**Motivations**

- Core-collapse supernovae of massive stars $M \geq 8M_\odot$ emit a huge number of neutrinos ($\sim 10^{56}$).
- The physics of matter under extreme conditions is strongly flavor-dependent (nucleosynthesis, neutron-proton ratio, spectrum splits...).
- Interesting quantum many-body problem governed by the weak interaction.
- Describing the full dynamics is very complicated due to the collective neutrino oscillations that make the equation non linear.

**Physical description**

- Two-flavors approximation (SU(2) model) to encode the flavor state in a qubit state: $|\nu_0\rangle \rightarrow |0\rangle$ , $|\nu_3\rangle \rightarrow |1\rangle$
- $N$ neutrinos encoded into $N$ qubits.
- The flavor Hamiltonian of $N$ neutrinos is:
\[
H = \sum_{i=1}^{N} b_i \cdot \sigma_i + \sum_{i<j} J_{ij} \sigma_i \cdot \sigma_j
\]

1-body term: vacuum mixing
\[
b_i = \frac{\delta m^2}{4E} (\sin(2\theta_{13}) \cdot 0 - \cos(2\theta_{13})).
\]

2-body term: $\nu\nu$-interaction
\[
J_{ij} = \frac{\rho}{N}(1 - \cos(2\theta_{12})), \quad \rho = \sqrt{\delta m^2}
\]

- Initial state for $N = 4$: $|\Psi_0\rangle = |0001\rangle$.
- We can look at the inversion probability:

**Unitary implementation**

- To perform the quantum simulation we need a quantum gate decomposition of the $U(t)$ operator ($2^N \times 2^N$ unitary matrix on the flavor basis):
- Divide 1-body and 2-body parts that commute:
\[
U(t) = U_{1body}(t) U_{2body}(t)
\]

- Approximate the 2-body part as a product of pair interactions.

**Multiple steps evolution**

- Evolve the system until $T$ applying $k = T/\Delta t$ Trotter steps:
\[
|\Psi(T)\rangle = U_{Trotter,k} |\Psi_0\rangle
\]

**Complexity algorithm scaling**

- The number of 2-qubit gates needed to evolve up to $T$ with an error $< \epsilon$ scales polynomially with $N$:
\[
\text{Decomposition type} \quad \text{Circuit complexity} \quad O\left(\frac{N^2}{\epsilon^2}\right)
\]

**Conclusion**

- Fully connected gates allow for freedom in gate composition.
- Quantum circuit optimization is a crucial step in order to perform simulations on a near-term quantum device.
- The complexity of the implementation of time evolution scales polynomially with the number of neutrinos.

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**References**