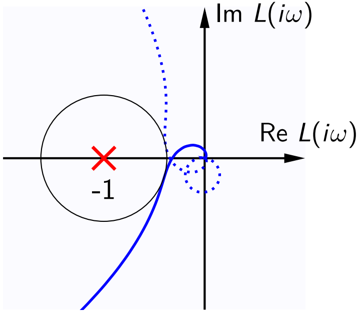
Set controller parameters for good phase of resonant "bubble"

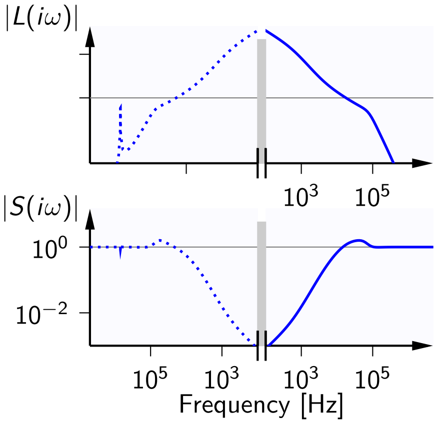




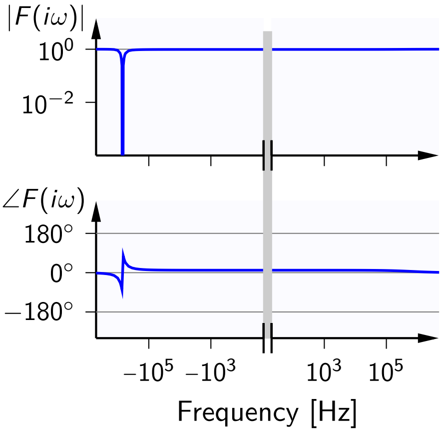
Example Control Strategies for Parasitic Modes (2/3)

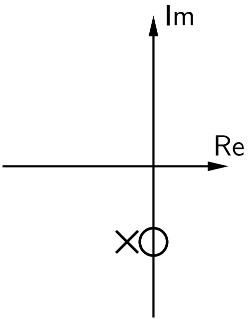
PI controller + 2nd order filter
Wide-band suppression of the "bubble"





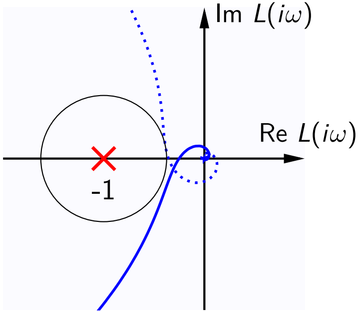
One-Sided Notch Filter

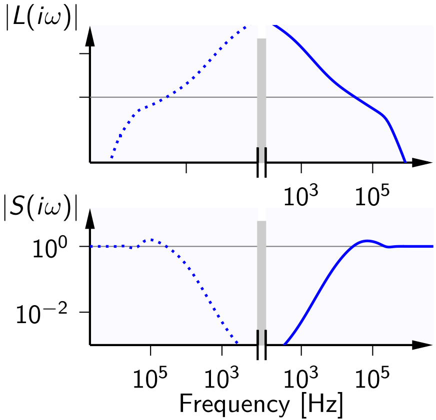




Example Control Strategies for Parasitic Modes (3/3)

PI controller + one-sided notch filter + 2nd order filter
Notch out the "bubble"





Summary

- Analyzing the field control loop as a complex-coefficient system is easier and gives more understanding. Particularly for loop-phase adjustment and parasitic modes.
- Energy-based cavity parametrization is more convenient and fundamental.
- There is a factor ≈ 2 in relative sensitivity to amplifier variations.
- Parasitic modes may give systematic control error since the controlled variable is not measured.

Summary

- Analyzing the field control loop as a complex-coefficient system is easier and gives more understanding. Particularly for loop-phase adjustment and parasitic modes.
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- There is a factor ≈ 2 in relative sensitivity to amplifier variations.
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More details in upcoming PhD thesis.

Thanks

Bo Bernhardsson
main Supervisor

Anders J Johansson (EIT)
Project Leader
co-supervisor

Rolf Johansson
co-supervisor

and people at ESS, SNS, DESY, LBNL