# Stasis in an Expanding Universe


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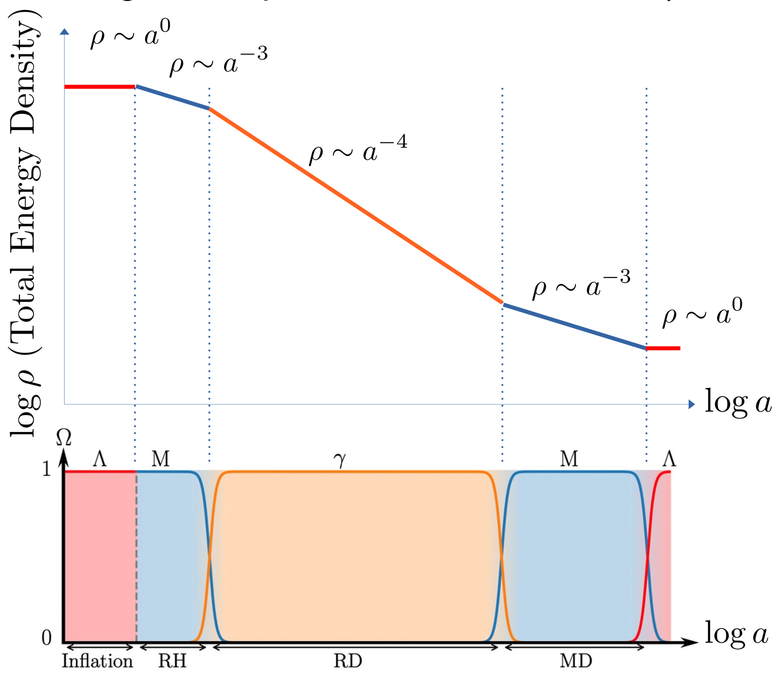
## Stable Mixed-Component Cosmological Eras?

- The energy densities $\rho_i$ associated with different ***cosmological components*** (matter, radiation, vacuum energy, etc.) with different equations of state scale differently with the scale factor $a$ under cosmic expansion.
- As a result, except during brief transition periods, the energy density of the universe is ***dominated by one such component***.
- This is the case not only in the standard cosmology, but typically in the case of ***modified cosmologies*** (e.g., with epochs of early matter- or vacuum-energy-domination) as well.

The Standard Cosmology (A Sequence of Single-Component-Dominated Eras)

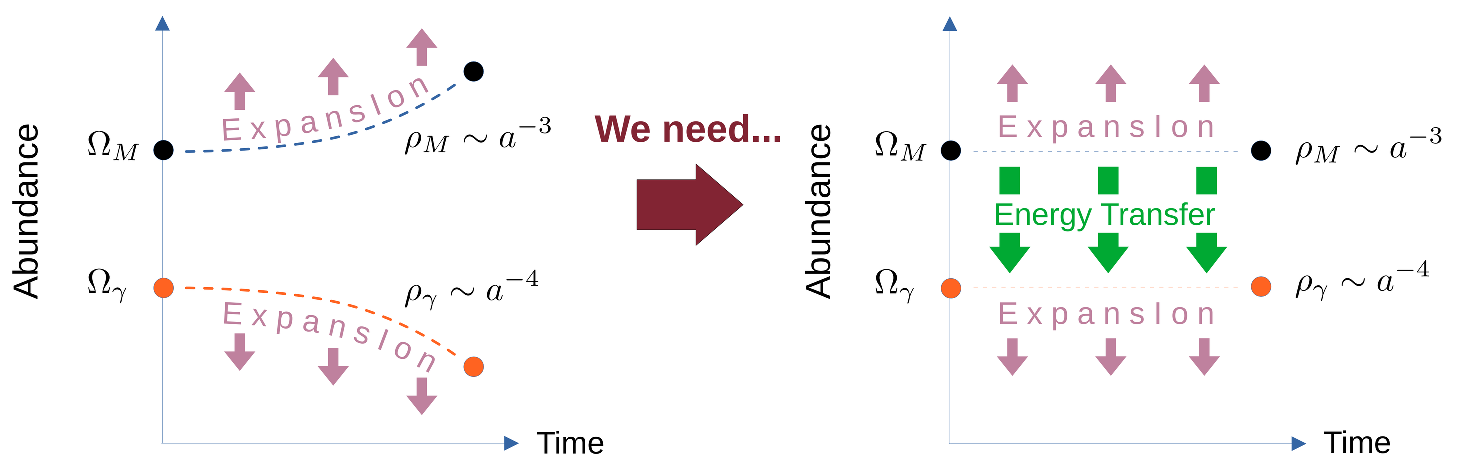



**Q** Is it possible to achieve a ***stable, mixed-component cosmological era*** in which multiple $\Omega_i$ maintain non-neglible, effectively constant values over an extended period? In other words, can we arrange for the paritioning of the cosmic pie to ***remain effectively fixed*** over an extended period, with sizable slices corresponding to components with different equations of state?

**A** At first glance, arranging this may seem impossible – or at least attainable only with a ridiculous amount of fine-tuning. However, the answer is yes! Moreover, such eras, which we call periods of ***cosmic stasis***, can be realized in a straightforward manner and in fact arise naturally in many extensions of the Standard model.

- To see how a stasis era can arise, let us consider a universe effectively consisting of matter and radiation alone, with all other $\Omega_i$ negligible. Since $\rho_M$ and $\rho_\gamma$ scale differently under cosmic expansion, $\Omega_M$ typically increases, while $\Omega_\gamma$ decreases. In order to compensate for this effect, what's needed is a ***continuous transfer of energy density*** from matter to radiation.

| Boltzmann Eqns. (Usual Case) | Boltzmann Eqns. (with Sources/Sinks) |
|---|---|
| $\frac{d\rho_M}{dt} = -3H\rho_M \qquad \frac{d\rho_\gamma}{dt} = -4H\rho_\gamma$ | $\frac{d\rho_M}{dt} = -3H\rho_M - S(t) \qquad \frac{d\rho_\gamma}{dt} = -4H\rho_\gamma + S(t)$ |




## A Model of Stasis

- ***Particle decays*** provide a natural mechanism for obtaining these source/sink terms. However, the exponential decay of a single matter species, which occurs over a relatively short time period, is insufficient for achieving stasis.
- What we need is a ***tower of matter states*** $\phi_\ell$, where $\ell = 0, 1, 2, \ldots, N-1$, whose decay widths $\Gamma_\ell$ and initial abundances $\Omega_\ell^{(0)}$ scale across the tower in such a way that the effect of decays on $\Omega_M$ and $\Omega_\gamma$ compensates for the effect of cosmic expansion over a extended period. In particular, we consider a tower of $N$ such states states with...

| Masses | Decay Widths | Initial Abundances |
|---|---|---|
| $m_\ell = m_0 + (\Delta m)\ell^\delta$ | $\Gamma_\ell = \Gamma_0 \left(\frac{m_\ell}{m_0}\right)^\gamma$ | $\Omega_\ell^{(0)} = \Omega_0^{(0)} \left(\frac{m_\ell}{m_0}\right)^\alpha$ |

- Towers of states with mass spectra of this form arise naturally in many extensions of the Standard Model, including those with ***extra spacetime dimensions*** or additional ***strongly-coupled gauge groups***.
- The Boltzmann equations for the individual $\rho_\ell$, in conjunction with the relevant Friedmann equation, yield an equation of motion for $\Omega_M$.

Boltzmann Equations
$$\frac{d\rho_\ell}{dt} = -3H\rho_\ell - \Gamma_\ell\rho_\ell \qquad \frac{d\rho_\gamma}{dt} = -4H\rho_\gamma + \sum_\ell \Gamma_\ell \rho_\ell$$

Friedmann Equation
$$H^2 = \frac{8\pi G}{3}(\rho_M + \rho_\gamma)$$

We want this to vanish for an extended period

$$\frac{d\Omega_M}{dt} = -\sum_\ell \Gamma_\ell \Omega_\ell + H(\Omega_M - \Omega_M^2)$$

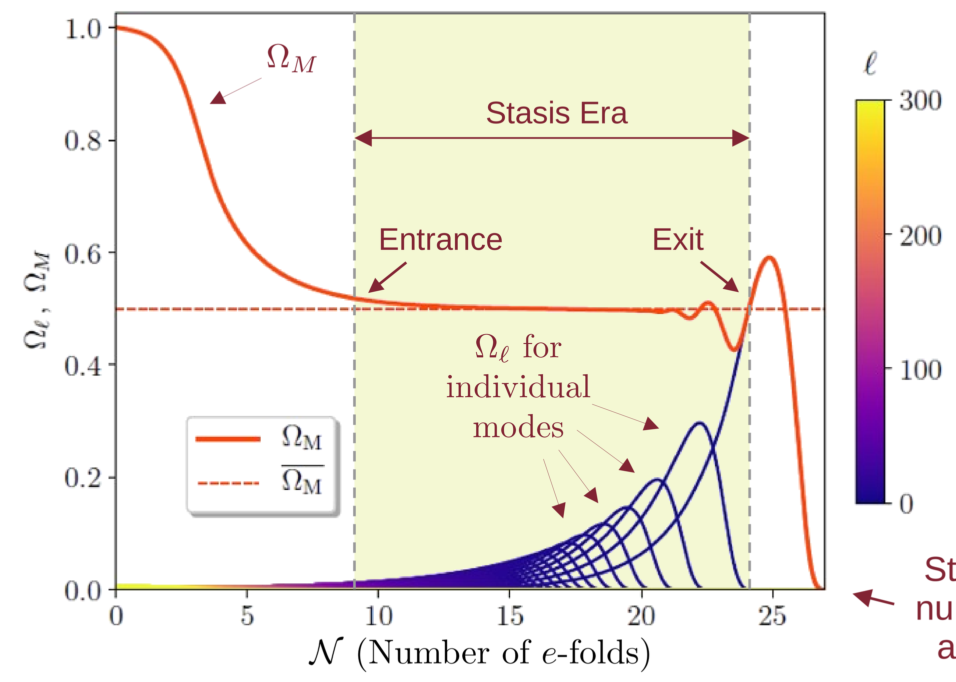
- In order to achieve an ***extended period*** of stasis, we need $d\Omega_M/dt$ to vanish over a significant range of $t$.

$$\frac{d\Omega_M}{dt} = 0 \;\Rightarrow\; \boxed{\sum_\ell \Gamma_\ell\Omega_\ell = H(\Omega_M - \Omega_M^2)} \;\xrightarrow{\text{Abundances constant}}\; \boxed{\sum_\ell \Gamma_\ell \Omega_\ell = \frac{2\overline{\Omega}_M(1-\overline{\Omega}_M)}{4-\overline{\Omega}_M}\frac{1}{t}}$$

## Achieving Stasis: Results

- Indeed, for any $\alpha$, $\gamma$, and $\delta$, we find that a tower of decaying matter states with the scaling relations above is capable of satisfying this criterion and gives rise to a period of stasis with matter and radiation abundances given by:

$$\overline{\Omega}_M = \frac{2\gamma\delta - 4(1+\alpha\delta)}{2\gamma\delta - (1+\alpha\delta)} \qquad \overline{\Omega}_\gamma = \frac{3(1+\alpha\delta)}{2\gamma\delta - (1+\alpha\delta)}$$




**Example Parameters**

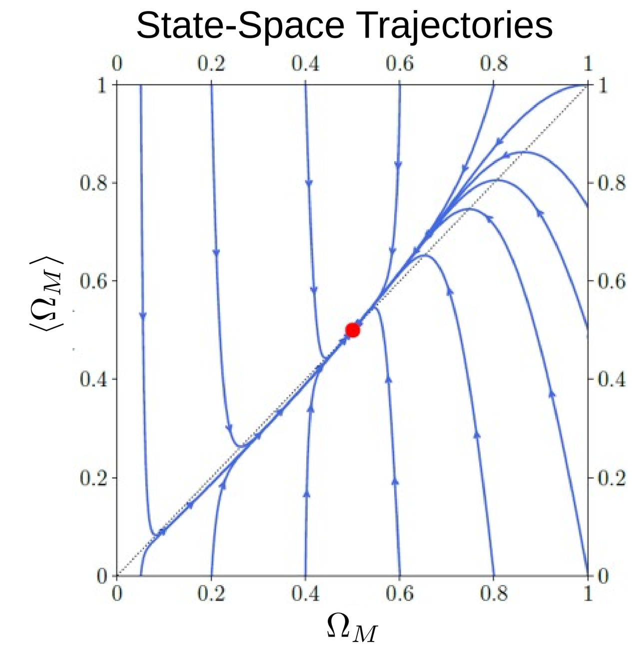
$\alpha = 1$
$\gamma = 7$
$\delta = 1$
$N = 300$
$\frac{m_0}{\Delta m} = 1$
$\frac{\Gamma_{N-1}}{H^{(0)}} = 0.01$

Stasis eras lasting larger numbers of $e$-folds can be arranged just as easily.

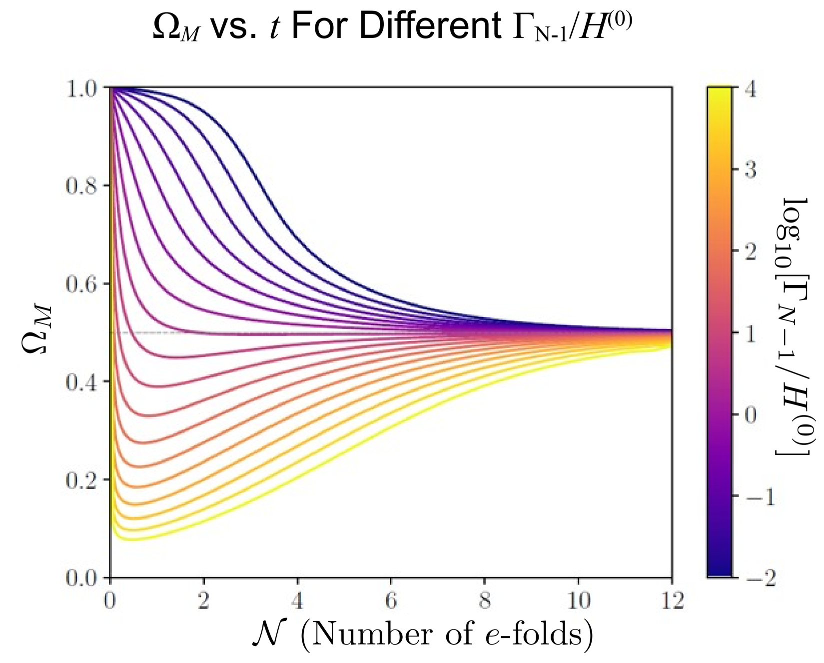
## Stasis Is a Global Attractor

- Cosmic stasis does not require a fine-tuning of the initial conditions for $\Omega_M$ and $\Omega_\gamma$, or for the ratio $\Gamma_{N-1}/H^{(0)}$. In fact, stasis is a ***global attractor***. Regardless of what $\Omega_M(t)$ and its time-average $\langle\Omega_M\rangle(t)$ from $t^{(0)}$ to $t$ are at a given $t \geq t^{(0)}$, $\Omega_M$ and $\Omega_\gamma$ will ***evolve toward their stasis values***. Stasis doesn't require any special $\Gamma_{N-1}/H^{(0)}$ value either.

State-Space Trajectories

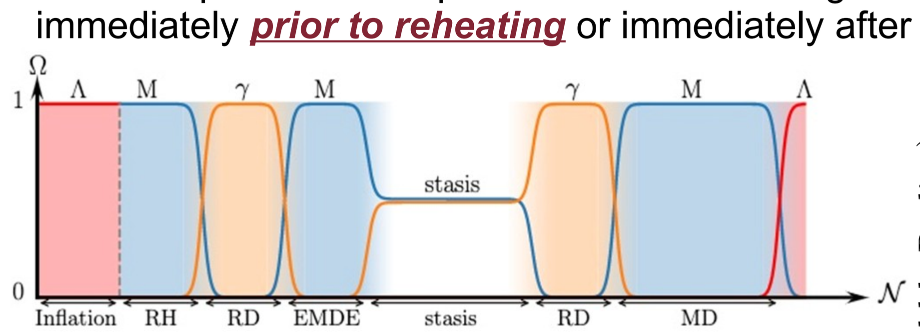



$\Omega_M$ vs. $t$ For Different $\Gamma_{N-1}/H^{(0)}$



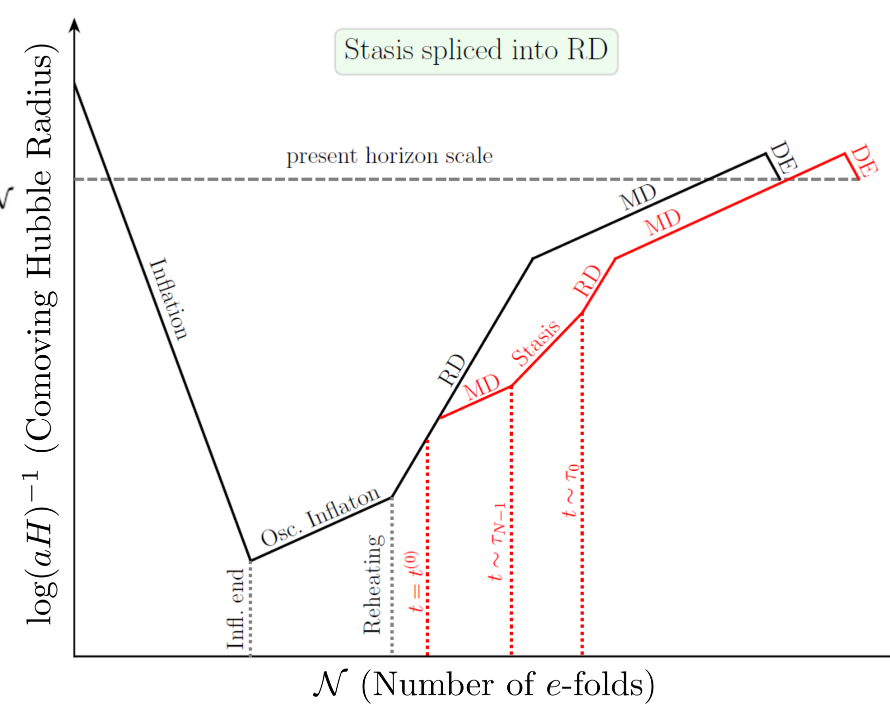

## Conclusions and Implications of Stasis

- A stasis epoch can be spliced into the cosmological timeline in a variety of places – for example, immediately ***prior to reheating*** or immediately after a period of ***early matter domination***.




- The comoving Hubble radius grows more slowly in cosmologies with a stasis era, so perturbation modes re-enter the horizon at a later time. This has implications for ***inflationary observables***.
- ***Density perturbations*** grow more quickly during stasis than in an radiation-dominated era. As a result, compact objects (e.g., compact minihalos) can potentially form during stasis, as they do in an early matter-dominated era.
- A stasis era can also be realized involving ***matter and vacuum energy***. The staggered transitions from overdamped to underdamped oscillation of scalars which acquire abundances from vacuum misalignment convert vacuum energy to matter.

[Dienes, Heurtier, Huang, Kim, Tait, BT: 2207.xxxxx]




- A population of ***primordial black holes***, which transfer energy density to radiation as they evaporate, can also yield a period of stasis.

[Dienes, Heurtier, Huang, Kim, Tait, BT: 2207.xxxx(x+1)]

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