Muon Momentum Estimation Using Multiple Coulomb Scattering

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Momentum Estimation via a Maximum Likelihood Method in ProtoDUNE

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Multiple Coulomb Scattering

- Process of charged particles scattering through matter.
- Benefits to momentum estimation using MCS:
  - Does not require full ionization track to be contained in the detector volume.
    - Unlike calorimetry and track length estimations.
  - From MicroBooNE analysis, momentum resolution is good to 5-10% for fully contained muon tracks with a small bias and 15% for exiting muons with momentum less than 2 GeV/c with at least 1 m of the track contained in the detector volume.
  - ICARUS noted average momentum resolution ~16% for 0.5 – 5 GeV/c muons.

How does a charged particle scatter?

- Angle of scatter has a gaussian distribution centered at 0, standard deviation given by momentum-dependent Highland formula:

β	v/c
z	Charge of muon, unity
x_0	Radiation Length (14 cm, LAr)
l	Length inside material
S_2	13.6 MeV, fit parameter
ε	0.038, fit parameter

$$\sigma_0^{HL} = \frac{S_2}{p\beta c} z \sqrt{\frac{l}{x_0}} \left(1 + \varepsilon * \ln\left(\frac{l}{x_0}\right)\right)$$

For $l = x_0$, $\sigma_0^{HL} = \frac{S_2}{p\beta c}$

Modifying the Highland Formula for LAr

- S_2 and ε were determined in 1991 using a global fit to MCS simulated data using a variety of thickness ranges and materials varying from Hydrogen with Z = 1 to Uranium with Z = 92.
- MicroBooNE refit the parameters using liquid Ar-muon simulations alone and $l \approx x_0$
  - The benefit of using $l \approx x_0$ was so only S_2 had to be found
- They found a function to replace S_2, $\kappa(p) = {}^{a}/_{p^2} + c$
  - The fit showed a = 0.105 and c = 11.004
- The resulting modified formula for the RMS uncertainty with $l = x_0$:
  - $\sigma_0^{RMS} = \sqrt{(\frac{\kappa(p)}{p\beta c})^2 + \sigma_0^{res\,2}}$, where σ_0^{res} is a detector inherent resolution term
  - So far in this analysis, we have used $\sigma_0^{res} = 0$ but MicroBooNE used $\sigma_0^{res} = 3$ mrad

Overview of Momentum Estimation

1. 3D track is divided into segments of 14 cm
2. Scattering angles between consecutive segments is measured
3. Angles are used with modified Highland formula to find the likelihood that the series of scattering angles would occur given the measured scattering angles
4. Cycle through 0.1 – 7.5 GeV/c in 1 MeV/c increments to build $L(p; \{\theta\}, \{l\})$ taking into account energy loss for each segment by using the Bethe-Bloch equation
5. Maximize $L(p; \{\theta\}, \{l\})$, use that as MCS estimate of p

Simulated Muon Sample Information

- 1000 event, single muon, 0 – 1 GeV (uniform distribution)
- 1000 event, single muon, 0.5 GeV monoenergetic sample
- 1000 event, single muon, 1.5 GeV monoenergetic sample
- 1000 event, single muon, 0 – 4 GeV (uniform distribution)
- For all of the above samples:
  - Starting Position: x = 7.966 cm, y = 460.84 cm, z = -191.6 cm
  - Starting Angle: θ_{XZ} = -11.844°, θ_{YZ} = -11.107°
- For muons < ~2 GeV/c, the tracks are fully contained

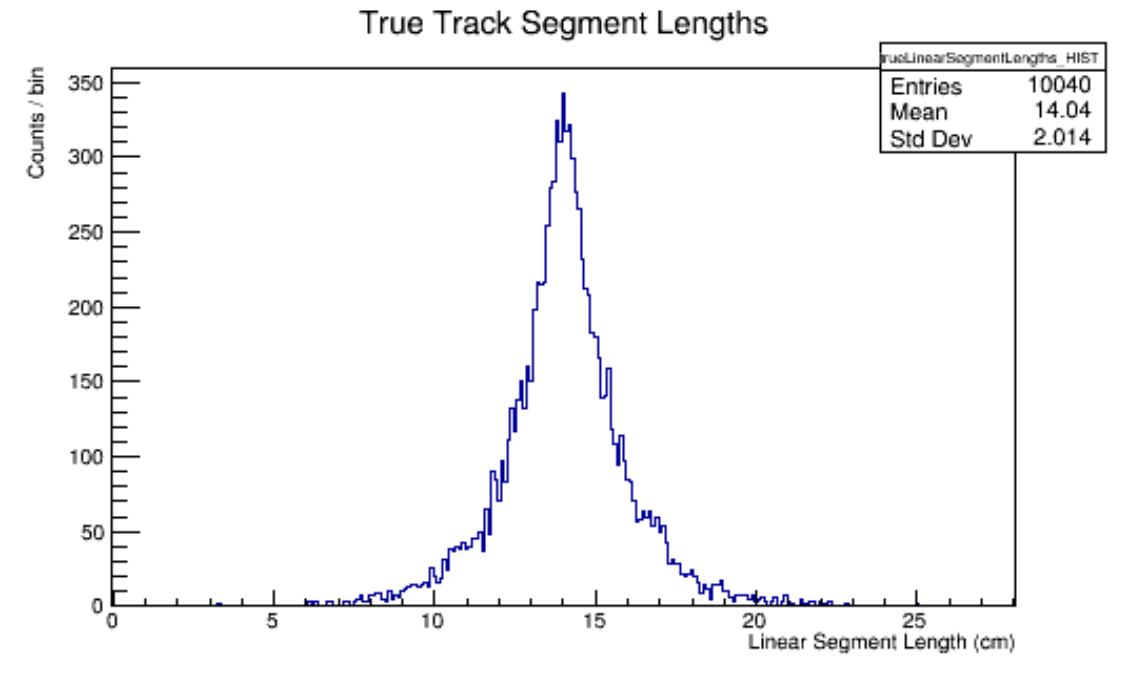
Selection of Events

- The criterium we used was if the length of the track was greater than 100 cm and if the track was created by a muon.
- We used other criteria that was based off of Backtracking, the process of matching a reconstructed track to a true track. (See Extra Slides for more information)
- Since all of our samples are simulations, we can preliminarily use Backtracking to aid in selection criteria.

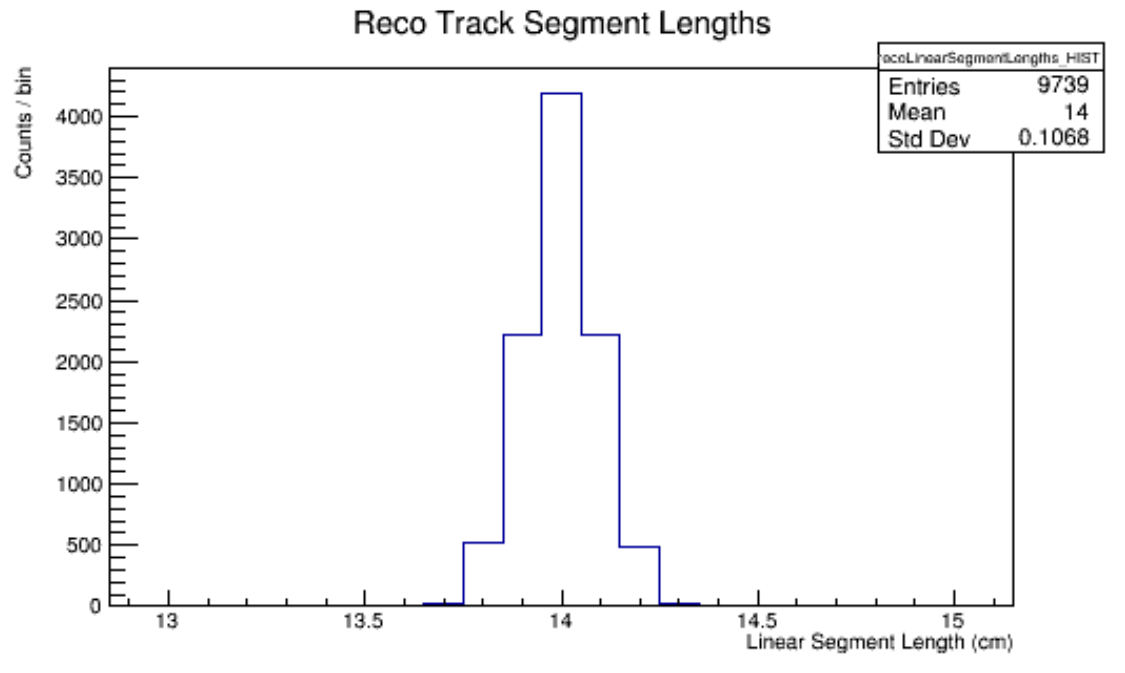
Forming the Track Segments

- The ‘track’ segments are formed by starting at the first trajectory point in the detector and adding the straight line distance between each point to form a segment that is as close to 14 cm as we can get.
- The position data can either be true position data or reconstructed position data.
  - We use true position data to validate the algorithm.
  - This presentation will focus on using reconstructed position data which is reconstructed by the pmtrack algorithm.
  - Recently switched to Pandora reconstruction (Work in progress – see later slide)
- A ‘track’ segment is a collection of trajectory points from the track. The start of one segment is the end of the previous segment to avoid gaps between segments.

Track Segment Lengths



Notice the wider distribution, std. dev. = 2 cm

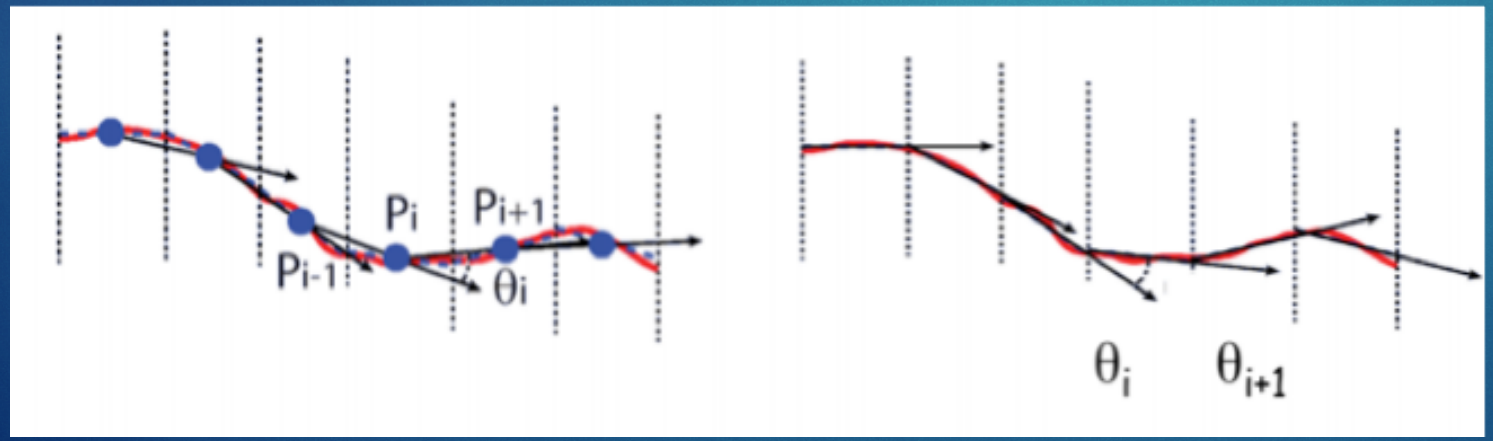


Notice the smaller distribution, std. dev. = 0.1 cm

- Length of track segments for true position data and reconstructed position data (pmtrack).
- Taken from a 0 – 1 GeV 1000 single muon sample. Starting position is from the beam line.

Getting Scattering Angles

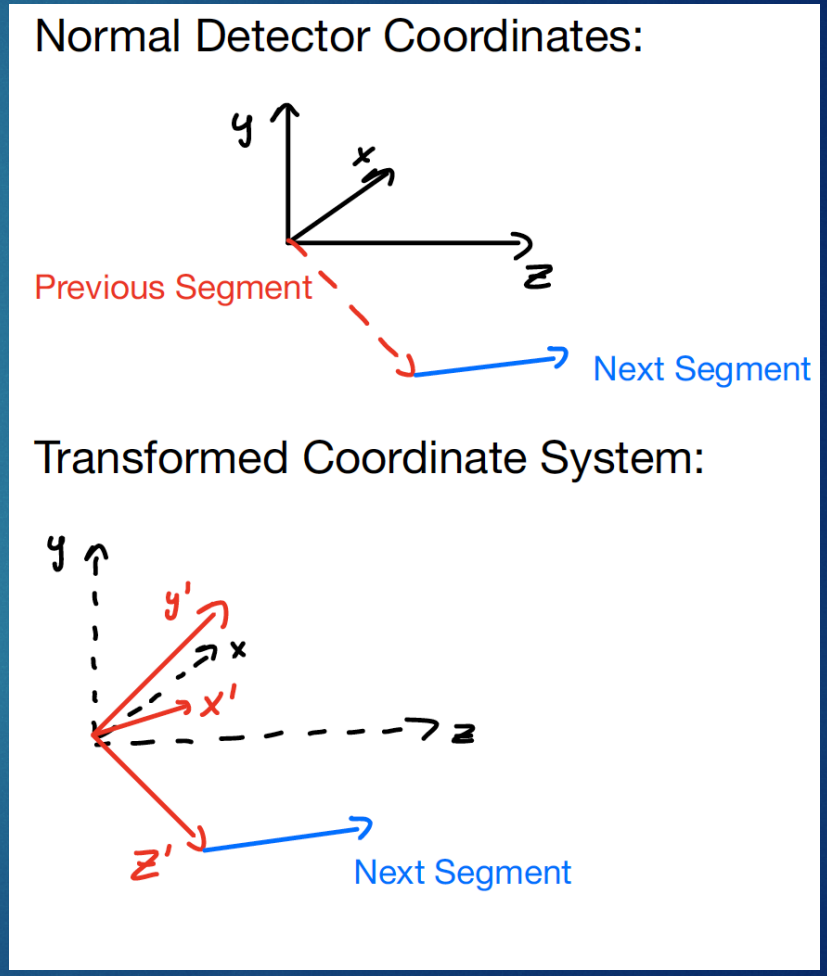
- There are two ways to get scattering angles:
  - Polygonal angles and Linear fit angles
  - Polygonal Angles: Angles between lines connecting the barycenters of track segments. (Used by ICARUS collaboration)
  - Linear fit angles: Angles between linear fits of each track segment. (Used by MicroBooNE and ICARUS)



- Left: Polygonal Angles, the black lines are the 'polygonal segments'
- Right: Linear Fit angles, black lines are the 'linear segments'

Getting Scattering Angles

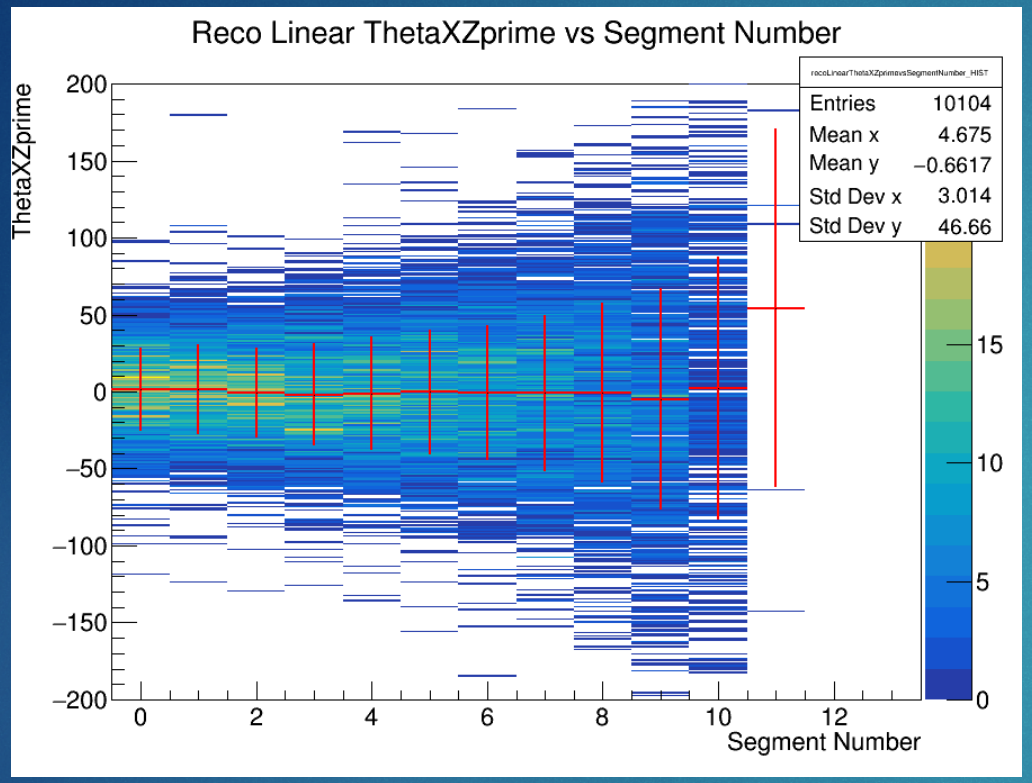
- First we get θ_{XZ} and θ_{YZ} which are defined by the previous segment in normal coordinates.
- Next, we transform our coordinate system to the primed coordinate system.
  - This is a rigid rotation to bring z -> z'
- We measure θ_{XZ}' and θ_{YZ}' of the next segment in this new coordinate system, which should follow a gaussian distribution centered at 0 with a standard deviation given by the Highland formula:
  - $$\sigma_0^{RMS} = \sqrt{\left(\frac{\kappa(p)}{p\beta c} z \sqrt{\frac{l}{x_0}}\left(1 + \varepsilon * \ln\left(\frac{l}{x_0}\right)\right)\right)^2 + (\sigma_0^{RES})^2}$$
  - At our current state, we're using $\sigma_0^{RES} = 0$

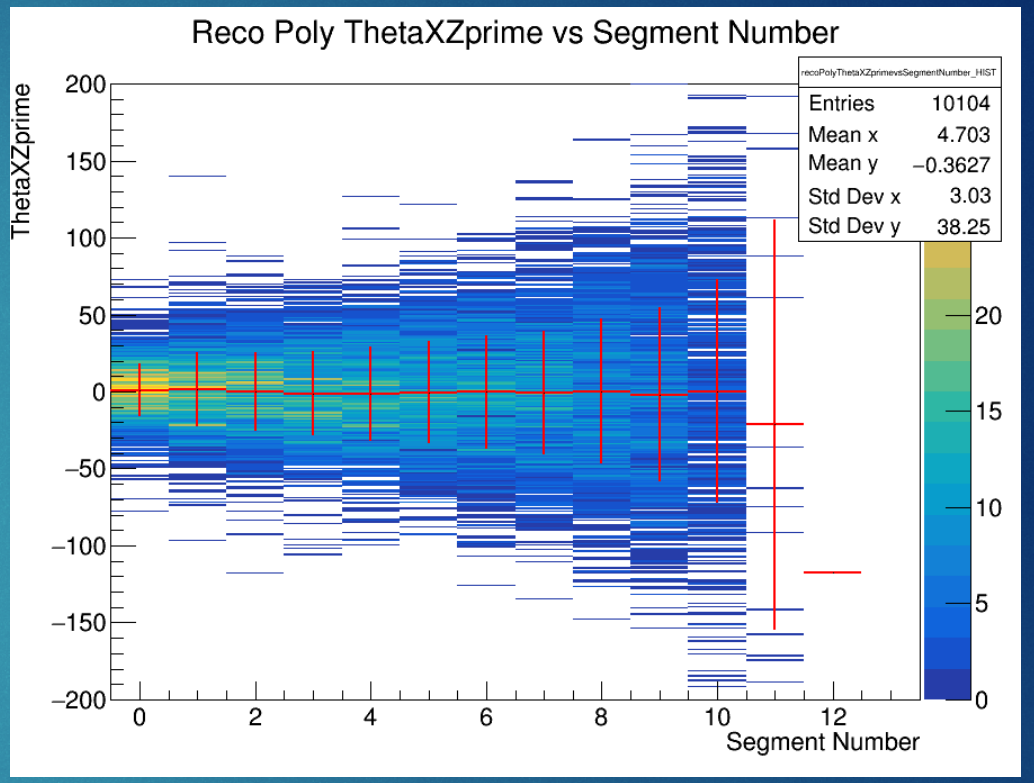


Notes on the first segment

- The first polygonal segment is the line connecting the first point in the detector to the first barycenter. (~7 cm in length)
- The first linear segment is the linear fit of the first track segment. (~14 cm in length)
- We use these to get our first θ_{XZ} and θ_{YZ} since we do not necessarily know the initial direction of the muons.
- This means our first θ_{XZ}' and θ_{YZ}' is for the second segment, so we actually estimate the muon momentum at the start of the second segment.
- After we estimate this momentum, we must account for the energy loss over the first segment to get our final momentum estimate.

Graphs of Scattering Angles



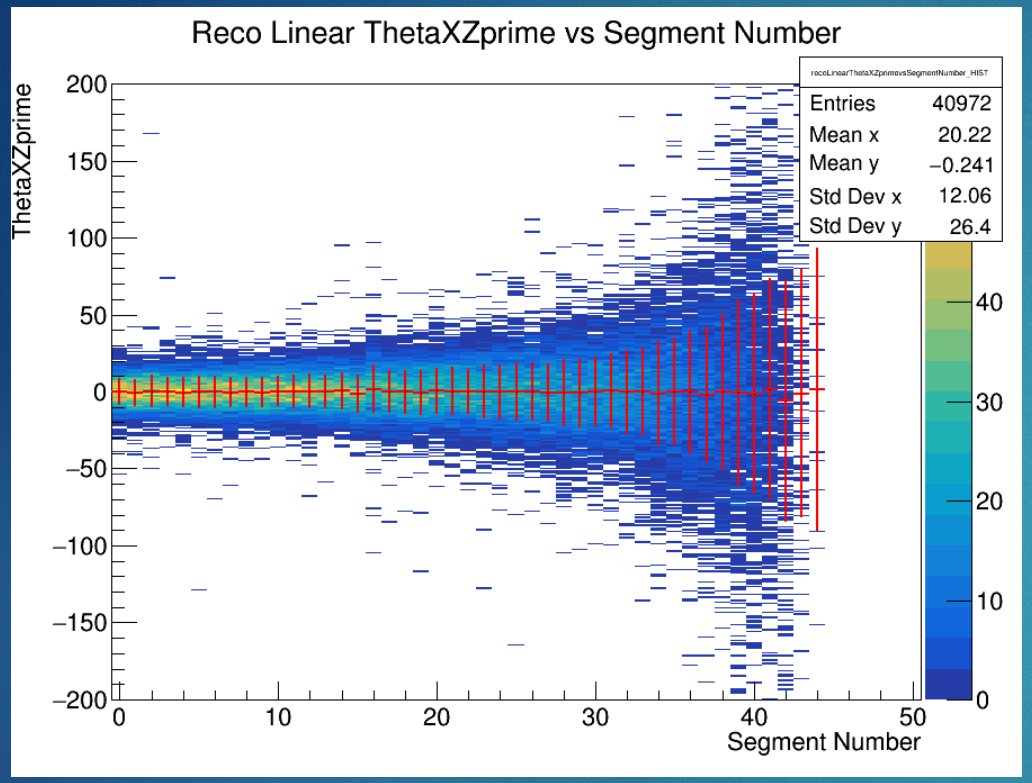


Reco Linear ThetaXZprime vs. Seg. #

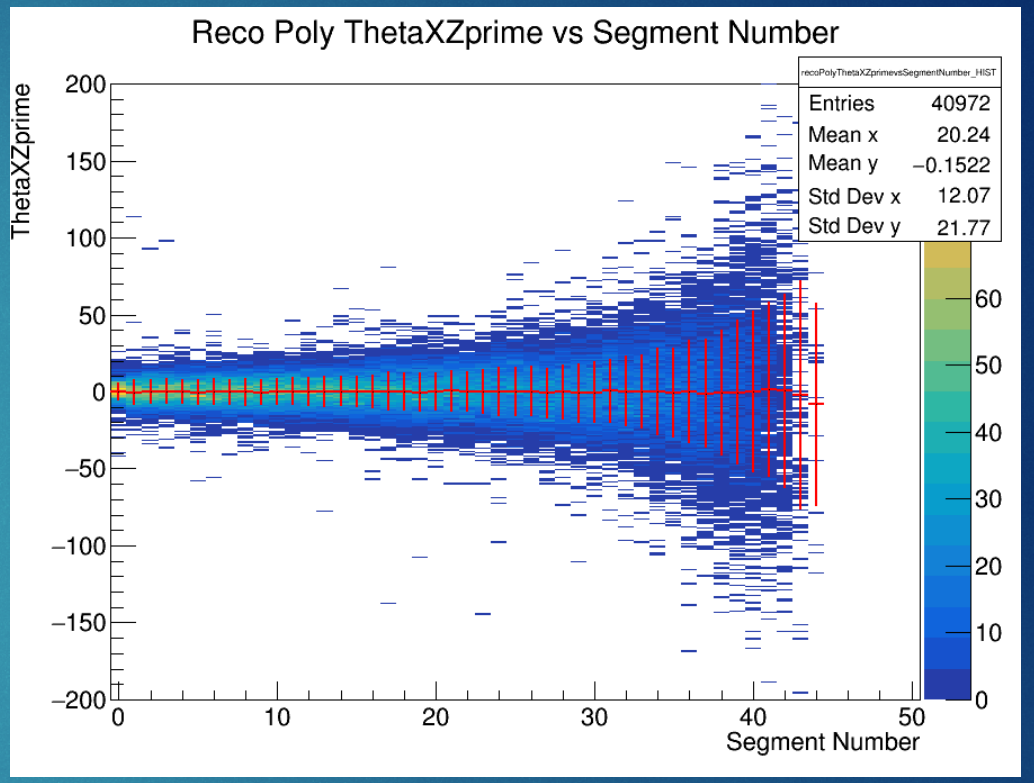
Reco Poly ThetaXZprime vs. Seg. #

- 0.5 GeV monoenergetic sample
- Comparison of reconstructed linear and polygonal angles
- See Extra Slides for ThetaYZprime vs Segment Number

Graphs of Scattering Angles



Reco Linear ThetaXZprime vs. Seg. #



Reco Poly ThetaXZprime vs. Seg. #

- 1.5 GeV monoenergetic sample
- Comparison of reconstructed linear and polygonal angles

The Maximum Likelihood Method

- The angle of scatter will follow a gaussian distribution of the form:

$$f(\theta) = \frac{1}{\sigma_0^{RMS}\sqrt{2\pi}} \exp[-\frac{1}{2}\left(\frac{\theta}{\sigma_0^{RMS}}\right)^2]$$

- The likelihood that the series of angles would occur:

$$L(\{\sigma_j\},\{\theta_j\}) = (2\pi)^{-\frac{n}{2}} \times \prod_{j=1}^{n} (\sigma_j)^{-1} \times exp\,[-\frac{1}{2}\sum_{j=1}^{n}(\frac{\theta_j}{\sigma_j})^2]$$

- It is computationally easier to compute:

$$-l(\{\sigma_j\},\{\theta_j\}) = -\ln(L) = \frac{n}{2}\ln(2\pi) + \sum_{j=1}^{n}\ln(\sigma_j) + \frac{1}{2}\sum_{j=1}^{n}(\frac{\theta_j}{\sigma_j})^2$$

- Rather than maximizing L we minimize -ln(L)

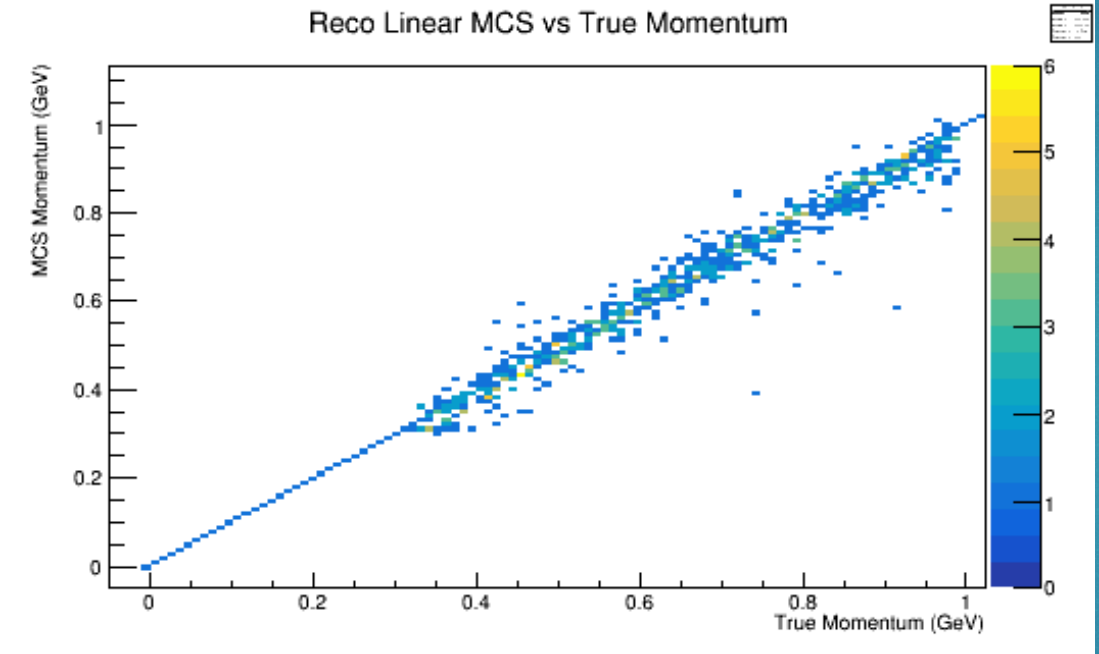
Accounting for Energy Loss

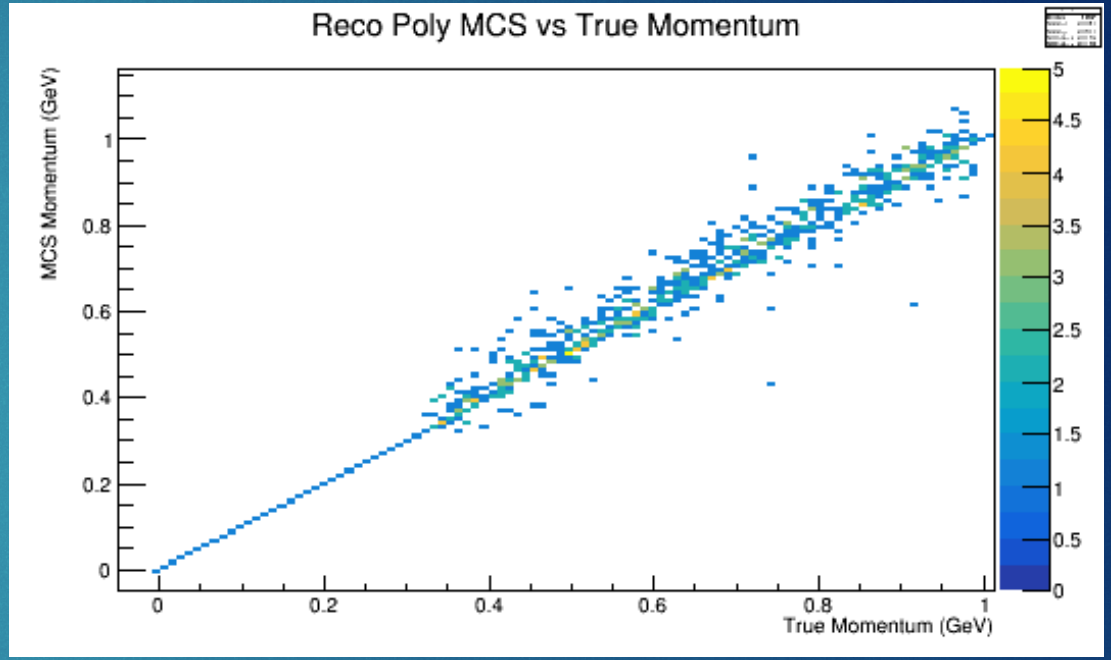
- While the muon travels through the detector it loses energy which must be accounted for in order to properly determine likelihood since σ_0^{RMS} is momentum dependent.
- We estimate energy loss by the Bethe-Bloch Equation:

$$-\left\langle \frac{dE}{dx} \right\rangle = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \left(\frac{2m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} \right) - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

- The main takeaway is that this formula is a function of p using certain parameters of muons and LAr

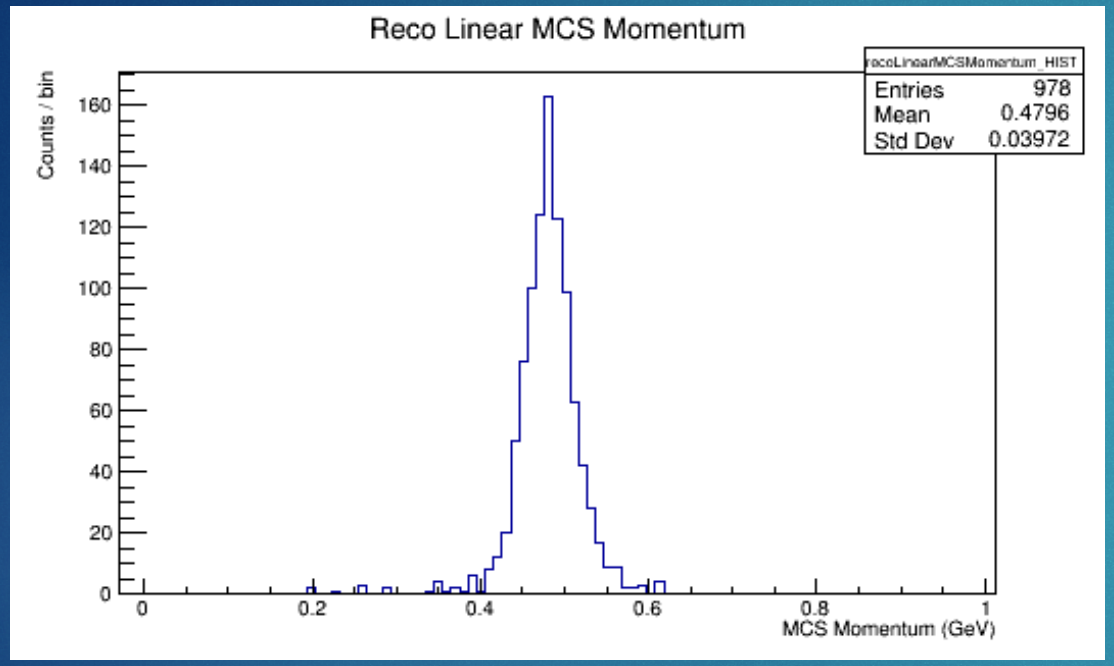
MCS Momentum vs True Momentum

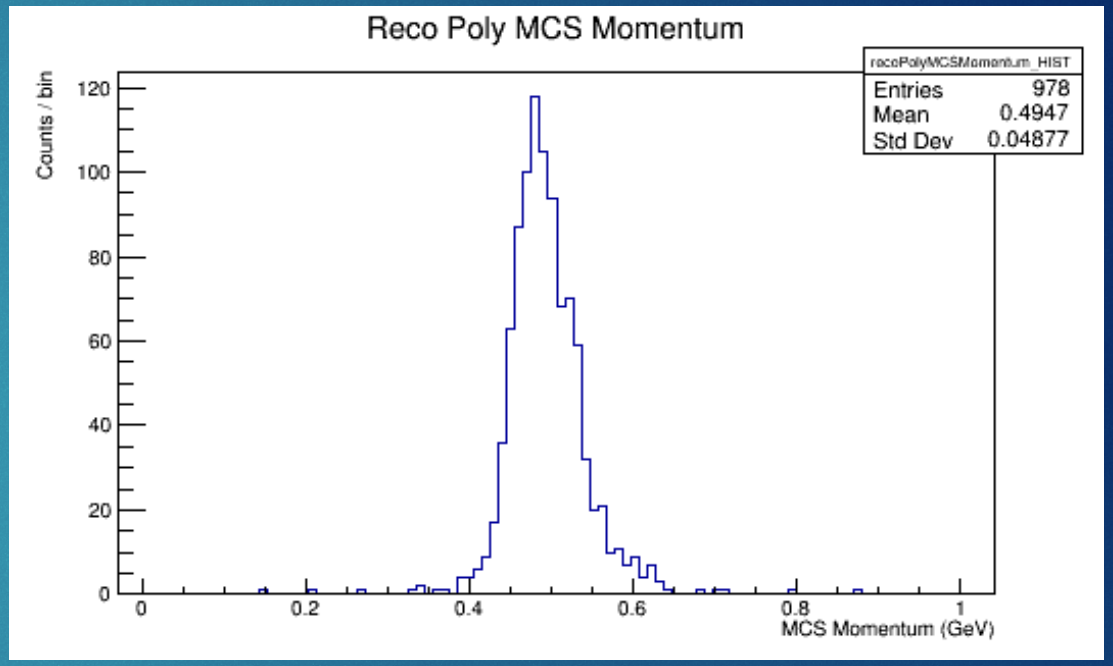




- 1000, 0 – 1 GeV single muon sample
- True Momentum is the true momentum at the first traj. point in the detector

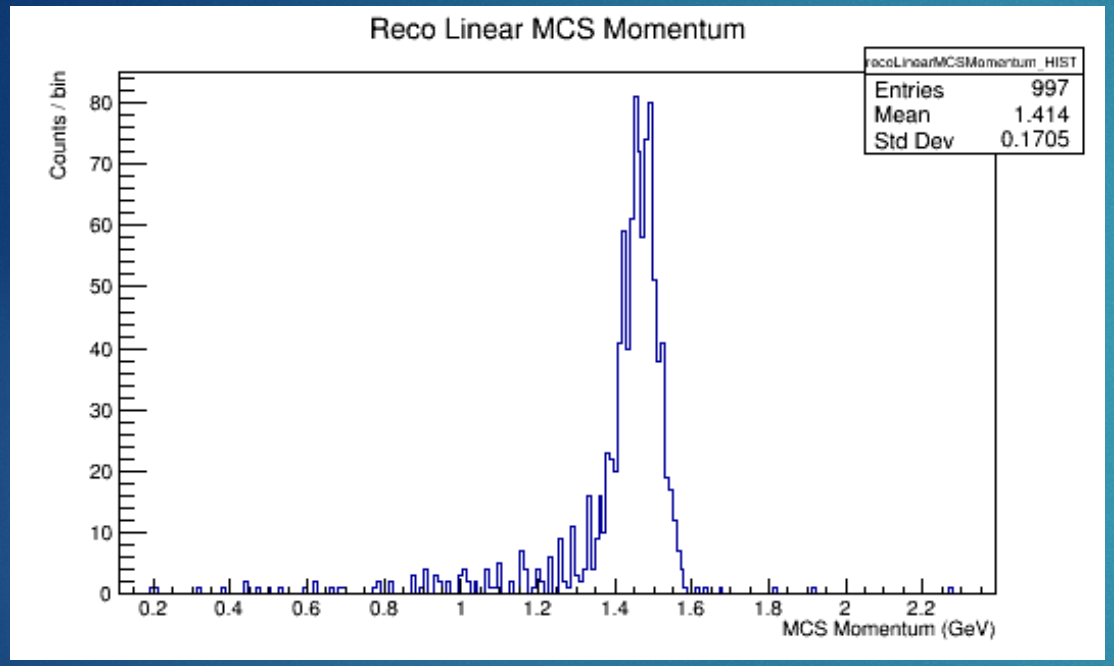
MCS Momentum Estimate

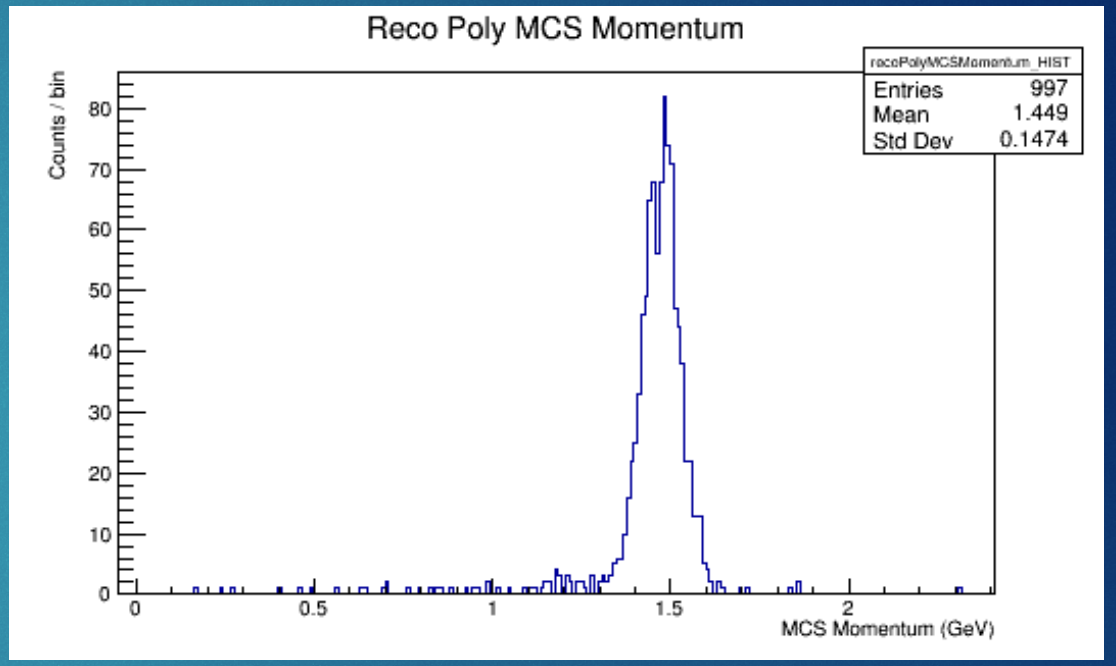




- 1000 0.5 GeV monoenergetic, single muon sample

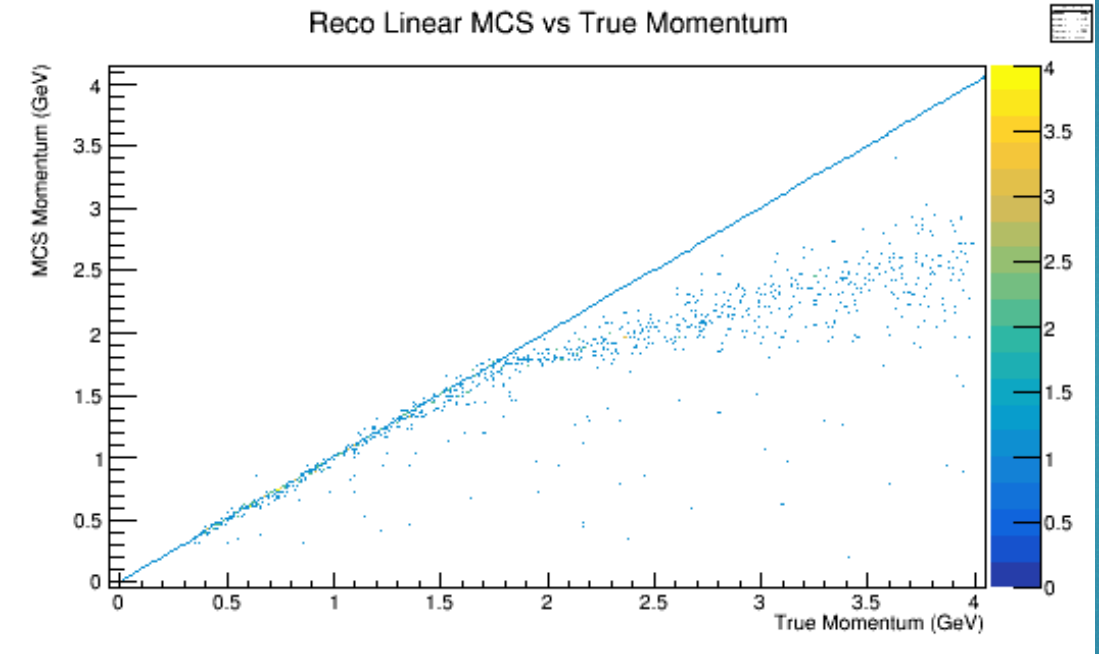
MCS Momentum Estimate

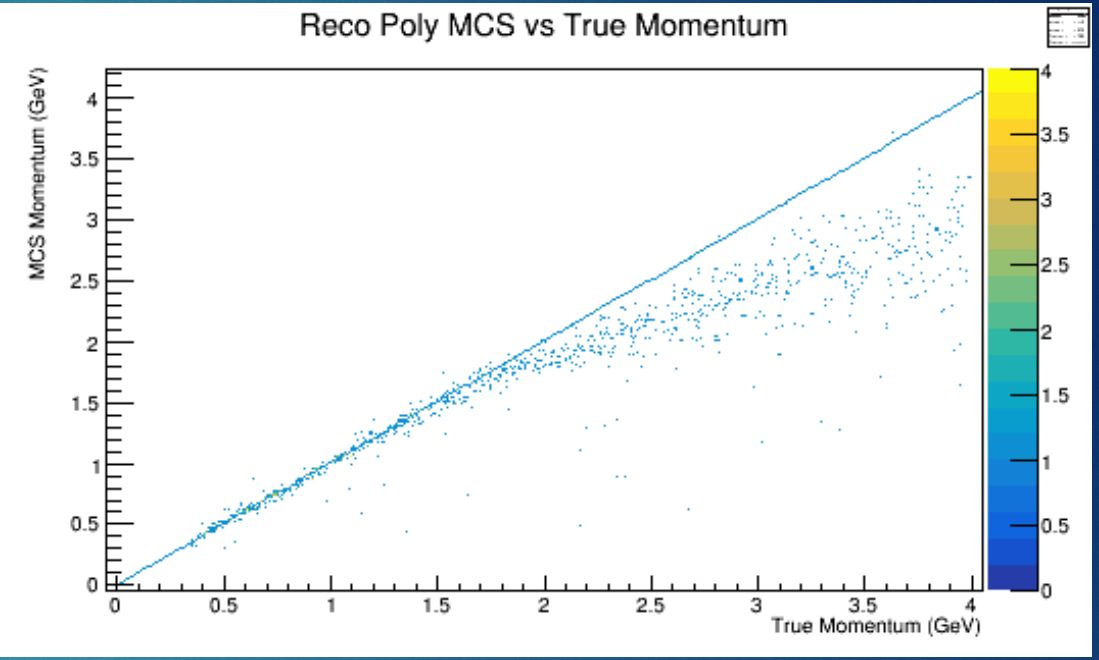




- 1000 1.5 GeV monoenergetic, single muon sample

MCS Momentum vs True Momentum

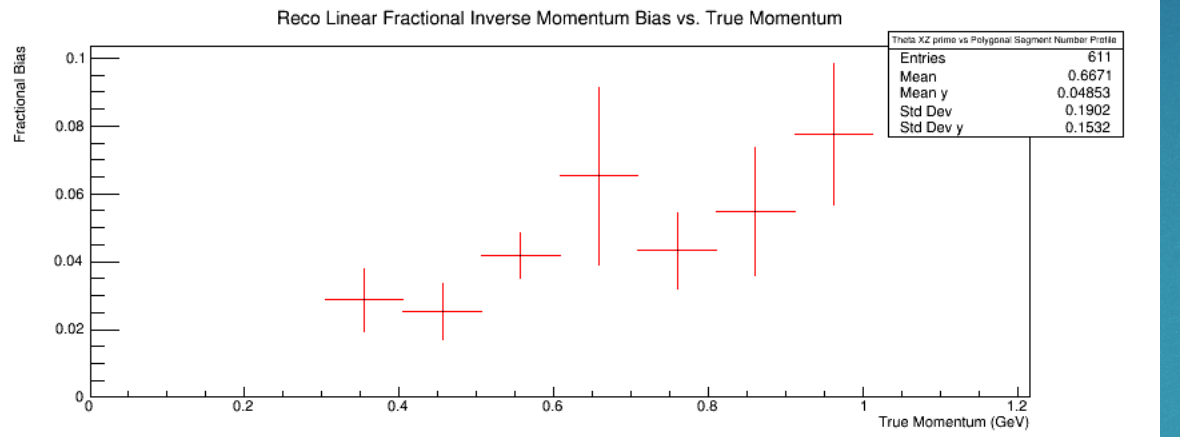


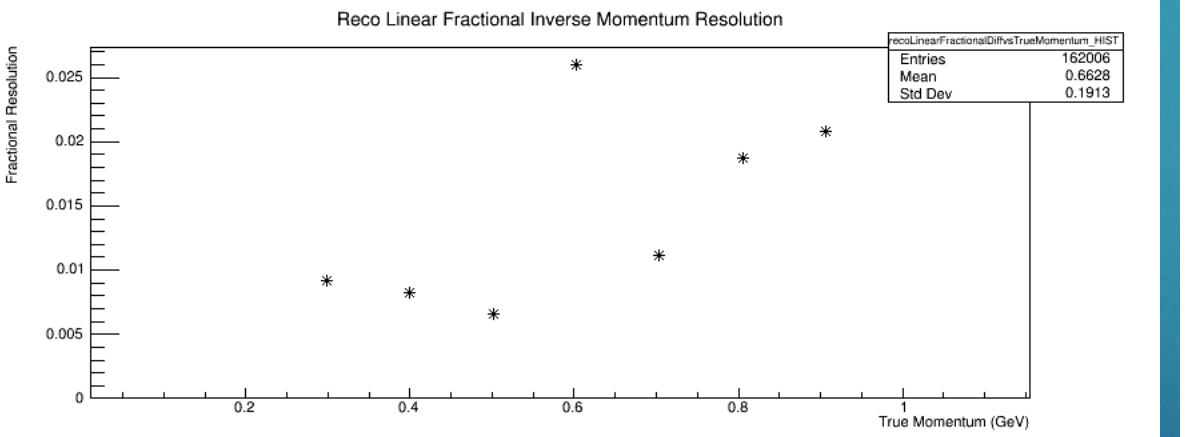


- 1000, 0 – 4 GeV single muon sample
- Detector size is artificially limited to 2.3 m in z-direction due to lack of track stitching in pmtrack reconstruction.
- True Momentum is the true momentum at the first traj. point in the detector

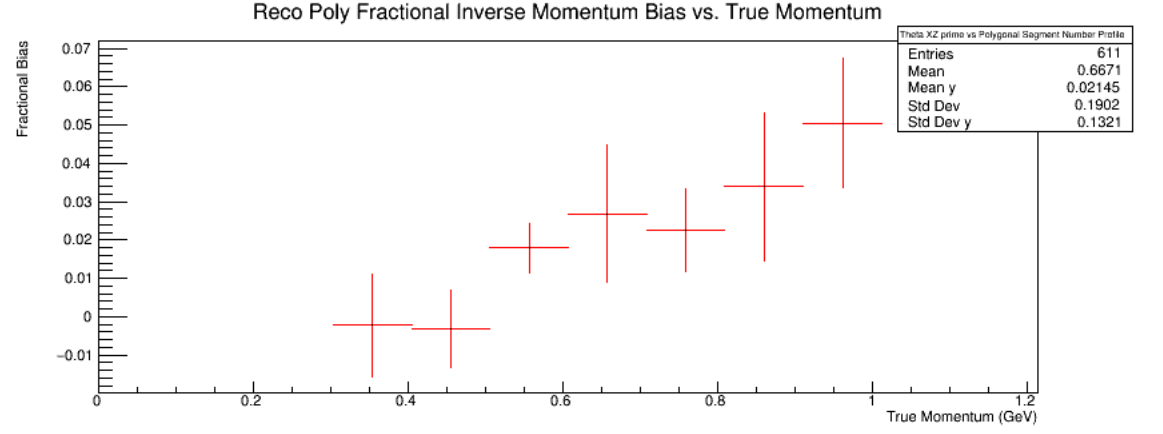
Fractional Bias and Resolution

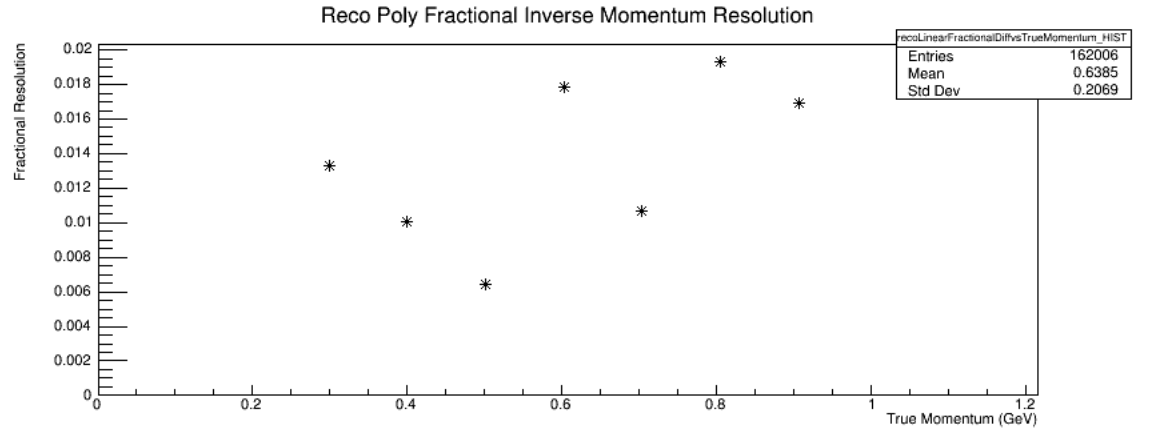
Linear Angle Method





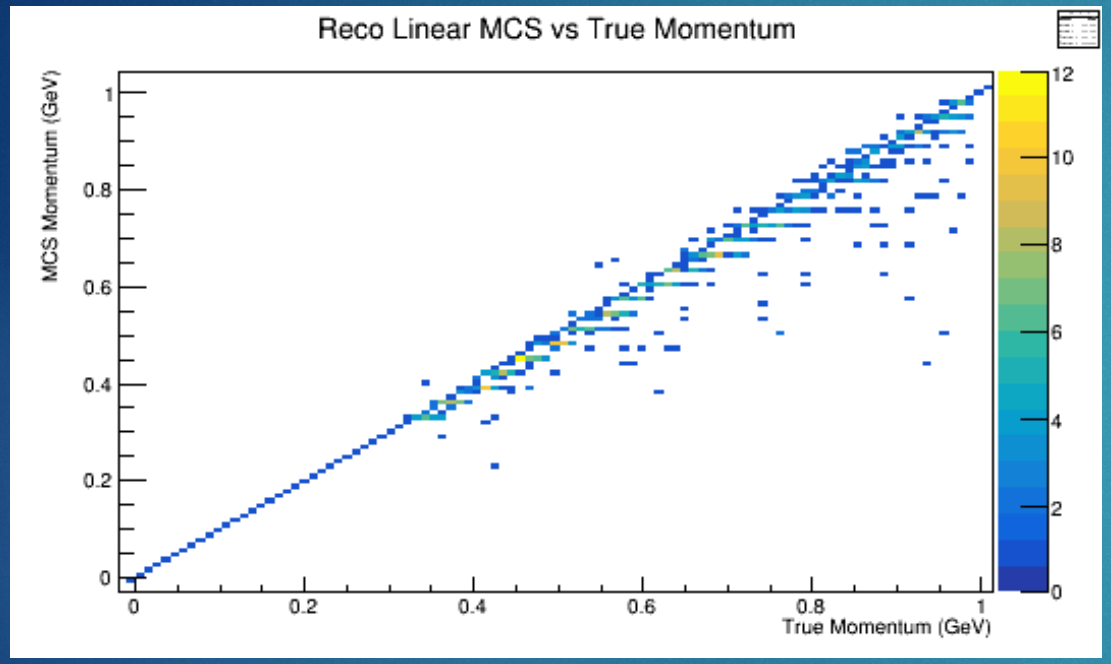
Polygonal Angle Method

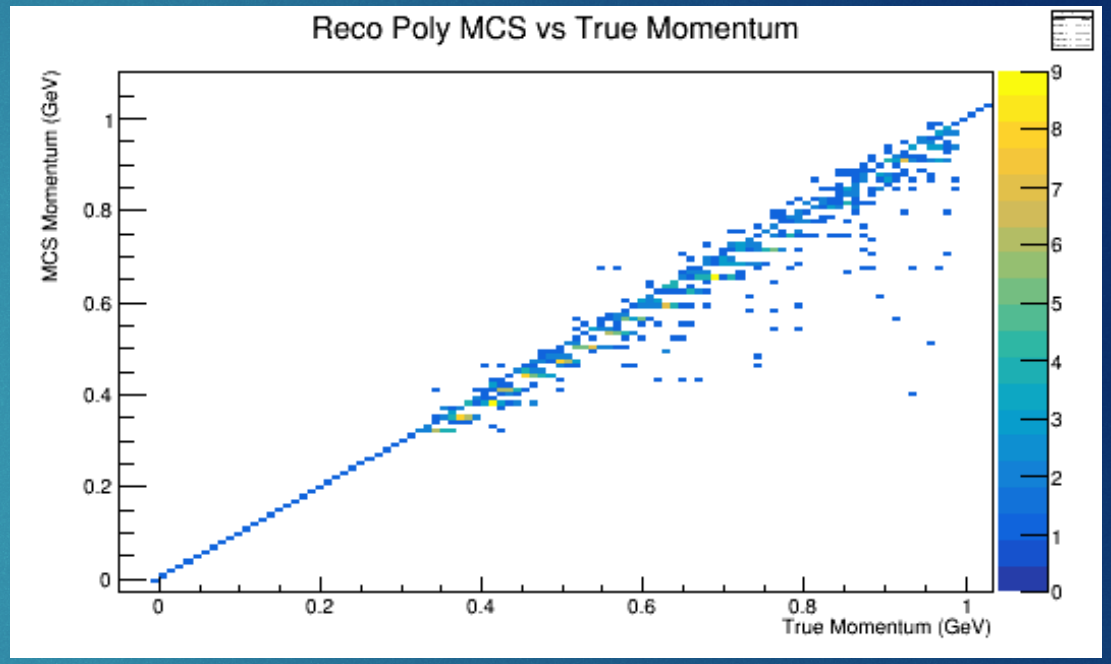




- Top Graphs: Fractional Bias, mean of inverse fractional differences
- Bottom Graphs: Fractional Resolution, std. dev. of inverse fractional differences

MCS Momentum Estimation Using Pandora Reconstruction





- Apparent binning in MCS Momentum
- Not yet understood, work in progress
- Currently using DUNE v07_13_00, simulations are also old, possible solution

To-do

- Determine a value for σ_0^{RES}
  - Compare measured σ_0^{RMS} to expected σ_0^{HL} of first segment for monoenergetic events that have precisely known starting direction
- Fix MCS Momentum estimation using pandora reconstruction
- Estimate Momentum Uncertainty
- Estimate momentum for every trajectory point
- Run on MCC11 samples
- Run on data
- Estimate pion momentum by MCS

References

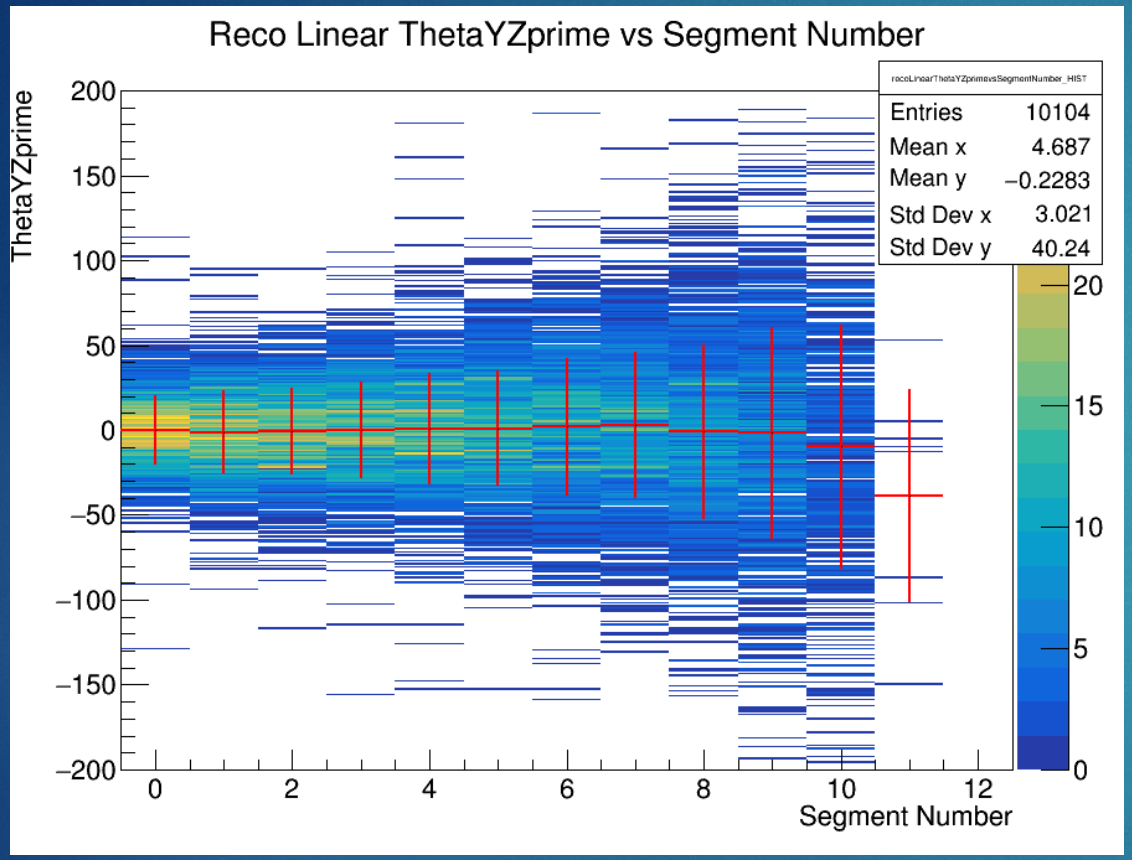
- MicroBooNE Collaboration, P. Abratenko *et al.*, "Determination of muon momentum in the MicroBooNE LArTPC using an improved model of multiple Coulomb scattering," *JINST* 12 no. 10, (2017) P10010, arXiv:1703.06187 [physics.ins-det].
- ICARUS Collaboration, M. Antonello *et al.*, "Muon momentum measurement in ICARUS-T600 LAr-TPC via multiple scattering in few-GeV range," *JINST* 12 no. 04, (2017) P04010, arXiv:1612.07715 [physics.ins-det].
- Groom, D.E. & Klein, S.R. Eur. Phys. J. C (2000) 15: 163. https://doi.org/10.1007/BF02683419
- Pawitan, Y. (2001). Elements of likelihood inference. In *In All Likelihood: Statistical Modelling and Inference Using Likelihood* (pp. 30-66). New York, NY: Oxford University Press.

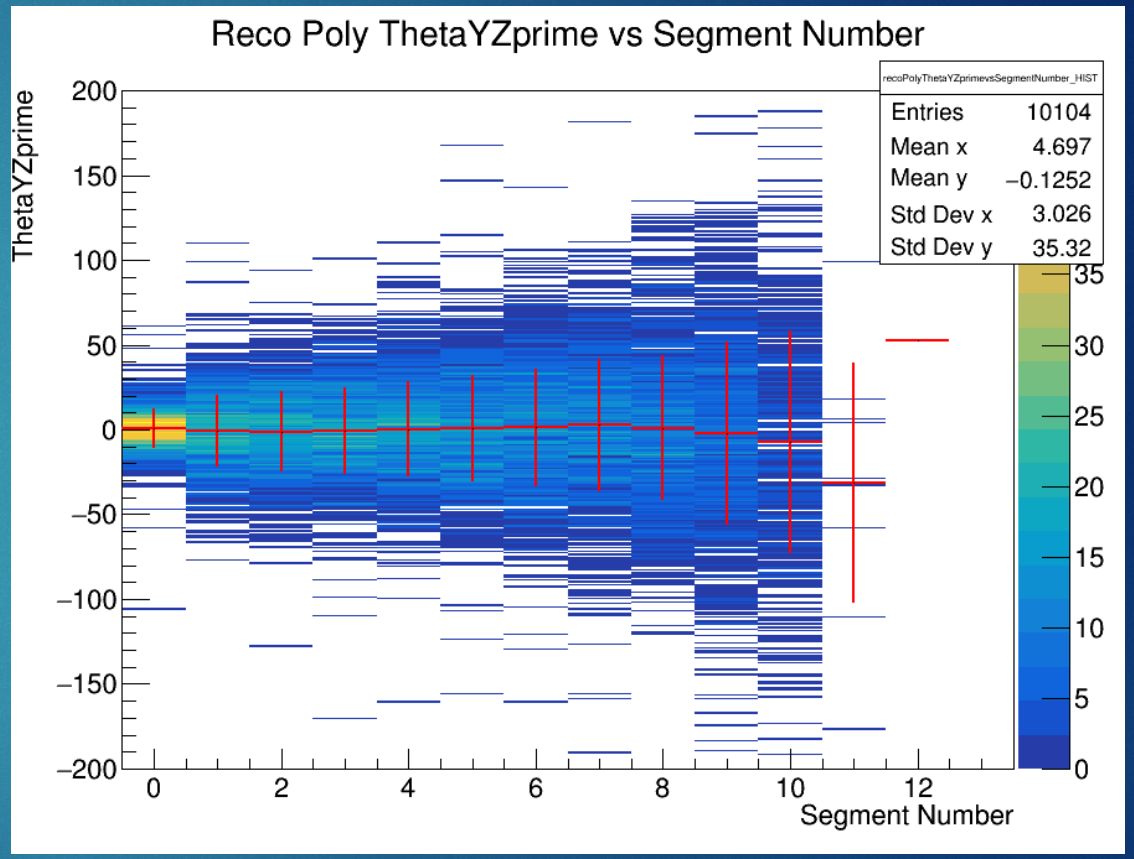
EXTRA SLIDES

Backtracking Reconstructed Tracks to MCParticles

- We only selected events that passed the following criteria:
  - If the length of the track is greater than 100 cm
  - If the reco track's backtracked true track id was not negative (meaning the hits came from an MCParticle as opposed to an EM shower)
    - This is not relevant in this study but we have included it anyways.
  - If the track was created by a muon
  - I keep count of the true track id's gotten from Backtracking each reconstructed track. If the current track's true track id was the track id from multiple reconstructed tracks, I disregard the event.
    - The reason for this is because one MCParticle can form multiple Track's while reconstructing using pmtrack. This is not an issue with pandora due to pandora's track stitching.

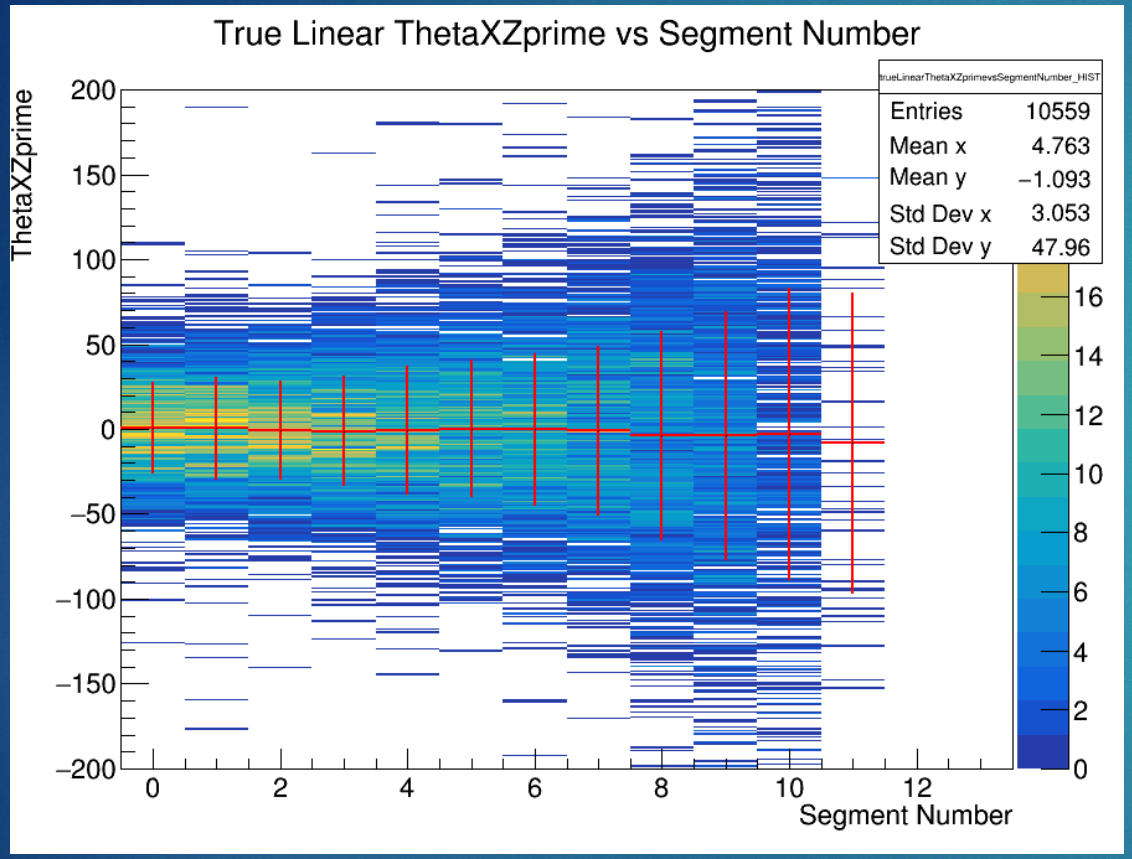
Graphs of Scattering Angles

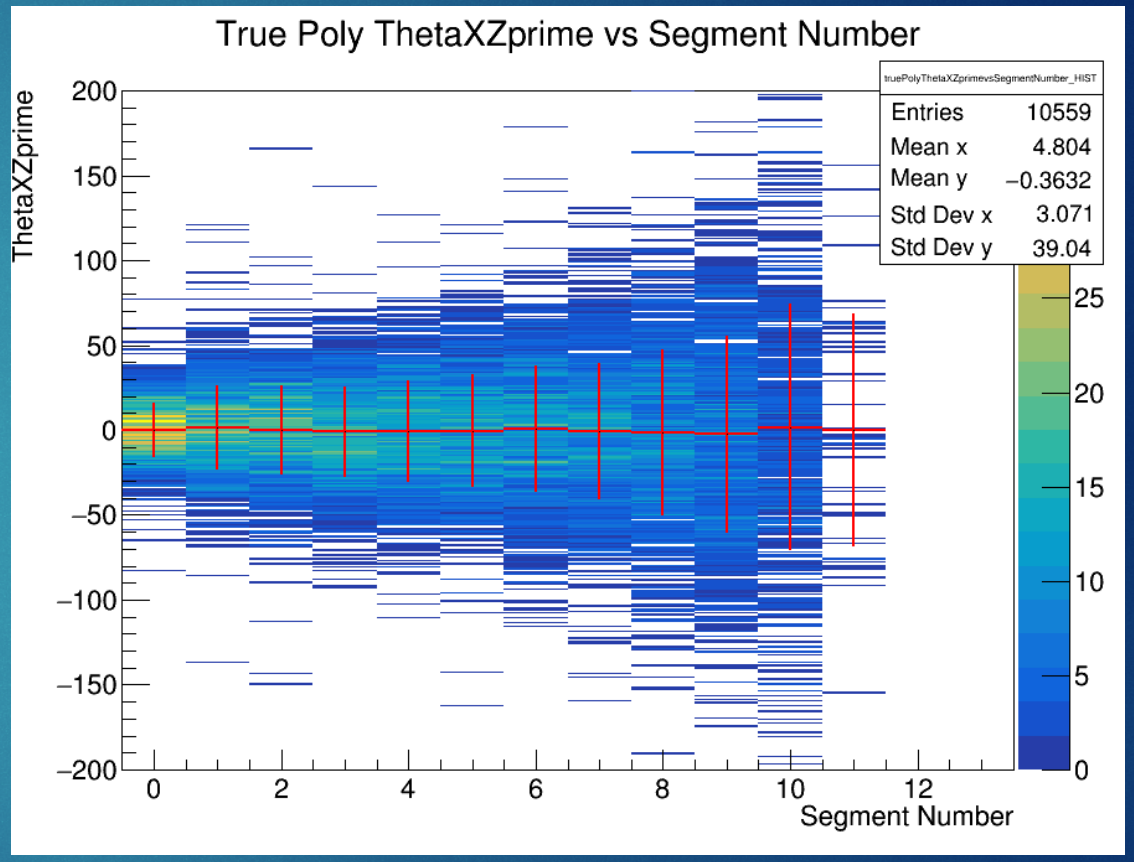




- 0.5 GeV monoenergetic sample
- Comparison of reconstructed linear and polygonal angles

Graphs of Scattering Angles





- 0.5 GeV monoenergetic sample
- Comparison of true linear and polygonal angles

Momentum Uncertainty

- Once we have our maximum likelihood estimate of p, p_{MCS}, we can approximate our uncertainty in our estimate by looking at our likelihood function's second derivative:

$$\sigma_{p_{MCS}} = \frac{1}{\sqrt{-l''(p_{MCS})}}$$

- We have yet to implement this into our analysis