QUANTUM COMPUTING SIMULATION FOR COLLECTIVE NEUTRINO OSCILLATIONS [1]

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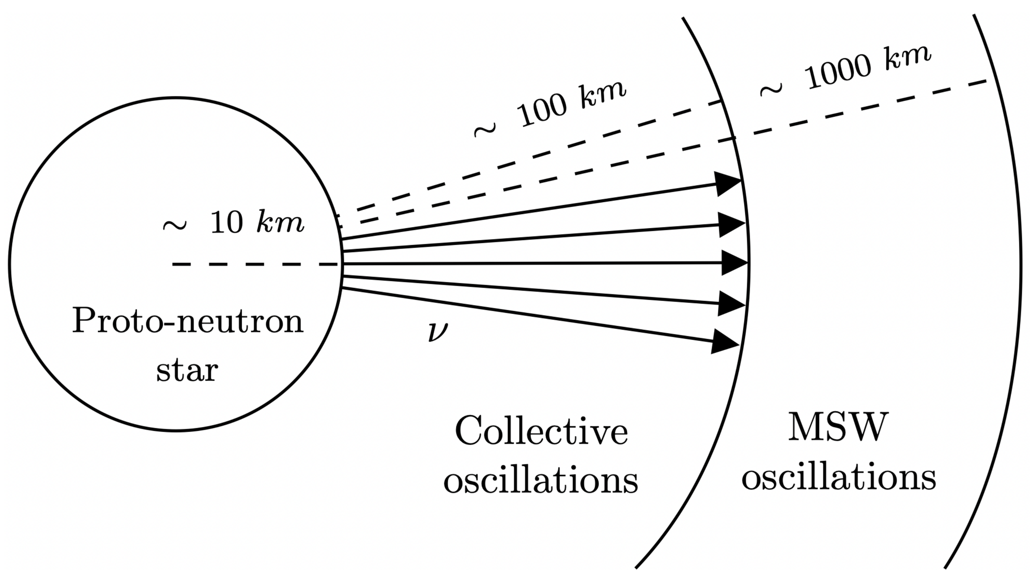
MOTIVATIONS

- **Core-collapse supernovae** of massive stars $M \gtrsim 8M_\odot$ emit a huge number of neutrinos ($\sim 10^{58}$).
- The physics of matter under extreme conditions is strongly **flavor-dependent** (nucleosynthesis, neutron-proton ratio, spectrum splits...).
- Interesting quantum many-body problem governed by the weak interaction.
- Describing the full dynamic is very complicated due to the **collective neutrino oscillations** that make the equation non linear.



- We want to simulate the real time evolution:

$$|\Psi(t)\rangle = U(t)\,|\Psi_0\rangle\ , \quad U(t) = e^{-iHt}\,. \tag{1}$$

PHYSICAL DESCRIPTION

- Two-flavors approximation ($SU(2)$ model) to encode the flavor state in a qubit state:

$$|\nu_e\rangle \longmapsto |0\rangle\ , \quad |\nu_x\rangle \longmapsto |1\rangle \tag{2}$$

- N neutrinos encoded into N qubits.
- The flavor Hamiltonian of N neutrinos is:

$$H = \sum_{i=1}^{N} \boldsymbol{b}\cdot\boldsymbol{\sigma}_i + \sum_{i<j}^{N} J_{ij}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j \tag{3}$$

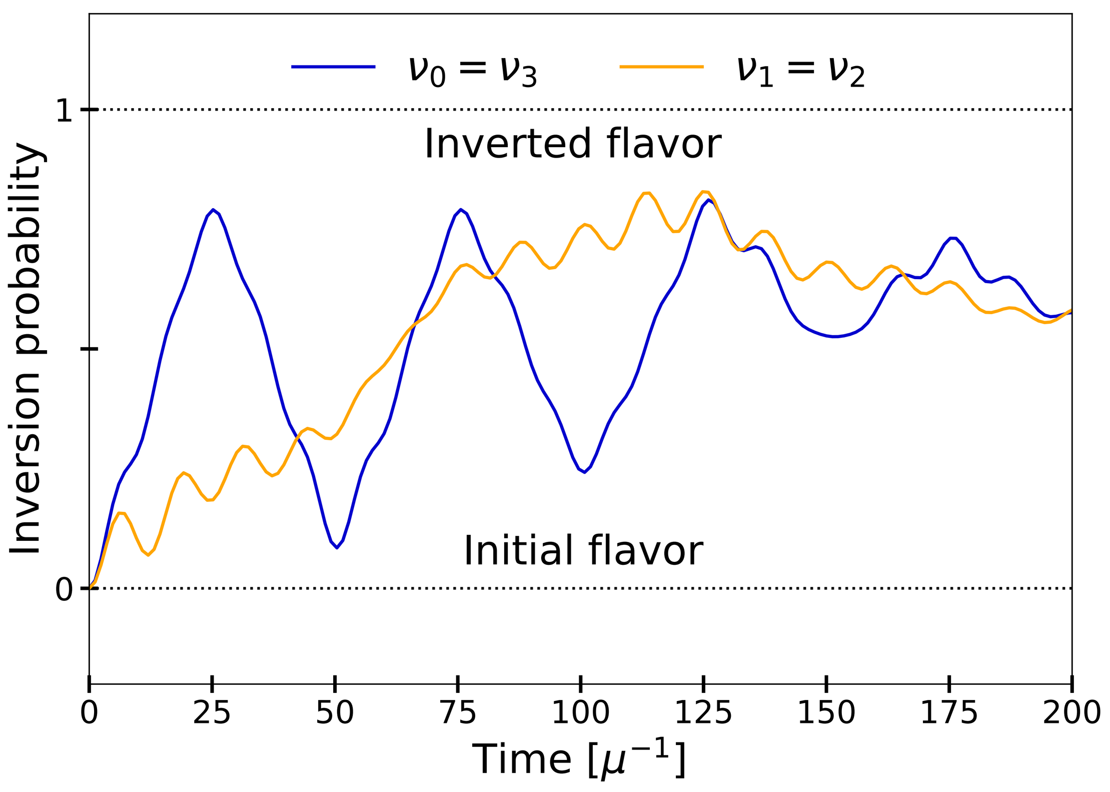
- **1-body term**: vacuum mixing

$$\boldsymbol{b} = \frac{\delta m^2}{4E_\nu}(\sin(2\theta_\nu), 0, -\cos(2\theta_\nu))\,.$$

- **2-body term**: $\nu\nu$-interaction

$$J_{ij} := \frac{\mu}{N}(1-\cos(\theta_{ij}))\ , \quad \mu = \sqrt{2}G_F\rho_\nu\,.$$

- Initial state for $N = 4$: $|\Psi_0\rangle = |0011\rangle$.
- We can look at the inversion probability:



- Symmetry under particle exchange:

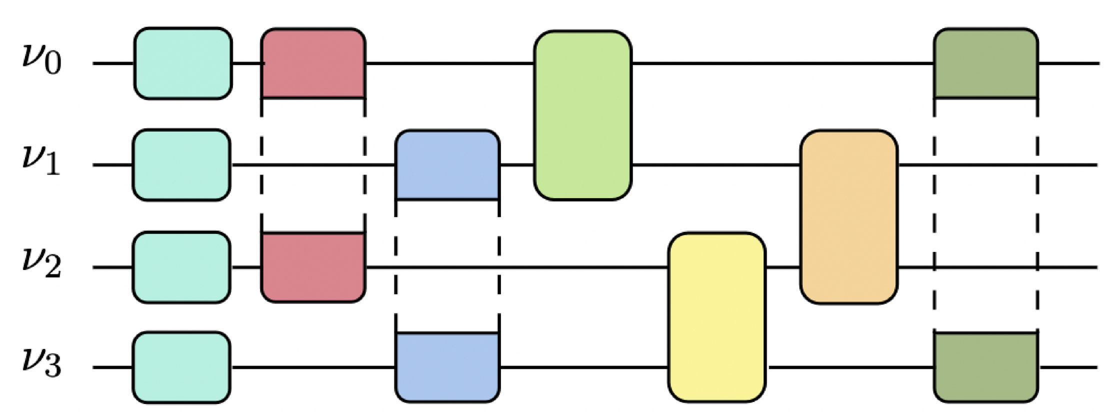
$$0 \longleftrightarrow 3\ , \quad 1 \longleftrightarrow 2\,. \tag{4}$$

UNITARY IMPLEMENTATION

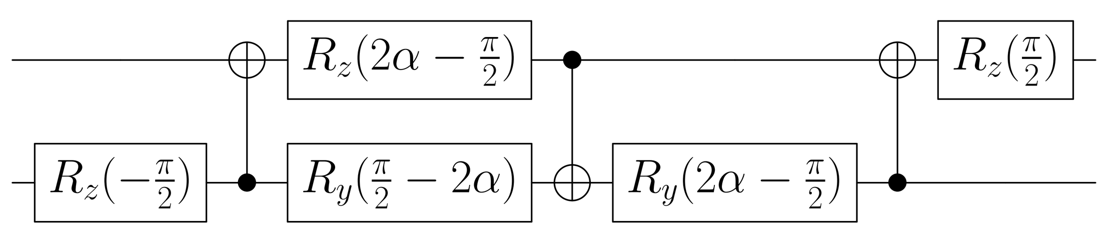
- To perform the quantum simulation we need a **quantum gate decomposition** of the $U(t)$ operator ($2^N \times 2^N$ unitary matrix on the flavor basis):
- Divide 1-body and 2-body parts that commute:

$$U(t) = U_2(t)U_1(t)\,. \tag{5}$$

- Approximate the 2-body part as a product of pair interactions.



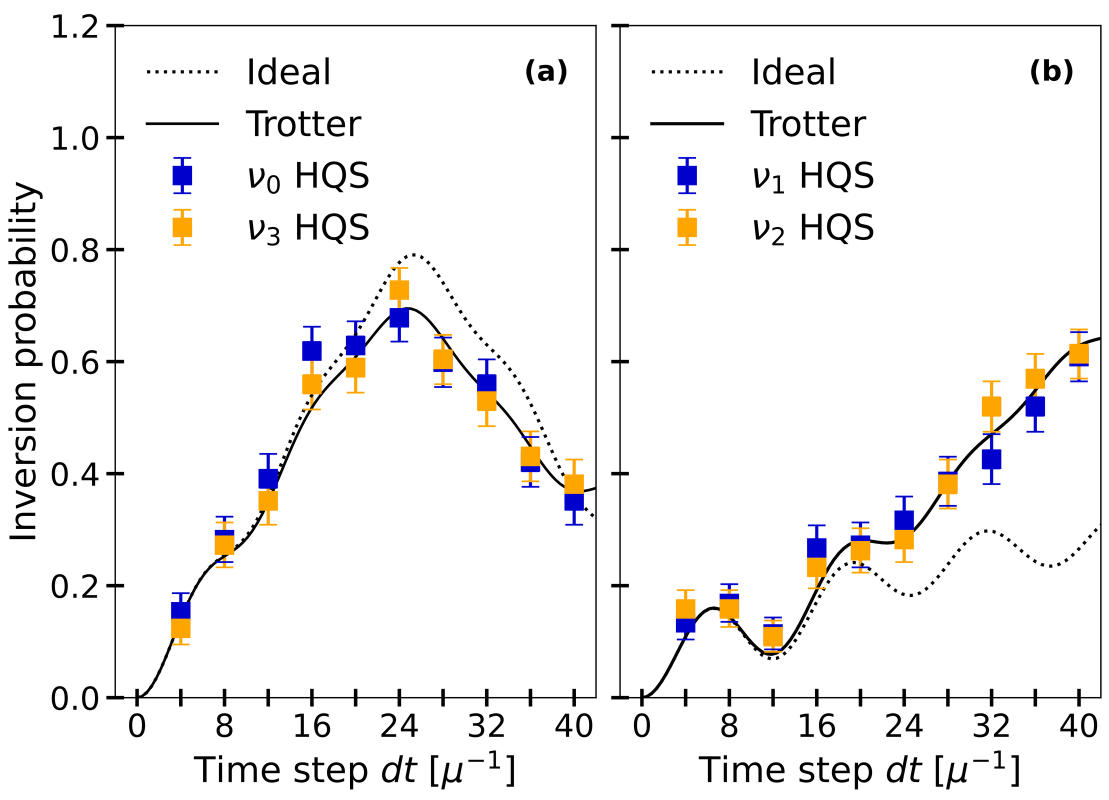
- Each 2-qubit gate $u_{ij} = e^{i\alpha(X\otimes X + Y\otimes Y + Z\otimes Z)}$ can be implemented as [3]:



- The order in which the pairs interact changes the error due to the commutators.
- The **swap network** proposed in Ref. [2] implements the interaction on a chain of linearly connected qubits.
- **All-to-all connectivity** allows for best ordering and lower circuit complexity.
- Machine-aware compilation.

SINGLE TROTTER STEP PROPAGATION

- The propagator is applied to the same initial state for different Trotter steps $dt = 4, 8, ..., 40\ \mu^{-1}$.
- Results obtained from the simulations on Quantinuum's trapped-ion device:

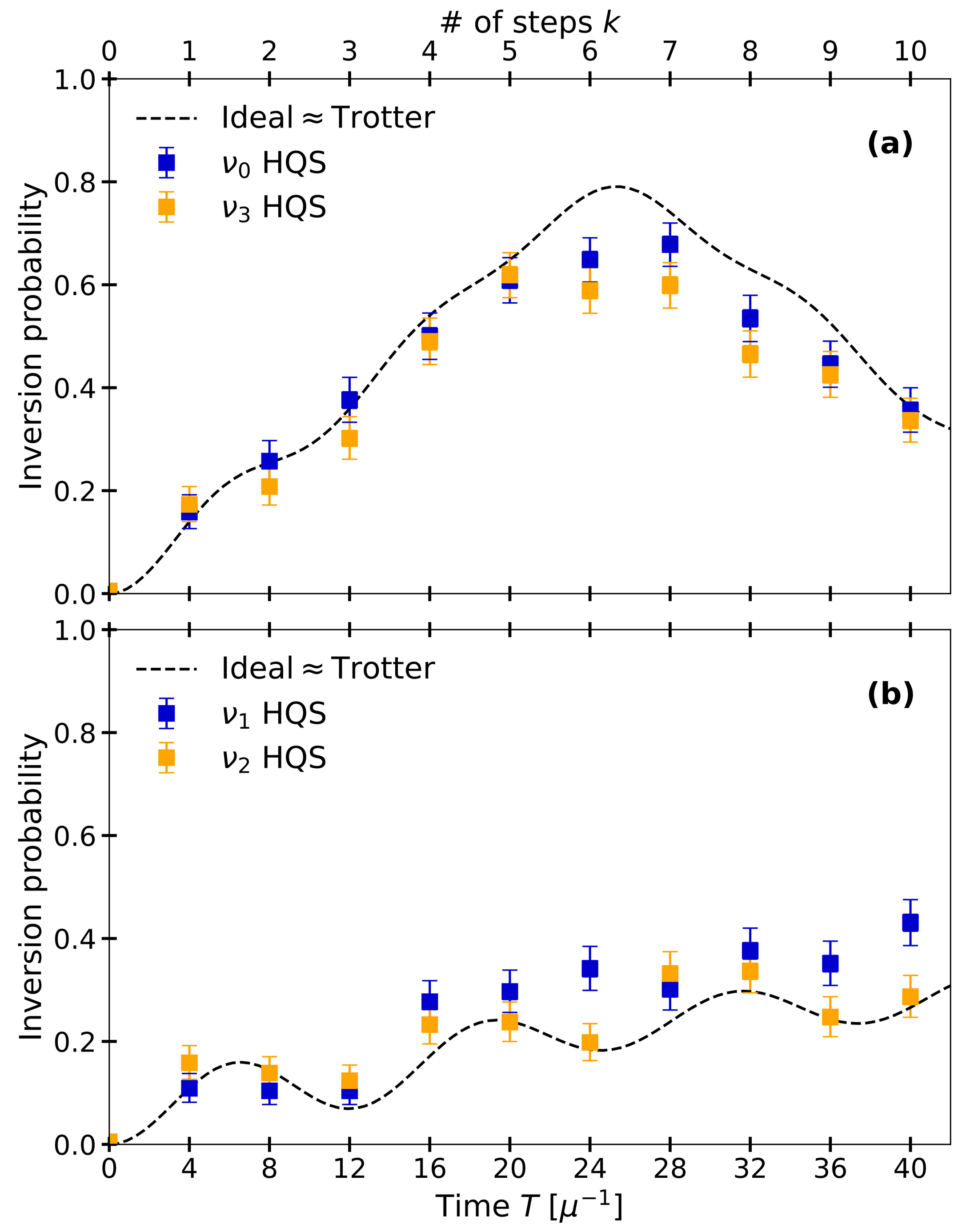


MULTIPLE STEPS EVOLUTION

- Evolve the system until T applying $k = T/dt$ Trotter steps:

$$|\Psi(T)\rangle = U_2(dt)^k U_1(dt)^k\,|\Psi_0\rangle\,. \tag{6}$$

- Real quantum machine results:



- Very long circuits with a huge number of gates:

k	1	2	3	4	5	6	7	8	9	10
$\#\,ZZ$	18	36	54	72	90	108	126	144	162	180
$\#\,SU(2)$	36	68	100	132	164	196	228	260	292	324

COMPLEXITY ALGORITHM SCALING

- The number of 2-qubit gates needed to evolve up to T with an error $< \epsilon$ scales polynomially with N:

Decomposition type	Circuit complexity
First order Trotter	$\mathcal{O}\left(\frac{T^2\mu^2}{\epsilon}N^3\right)$
Second order trotter	$\mathcal{O}\left(\frac{T^{3/2}\mu^{3/2}}{\sqrt{\epsilon}}N^{5/2}\right)$
Qubitization	$\mathcal{O}\left(T\mu N^3 + \log(1/\epsilon)N^2\right)$

CONCLUSION

- Fully connected qubits allow for more freedom in gate decomposition.
- Quantum circuit optimization is a crucial step in order to perform simulations on a near-term quantum device.
- The complexity of the implementation of time evolution scales polynomially with the number of neutrinos.

References

[1] V. Amitrano et al. "Trapped-Ion Quantum Simulation of Collective Neutrino Oscillations". In: *arXiv:2207.03189* (2022).

[2] B. Hall et al. "Simulation of Collective Neutrino Oscillations on a Quantum Computer". In: *Phys. Rev. D 104, 063009* (2021).

[3] F. Vatan et al. "Optimal Quantum Circuits for General Two-Qubit Gates". In: *Phys. Rev. A 69, 032315* (2004).

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INFN Istituto Nazionale di Fisica Nucleare

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Q@TN Quantum Science and Technology in Trento