

# Enabling Precision Quark- and Lepton-flavor Physics with Lattice **QCD**

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Snowmass Community Summer Study | Seattle  
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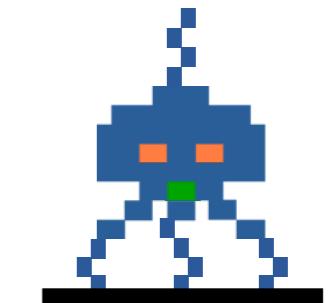
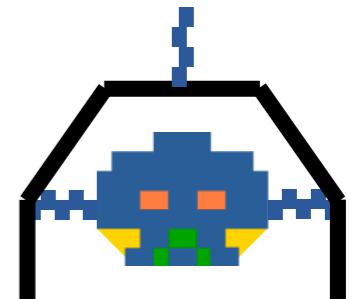
# Need for Precision

- Muon anomalous magnetic moment [Muon g-2 Theory Initiative, [SnowWP](#)].
  - Precision  $a_\mu = (g-2)_\mu/2 -$ 
    - QED: 1 ppb;
    - SM total: 370 ppb;
    - Experiment: 350 ppb, aiming for 140 ppb.
  - SM prediction limited by hadronic contributions (next slide).
  - Electron  $g_e$ : 28 ppt ( $a_e$ : 0.2 ppb).
- |                              | $10^{11}[a_\mu = (g-2)_\mu/2]$ | uncertainty now | uncertainty target |
|------------------------------|--------------------------------|-----------------|--------------------|
| QED                          | 116 584 718 .93(10)            |                 |                    |
| EW                           | 153 .6(1.0)                    |                 |                    |
| HVP<br>LO+NLO+NNLO           | 6845 (40)                      | (13)            |                    |
| HLbL<br>LO+NLO               | 95 (18)                        | (9)             |                    |
| SM total                     | 116 591 810 (43)               | (16)            |                    |
| BNL 821 $\oplus$<br>FNAL 989 | 116 592 061 (41)               | (16)            |                    |
| Expt - SM                    | 251 (59)                       | (22)            |                    |

# Hadronic Contributions

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- Two virtual processes:
  - hadronic vacuum polarization (HVP)—
    - $6845(40) = 6931(40) - 98.3(7) + 12.4(1);$
  - hadronic light-by-light scattering (HLbL)—
    - $92(18) = 90(17) + 2(1).$
- Two methods: experimental data + dispersion theory OR lattice QCD:
  - Can we get from  $6931(40)$  [actually  $7075(55)$ ] to  $7000(13)$ ?
  - Has anything like this happened with lattice gauge theory already?



# Outline

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- Need for precision
- QCD and lattice gauge theory
- Some precise results from lattice QCD
  - Masses—hadron and quark
  - Quark flavor
  - Muon g-2
- Enabling precision lattice QCD

Lattice QCD

# QCD Lagrangian

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- SU(3) gauge symmetry and  $1 + n_f + 1$  parameters:

$$\begin{aligned}\mathcal{L}_{\text{QCD}} = & \frac{1}{g_0^2} \text{tr}[F_{\mu\nu}F^{\mu\nu}] && M_\Omega \text{ or similar, ;} \\ & - \sum_f \bar{\Psi}_f (\not{D} + m_f) \Psi_f && M_\pi, M_K, M_{J/\psi}, M_Y, \dots; \\ & + \frac{i\theta}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{tr}[F_{\mu\nu}F_{\rho\sigma}] && \theta = 0.\end{aligned}$$

- Gauge coupling  $g_0$  and quarks masses  $m_f$  are not directly measurable:
  - quarks bound into hadrons: use meson masses to fix  $m_f$ ,
  - “dimensional transmutation”: use another mass to eliminate  $g_0$ .

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Humankind's most perfect theory—Wilczek

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# QCD Functional Integral

- Everything from integrals:

$$\langle \mathcal{O}(U, \psi, \bar{\psi}) \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{O}(U, \psi, \bar{\psi}) e^{-S}$$

The diagram illustrates the decomposition of the path integral measure  $\mathcal{D}$  into two components: gluons and (anti)quarks. A blue line labeled "gluons" originates from the first term  $\mathcal{D}U$  in the measure. A green line labeled "(anti)quarks" originates from the second term  $\mathcal{D}\psi \mathcal{D}\bar{\psi}$ . These lines converge at the exponential factor  $e^{-S}$ .

- Infinite spacetime lattice makes set of integration variables countable; finite lattice makes them finite.
- Markov chain Monte Carlo with important sampling:

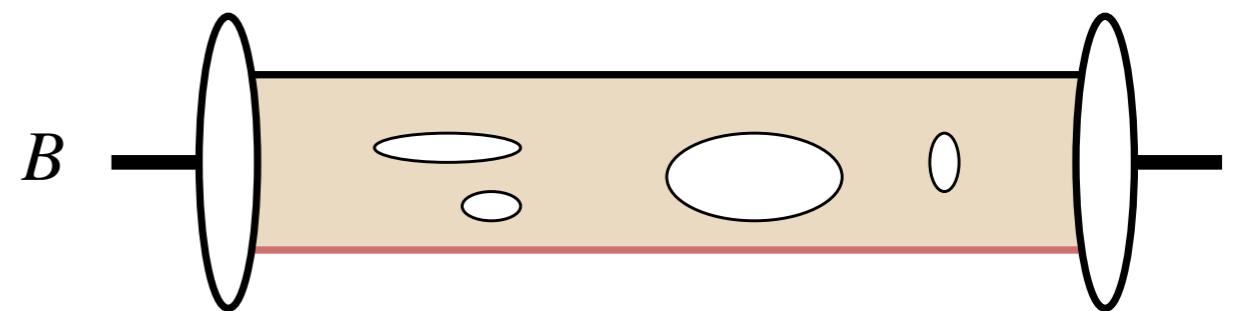
$$\langle \mathcal{O}(U, G) \rangle = \frac{1}{C} \sum_{c=0}^{C-1} \mathcal{O}(U, G)$$

Hybrid Monte Carlo (HMC)  
Duane, Kennedy, Pendleton, Roweth  
[Phys. Lett. B 195 \(1987\) 216](#)

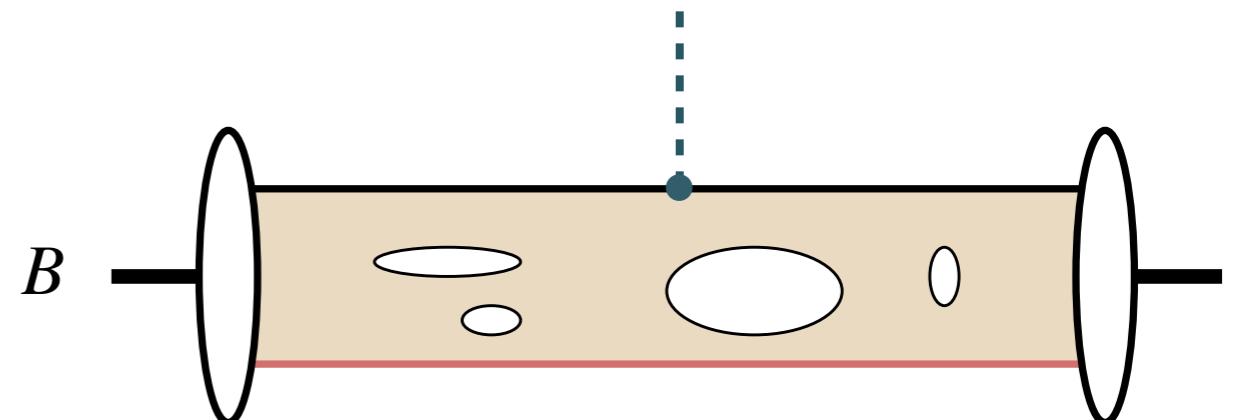
# QCD Correlation Functions

- Everything (almost) from correlation functions:

- Masses, annihilation matrix elements:



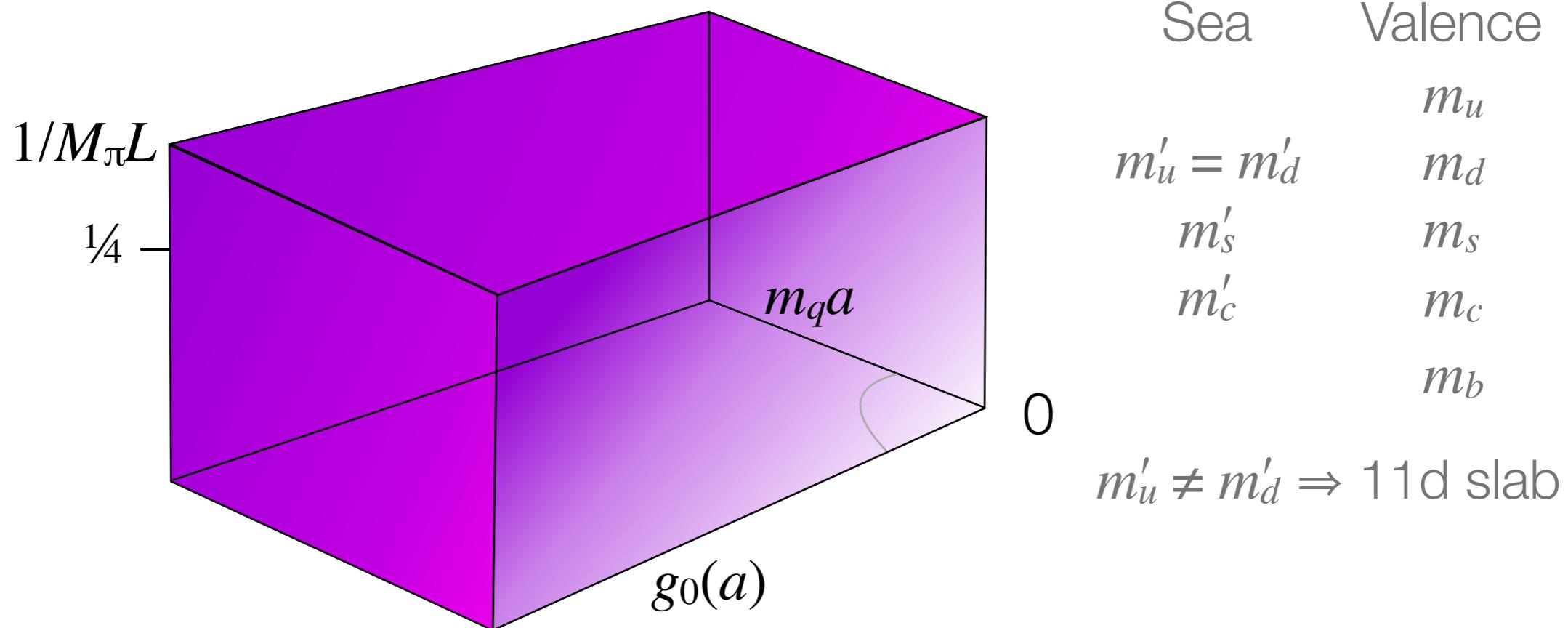
- Decay and mixing matrix elements:



- Valence & sea quarks: different parts of code, so masses can differ.

# Lattice **QCD** Data

- Computer generates data in a slab of a 10-dimensional parameter space:



- Combine data with effective field theories (EFT) [e.g., [hep-lat/0205021](https://arxiv.org/abs/hep-lat/0205021)].



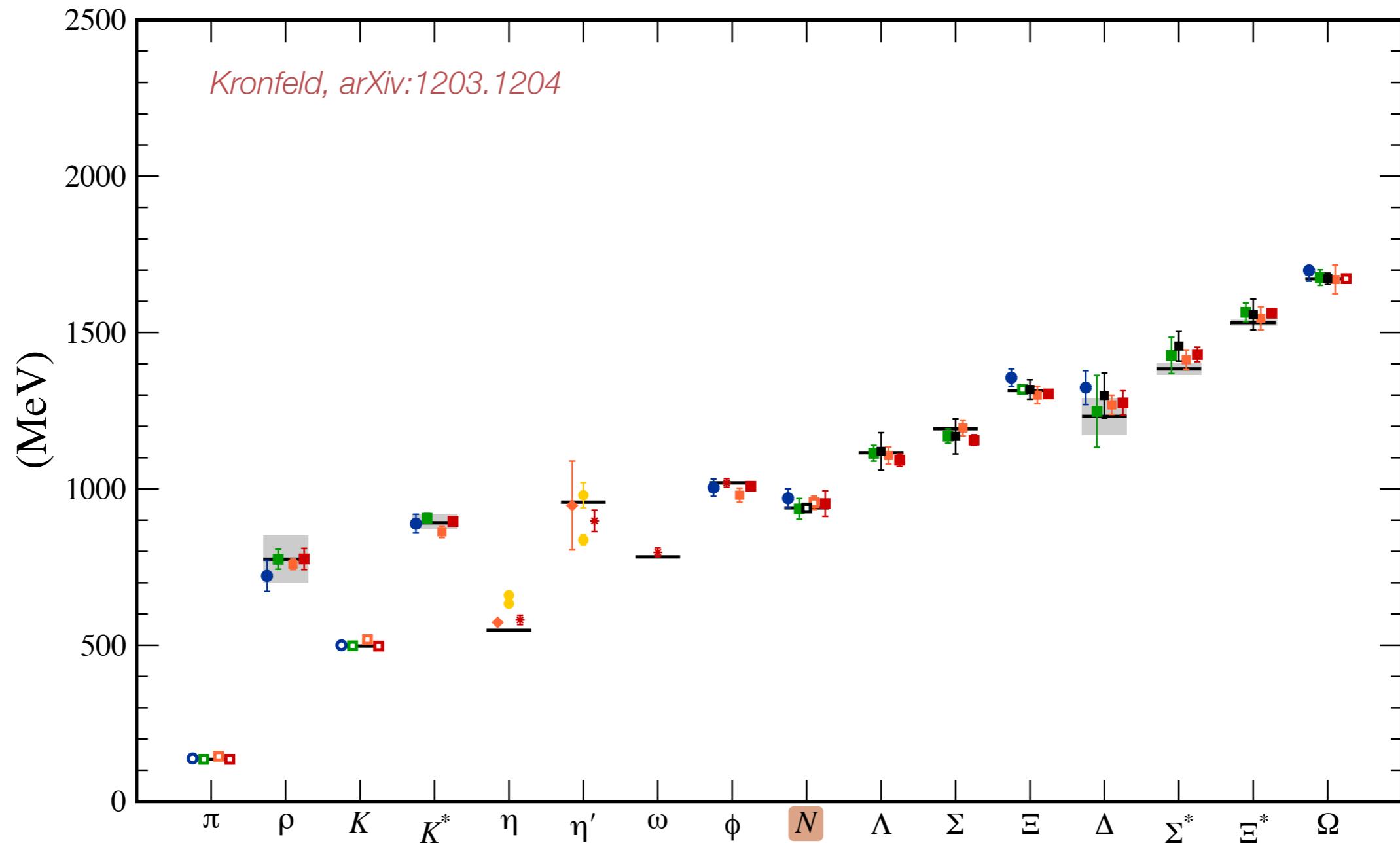
red EFT

- Effective field theories: Symanzik ( $a \rightarrow 0$ ), chiral perturbation theory ( $m'_q, m_q \rightarrow m_u, m_d$ ), heavy quark theory ( $m_Q \rightarrow m_b, m_c$ ).

# Masses

$\pi \dots \Omega$ : BMW, MILC, PACS-CS, QCDSF; ETM (2+1+1);  
 $\eta - \eta'$ : RBC, UKQCD, Hadron Spectrum ( $\omega$ ).

# Hadron Spectrum



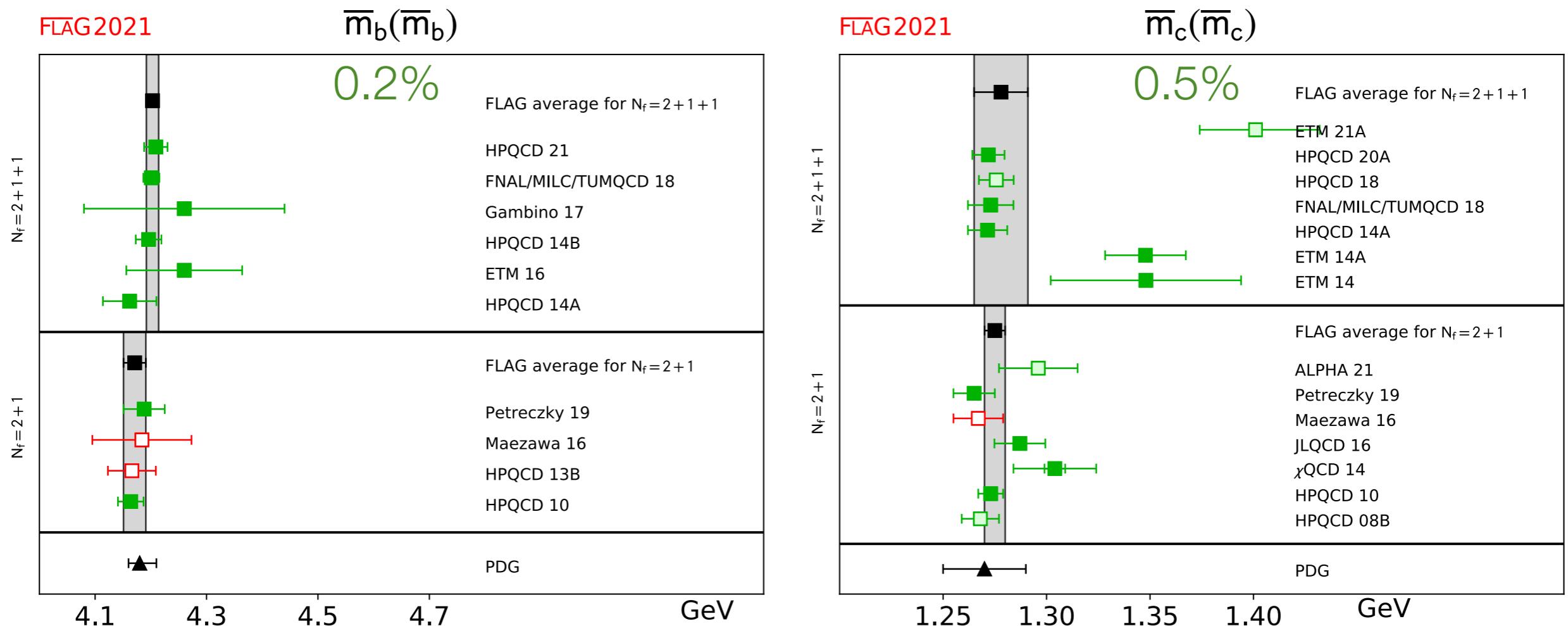
# Hadron Spectroscopy

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- Not much interest in normal hadron masses these days:
  - even  $M_n - M_p$  studied vs.  $m_d - m_u$  &  $\alpha$  a while ago [[arXiv:1406.4088](#)].
- Lots of interest in exotic hadrons (XYZ, tetraquarks, pentaquarks): lattice QCD  $\leftrightarrow$  structure [[USQCD-WP](#), [SnowWP](#)].
- Most precisely calculable masses are pseudoscalar meson masses:
  - adjust bare quark masses until  $n_f$  of them agree with experiment —  $\pi$ ,  $K^0 \pm K^+$ ,  $D_{(s)}$  ( $\eta_c$ ),  $B_{(s)}$  ( $\eta_b$ );
  - convert them to quark masses in renormalization schemes used in continuum QCD ( $\overline{\text{MS}}$ , RGI).

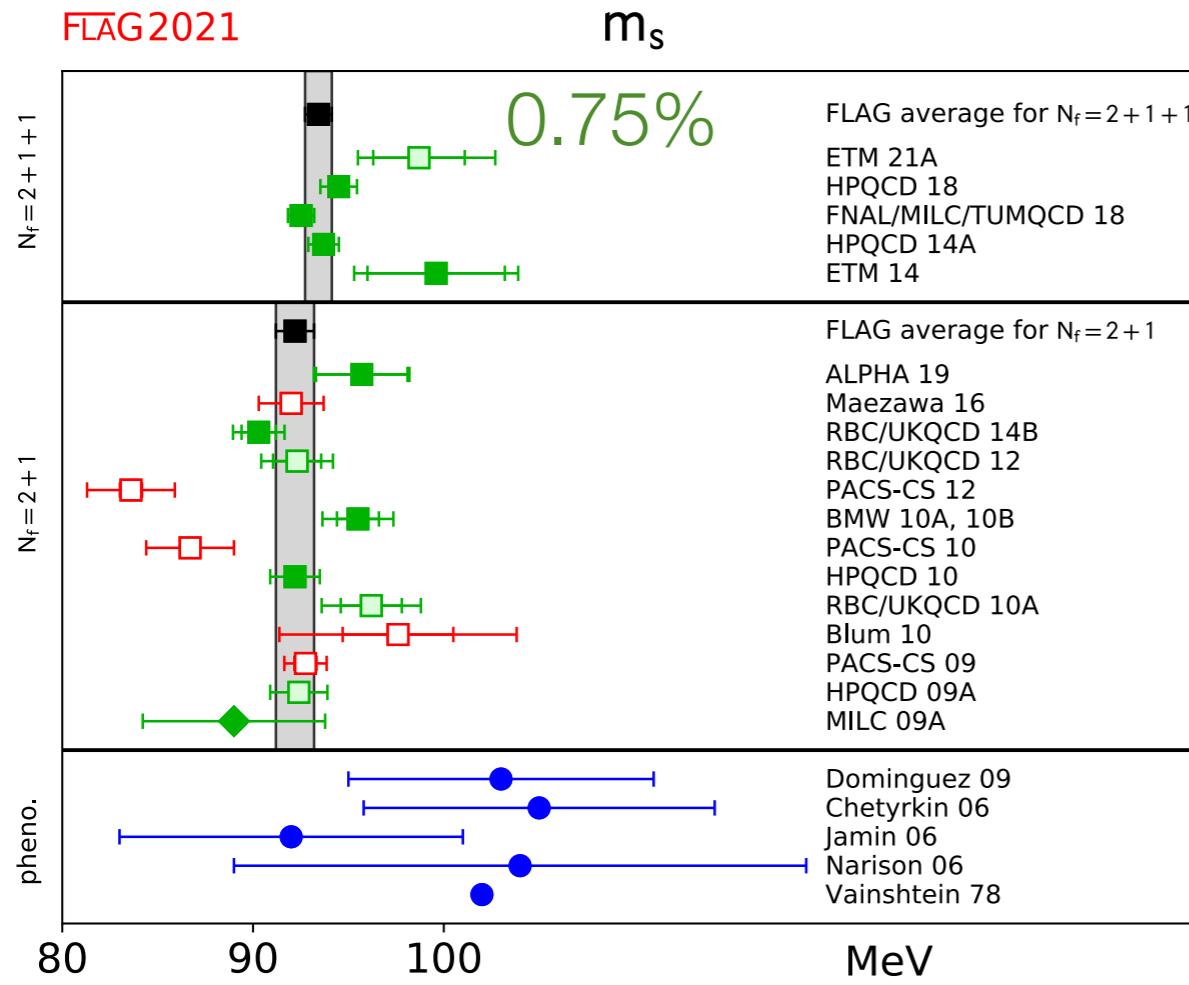
# Quark Masses: bottom, charm

- Flavor Lattice Averaging Group ([FLAG](#)), [arXiv:2111.09849](#).



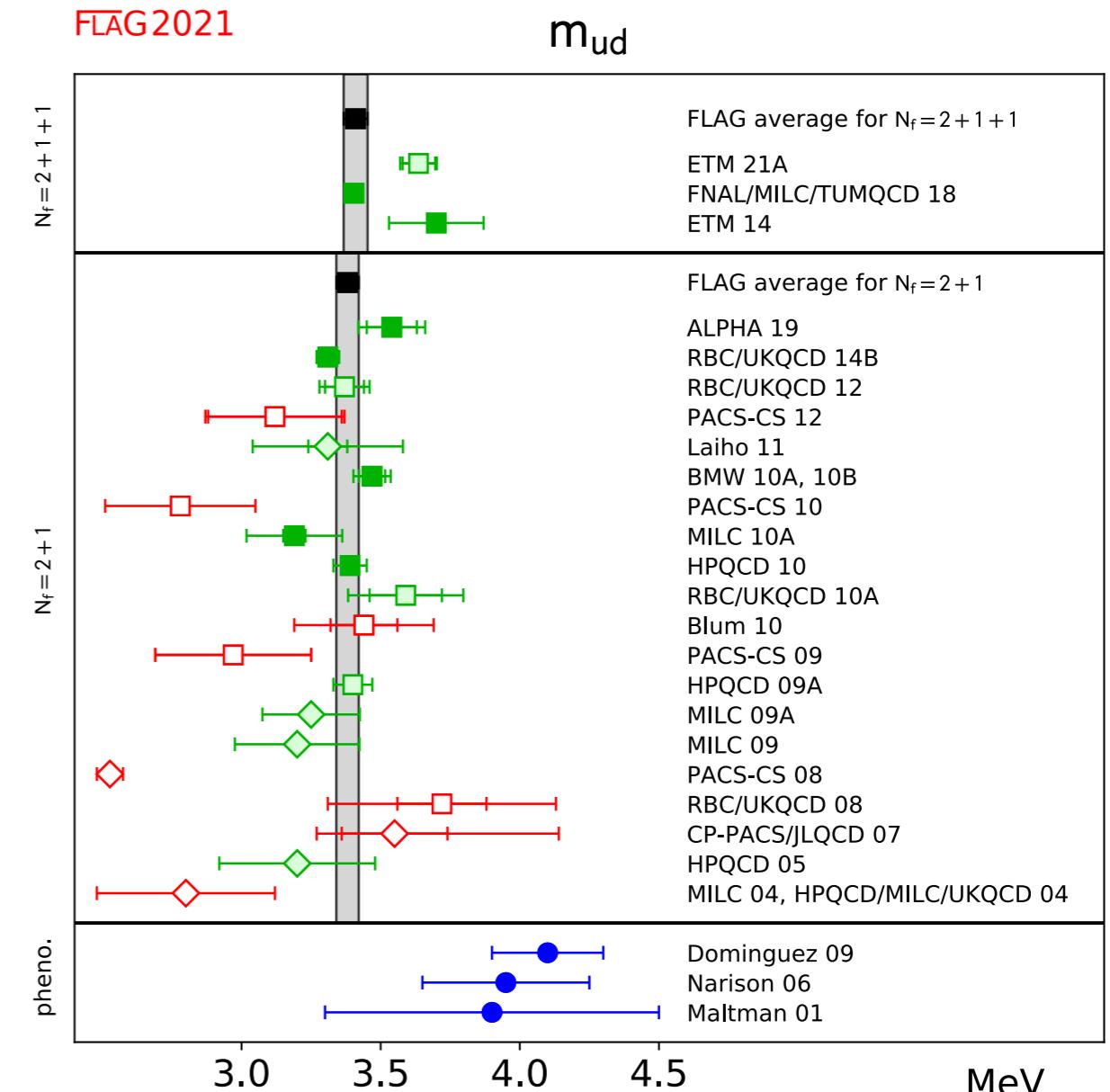
- Green passes all quality criteria; filled enters average (open superseded).

# Quark Masses: strange, down/up



$$m_{u,\overline{\text{MS}}}(2 \text{ GeV}) = 2.130(41) \text{ MeV}$$

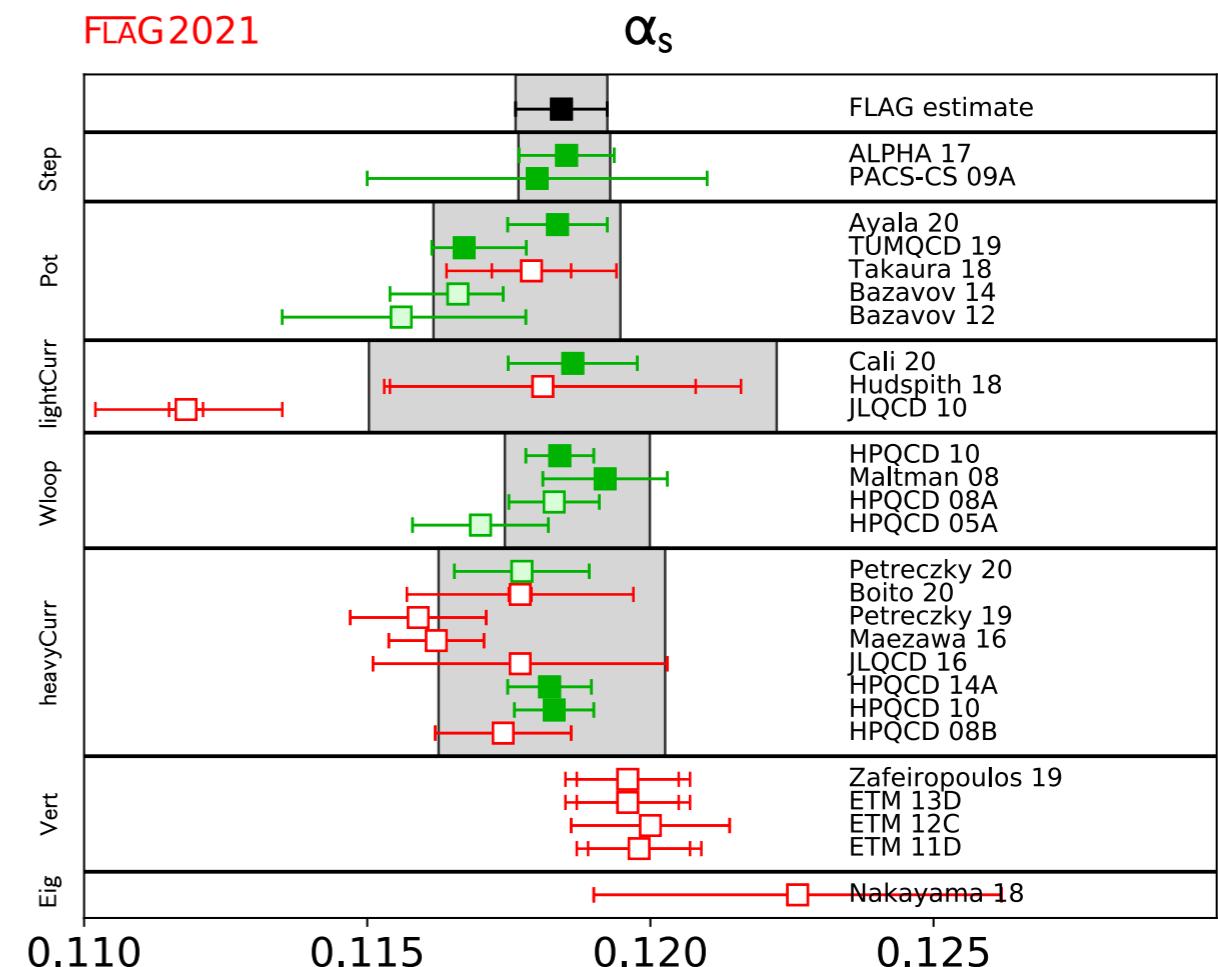
$$m_{d,\overline{\text{MS}}}(2 \text{ GeV}) = 4.675(56) \text{ MeV}$$



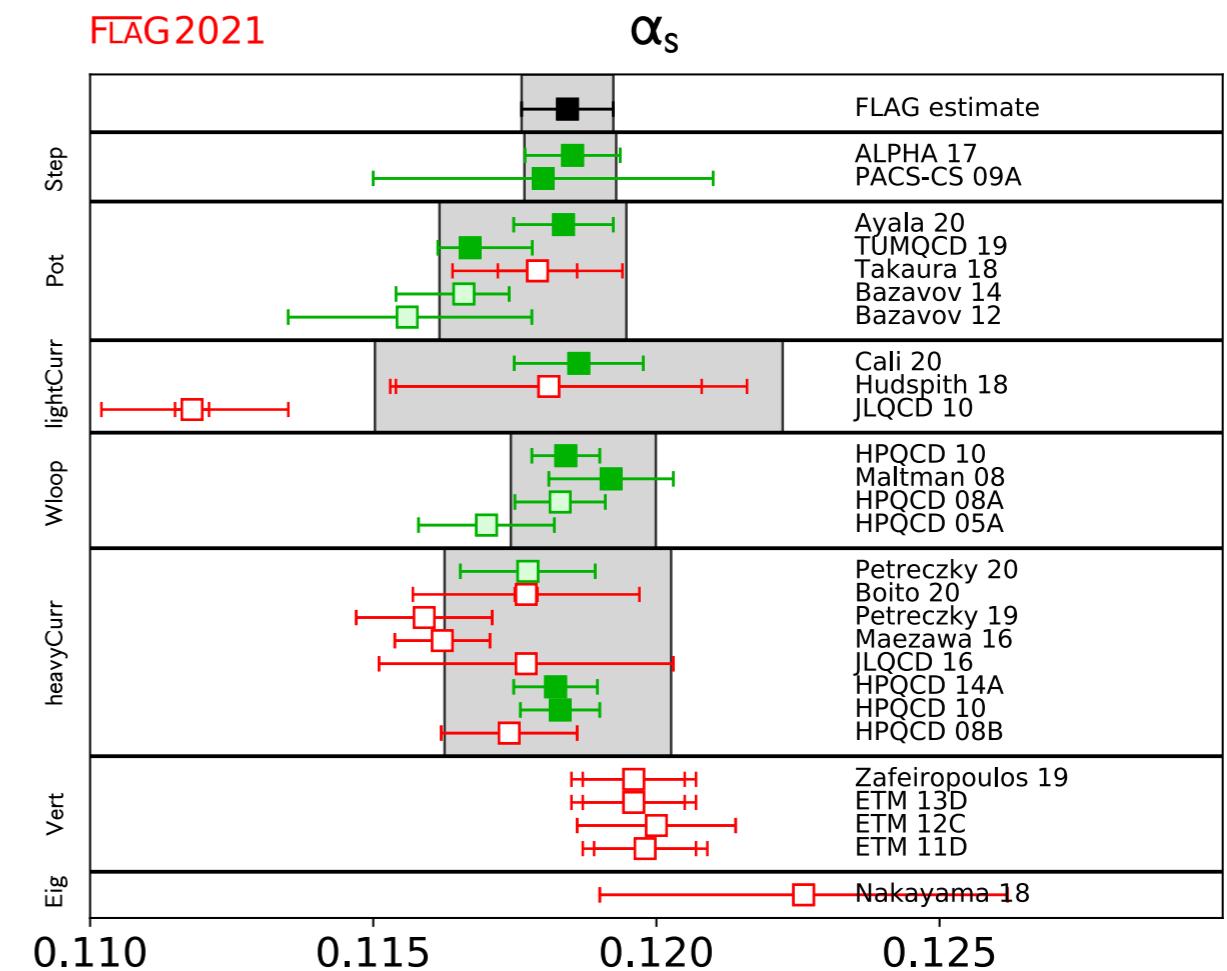
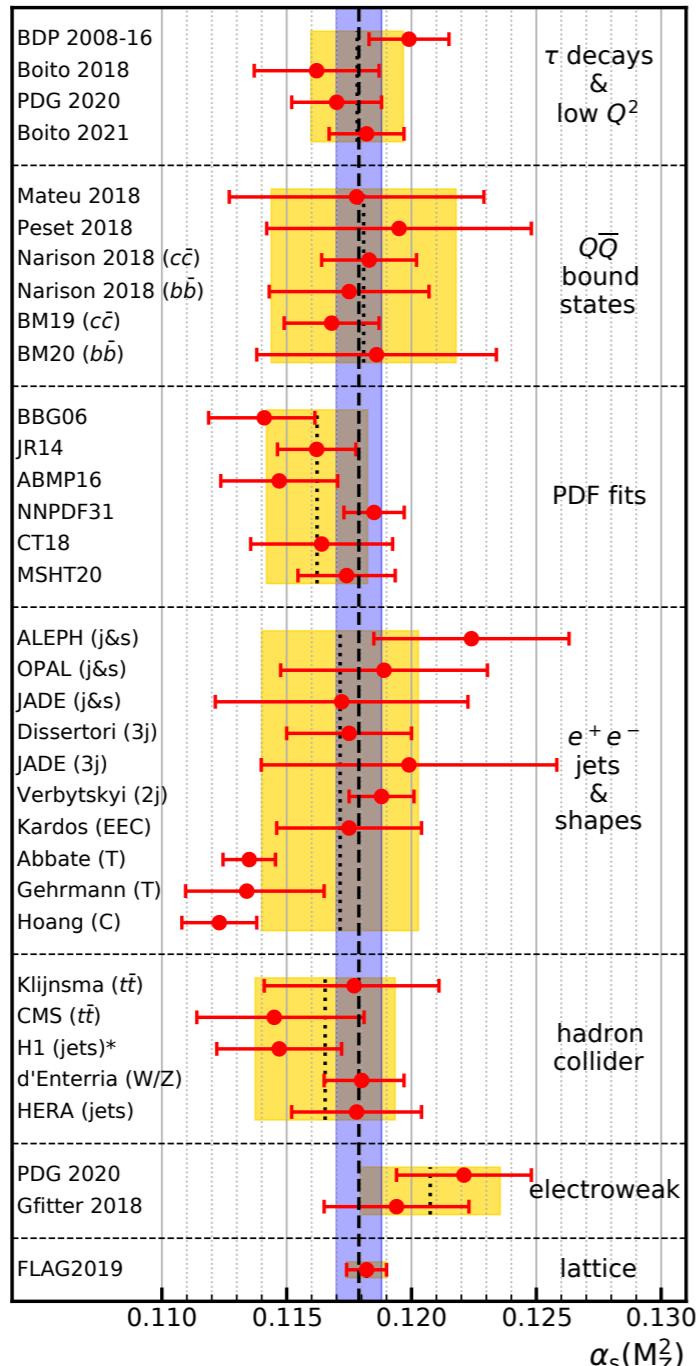
- arXiv:1802.04248, arXiv:1805.06225

# Strong Coupling $\alpha_s(\mu)$

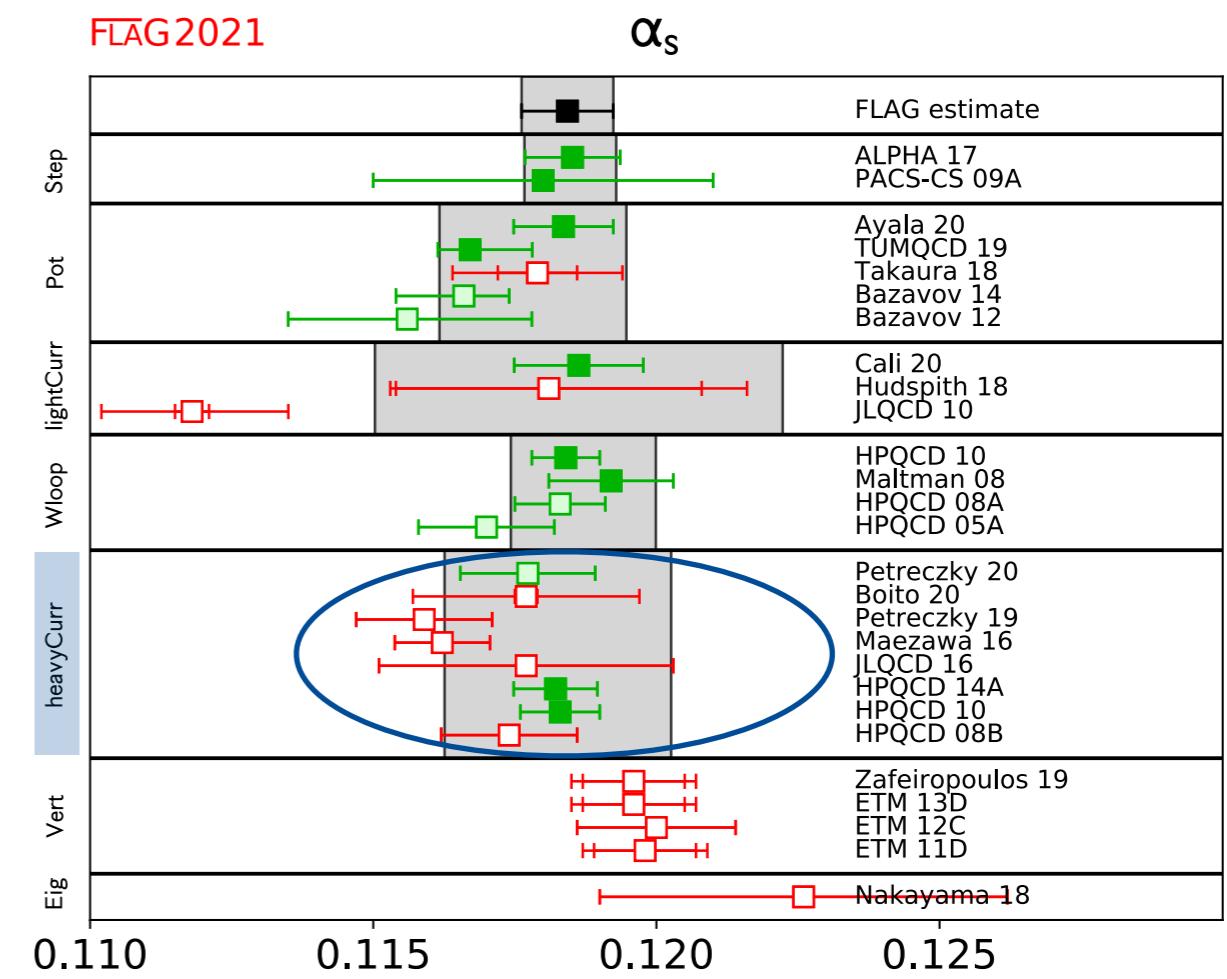
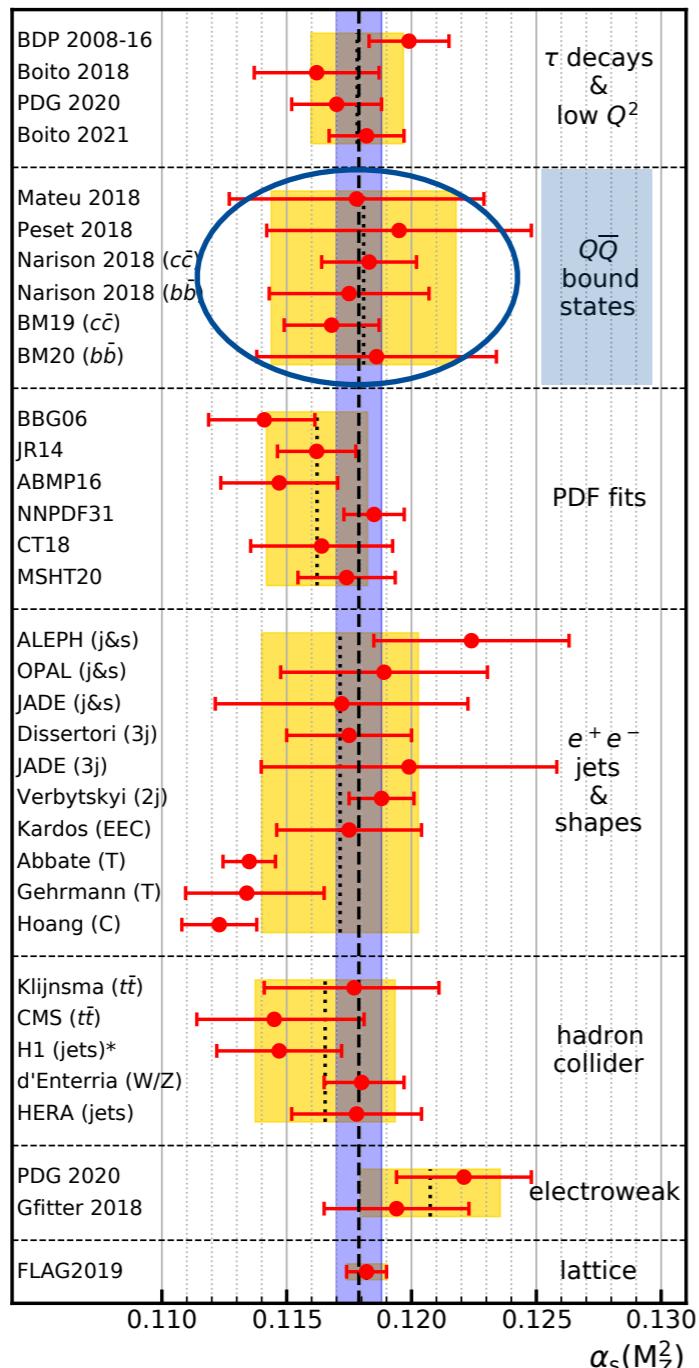
- Numerous methods:
  - different observables;
  - different probes;
  - different systematics.
- Consider “heavyCurr” 
- same PT as in  $e^+ e^- \rightarrow Q\bar{Q}$  determination.
- SnowWP on  $\alpha_s$ .



# Strong Coupling $\alpha_s(\mu)$

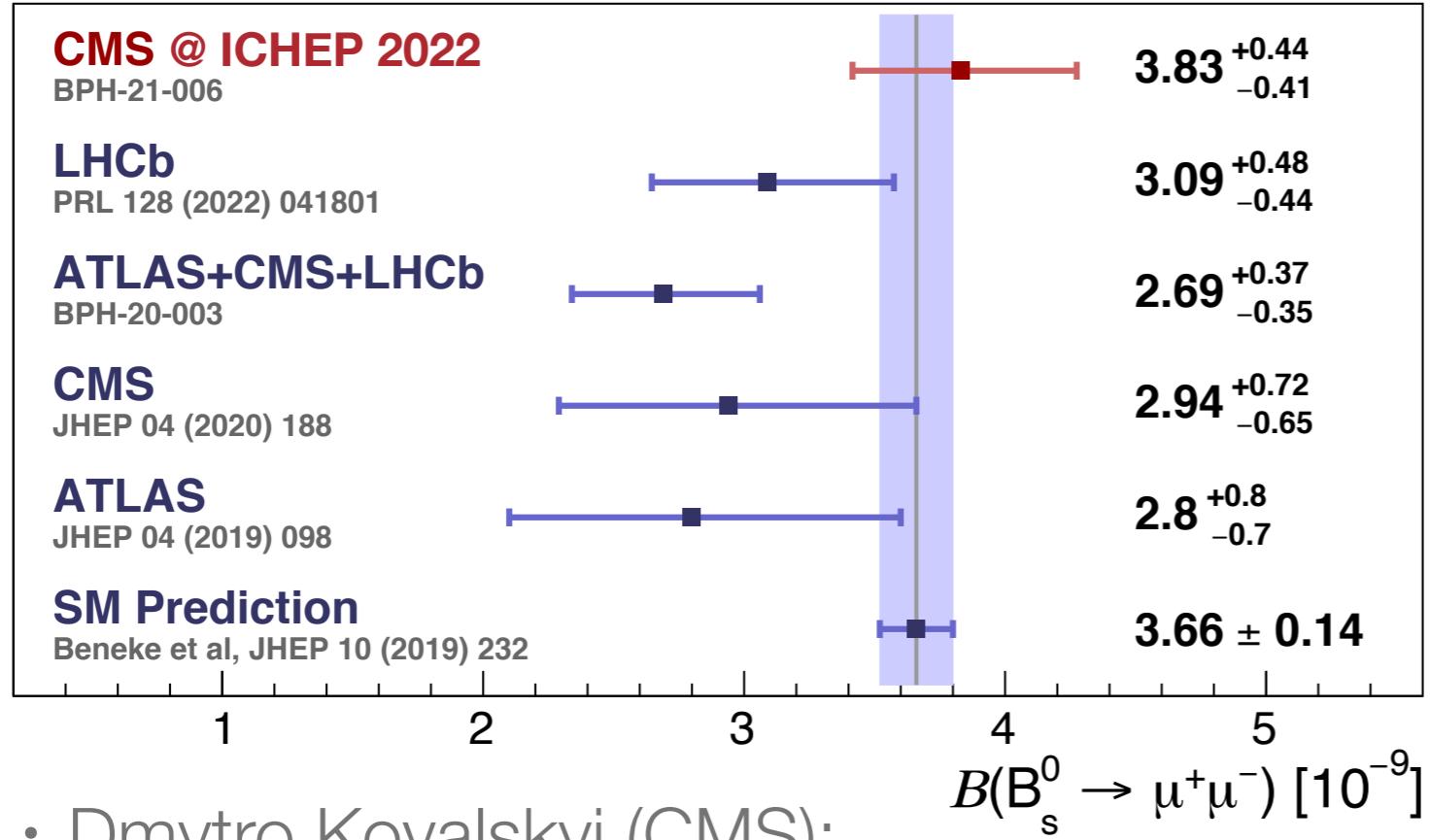


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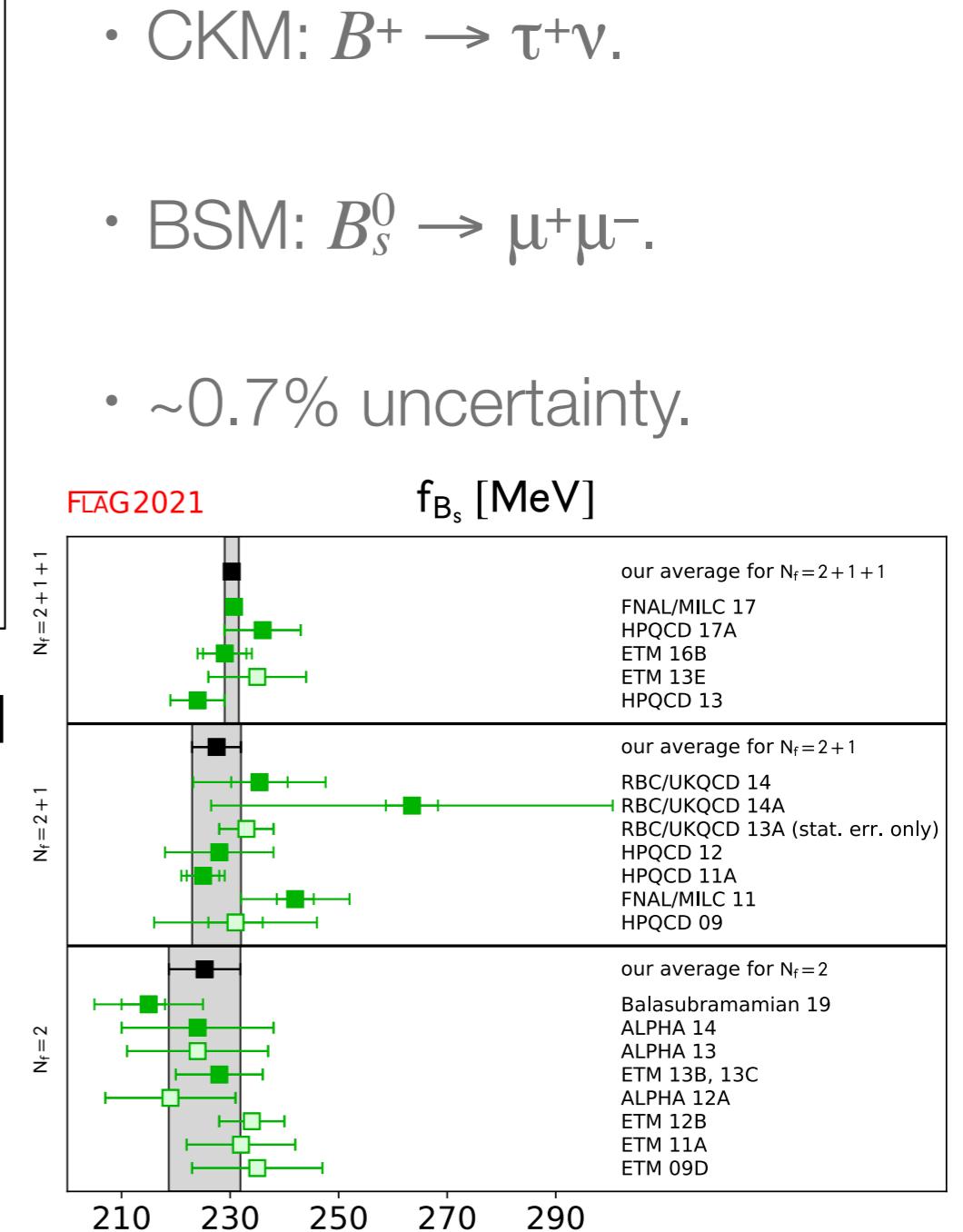


# Matrix Elements

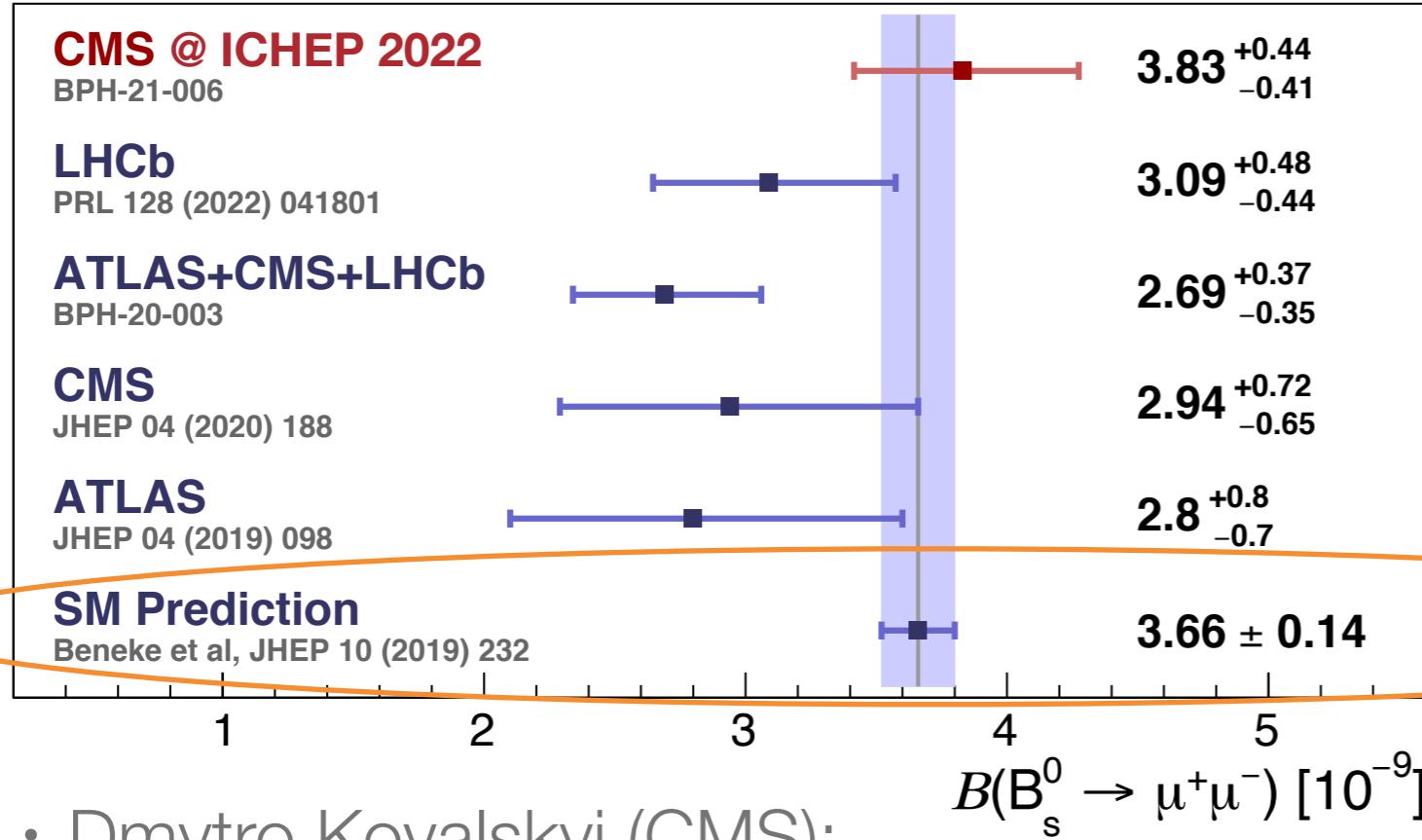
# Leptonic Decays



- Dmytro Kovalskyi (CMS):
  - “Theoretically clean
  - nonperturbative contributions are in  $[f_{B_s}]$ ; well known from lattice QCD.”

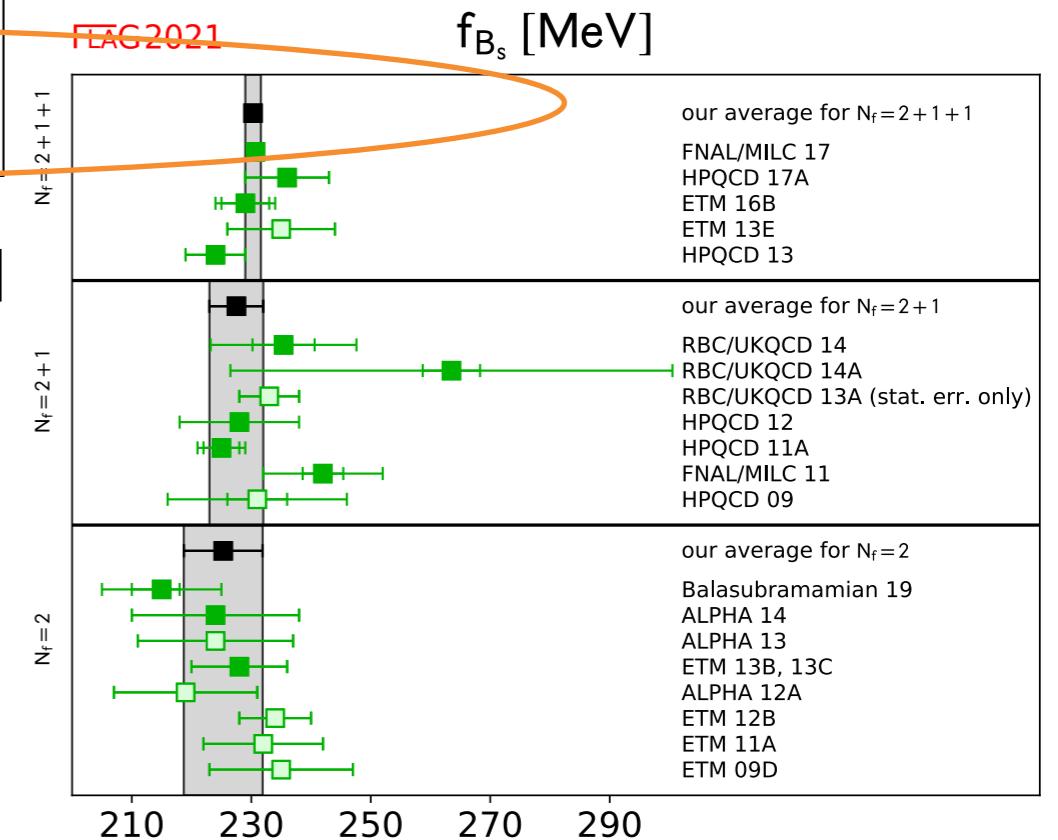


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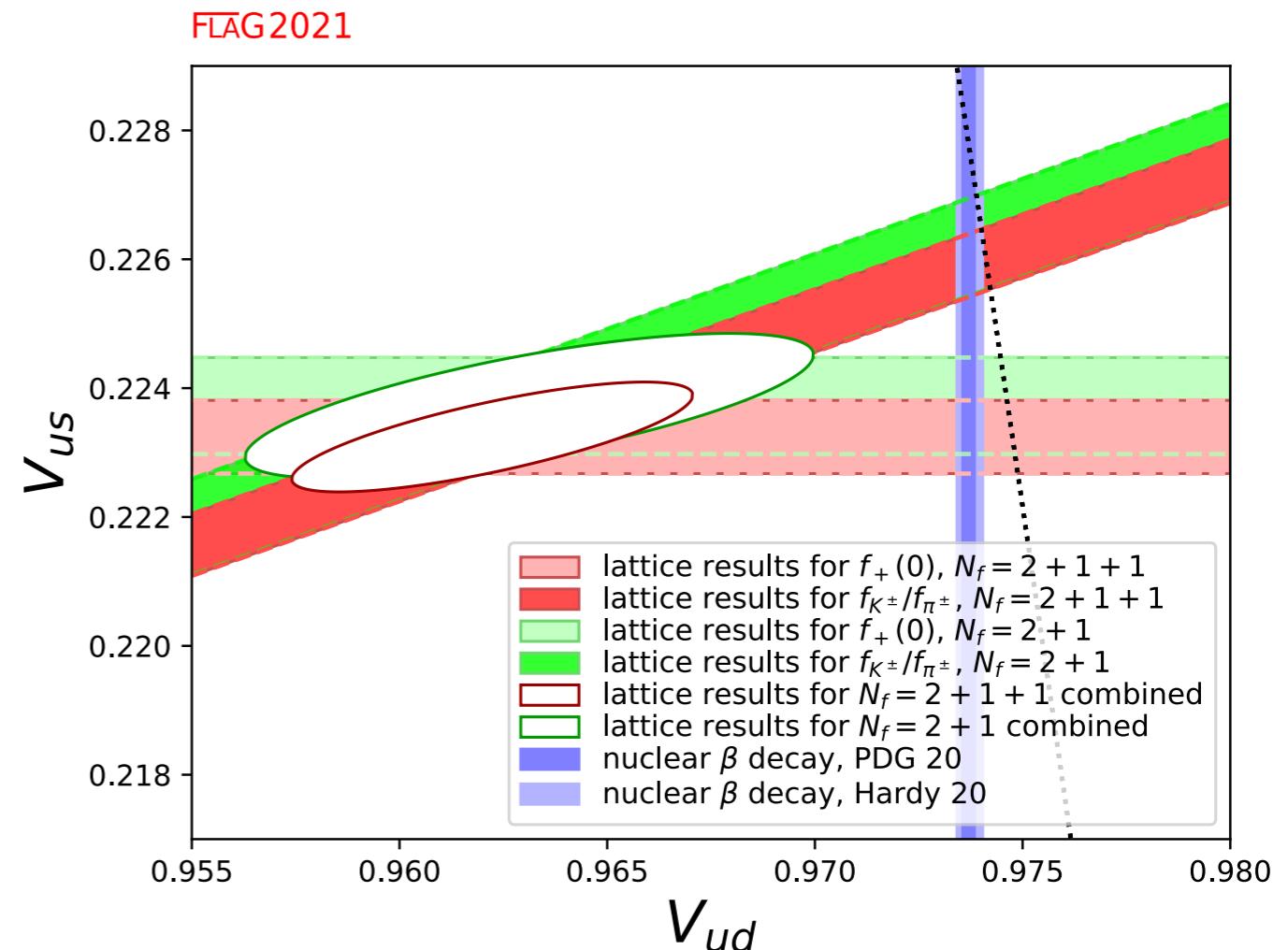
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- CKM:  $B^+ \rightarrow \tau^+\nu_\tau$ .
- BSM:  $B_s^0 \rightarrow \mu^+\mu^-$ .
- ~0.7% uncertainty.



# Cabibbo Angle Anomaly

- Leptonic decays  $\pi_{l2}/K_{l2}$  with  $f_K/f_\pi$  yield  $|V_{ud}|/|V_{us}|$ .
- Semileptonic decay  $K_{l3}$  with  $f_+(0)$  yields  $|V_{us}|$ .
- Superallowed nuclear decay with nuclear theory yields  $|V_{ud}|$ .
- Unitarity crisis ( $3.2\sigma$ ):



$$1 - |V_{ud}|_{0^+}^2 - |V_{us}|_{K_{l3}}^2 = 0.0021(2)(2)(2)(2)(5)_{\text{NS}} \quad [\text{arXiv:2107.14708}]$$

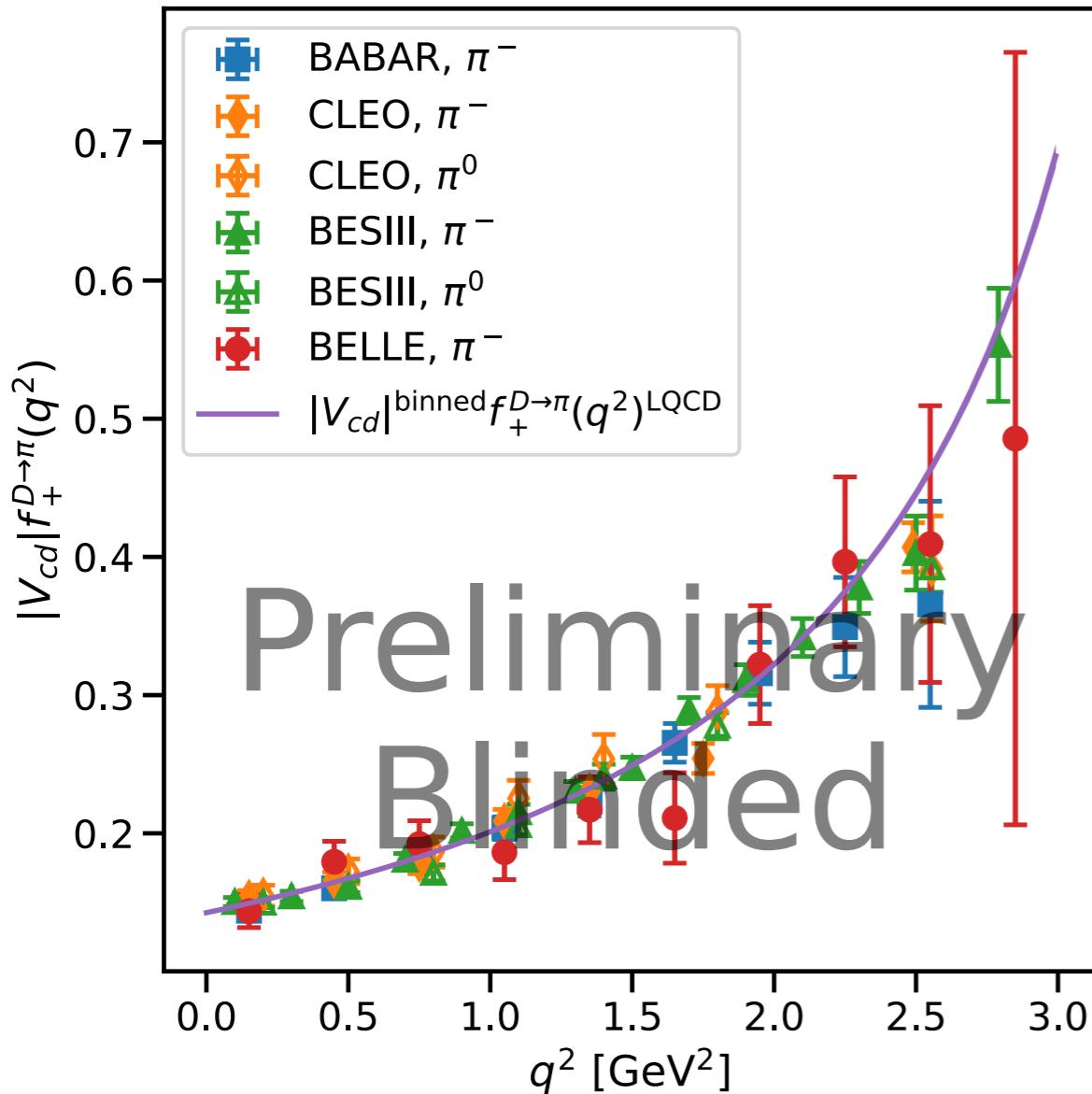
- Improve NS or (why not?) just use  $\pi$  &  $K$ .

nuclear structure

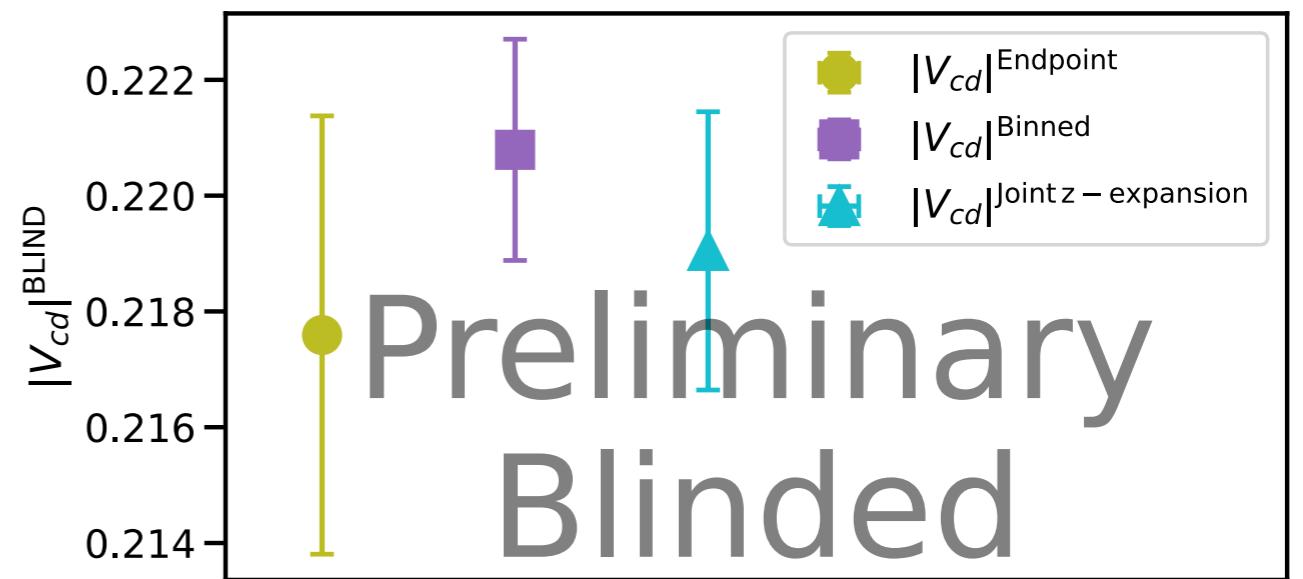


# Coming Soon: $|V_{cd}|$ and $|V_{cs}|$

- Semileptonic decays: great way to determine CKM: check if shapes agree & the fit relative normalization.  
Plots by Will I. Jay.



- $|V_{cd}|$  &  $|V_{cs}|$  to  $\sim 1\%$ .
- Same approach for  $|V_{cb}|$  &  $|V_{ub}|$ :  $(<)1\%$  for  $|V_{ub}|$  ( $|V_{cb}|$ ) in  $\sim 2$  years.



# Flavor Anomalies

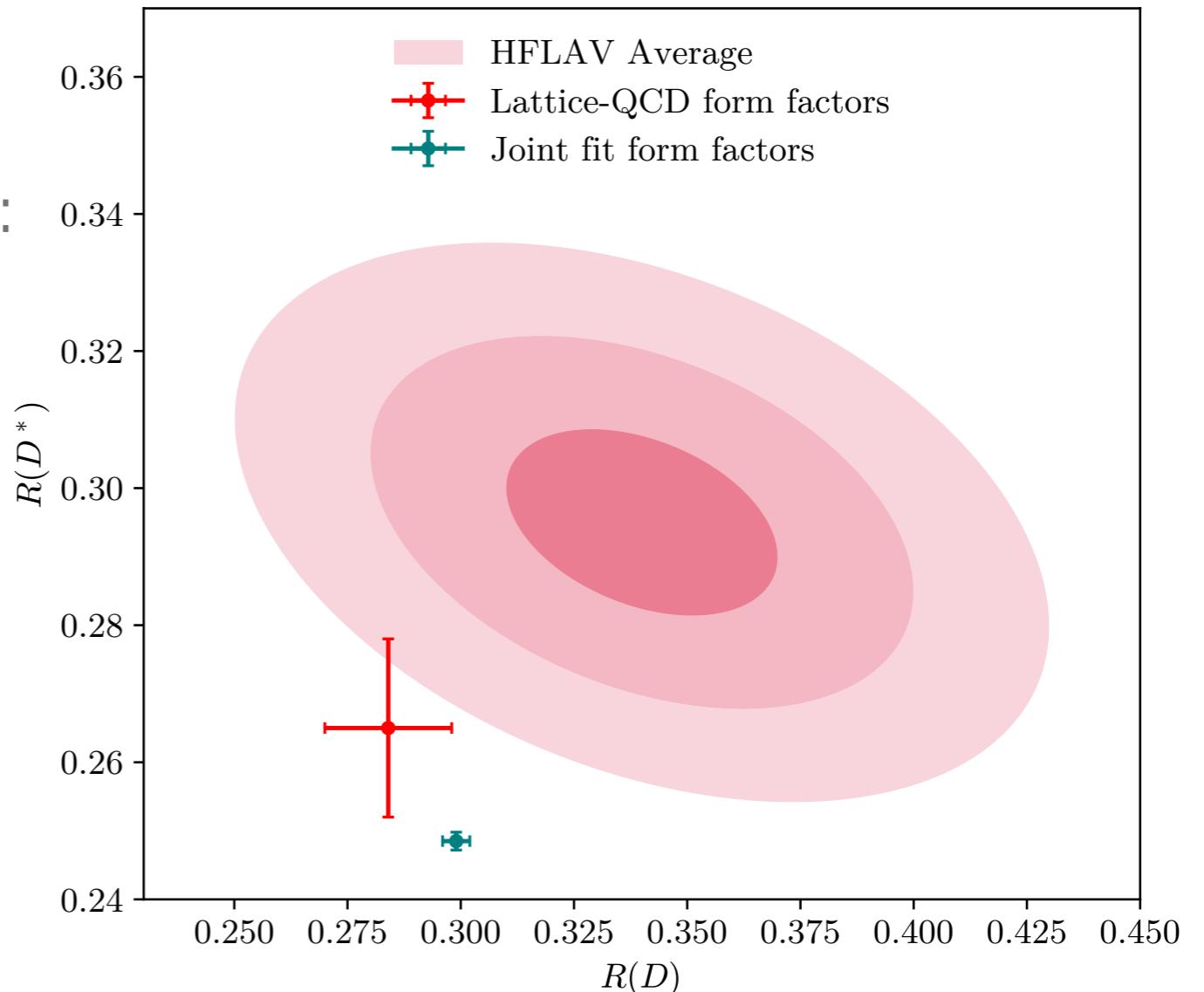
[arXiv:2105.14019](https://arxiv.org/abs/2105.14019)



- Plot by Alejandro Vaquero.
- Lepton-flavor universality violation:

$$R(D^{(*)}) = \frac{\mathcal{B}(B \rightarrow D^{(*)} \tau \nu_\tau)}{\mathcal{B}(B \rightarrow D^{(*)} \ell \nu_\ell)}$$
$$\ell = e, \mu$$

- Lattice-QCD only form factors: errors comparable to data.
- Form factors from  $|V_{cb}|$  fit: errors negligible:  $\sim 4\sigma$  discrepancy.
- Also  $10^3|V_{cb}| = 38.40(68_{\text{th}})(0.34_{\text{expt}})(0.18_{\text{EM}})$ , still in tension with inclusive.



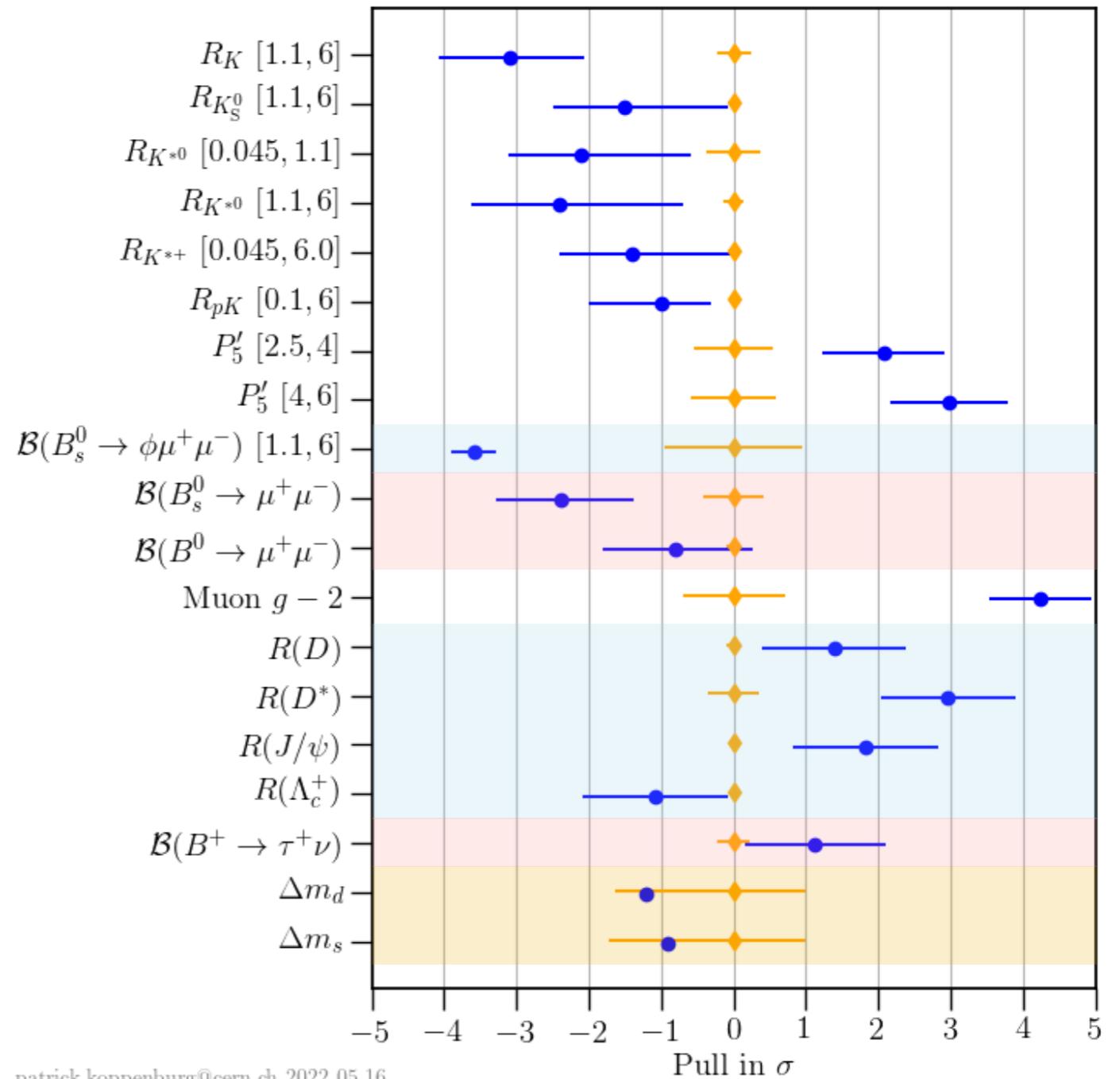
# An Error Budget

[arXiv:2105.14019](https://arxiv.org/abs/2105.14019)

Source	$h_V(\%)$	$h_{A_1}(\%)$	$h_{A_2}(\%)$	$h_{A_3}(\%)$
Chiral-continuum fit error	4.2	2.0	17.4	6.9
(Statistics)	(3.7)	(1.2)	(16.9)	(6.3)
(Chiral-continuum extr.)	(0.8)	(0.9)	(1.7)	(0.5)
(LQ and HQ discretization)	(2.6)	(1.3)	(9.7)	(4.4)
(HQ mistuning)	(0.0)	(0.0)	(1.7)	(0.0)
(Matching $O(am_c \alpha_s)$ )	(0.3)	(0.2)	(1.7)	(0.5)
LQ mistuning	0.0	0.0	0.1	0.0
Matching $O(\alpha_s^2)$	0.7	0.3	0.5	0.3
Scale setting	0.0	0.0	0.3	0.1
Isospin effects	0.1	0.1	0.4	0.2
Finite volume	—	—	—	—
Total error	4.3	2.0	17.4	6.9

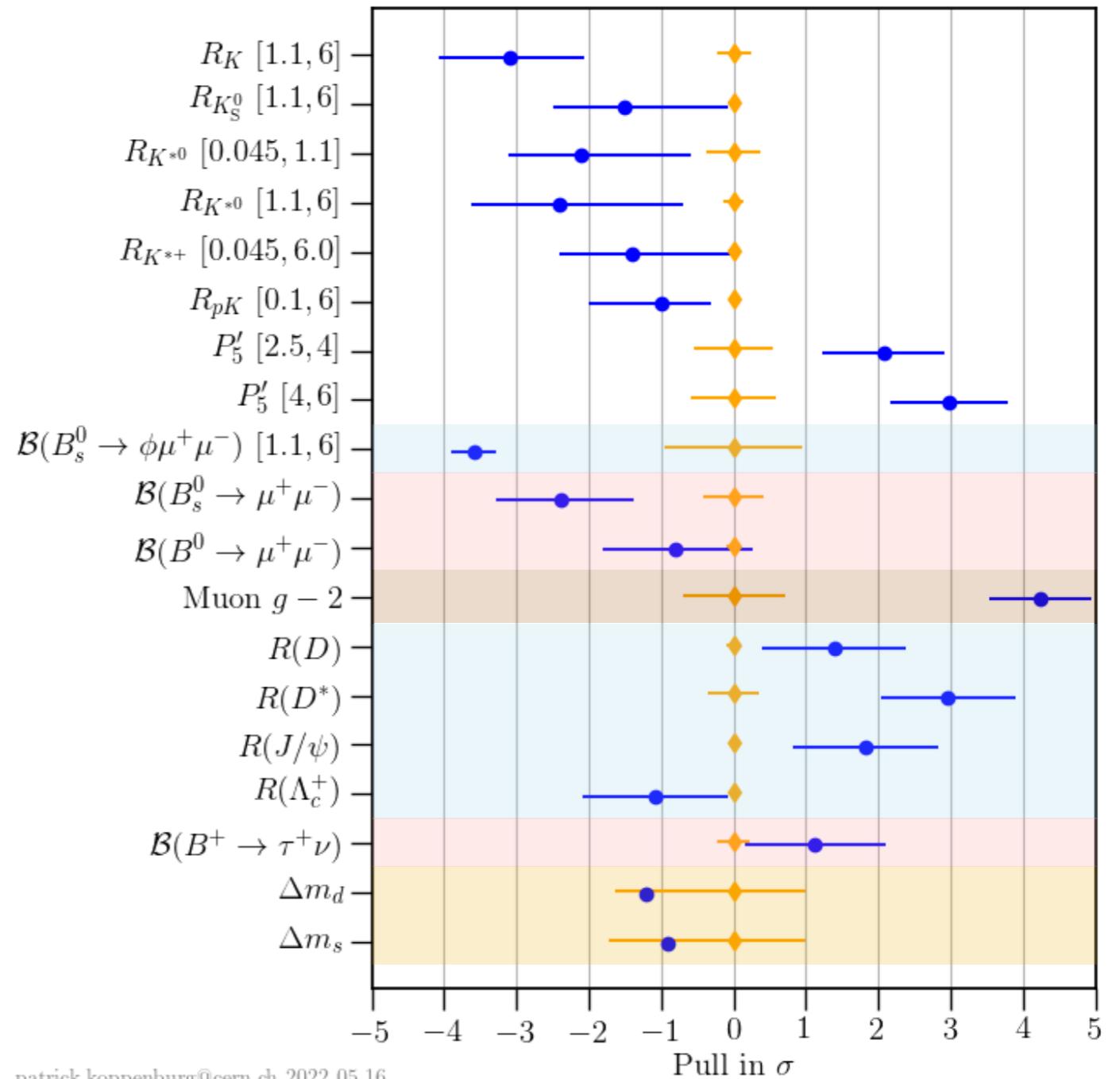
# Flavor Anomalies—Summary

- Many rely on QCD input:
  - decay constants;
  - form factors;
  - four-quark operators.
- (Angular observables and LFUV profit from, but don't rely on, form factors.)
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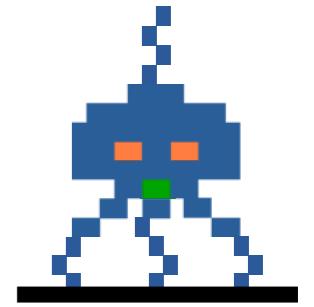


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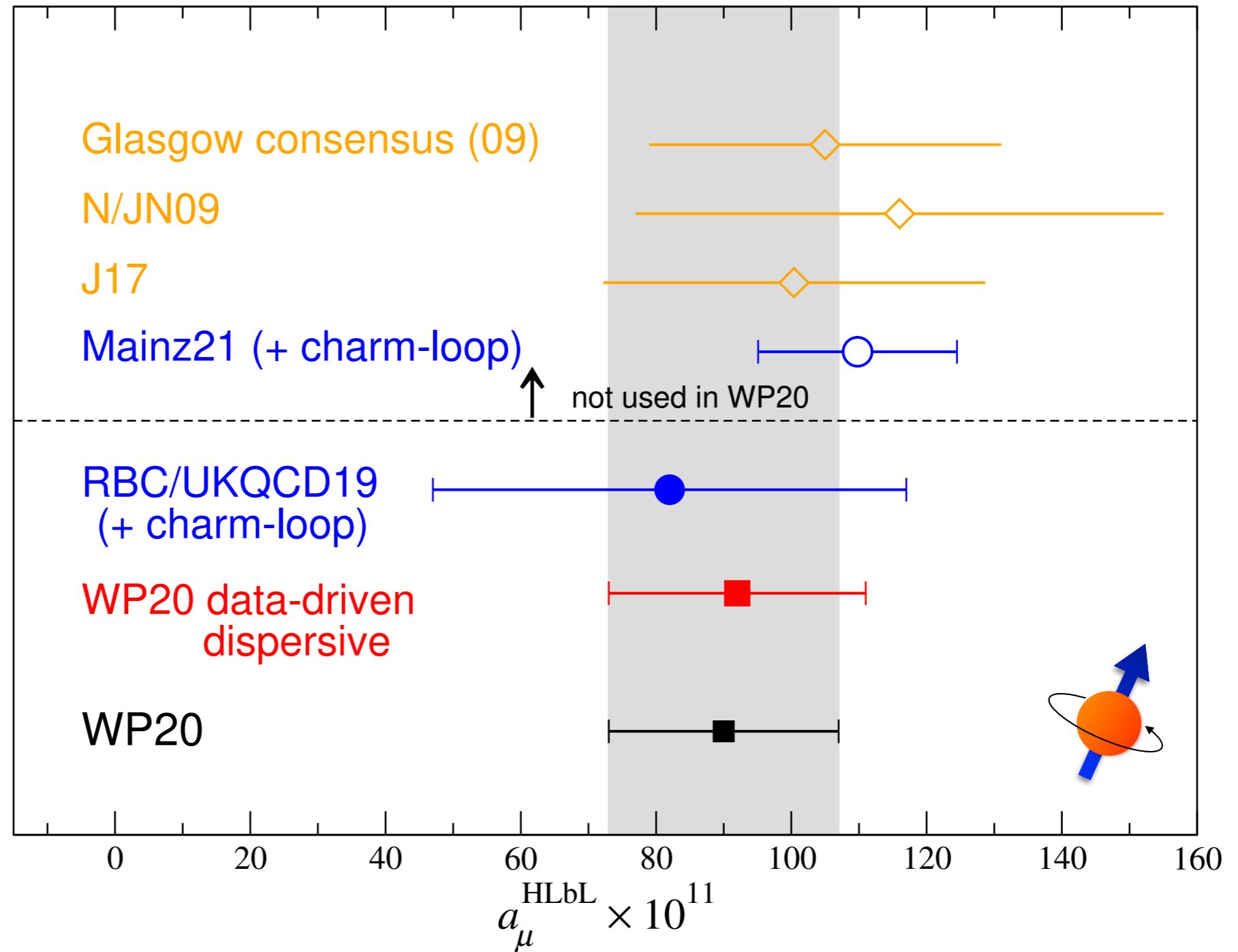


Muon  $g-2$  Again

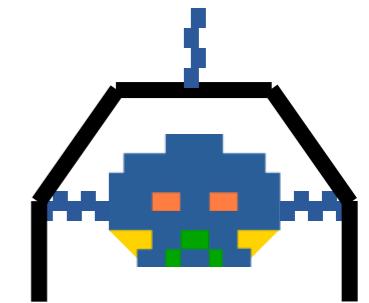


# Hadronic Light-by-Light

- Lattice QCD and dispersive method both support model estimates.
- Agreement is too good and error is too small for HLbL to explain the discrepancy.
- Plot from Mg-2TI [SnowWP](#).

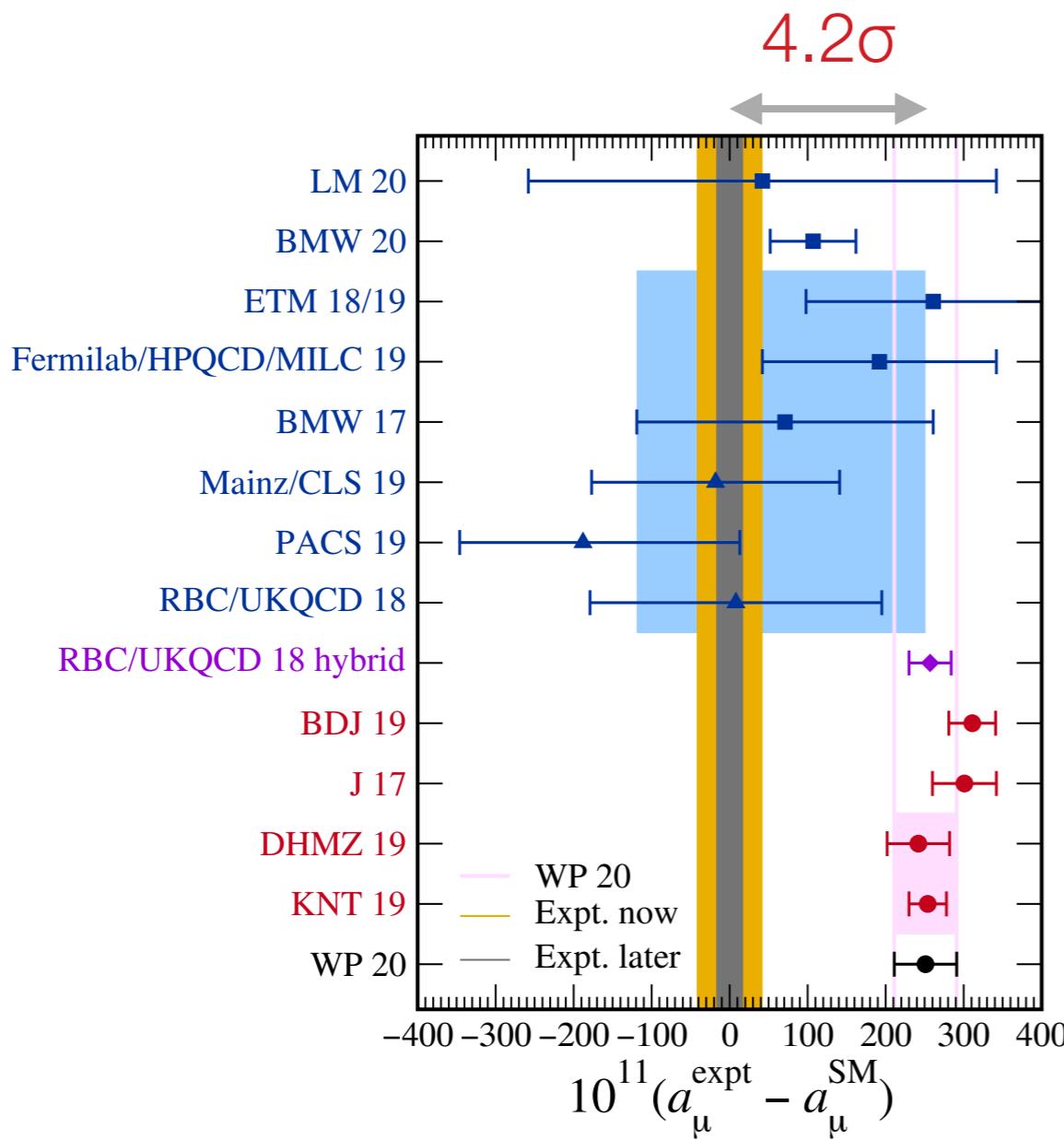


# Hadronic Vacuum Polarization



- Discrepancy: HVP vs. BSM?!?

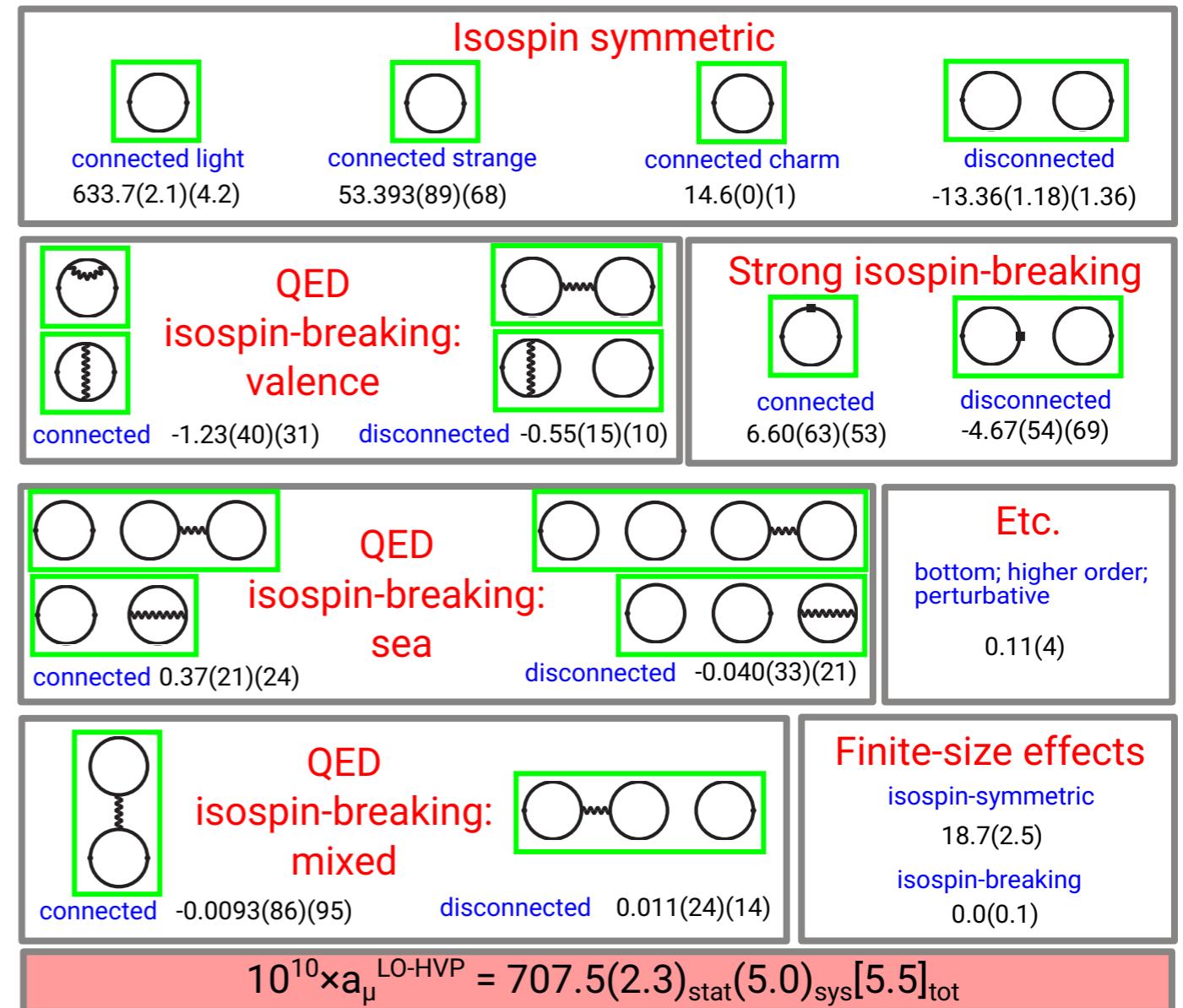
[Mg–2TI, [SnowWP](#)]



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Expt – SM	251 (59)	(22)	

# Many Contributions

- $u, d, s$  connected dominates.
- Also need conversion from QCD units to muon units:  $m_\mu/M_\Omega$ .
- Need independent lattice-QCD calculations:
  - Fermilab Lattice+MILC+ HPQCD; Aubin *et alia*;
  - xQCD; RBC+UKQCD;
  - Mainz/CLS; ETM;
  - BMW.



BMW arXiv:2002.12347

# Road to Precision

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- Goal is 0.2%, which will be  $0.5\%/\sqrt{\text{few}}$ .
- Road to precision will involve jumping through windows [[arXiv:1801.07224](#)]:

$$a_\mu = a_\mu^{\text{SD}}(t_0, \Delta) + a_\mu^{\text{W}}(t_0, t_1, \Delta) + a_\mu^{\text{LD}}(t_1, \Delta)$$

- TI put forward intermediate window as a useful point of comparison.
- Also amenable to  $R(e^+e^-)$  via Laplace transform.
- Very active: [arXiv:2111.15329](#) (ETM),  
[arXiv:2204.01280](#) ( $\chi$ QCD),  
[arXiv:2206.06582](#) (Mainz/CLS),  
[arXiv:2206.15084](#) (ETM),  
[arXiv:2207.04765](#) (Fermilab/HPQCD/MILC).

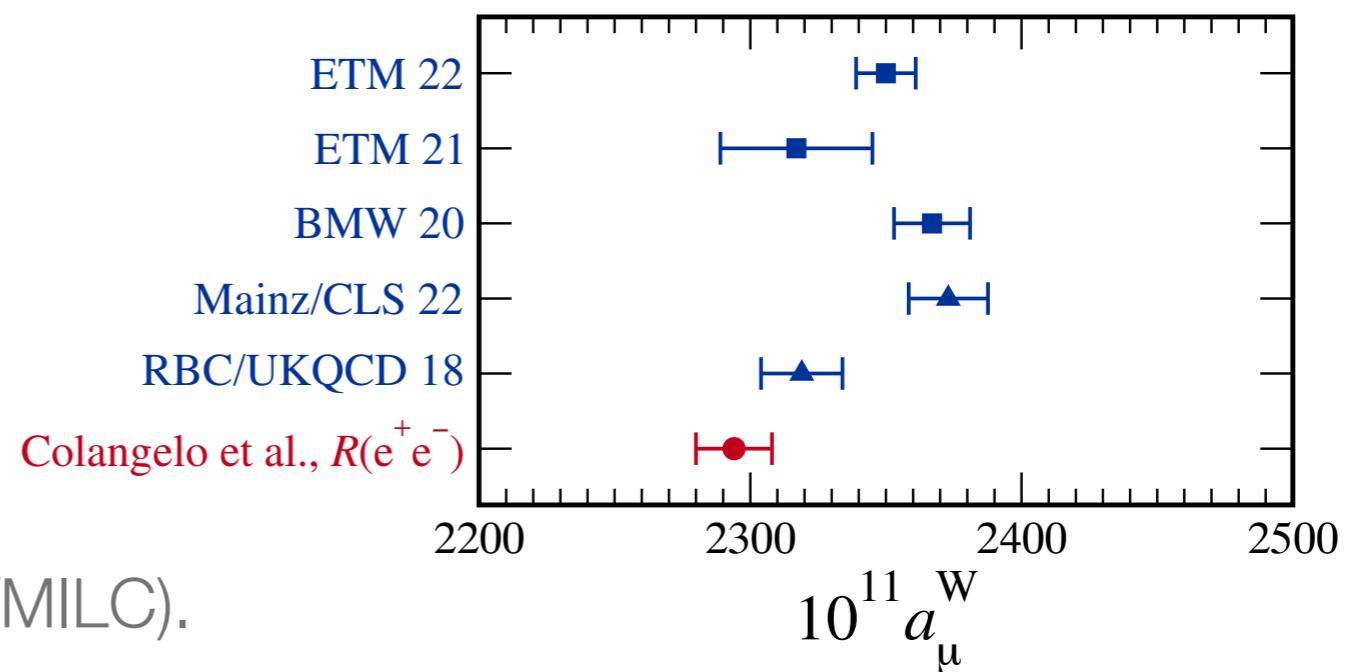
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# Enabling Precision Lattice **QCD**

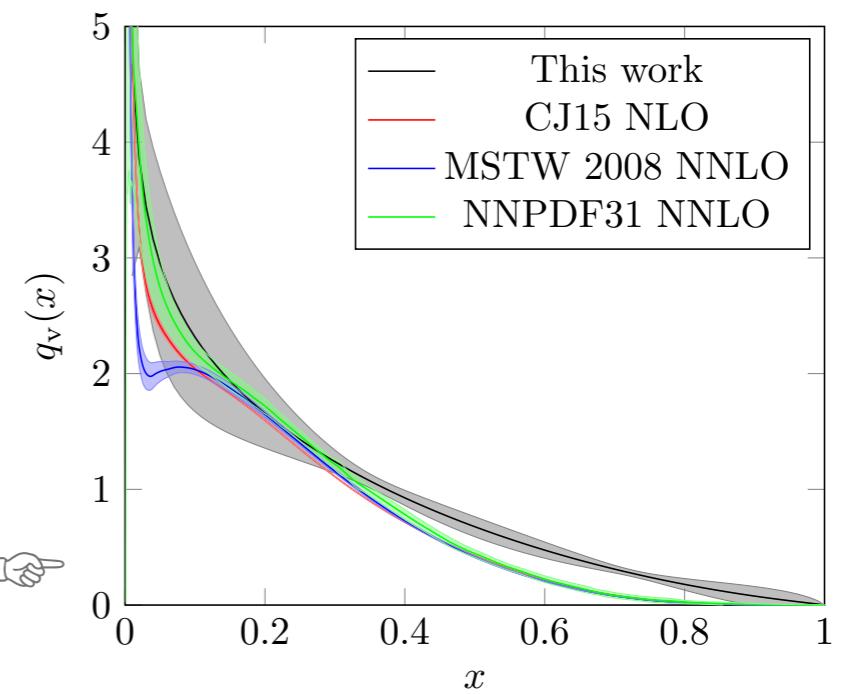
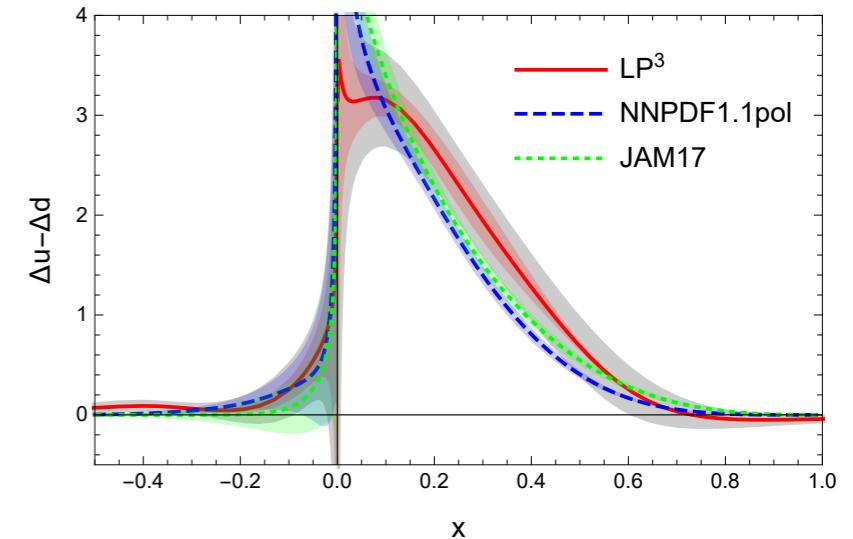
# Future Precision Topic I

- Parton distribution functions [[SnowWP](#)]:

- first few moments for several years;
- explosion of numerical and theoretical work:
  - starting w/ Ji's introduction of LaMET [[arXiv:1305.1539](#)], a framework to connect space-like to light-cone distributions in Bjorken  $x = -q^2/2p \cdot q$ ;
  - quasi-PDFs, pseudo-PDFs, hadron tensor,  
....

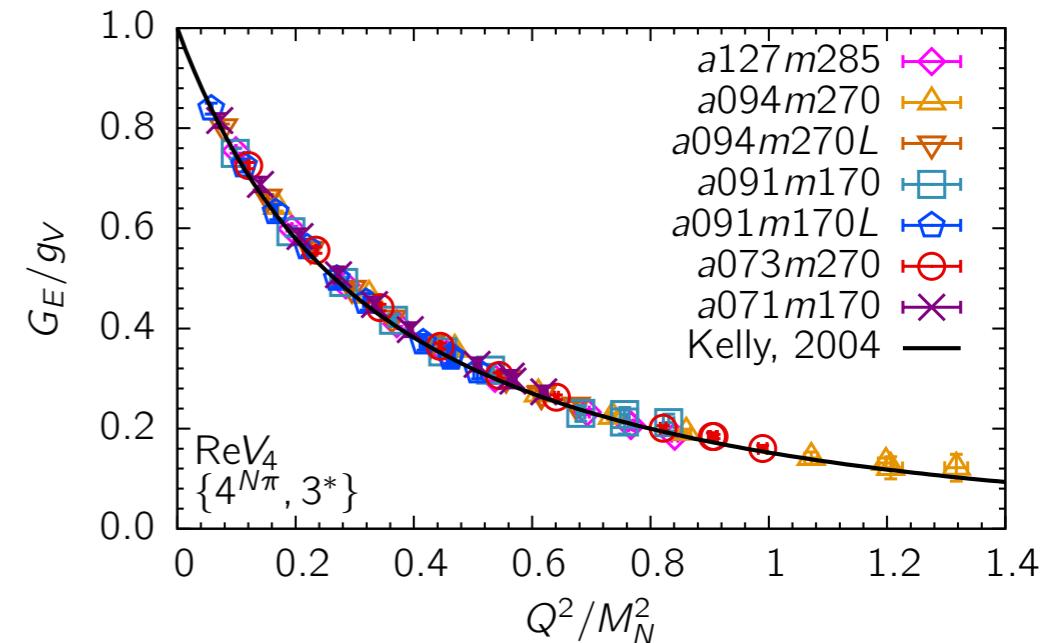
HadStruct [arXiv:2004.01687](#)

LP3 [arXiv:1807.07431](#) ↴

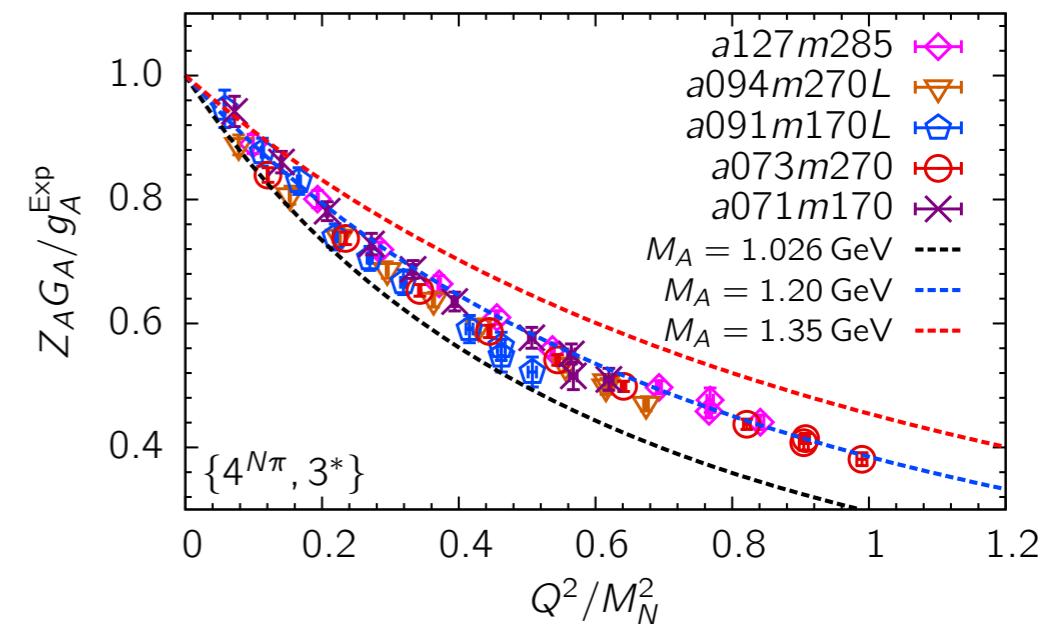


# Future Precision Topic II

- Nucleon form factors [[SnowWP](#)]:
  - nucleon electric form factor agrees well with measurements from  $ep$  scattering, even before continuum extrapolation;
  - nucleon axial form factor is needed in **vA scattering** yet not directly accessible: use, e.g., **vD** data.
- Both on cusp of results with full error budget, but then ...
  - ... combine with nuclear theory ([next talk](#)).



NME [arXiv:2103.05599](#)



# Enabling Precision

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- The push beyond 1% requires QED ( $q_d \neq q_u$ ) and strong isospin breaking ( $m_d \neq m_u$ ). QED requires theoretical development as well as computational.
- Lattice QCD is (mostly) carried out by collaborations with a mix of skills:
  - computing, phenomenology, theory;
  - on the computing side, we have been generously supported by the Exascale Computing Project (ECP from DOE ASCR), but post-ECP (FY24) not so clear.
- Lattice gauge theory (QCD and BSM) is a tool valuable in a wide variety of topics in particle physics, nuclear physics, and astrophysics.

Thank you for your attention!

**USQCD @CompF: [arXiv:2204.00039](#)**  
**USQCD @TF: [arXiv:2207.07641](#)**

# Subset of Topics Not Covered

- Kaon mixing:  $\Delta M_K$  and  $\varepsilon'/\varepsilon$ —see Chris Kelly.
- Hot, dense QCD: heavy-ion collisions, early universe.
- Hadron spectroscopy: LHCb, Belle II, BES III, GlueX; also widths, scattering lengths, etc., from FV effects.
- BSM spectroscopy:
  - composite dark matter such as dark glueballs condensing out of a DGP in a first order phase transition;
  - composite Higgs models—light “impostor” with rest of spectrum just out of LHC reach.
- Topology vs. temperature for axion thermal mass as universe cools.

