Operational Islands and Black Hole Dissipation in JT Gravity

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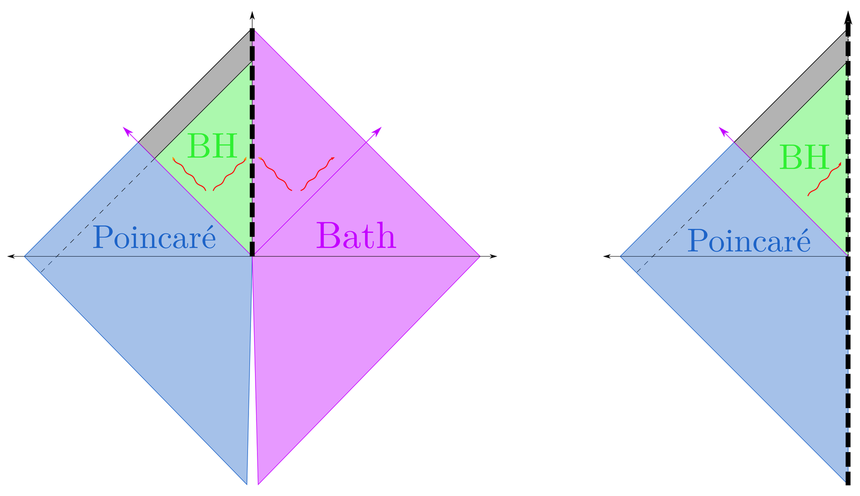
Introduction

The black hole information paradox remains one of the key problems in quantum gravity. One of the proxies for solving this problem is showing that unitarity is preserved through the Page curve. In recent years, significant attention has been given to the island conjecture which also solves the AMPS paradox. In this paper we

- Retake the calculation in JT gravity with two adjustments: a renormalised entropy and no heat bath. Absorption is into a fiducial detector.
- We also solve the semiclassical analysis for JT gravity coupled to charged matter, motivated by the supersymmetric model.



Operational islands

- The boundary observer shoots in light rays $u = t + z,\ v = t - z$: sends at $v = t_1$ w.r.t. their clock and collects back at $u = t_2$.
- From Poincaré patch: sends at $F_1 = F(t_1)$ and receives at $F_2 = F(t_2)$, to every bulk point they associate $X^- = F_1,\ X^+ = F_2$ [2]. Construct unique bulk frame giving an operational definition for islands

$$X^+(u) = F(u), \quad X^-(v) = F(v), \quad ds^2(F) = \frac{F'(u)F'(v)}{[F(u) - F(v)]^2}(dz^2 - dt^2).$$

Modelling evaporation

Send in classical pulse with energy E_0 along null line $v = 0$ to form a black hole

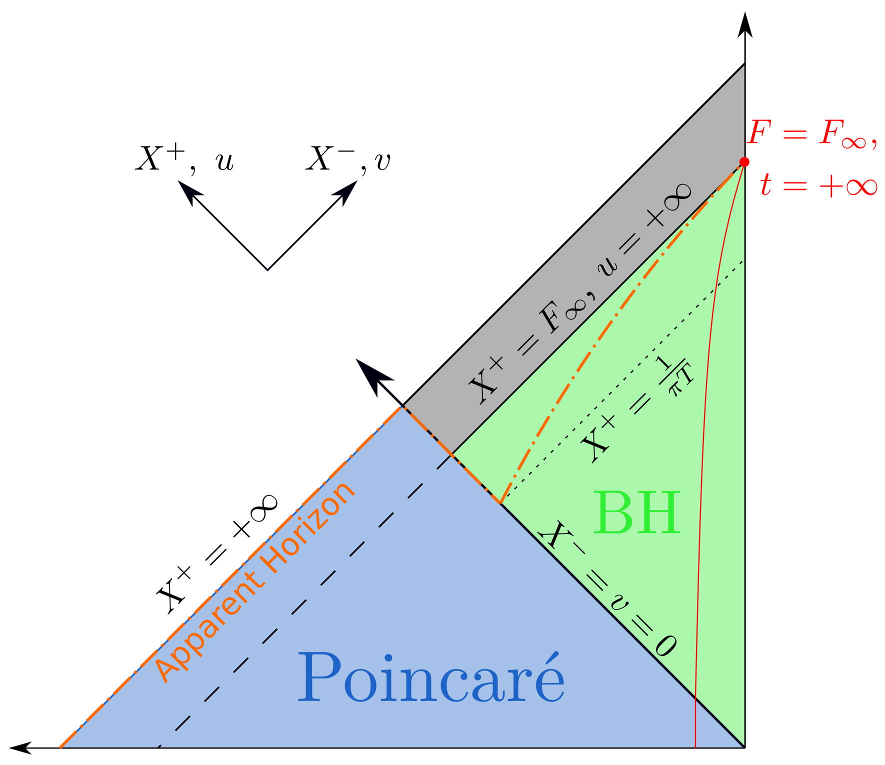
$$\frac{\mathrm{d}E}{\mathrm{d}t} = E_0\delta(t) + \langle :T_{vv}(t): \rangle - \langle :T_{uu}(t): \rangle\,.$$

Let boundary observer perfectly absorb all Hawking radiation

$$\langle :T_{vv}: \rangle = 0, \quad \langle :T_{uu}: \rangle = -\frac{c}{24\pi}\{F(t), t\}.$$

modelled by coupling to 2d matter CFT with central charge c.
Find time reparametrisation $F(t)$ from

$$E(t) = E_0 \exp(-kt) = -C\{F(t), t\}, \quad C = \frac{\phi_r}{8\pi G_{\mathrm{N}}}, \quad k = \frac{cG_{\mathrm{N}}}{3\phi_r}.$$



Renormalised islands

Use quantum extremal surface/island $\mathcal{X}$ formula [1]

$$S(\mathcal{X}) = \underset{\mathcal{X}}{\text{Min}}\ \underset{\mathcal{X}}{\text{Ext}}\left[\frac{A(\mathcal{X})}{4G_{\mathrm{N}}} + S_{\text{ren}}(\mathcal{X})\right].$$

- Dilaton gives area contribution

$$\phi = 2\phi_r\left[\frac{1}{2}\frac{F''(v)}{F'(v)} + \frac{F'(v)}{F(u) - F(v)}\right].$$

- Finite, renormalised entropy (Poincaré - boundary frame) [3]

$$S_{\text{ren}} \equiv S_{\text{bare}} - S_{\text{ref}}$$
$$= \frac{c}{12}\ln\frac{(F(u_1) - F(u_2))^2}{F'(u_1)F'(u_2)(u_1 - u_2)^2} + (u_i \to v_i).$$

Differs from renormalisation $1/G_{\text{N,ren}} \equiv 1/G_{\text{N,bare}} + \#/\epsilon^2$ due to 1-loop matter contribution.

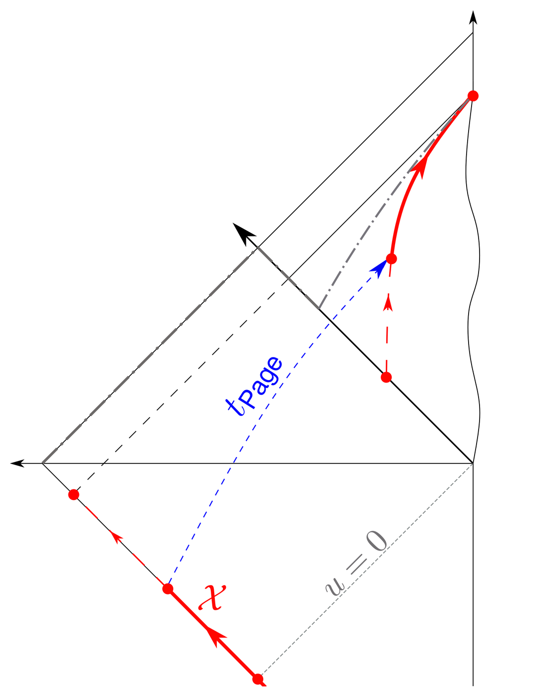
Page curve from operational islands

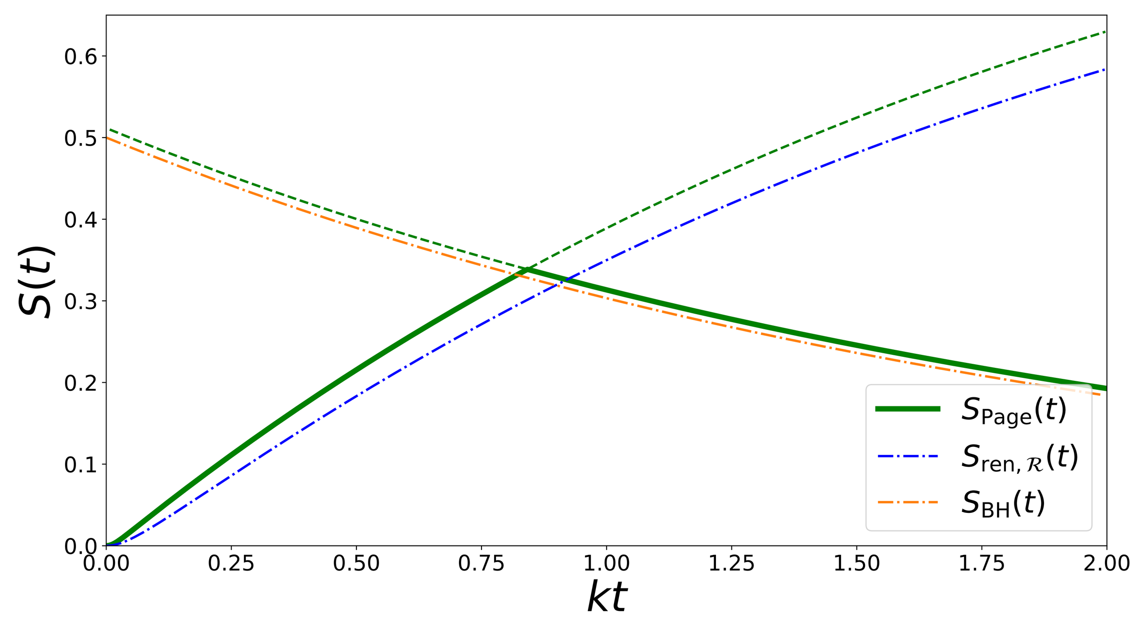
- $t \leq t_{\text{Page}}$: trivial island on past horizon without area contribution, purely matter and matches Hawking result in macroscopic limit (large black hole $E_0/k \gg 1$)

$$S_{\text{pre}}(t) = -\frac{c}{12}\ln F'(t) \approx S_{\text{ren},\mathcal{R}}(t) = \frac{c}{12}\ln\frac{F(t)^2}{F'(t)t^2} = 4\pi\sqrt{2CE_0}\left[1 - \exp\left(-\frac{k}{2}t\right)\right].$$

- $t \geq t_{\text{Page}}$: nontrivial island in front of horizon, almost no matter contribution, and matches Bekenstein–Hawking result in macroscopic limit

$$S_{\text{post}}(t) \approx S_{\text{BH}}(t) = 2\pi\sqrt{2CE_0}\exp\left(-\frac{k}{2}t\right).$$





Charged dissipation & superradiance

Introducing charge leads to the Schwarzian model coupled to a U(1) BF theory

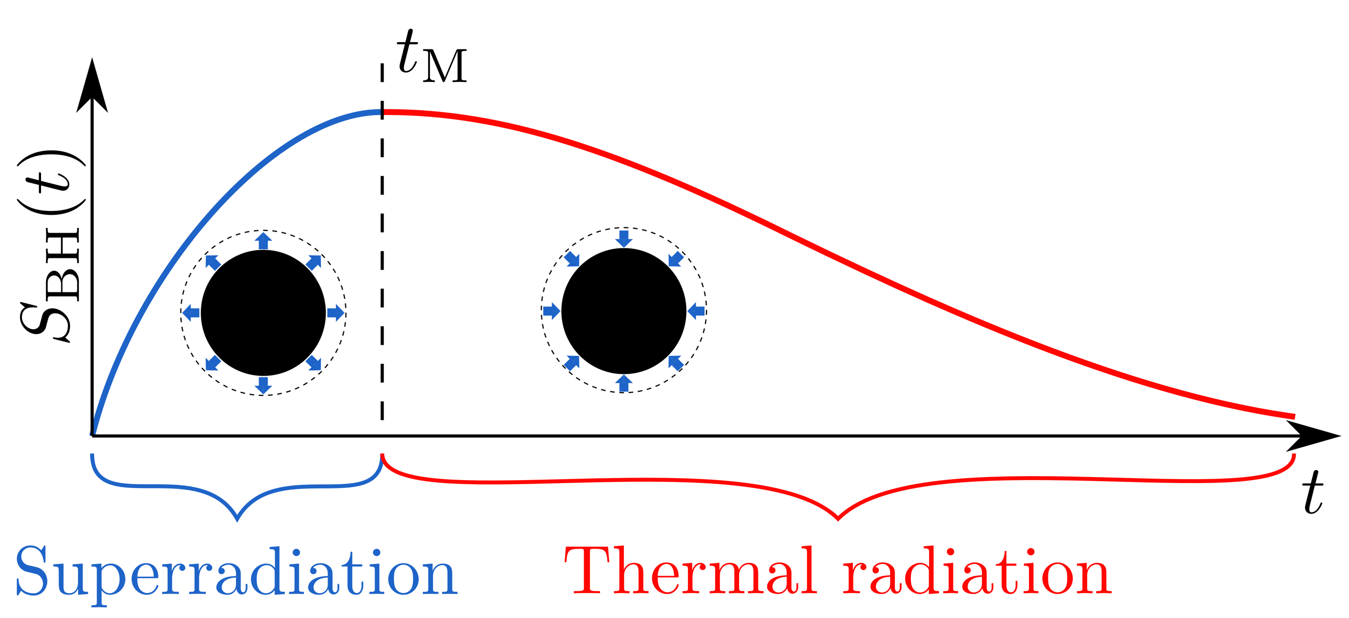
$$S[F, \sigma] = -C\int d\tau\,\{F(\tau), \tau\} + \frac{K}{2}\int d\tau\,\sigma'(\tau)^2.$$

Inject a pulse (E_0, Q_0) and absorb emitted radiation modelled by massless complex scalar with charge q

$$E_{\text{BH}}(t) = E_0\exp(-kt) - \frac{Q_0^2}{4K}\exp\left(-\frac{q^2}{\pi K}t\right), \quad Q(t) = Q_0\exp\left(-\frac{q^2}{2\pi K}t\right),$$

Solvable to $F(t)$ for $6q^2C = K$ motivated by the $\mathcal{N} = 2$ supersymmetric Schwarzian system.
Eventually, we get an entropy going as

$$S_{\text{BH}}(t) = 2\pi\sqrt{2C}\sqrt{E(t) - \frac{Q(t)^2}{2K}} = 2\pi\sqrt{2C}\sqrt{E_{\text{BH}}(t)}.$$



- $Q_0^2 \leq E_0K$: the black hole emits superradiant modes first and starts to grow and heat up until thermal modes become more densely populated at t_M. Afterwards, the black hole starts to evaporate, cool down, and shrink. A monotonically decaying charge prevents superradiant modes from dominating again.
- $Q_0^2 \geq E_0K$: superradiant modes are suppressed, the black hole immediately emits thermal radiation and evaporates.

Conclusion

- Operational islands with a renormalised entropy result in a Page curve.
- Generalisation to charged dissipation is possible.

More info

J. De Vuyst and T. G. Mertens, arXiv:2207.03351 [hep-th]

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References

[1] Almheiri, A. et al. 2019, JHEP, 12, 063
[2] Blommaert, A. et al. 2019, JHEP, 09, 060
[3] Holzhey, C. et al. 1994, Nucl. Phys. B, 424, 443