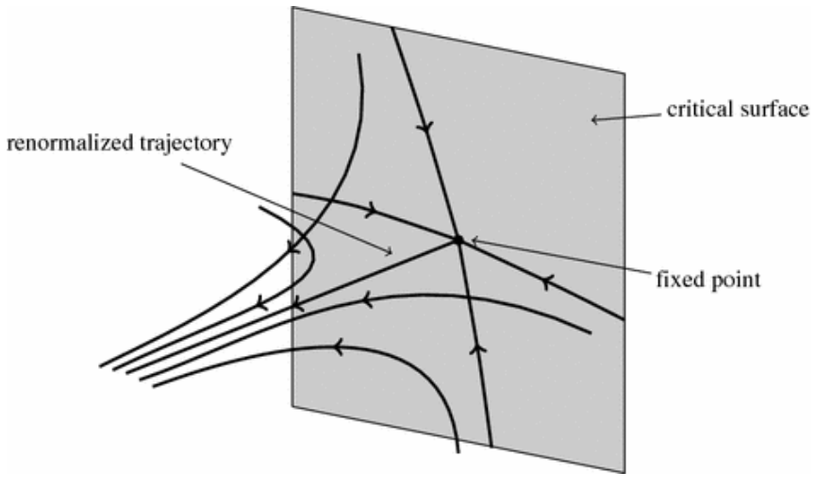
String theory, QFT, and all that…

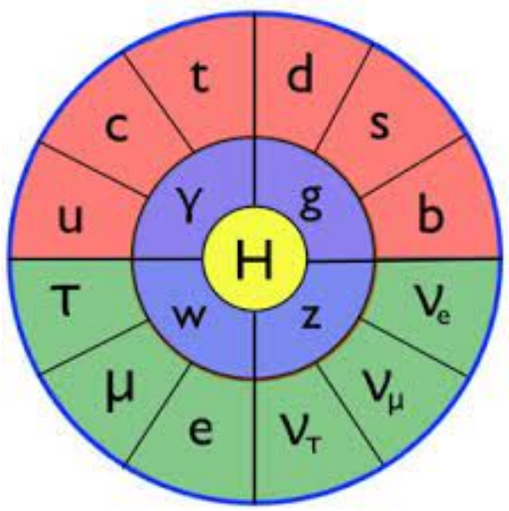
Snowmass @ Seattle

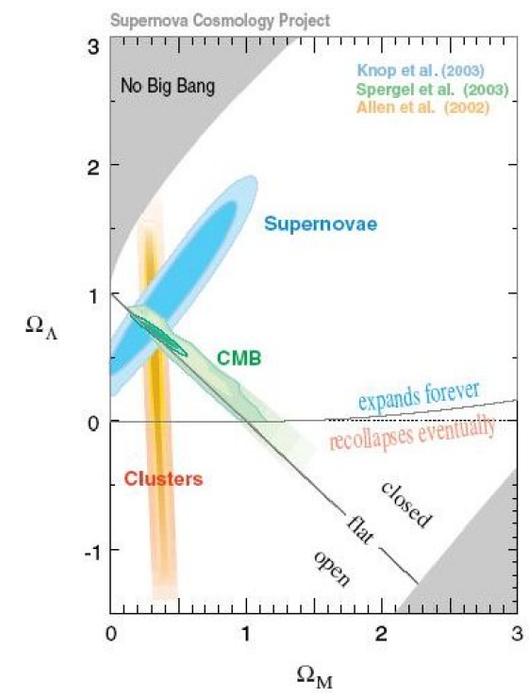


Shamit Kachru
Stanford

Introduction

“Fundamental” physics — whatever it may mean — has made tremendous progress in the last ~ century.





The Standard Model and the Concordance Cosmology can explain a bewildering host of phenomena at scales ranging from subnuclear to our present Hubble volume.

Every answer raises more questions.

— What is the origin of the Standard Model gauge group and charged particle content?

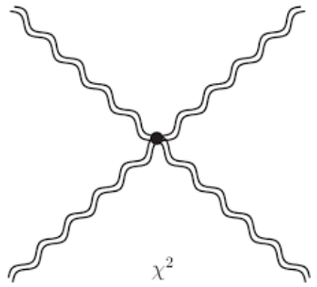
— Is the dark energy a cosmological constant (as measurements of w hint)? Can we understand its value?

— Does cosmic inflation explain the approximate flatness of todays Universe? Inflation at what scale, with what properties?

— How do we understand the hierarchies visible in Yukawa couplings of the Standard Model?

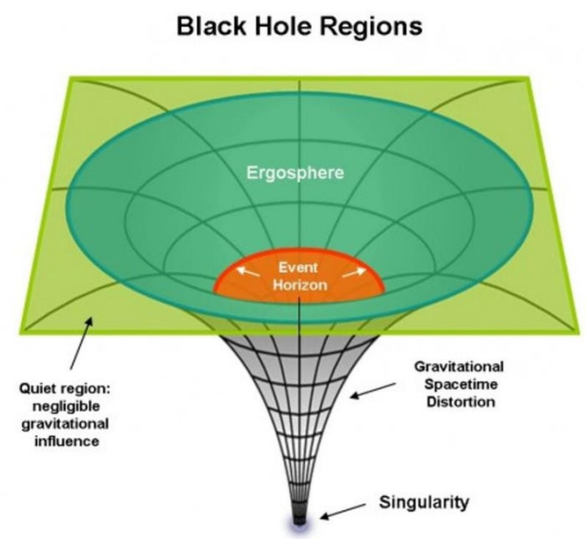
— Why/how is the Higgs boson so light?

These are questions directly motivated by data. But the same framework also suggests more qualitative questions.



Sewing together graviton vertices to make a perturbation theory for quantum gravity leads to UV divergences. How to cure?

General relativity also has black hole solutions. There are qualitative puzzles associated with these objects.



There are singularity theorems governing relativistic cosmology. How do we understand the initial singularity?

We don't have any examples of theories which we know to satisfactorily answer the full second set of questions (even at the level of "toy models" that do not accurately describe Nature).

All these questions have helped motivate an intense research program on quantum field theory and string theory over the past decades.

Today I will try to report on where some of that has gone / is going. The community of researchers is large and diverse in origin, interests, and opinions. The talk will be guided by my own biases and limited competence.

The rest of the talk will consist of…

II. Some lessons about quantum field theory
III. Some lessons about gravity
IV. Some important directions

II. Some lessons in quantum field theory

When we take a quantum field theory class, we (used to?) learn about QFT by starting with an expansion of a free field in terms of harmonic oscillator "creation and annihilation operators" that create or destroy single quanta:

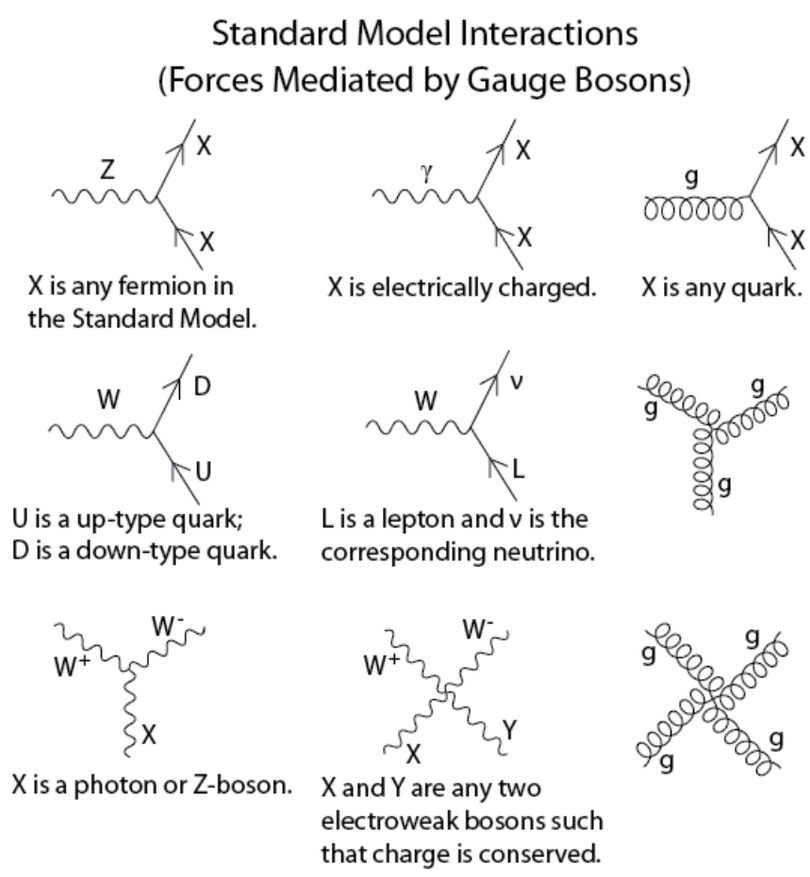
$$\phi \sim \sum_k a_k e^{ikx} + a_k^\dagger e^{-ikx} \ .$$

Then treating interactions in perturbation theory, we could study theories like that with the Lagrangian

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!} \phi^4 + \cdots$$

This leads to a set of scattering processes whose amplitudes are computable in terms of Feynman diagrams.

It is a brief hop, skip, and jump to the Feynman rules of the Standard Model:



Of course perturbation theory has enjoyed tremendous success in this setting!

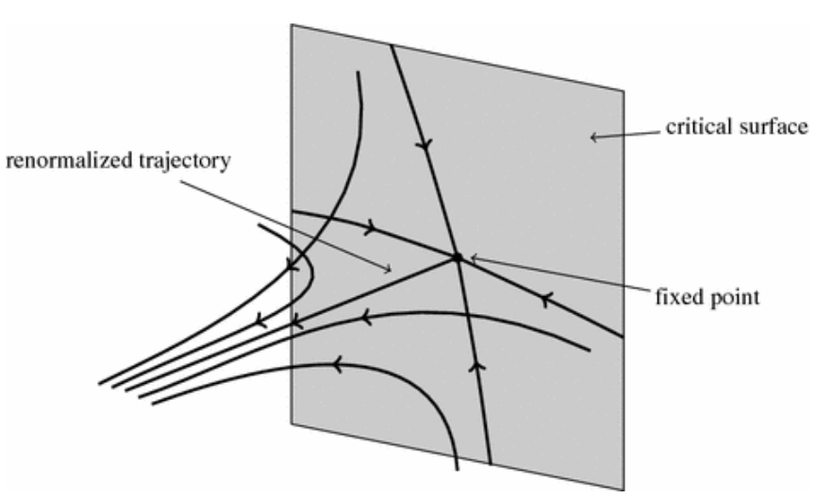
An old-fashioned viewpoint was that particular Lagrangians — those for "renormalizable theories" — were especially preferred.

A more modern viewpoint is that of effective field theory. We keep terms that are naively non-renormalizable, suppressed by appropriate powers of a high scale.

Classifying operators in L by their scaling dimensions under the rescaling of space and time

$$t \to \lambda t, x \to \lambda x$$

we learn that at low energy / long distances, we can (often) ignore operators which scale to zero ("irrelevant operators"), while focusing on those which are relevant or marginal.



This leads to the Wilsonian picture on my opening slide, depicting some renormalization group trajectories — studies of how the couplings in a QFT flow as one goes to long distance.

The fixed points of this flow are then natural QFTs to consider. They are the starting or ending points of renormalization group flows. They contain the answers to all "universal" questions.

In particular, they capture (universal properties of) second order phase transitions in real statistical systems.

A great deal of effort has gone towards understanding and classifying little patches of the space of conformal field theories.

Taxonomy (the science of classification) is often undervalued as a glorified form of filing-with each species in its folder, like a stamp in its prescribed place in an album; but taxonomy is a fundamental and dynamic science, dedicated to exploring the causes of relationships and similarities among organisms. Classifications are theories about the basis of natural order, not dull catalogues compiled only to avoid chaos.

Stephen Jay Gould

QuoteMaster.org

Examples:

- **2d CFTs**
- **the conformal bootstrap**
- **the modular bootstrap**
- **supersymmetric CFTs**

....

Let me discuss a few lessons from these studies.

Specialization to d=2 space-time dimensions

In 2d, the conformal group becomes infinite dimensional. The power of this bigger group of symmetries is considerable.

Example: the IR behavior of theories such as

$$L = (\partial\phi)^2 - g\phi^{2n}$$

can be understood and solved exactly (for all n). All correlation functions of the various operators in the quantum theory are determined by well specified solutions to concrete differential equations.

The 2d case is not an irrelevant peculiarity for two reasons.

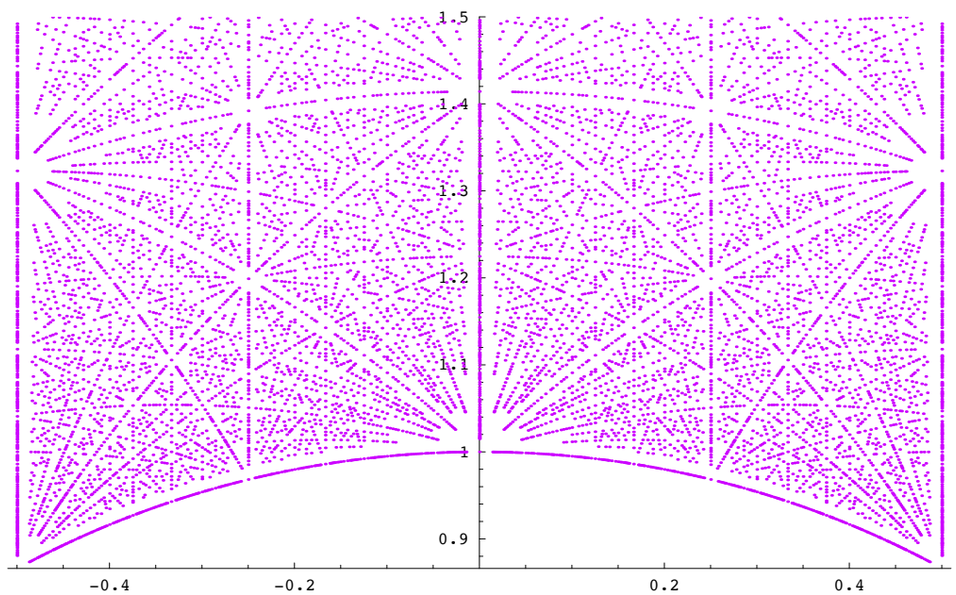
— critical phenomena in 2d arise in labs

— this is the case relevant to the world sheet of fundamental strings in string theory! (It is an old miracle that the equations for conformal invariance of the 2d world-sheet theory, are the space-time equations of motion.)

A 2d CFT, e.g. arising from a theory of maps from the worldsheet to a curved target space manifold, lives here.

It is not true that general 2d CFTs can be exactly solved. E.g., the sigma models mapping the string worldsheet to standard manifolds used in string compactification (Calabi-Yau manifolds) are rarely soluble.

Early on theorists speculated that perhaps the "space of all 2d quantum field theories" provides a suitable off-shell configuration space for string theory, with the CFTs cutting out on-shell configurations.



There was even an optimistic conjecture that exactly soluble 2d CFTs would be "dense" in the space of 2d CFTs relevant for strings.

This gives an illustration of how ideas about the full space of (conformal) field theories can potentially be important.

The conformal bootstrap

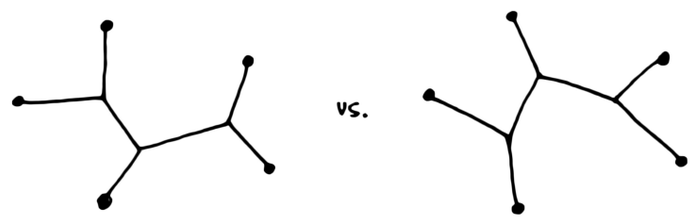
Given a CFT, one can give a list of its operators. A special set called the "primary operators" are most important.

A fundamental role is played by the operator product expansion of the (primary) operators of the theory:

$$\mathcal{O}_i(x_1)\mathcal{O}_j(x_2) = \sum_k C_{ijk}(x_{12}, \partial_2)\mathcal{O}_k(x_2),$$

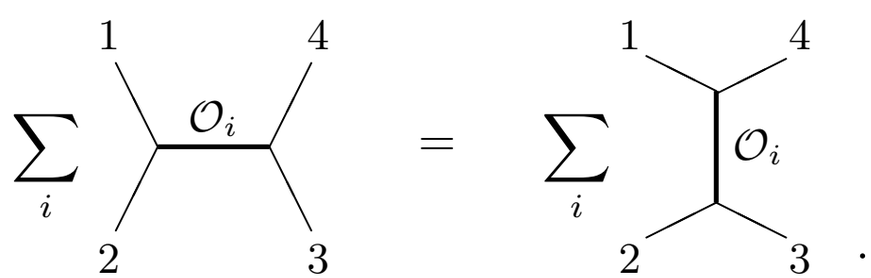
Very roughly, one can totally specify the CFT by giving the following data:

- **a list of primary operators and their scaling dimensions**
- **three-point correlation functions of those operators (which boils down to the C_{ijk})**



But not all sets of data are allowed! There are crossing conditions, which basically impose that one should be able to take OPEs of different operators in different orders and still get the same answer.

These equations can be summarized as:



Consider a four-point function of four identical scalar operators.

Concretely, for a given operator appearing in the intermediate channel, we get a contribution to the four-point function governed by a particular function of the (cross-ratios of the) four points of insertion:

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{g(u,v)}{x_{12}^{2\Delta_\phi} x_{34}^{2\Delta_\phi}}$$

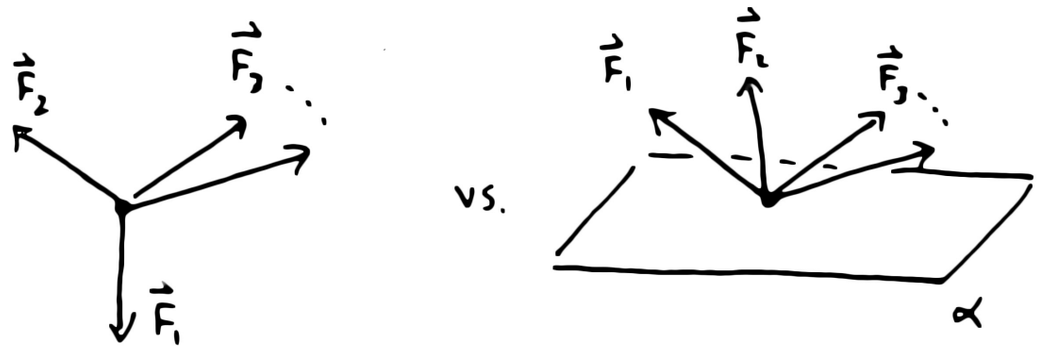
$$g(u,v) = \sum_{\mathcal{O}} f^2_{\phi\phi\mathcal{O}} g_{\Delta,\ell}(u,v)$$

We can actually write that equation two ways, by either fusing the operators at locations 1,2 and 3,4, or at 1,4 and 2,3. So we see:

$$\sum_{\mathcal{O}} f^2_{\phi\phi\mathcal{O}} \underbrace{\left(v^{\Delta_\phi} g_{\Delta,\ell}(u,v) - u^{\Delta_\phi} g_{\Delta,\ell}(v,u)\right)}_{F^{\Delta_\phi}_{\Delta,\ell}(u,v)} = 0.$$

positive!

vectors in some big space of functions



You can’t always get a bunch of vectors to sum to zero with positive coefficients.

This allows one to test proposed CFT spectra numerically in a very precise way.

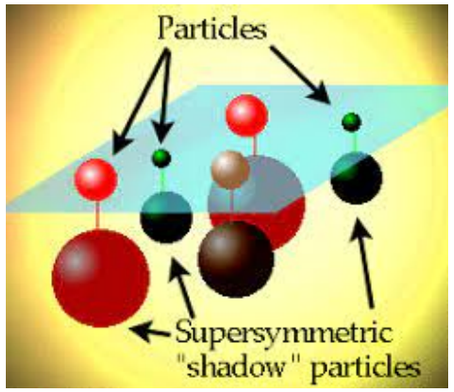
If one can find positive linear functionals acting on the (combinations of) blocks F appearing on the LHS, then the proposed OPE is inconsistent.

Generalizing this strategy has led to strong constraints on which simple CFTs can exist, including constraints on the precise OPE coefficients.

The really salient point here is that modern computing has grown so powerful that “brute-force” approaches likes this can now teach us a lot!

One goal of condensed matter physics has been to classify phases of matter. Classifying the possible critical theories would be a meaningful step in this direction. While neither the special power of 2d nor the bootstrap allow a complete solution of this problem, our knowledge of theory space has been considerably enlarged by both.

Last example: Supersymmetry



Supersymmetry has been a very well studied candidate for BSM physics.

There is still a lot of phase space to explore, and we have our

Supersymmetry has taught us a lot about theory space, regardless.

Maybe the prototype supersymmetric theory — in some sense, the simplest interacting QFT in four dimensions — is maximally supersymmetric Yang-Mills theory:

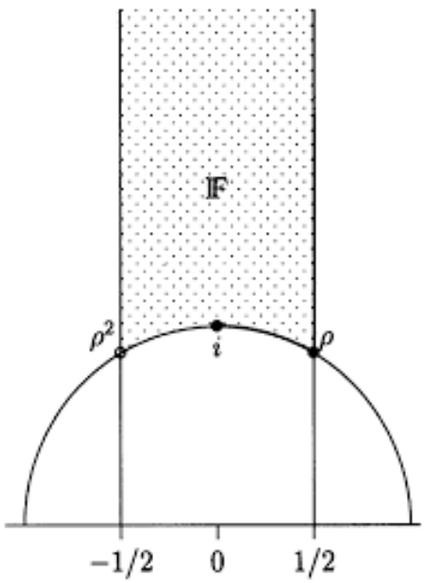
$$\text{Fields} : A_\mu, \phi, \lambda$$

$$L = \frac{1}{g^2} tr(F_{\mu\nu}F^{\mu\nu}) + tr(D_\mu\phi)^2 + \theta\ tr(F\tilde{F}) + \ldots$$

Remarkably, this theory has a full half-plane of conformal fixed points — one for each value of the Yang-Mills coupling and the theta angle.

First lesson: Conformal manifolds can exist and are interesting (though very non-generic).

Even more remarkably, what I told you is inaccurate in a beautiful way.



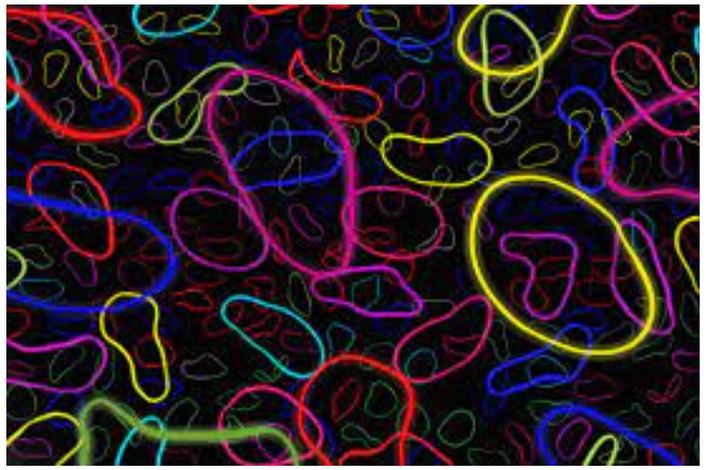
The theory enjoys a remarkable "duality" symmetry: the dualities relate different points on the plane of couplings $(\frac{1}{g^2}, \theta)$ **.**

By varying the couplings and the gauge group (e.g. SU(N) for any value of N), one finds in this way many theories whose properties differ significantly from anything one would see via perturbation theory from a free fixed point.

III. Lessons in gravity

We now segue to a discussion of gravity. This may seem like an abrupt change; I promise a connection will emerge.

It has been known for many years that there is a miraculous theoretical structure that is called "string theory". The name is inherited from the fact that in limits of its parameter space, it is well described by perturbative strings.

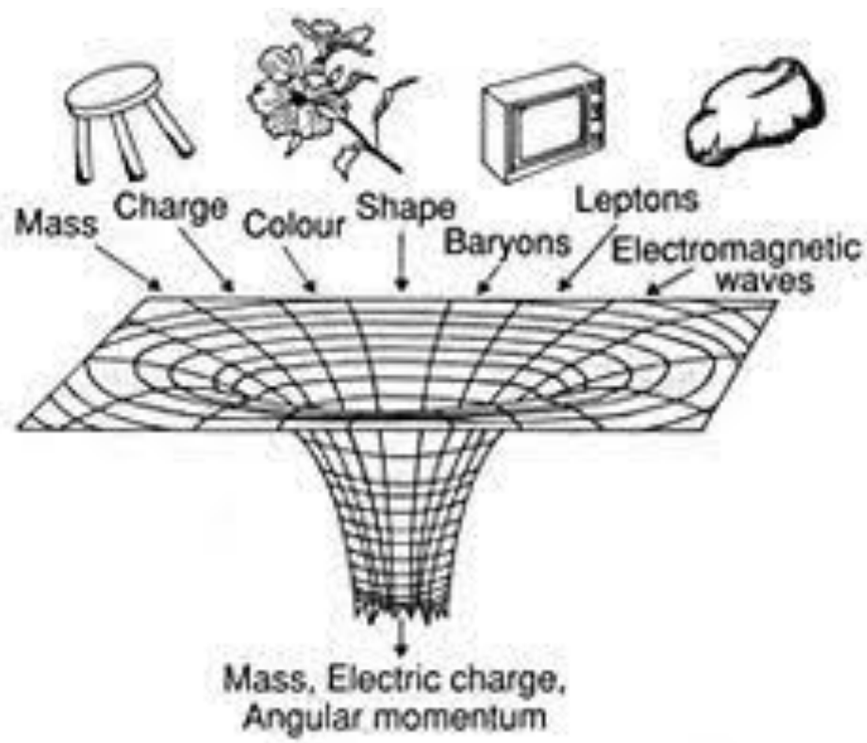


Some of the most exciting lessons of recent decades have come by trying to take the quest to find non-perturbative formulations seriously.

A major step was taken by thinking carefully about the entropy of black holes.

Work by general relativists in the 1970s taught us some striking facts:

1 — Black holes have no hair



2 — The space of black solutions in general relativity obeys a set of identities…

$$dM = \kappa \; dA + \mu \; dQ + \Omega \; dJ$$

$$dA \geq 0$$

…with notable similarity to the laws of thermodynamics.

In this analogy, the area of the event horizon "A" plays the role of an entropy. (Its coefficient above is the Hawking temperature!)

This is striking, as a black hole encloses a volume's worth of space, and conventional statistical systems have extensive entropy.

Careful study of higher-dimensional black holes in string theory has leveraged this observation into a remarkable tool — the "AdS/CFT" duality.

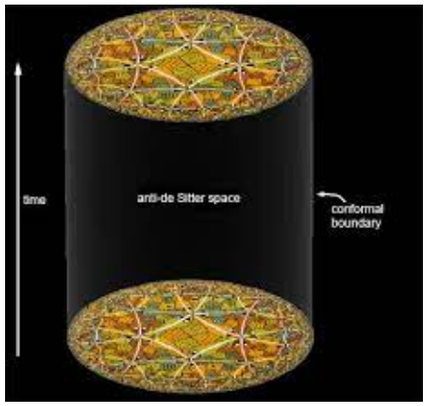
$$L \sim \sqrt{-g}\,(R - 2\Lambda) + \cdots$$

The starting point for what most of us mean by gravity is the Einstein-Hilbert action, perhaps with a cosmological term. With negative vacuum energy, some basic facts are:

— the maximally symmetric gravity solution is AdS space:

$$ds^2 = r^2(\eta_{\mu\nu}dx^\mu dx^\nu) + \frac{dr^2}{r^2}$$

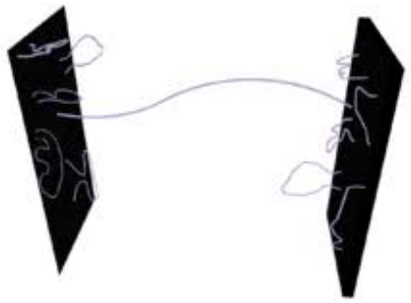
The (d+1)-dimensional AdS space has the structure of a can:



There is a peculiar coincidence: the symmetries of its metric are in accord with the conformal group in d space-time dimensions.

This coincidence is explained by the following observations:

In various examples, string theory has d-dimensional extended "branes" whose worldvolume supports a QFT.



Consider for specificity branes of spatial dimension three, whose world-volume fields are governed by a 3+1 dimensional quantum field theory.

These branes carry stress-energy. They source a solution of the Einstein equations in 10-dimensional string theory.

If the number of branes is large, the solution stays "weakly curved" near its core (where the branes are), and in fact in a suitable "close to the brane" limit it is precisely of the form

$$AdS_5 \times S^5$$

Heuristically, the "radial" direction in the cylinder picture takes us towards the branes (at the center of the AdS).

What is the field theory on the D3-branes? It is

$$\mathcal{N} = 4 \text{ super Yang} - \text{Mills}, \ \mathcal{G} = SU(N), \ g_{YM}^2 = g_s \ .$$

These observations led to the conjecture that IIB string theory in AdS5 and maximally supersymmetric YM theory are two descriptions of the same object!

The SYM is the theory governing the horizon entropy, in analogy to our earlier discussions of black holes.

The curvature of the AdS space is:

$$R^2 \sim g_{YM}^2 N$$

To get a description of this theory in the regime where curvatures are weak and string theory effects are small corrections to supergravity, we want:

$$N \to \infty, \;\; g_{YM}^2 N \text{ fixed, large}$$

These field theories are some of the peculiar beasts on the conformal manifold of maximally SUSY Yang-Mills theory!

The peculiar properties of these QFTs include:

- a large number of degrees of freedom
- a sparse spectrum of operators at low conformal dimensions
- large anomalous dimensions for operators not protected by SUSY

This full list is never true of CFTs we access by doing perturbation theory around free fixed points.

But by learning about these objects, we have stumbled upon a new type of free description for a QFT: in terms of higher dimensional supergravity!

In addition to providing an amazing example of emergence, this duality provides us with a candidate non-perturbative definition of string theory, with very particular asymptotics.

Two significant foci of recent work:

- obtaining a deeper understanding of how this kind of duality works
- pushing to expand to more interesting space-times

IV. Some important directions

Here I discuss, in cartoon terms, two directions that I find important.
There are others; my time is limited.

A. Nuts and bolts of AdS/CFT

A correspondence between a d-dimensional theory (without gravity) and a d+1-dimensional theory (with gravity) obviously begs certain questions.

How are degrees of freedom of the gravity theory encoded in the QFT?

Can we engineer QFTs to emerge designer gravity theories of our choice?

Let me just describe two qualitative insights in this direction.

1. Quantum entanglement is a natural object in the bulk

After the beautiful thought experiment of Einstein-Podolsky-Rosen and the discovery of the Bell inequalities, most of us have heard about quantum entanglement as a central interesting feature of QM:

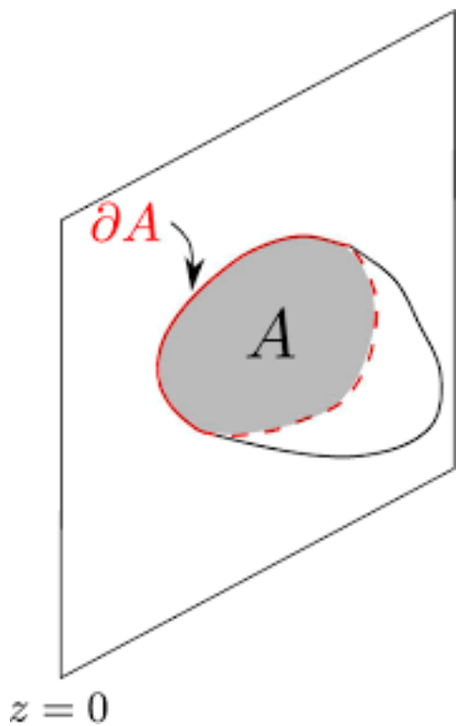
$$(|\uparrow\rangle + |\downarrow\rangle) \otimes (|\uparrow\rangle + |\downarrow\rangle)$$

vs

$$|\uparrow\rangle \otimes |\uparrow\rangle + |\downarrow\rangle \otimes |\downarrow\rangle$$

The entanglement entropy of two subsystems — which is a good simple measure — is defined by a suitable (partial) trace of the density matrix.

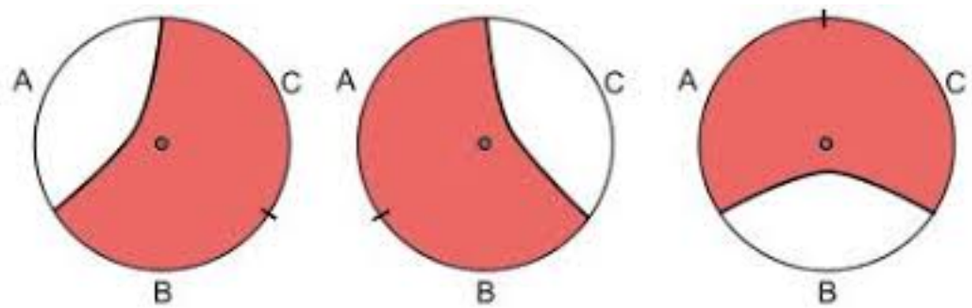
We can ask the same about the quantum state of two "regions" in a QFT. In holography there is an elegant statement:



The entanglement entropy between subregion A and its complement is measured by the area of a minimal surface extending from the boundary of A into the bulk.

This serves as a launching point for many investigations into "which region in a boundary field theory encodes which part of the bulk geometry."

One simple qualitative idea that comes out of this:



Easy to find situations where the physics of a point in the bulk could be encoded by an operator supported on any of

$$A \cup B, A \cup C, B \cup C$$

This indicates a duplication of information in how the bulk is encoded in the boundary, and is thought to be an indication that "quantum error-correcting codes" play a key role in the dictionary enabling bulk space reconstruction in holography.

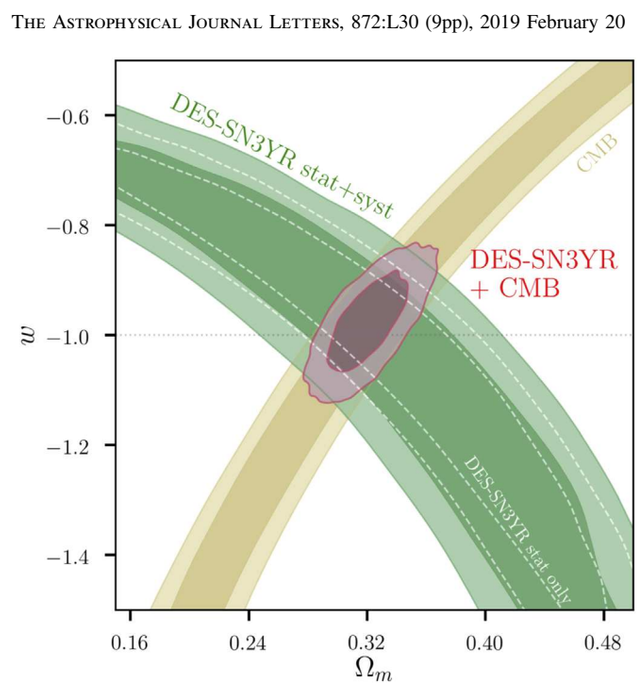
"logical qubits" $\longleftrightarrow$ **bulk physics**

"physical qubits" $\longleftrightarrow$ **boundary encoding**

2. More interesting space-times

AdS space is a great playground for theorists. The boundary locks down fluctuations, and nothing too wild can happen. AdS space plays well with supersymmetry, as do theorists.

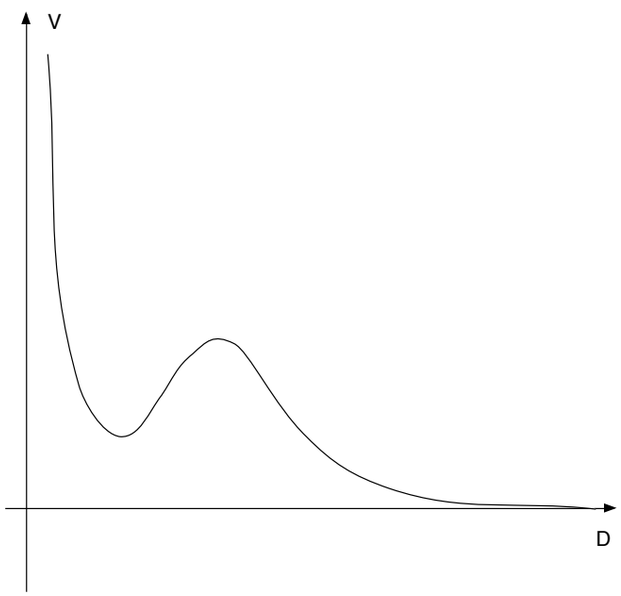
But for better or worse, we do not live in AdS space, and key features of both Minkowski and de Sitter space are different from AdS.



Given this plot (and many others), the case of accelerated expansion in general and de Sitter space in particular seems most urgent.

There has been a lot of work on obtaining semi-classical de Sitter solutions from string theory. There is nothing as simple as the supersymmetric AdS solutions. Approximations / effective field theory play an important role.

One lesson from these constructions is that de Sitter space will almost surely be metastable in string theory:



One line of research therefore seeks to embed de Sitter holography in an asymptotic boundary of the space-time decay product (e.g. an FRW space).

A more recent approach — particularly effective in 2+1 dimensional toy models — uses the remarkable properties of a new tractable deformation of QFT.

$$\mathcal{L}^{(\lambda+\delta\lambda)} = \mathcal{L}^{(\lambda)} + \delta\lambda\, T\bar{T}$$

$$\frac{dS(\lambda)}{d\lambda} = \int d^2x\, T\bar{T}(x).$$

Normally, one cannot write a perturbation of a Lagrangian by an irrelevant operator and make sense of integrating up the flow! The special properties of stress tensors in 2d CFT make this tractable.

Recent work starts with a 2d CFT dual to an AdS3 gravity theory, and finds the CFT state counts governing the entropy of the black holes there.

Deforming both sides by the $T\bar{T}$ flow (+…), it is argued that a patch of the geometry containing the black hole horizon morphs to a patch of dS space containing a cosmological horizon.

The Gibbons-Hawking entropy of de Sitter space (including a recently computed one-loop correction) emerges from this state count.

I believe that a more fundamental understanding of the technology to generate solutions, of the mechanisms and predictions underlying models of accelerated expansion in string theory, and of the conceptual issues of "non-perturbative formulations" in this setting, will be areas of well-motivated interest going forward.

Summary: where are we going?

In learning about theory space, I find this globe (the Hunt-Lenox globe from 1510) a good way to think about the state of our knowledge.

I'll bet continued exploration will reveal qualitatively new regions of theory space, and striking new phenomena in theoretical physics.

There are big questions about Nature we cannot answer today. It may prove worthwhile to understand theory space better.