α_s: state-of-the-art & the decade ahead

Snowass Summer Meetg. 2022

Seattle, 18th July 2022

David d'Enterria

CERN

The strong coupling constant: State of the art and the decade ahead

[arXiv:2203.08271 [hep-ph]]

Editors: D. d'Enterria,[1] S. Kluth,[2] G. Zanderighi,[2]

Authors: C. Ayala,[3] M.A. Benitez-Rathgeb,[4] J. Blümlein,[5] D. Boito,[4,6] N. Brambilla,[7] D. Britzger,[2] S. Camarda,[1] A. M. Cooper-Sarkar,[8] T. Cridge,[9] G. Cvetič,[10] D. d'Enterria,[1] M. Dalla Brida,[11] A. Deur,[12] F. Giuli,[1] M. Golterman,[13,14] A.H. Hoang,[4,15] J. Huston,[16] M. Jamin,[4,17] S. Kluth,[2] A. V. Kotikov,[18] V. G. Krivokhizhin,[18] A.S. Kronfeld,[19] V. Leino,[7] K. Lipka,[20] T. Mäkelä,[20] B. Malaescu,[21] K. Maltman,[22,23] S. Marzani,[24] V. Mateu,[25,26] S. Moch,[27] P. F. Monni,[11] P. Nadolsky,[28] P. Nason,[2,29] A.V. Nesterenko,[18] R. Pérez-Ramos,[30,31] S. Peris,[14] P. Petreczky,[32] A. Pich,[33] K. Rabbertz,[34] A. Ramos,[33] D. Reichelt,[35] A. Rodríguez-Sánchez,[36] J. Rojo,[37,38] M. Saragnese,[5] L. Sawyer,[39] M. Schott,[40] S. Schumann,[41] B. G. Shaikhatdenov,[18] S. Sint,[42] G. Soyez,[43] D. Teca,[10] A. Vairo,[7] M. Vos,[33] C. Waits,[39] J. H. Weber,[44] M. Wobisch,[39] K. Xie,[45] and G. Zanderighi[2]

α_s(2022) workshop: https://indico.cern.ch/e/alphas2022

α_s Snowmass White Paper

- Overarching themes: What is the current state-of-the-art and ultimate theoretical & experimental precision of current α_s extraction methods? What needs to be achieved in order to reach a $\mathcal{O}(0.1\%)$ precision?

- Each contributor was requested to provide a few pages summary of their work addressing the following questions:

  - Theory: What is the current state-of-the-art with regards to higher-order corrections (pQCD, mixed QCD-EW) of your calculations of α_s-dependent observables? What is the impact of non-pQCD corrections/uncertainties? (Are there new techniques to reduce them?) Provide your personal wish-list in theory/data developments needed to reach your ultimate α_s precision.

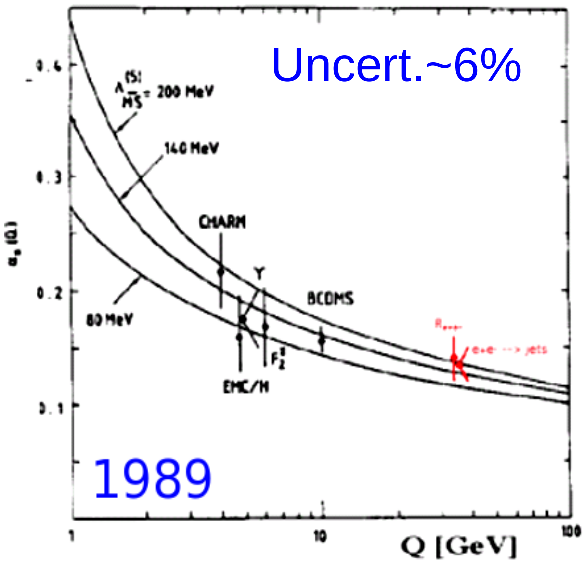
  - Experiment: What are the current leading syst./stat. uncertainties of your favorite α_s-dependent observable? What are the future reductions of syst./stat. uncertainties expected with current and future (e^+e^-, e-p, p-p) machines? (Are there new observables being considered?) Provide your personal wish-list in data/theory developments needed to reach your ultimate α_s precision.

α_s white paper contents

- αs(2022) workshop at ECT*-Trento (Feb. 2022) with S. Kluth & G. Zanderighi
  - Output white paper with 60+ authors, 130+ pages, 80+ figures:

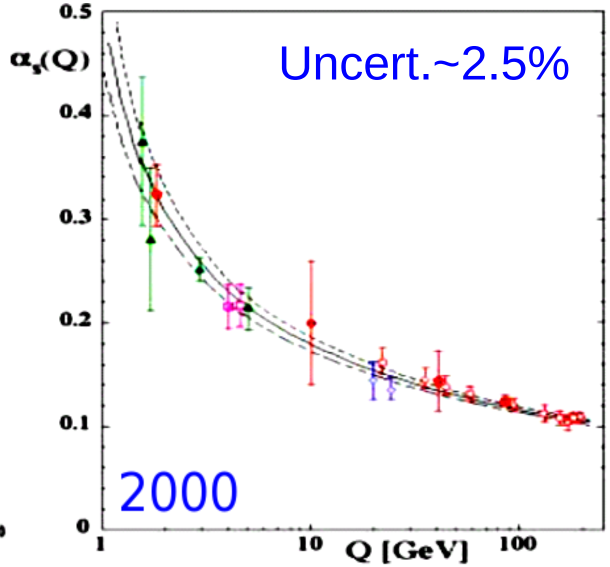
Motivation: QCD coupling α_s

- Determines strength of the strong interaction between quarks & gluons.
- Single free parameter of QCD in the $m_q = 0$ limit.
- Determined at a ref. scale ($Q=m_Z$), decreases as $\alpha_s \sim \ln(Q^2/\Lambda^2)^{-1}$, $\Lambda \sim 0.2$ GeV



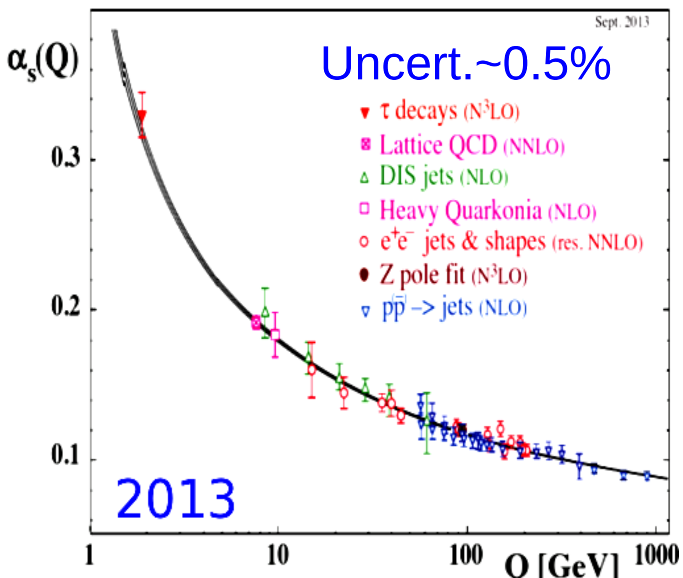
$\alpha_s(M_z) = 0.110^{+0.006}_{-0.008}$ (NLO)

G. Altarelli, Ann. Rev. Nucl. Part. Sci. 39, 1989



$\alpha_s(M_z) = 0.1184 \pm 0.0031$ (NNLO)

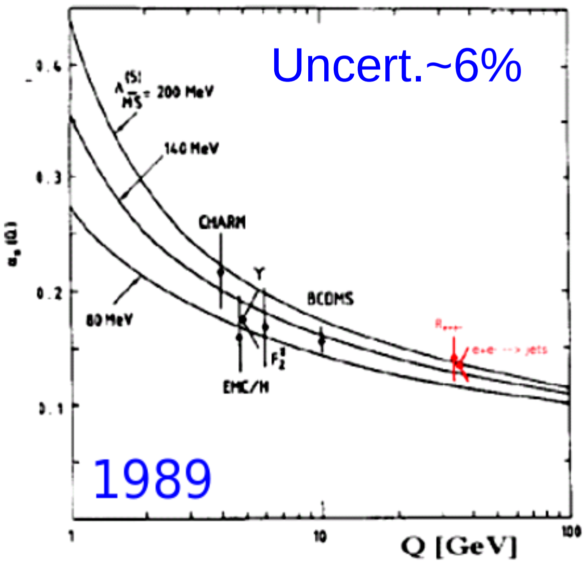
S. B. , J. Phys. G 26, 2000

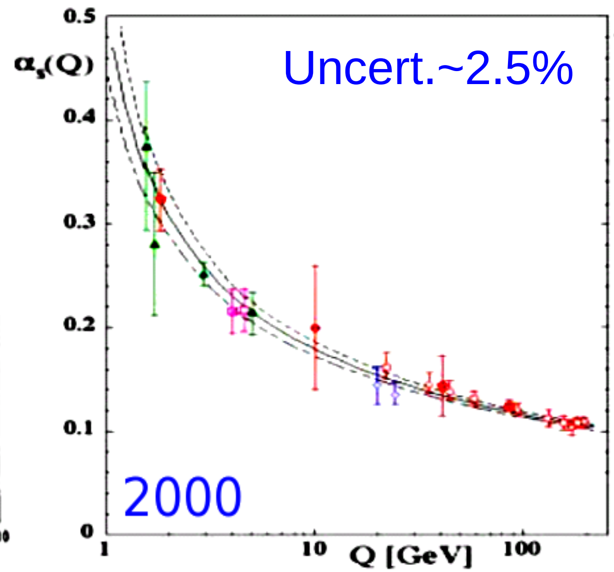


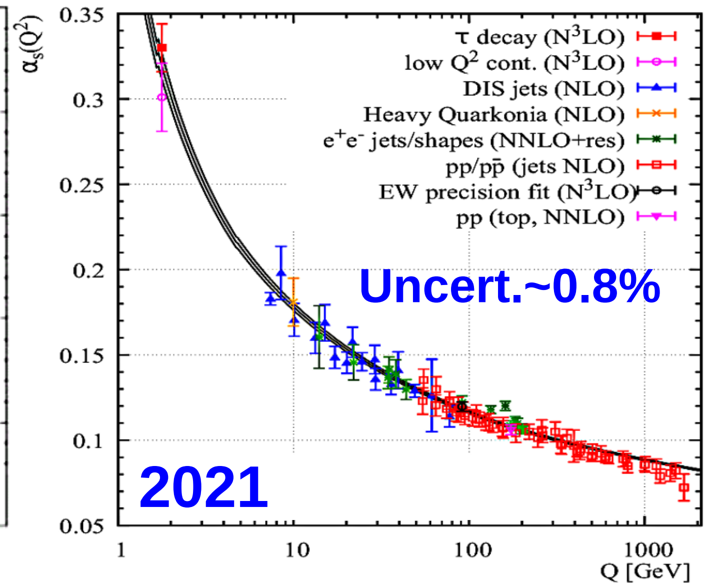
$\alpha_s(M_Z) = 0.1185 \pm 0.0006$ (NNLO)

Motivation: QCD coupling α_s

- Determines strength of the strong interaction between quarks & gluons.
- Single free parameter of QCD in the $m_q = 0$ limit.
- Determined at a ref. scale ($Q=m_Z$), decreases as $\alpha_s \sim \ln(Q^2/\Lambda^2)^{-1}$, $\Lambda \sim 0.2$ GeV







$\alpha_s(M_z) = 0.110^{+0.006}_{-0.008}$ (NLO)

G. Altarelli, Ann. Rev. Nucl. Part. Sci. 39, 1989

$\alpha_s(M_z) = 0.1184 \pm 0.0031$ (NNLO)

S. B. , J. Phys. G 26, 2000

$\alpha_s(M_Z^2) = 0.1179 \pm 0.0010$

➧ Least precisely known of all interaction couplings !

$\delta\alpha \sim 10^{-10} \ll \delta G_F \ll 10^{-7} \ll \delta G \sim 10^{-5} \ll \delta\alpha_s \sim 10^{-3}$

Motivation: α_s importance beyond QCD

■ Precision calculations of Higgs hadronic x-sections/decays, top mass, EWPO:

Process	σ (pb)	$\delta\alpha_s(\%)$	PDF $+\alpha_s(\%)$	Scale(%)
ggH	49.87	$\pm$ 3.7	-6.2 +7.4	-2.61 +0.32
ttH	0.611	$\pm$ 3.0	$\pm$ 8.9	-9.3 +5.9

Msbar mass error budget (from threshold scan)

$(\delta M_t^{\mathrm{SD-low}})^{\mathrm{exp}}$	$(\delta M_t^{\mathrm{SD-low}})^{\mathrm{theo}}$	$(\delta\overline{m}_t(\overline{m}_t))^{\mathrm{conversion}}$	$(\delta\overline{m}_t(\overline{m}_t))^{\alpha_s}$
40 MeV	50 MeV	7 – 23 MeV	70 MeV

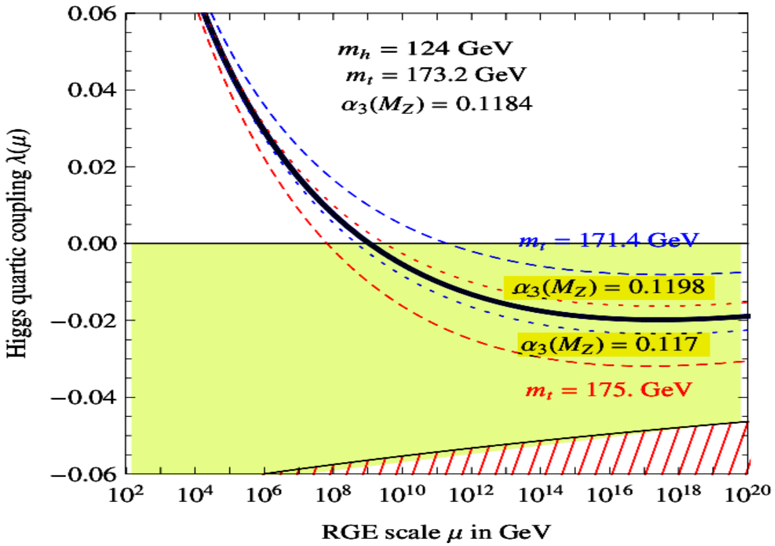
$\Rightarrow$ improvement in α_s crucial $\delta\alpha_s(M_z) = 0.001$

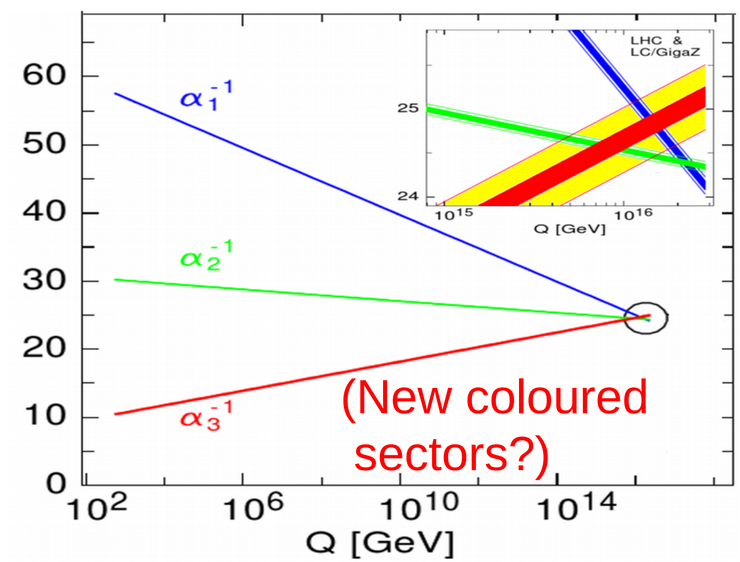
Partial width	intr. QCD	para. m_q	para. α_s
$H \to b\bar{b}$	$\sim 0.2\%$	1.4%	0.4%
$H \to c\bar{c}$	$\sim 0.2\%$	4.0%	0.4%
$H \to gg$	$\sim 3\%$	$< 0.2\%$	3.7%

Quantity	FCC-ee	future param.unc.	Main source
Γ_Z [MeV]	0.1	0.1	$\delta\alpha_s$
R_b $[10^{-5}]$	6	< 1	$\delta\alpha_s$
R_ℓ $[10^{-3}]$	1	1.3	$\delta\alpha_s$

Sven Heinemeyer – 1st FCC physics workshop, CERN, 17.01.2017

■ Impacts physics approaching Planck scale: EW vacuum stability, GUT

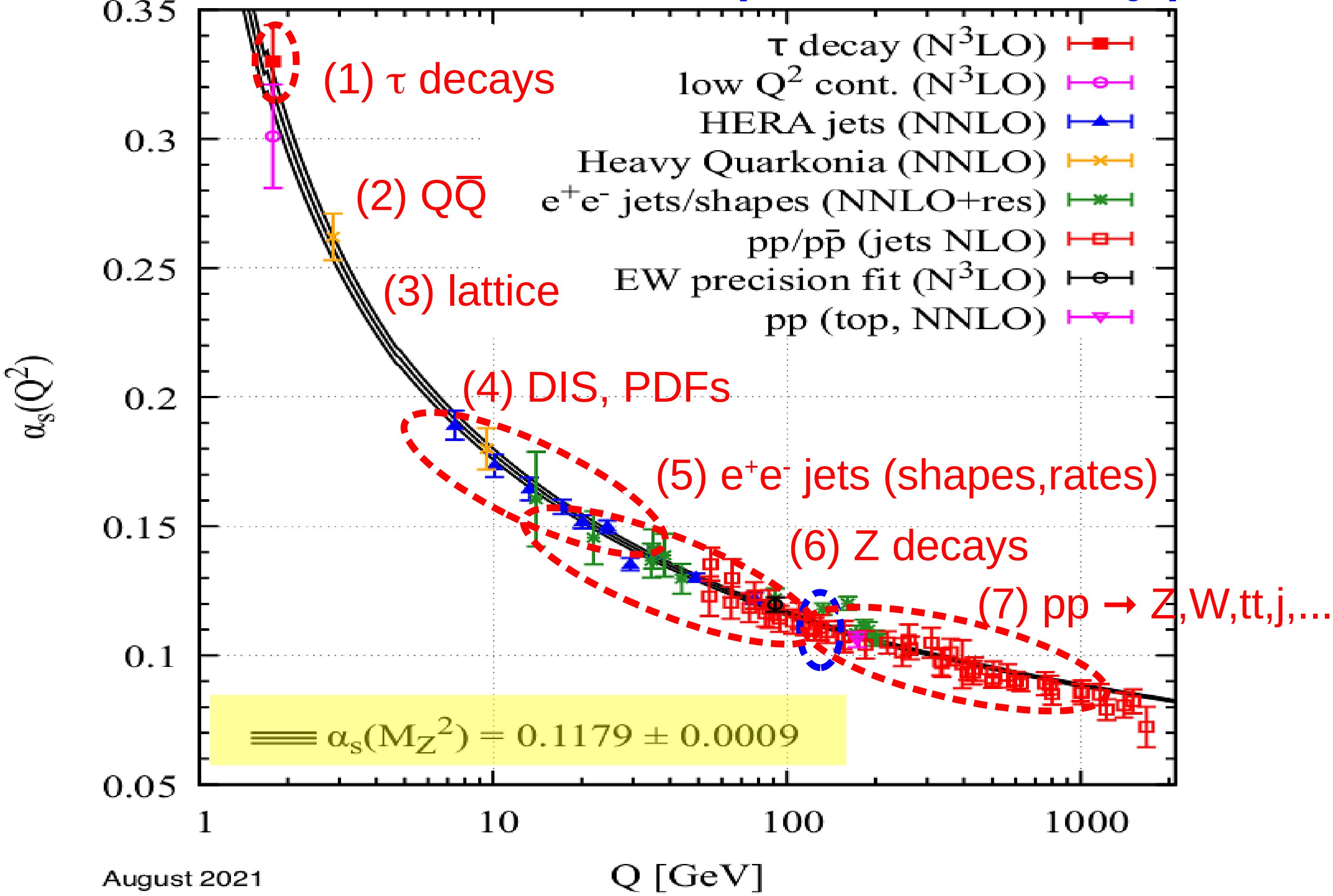




World α_s determination (PDG 2021)

- Determined today by comparing 7 experimental observables to pQCD NNLO,N^{3}LO predictions, plus global average at the Z pole scale:

[Huston/Rabbertz/Zanderighi]



World-average α_s (PDG 2021)

- Average of pre-averages from 7 categories of observables:

  $\alpha_s(M_z) = 0.1179 \pm 0.0009$ (±0.8%)

  Hadronic tau decay (4 values):

  $\alpha_s(M_z) = 0.1178 \pm 0.0019$ (±1.6%)

  Quarkonia properties (4 values):

  $\alpha_s(M_z) = 0.1181 \pm 0.037$ (±3.3%)

  DIS & PDFs fits (6 values):

  $\alpha_s(M_z) = 0.1162 \pm 0.0020$ (±1.7%)

  $e^+e^- \rightarrow$ hadrons final states (10 values):

  $\alpha_s(M_z) = 0.1171 \pm 0.0031$ (±2.6%)

  Hadron collider measurements (5 values):

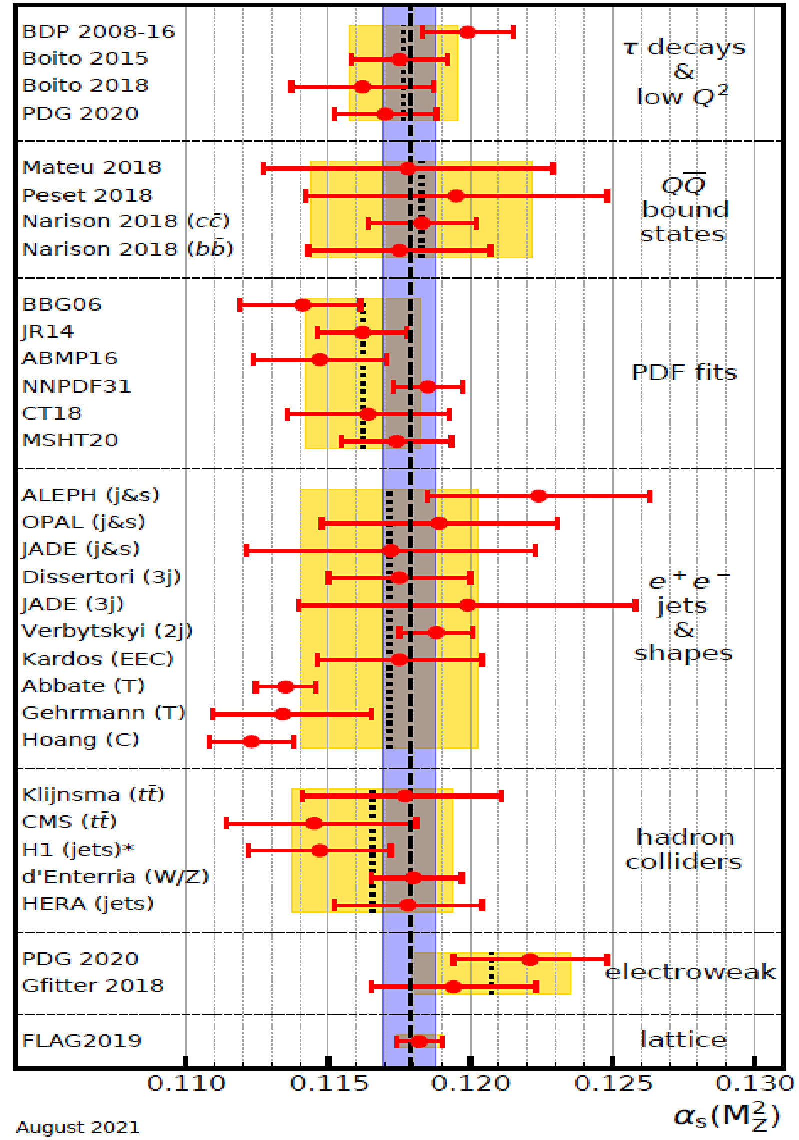
  $\alpha_s(M_z) = 0.1165 \pm 0.0028$ (±2.4%)

  Electroweak precision fits (2 values):

  $\alpha_s(M_z) = 0.1208 \pm 0.0028$ (±2.3%)

  Lattice-QCD (1 FLAG value):

  $\alpha_s(M_z) = 0.1182 \pm 0.0008$ (±0.7%)

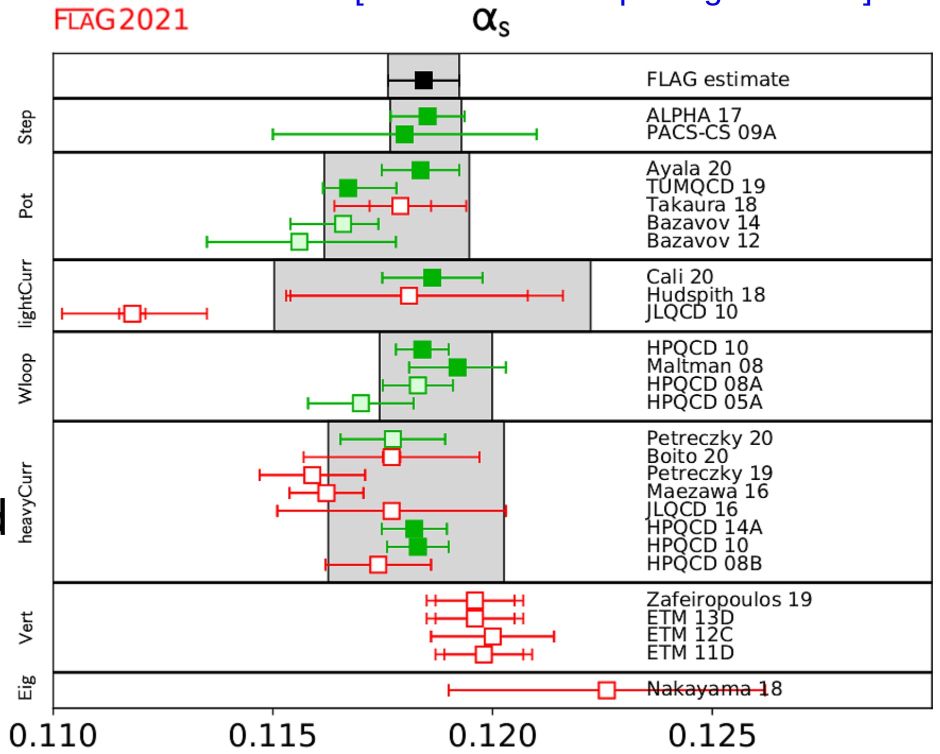


(1) α_s from lattice QCD

- Comparison of short-distance quantities (QCD static energy/force, light-q & heavy-Q currents, quarkonium,..) computed at $N^{2,3}LO$ in pQCD to lattice data with m_{had}, f_{had} experimental constraints:

$$K^{NP} = K^{PT} = \sum_{i=0}^{n} c_i \alpha_s^i$$

[FLAG Collab. http://flag.unibe.ch]



$\alpha_s = 0.1182 \pm 0.0008$ (±0.7%)

- Community-agreed (FLAG) criteria based on: renorm. scale, pQCD behaviour, continuum limit, peer-reviewed results.

- Current uncertainties driven mostly by pQCD truncation & matching, and continuum limit (lattice spacing & computing stats).

➧ Future prospects:

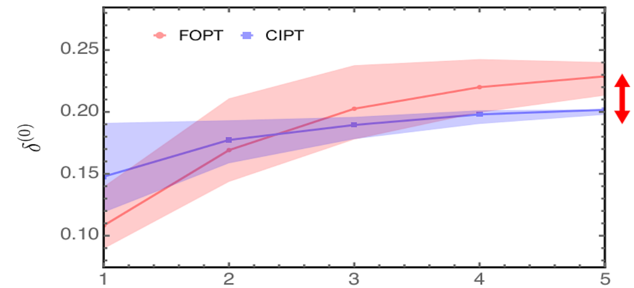
– Uncertainty in α_s halved with reduced latt. spacing, $N^{3,4}LO$ pQCD, active charm quark, extension of step-scaling method to more observables.

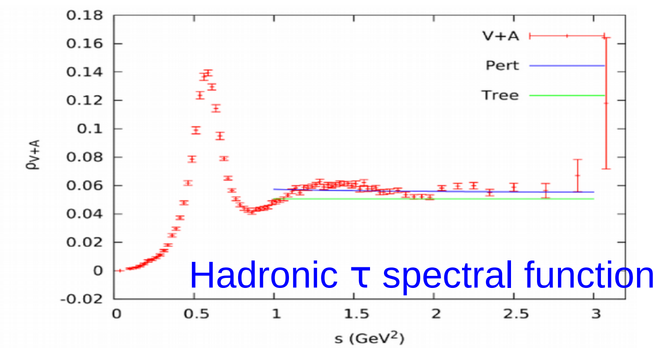
(2) α_s from hadronic τ-lepton decays

- Computed at N^{3}LO: $R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = S_{\rm EW} N_C \left(1 + \sum_{n=1}^{4} c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_{\rm np}\right)$
- Experimentally: $R_{\tau,\rm exp} = 3.6355 \pm 0.0081\ (\pm 0.22\%)$

- Uncertainty driven today by:
  - Differences in pQCD approaches. FOPT vs CIPT OPE expansions:
  - Treatment of non-pQCD corrections (duality violations): Note: $(\Lambda/m_\tau)^2 \sim 2\%$

Moment's perturbation series:





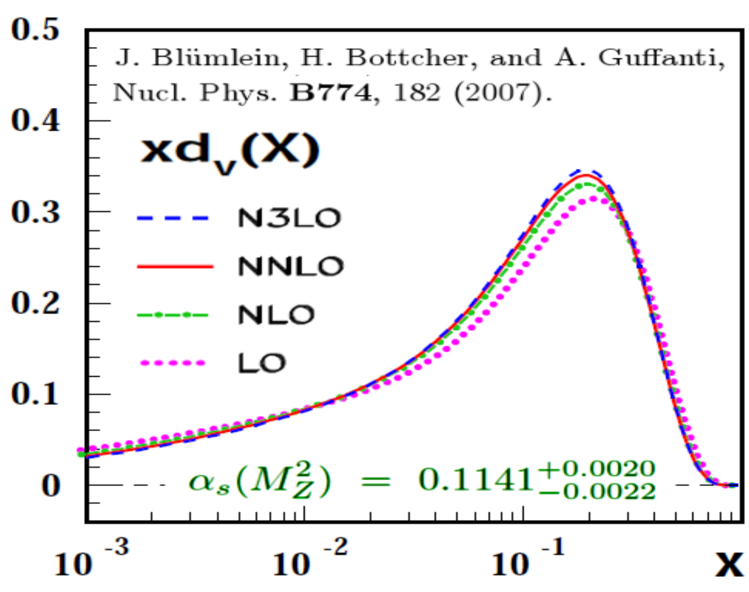
$\alpha_s = 0.1178 \pm 0.0019\ (\pm 1.6\%)$

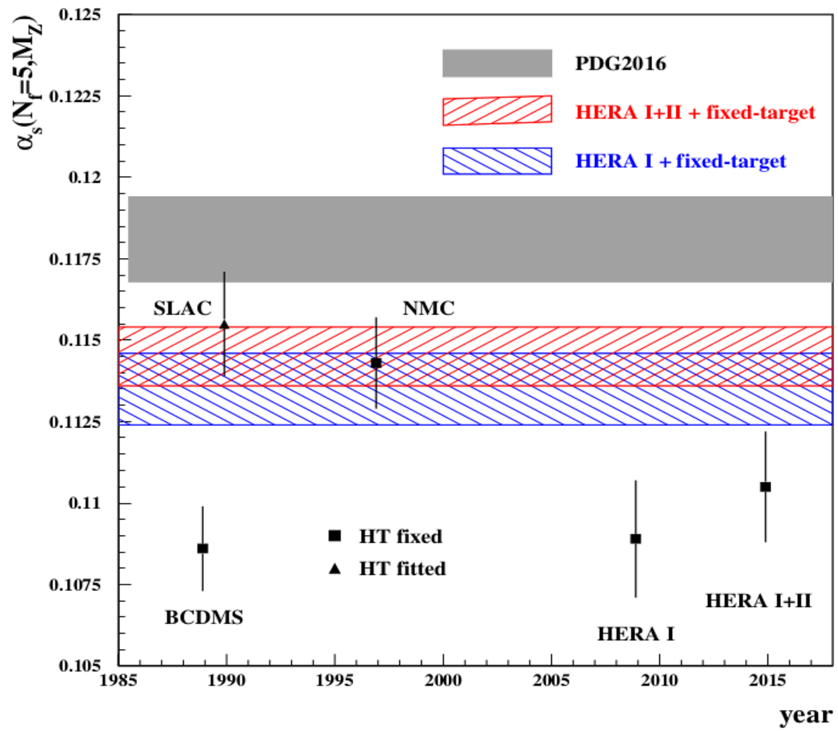
➧ Future prospects:
- N^{4}LO calculations.
- Reconciling FOPT vs CIPT results (IR renormalon-free gluon condensate)
- Better spectral functions needed: BELLE-II (~now). Longer future: $\mathcal{O}(10^{11})$ $Z \to \tau\tau$ at FCC-ee(90) !

(3) α_s from DIS struct. functions & PDF fits

- N^3LO/NNLO analysis of (non)singlet struct. functions (BBG, JR14) (and NNLO global PDF AMBP fit) tend to give "lowish" $\alpha_s(M_z) \approx 0.1150$

$$F_2(x,Q^2) = x\sum_{n=0}^{\infty}\frac{\alpha_s^n(\mu_R^2)}{(2\pi)^n}\sum_{i=q,g}\int_x^1\frac{dz}{z}C_{2,i}^{(n)}(z,Q^2,\mu_R^2,\mu_F^2)\,f_{i/p}\left(\frac{x}{z},\mu_F^2\right)+\mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$

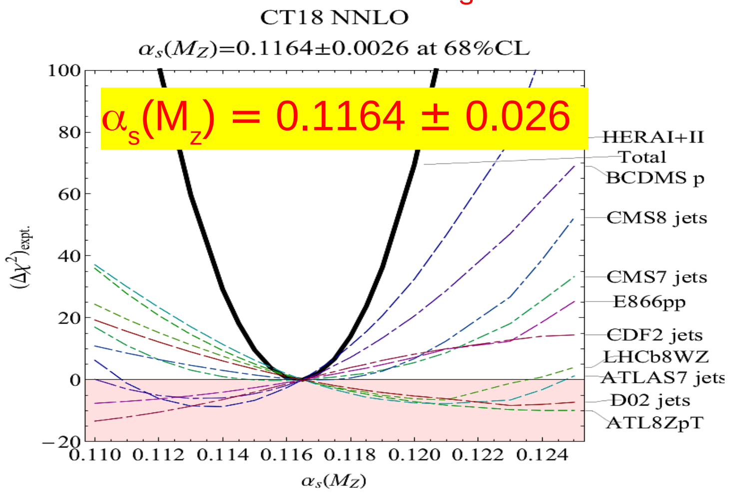


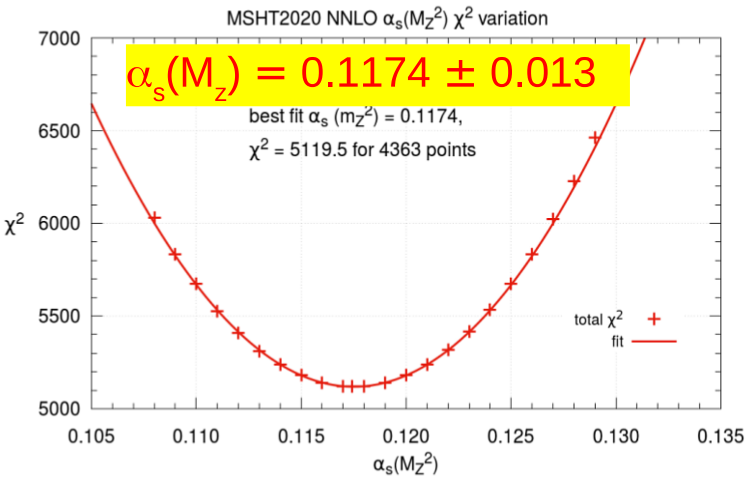


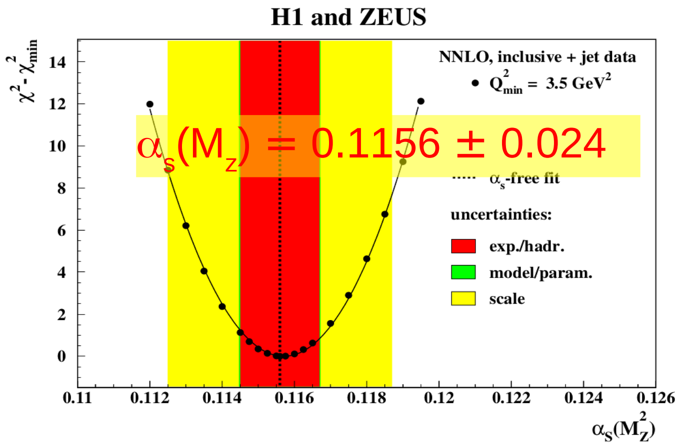
- Neglect of singlet contribs. for x>0.3 in NS fits? Size of higher-order corrs.?
- ➧ Future: New high-precision $F_i(x,Q^2)$ & polarized $g_i(x,Q^2)$ at EIC.

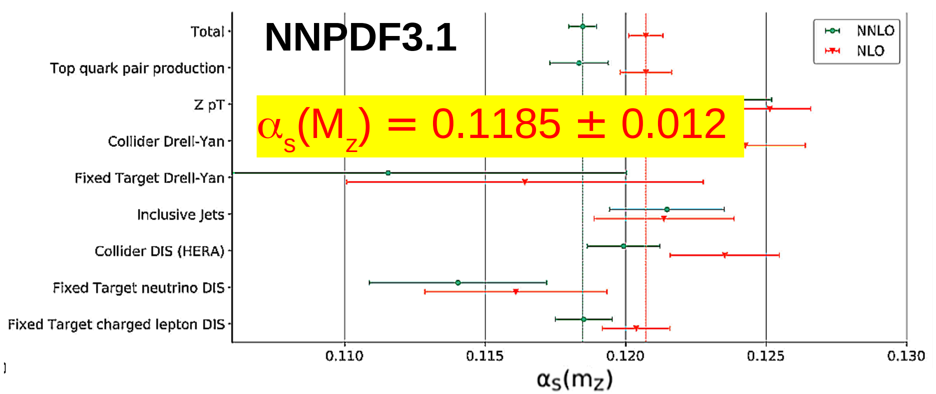
(3) α_s from DIS struct. functions & PDF fits

- NNLO global PDF+α_s fits: CT18, HERAPDF2.0+j, MSTH2020, NNPDF3.1









- DIS/FT (LHC) data tend to prefer lower (higher) values of $\alpha_s(M_z)$.
- Size of missing HO corrections? Global fits at N^{3}LO needed.

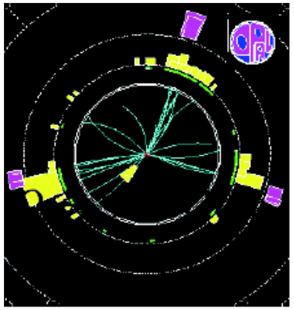
➧ Future: ±0.2% at LHeC/FCC-eh

(4) α_s from e^+e^- event shapes & jet rates

- Computed at N[2,3]LO+N[(2)]LL accuracy.
- Experimentally (LEP):
  Thrust, C-parameter, jet shapes
  3-jet x-sections

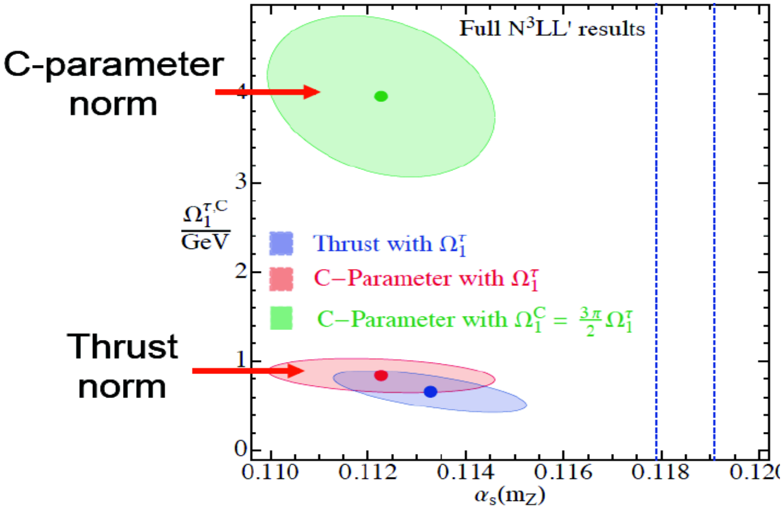
$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

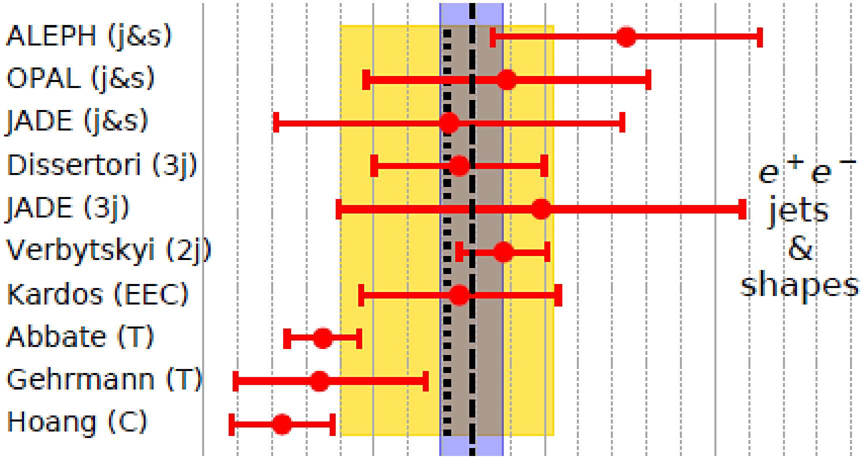
$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2\theta_{ij}}{(\sum_i |\vec{p}_i|)^2}$$



OPAL 3 jet event

- Results sensitive to non-pQCD (hadronization) accounted for via MCs or analytically:





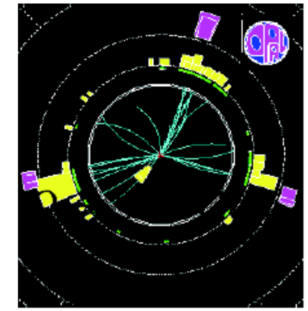
α_s = 0.1171 ± 0.0031 (±2.6%)

(4) α_s from e^+e^- event shapes & jet rates

- Computed at $N^{2,3}LO+N^{(2)}LL$ accuracy.
- Experimentally (LEP):
  Thrust, C-parameter, jet shapes
  3-jet x-sections

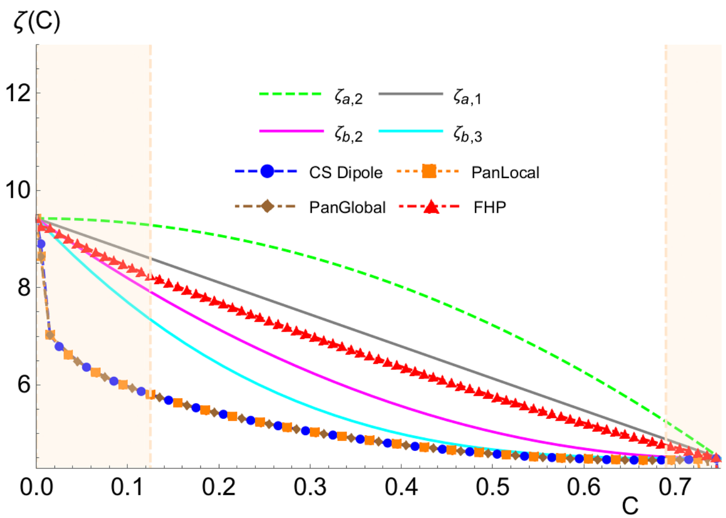
$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2\theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2}$$

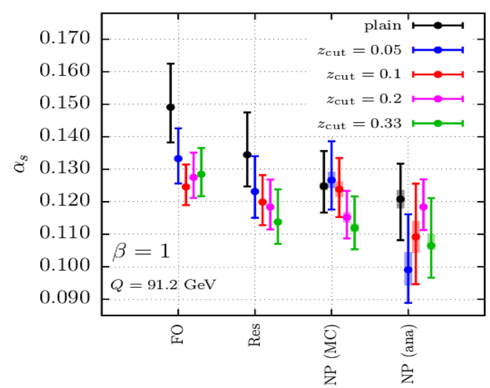


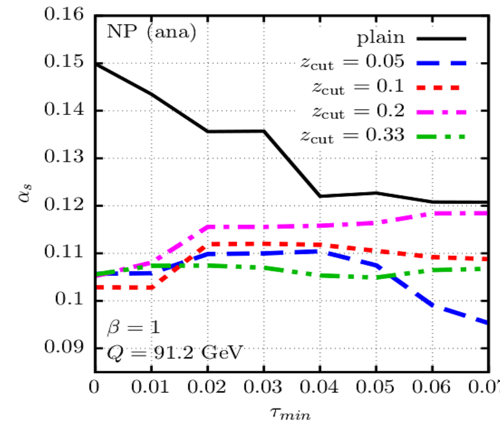
OPAL 3 jet event

- Results sensitive to non-pQCD (hadronization) corrections.
  - ➧ Improved evt-shape power-corrs:



➧ Modern jet substructure techniques: "Soft drop" can help reduce non-pQCD corrections for thrust:





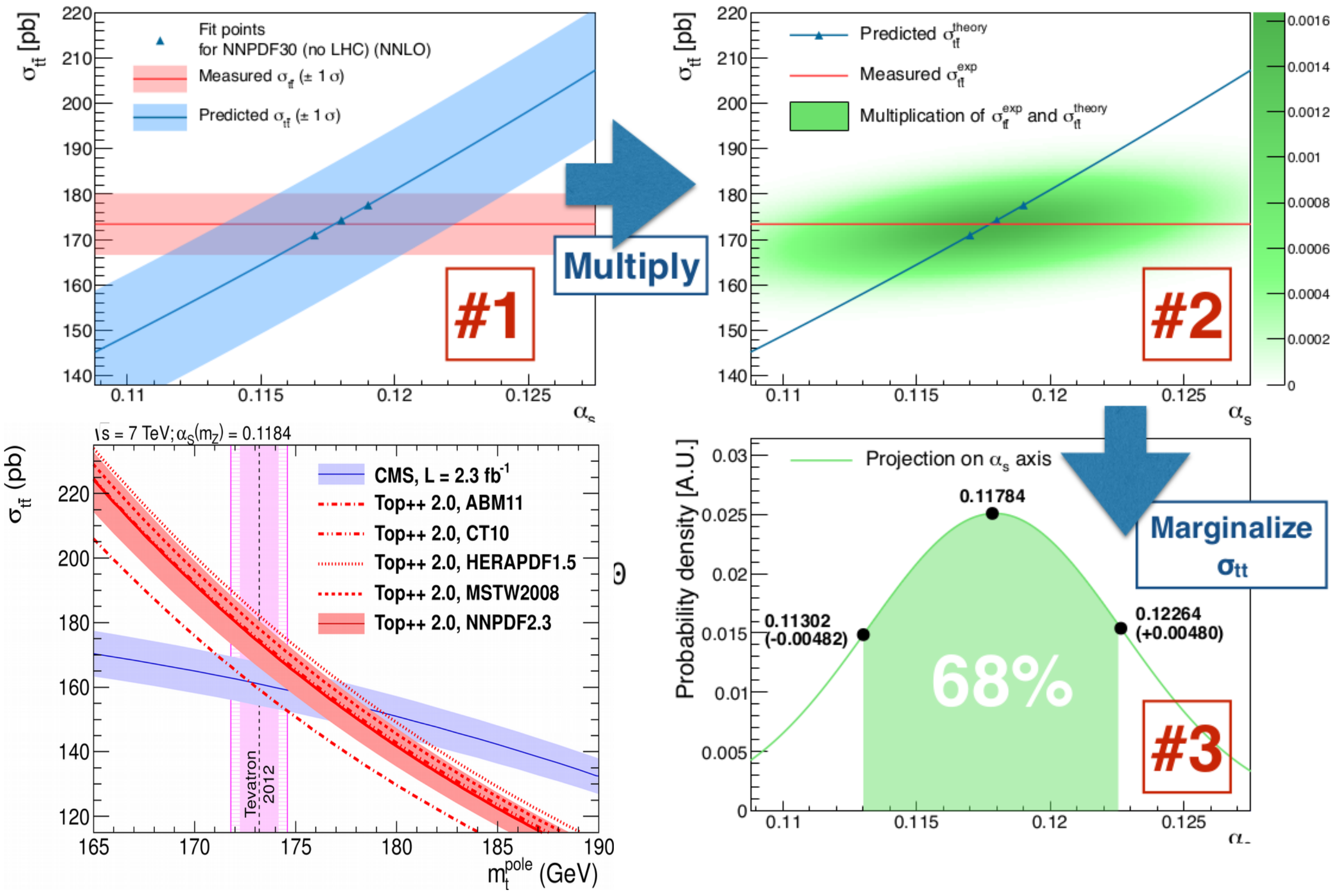
➧ Future:

– Power-corrections for shapes, ($N^{2,3}LL$) resummation for rates. Grooming.

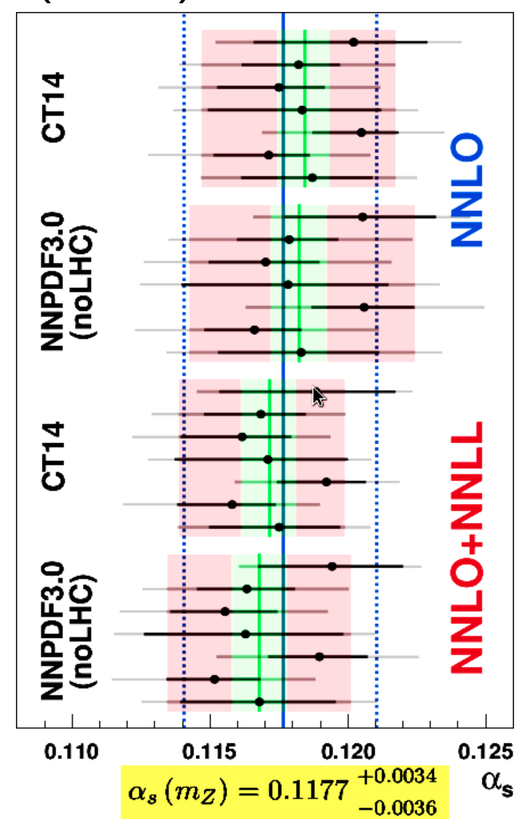
– New e^+e^- data at lower-√s (Belle-II) & higher-√s (Higgs factories) needed.

(5) α_s from hadron collider x-sections (ttbar)

- So far, only top-pair and W,Z boson x-sections available at NNLO.
  (Jets also available at NNLO since couple of years: (long) analysis ongoing).
- Method: Compare σ(exp) to σ(NNLO) computed w/ diff. PDFs/α_s: Extract α_s
- σ(tt) [dis]advantages: Direct sensitivity to α_s [uncertainties: ~5% exp./th., m_{top}]

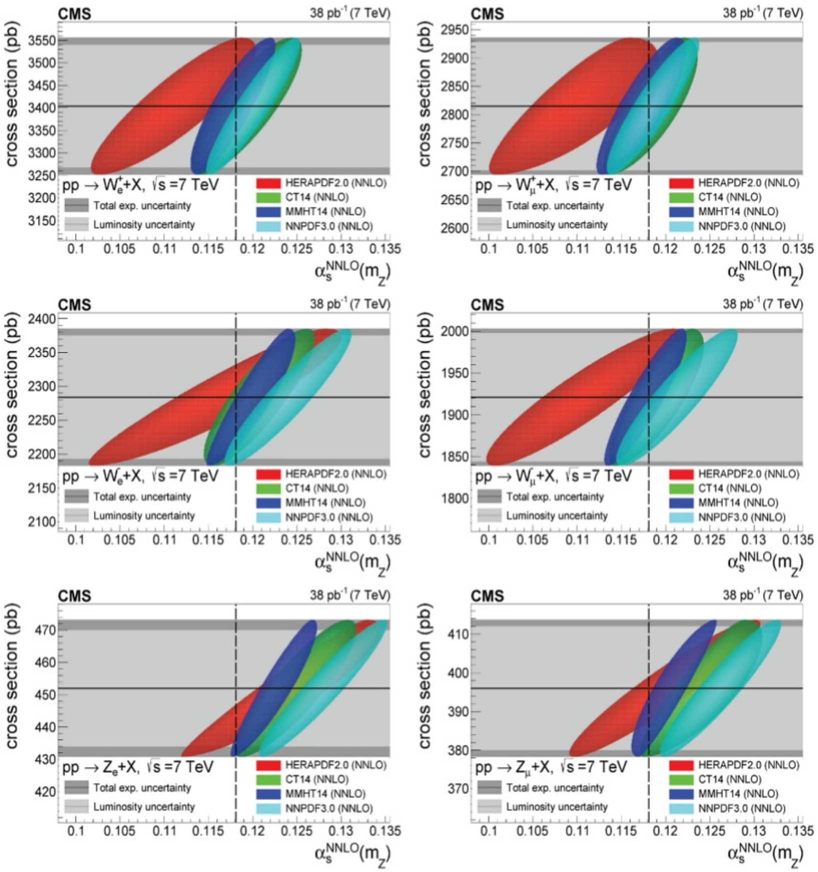


Analysis of 8 data sets (<2018)

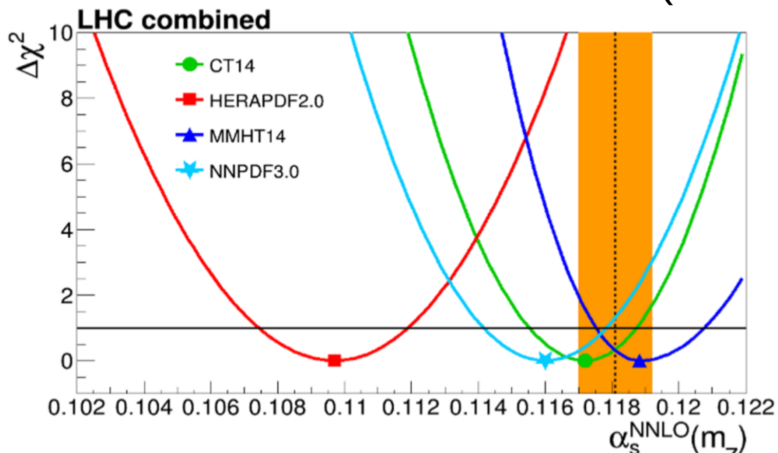


(5) α_s from hadron collider x-sections (W,Z)

- So far, only top-pair and W,Z boson x-sections available at NNLO.
  (Jets also available at NNLO since couple of years: (long) analysis ongoing).
- Method: Compare σ(exp) to σ(NNLO) computed w/ diff. PDFs/α_s: Extract α_s
- σ(W,Z) [dis]advantages: ~1–2% th./exp. uncertainties [not LO sensitivity to α_s]



Combined fit of 28 LHC data sets (<2019)



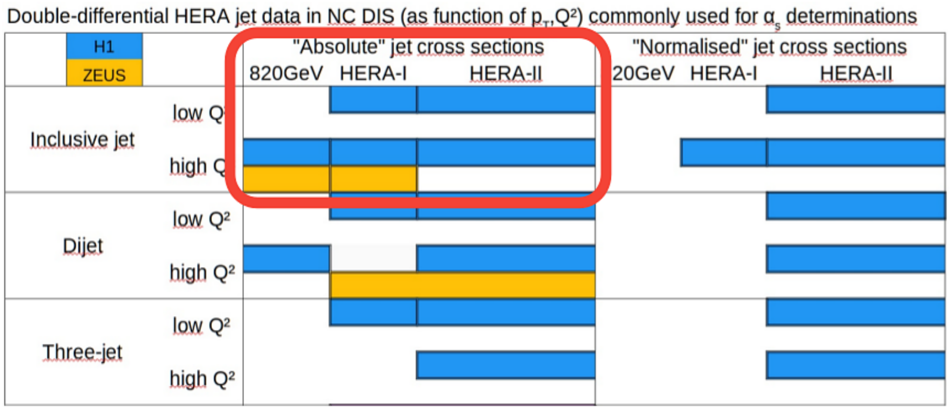
PDF	$\alpha_s(m_Z)$
CT14	$0.1172^{+0.0015}_{-0.0017}$
MMHT14	$0.1188^{+0.0019}_{-0.0013}$
HERAPDF2.0	$0.1097^{+0.0022}_{-0.0023}$
NNPDF3.0	0.1160 ± 0.0018

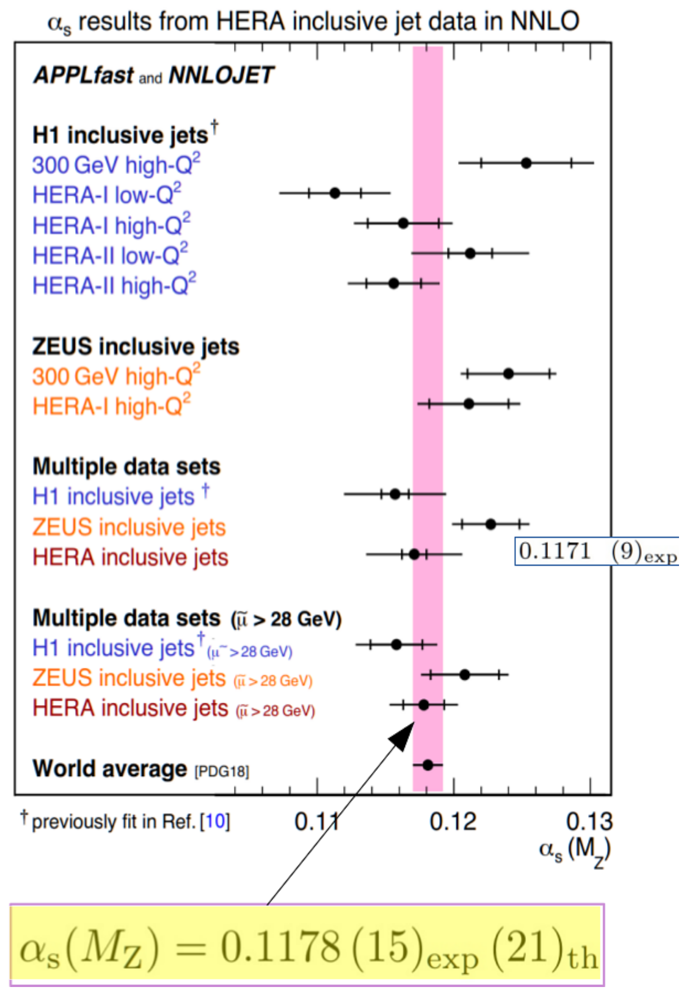
α_s = 0.1180 ± 0.0016 (±1.3%)

➧ Future: Incorporate σ(tt), σ(W,Z), σ(j), x-section ratios into global PDF+α_s fits.

(5) α_s from hadron collider x-sections (HERA jets)

- DIS HERA jet x-sections employed for α_s extractions via NNLOjet:





- α_s from inclusive jet cross sections in NC DIS
- NNLO pQCD w/ non-pert. hadronisation corrections
- H1 and ZEUS consistent
- Sizeable scale uncertainties (MHOU) since data are at comparably low scales
- Highest precision obtained in fit to data with $\mu > 28$ GeV

- Largest uncertainty from missing HO corrs.

➧ Future: Nice testbed for upcoming LHC NNLO jet x-sections-based extractions

(6) α_s from EW precision fits

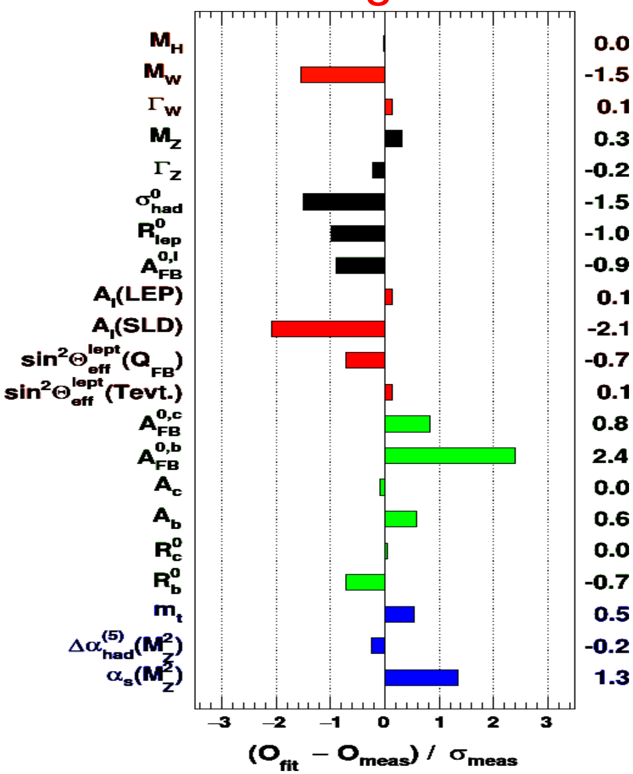
- Z-boson decays known at N³LO, no NP uncerts. (but only ~4% sensitivity to α_s):

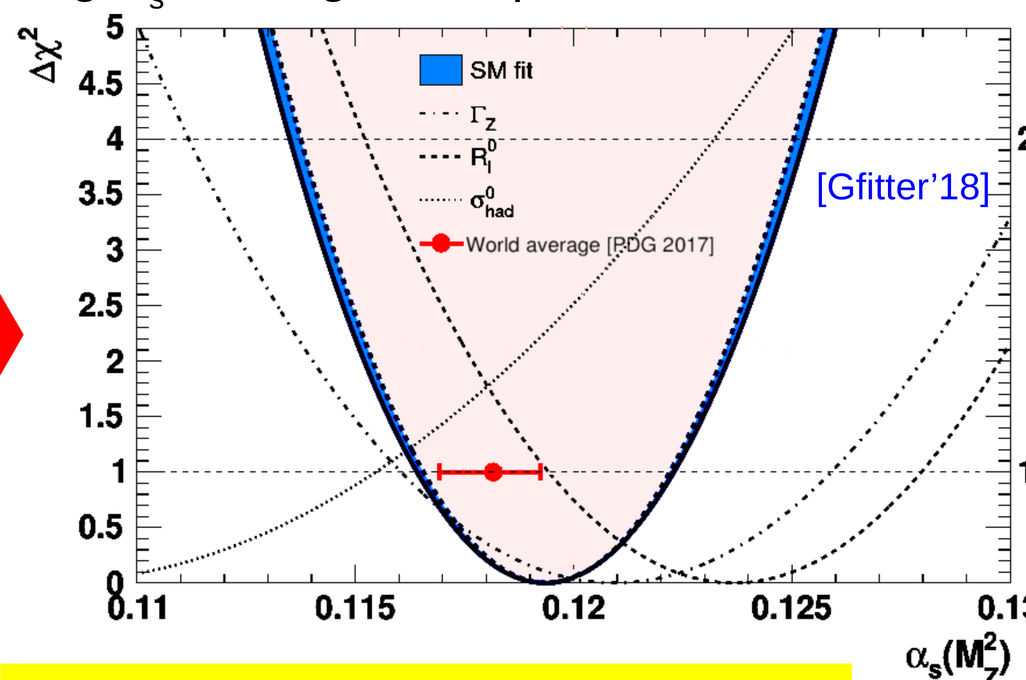
$$R_l^0 \equiv \frac{\Gamma(Z \to \mathrm{h})}{\Gamma(Z \to l)} = R_Z^{\mathrm{EW}} N_C \left(1 + \sum_{n=1}^{4} c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_{\mathrm{m}} + \delta_{\mathrm{np}}\right)$$

- Extraction from three Z-peak pseudo-observables (LEP, SLC):

Γ_Z = 2.4952±0.0023 GeV (±0.1%) ➡ α_s (M_Z) = 0.1221 ± 0.0027 (±2.3%)

- Also from the global EW fit leaving α_s as single free parameter:



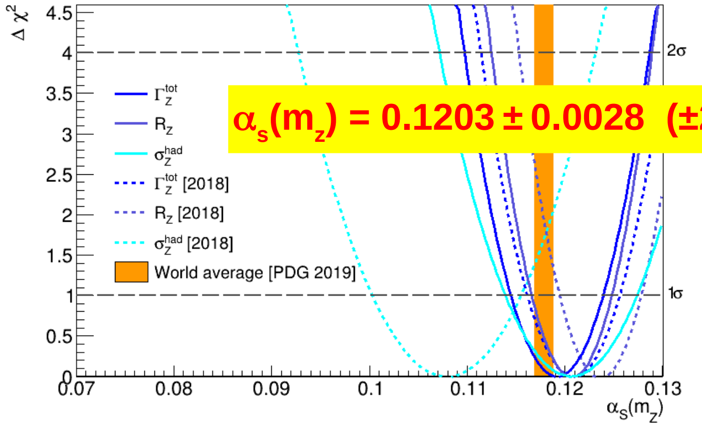


α_s (M_Z) = 0.1194 ± 0.0029 (±2.4%)

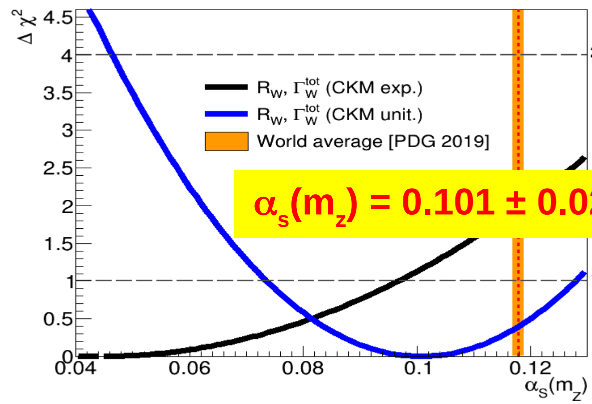
(6) α_s from hadronic EW bosons decays

Updated Z,W-based $\alpha_s(m_z)$ extractions:

New fit with HO EW corrs. + corrected Z LEP data. New N^{3}LO fit to Γ_W, R_W

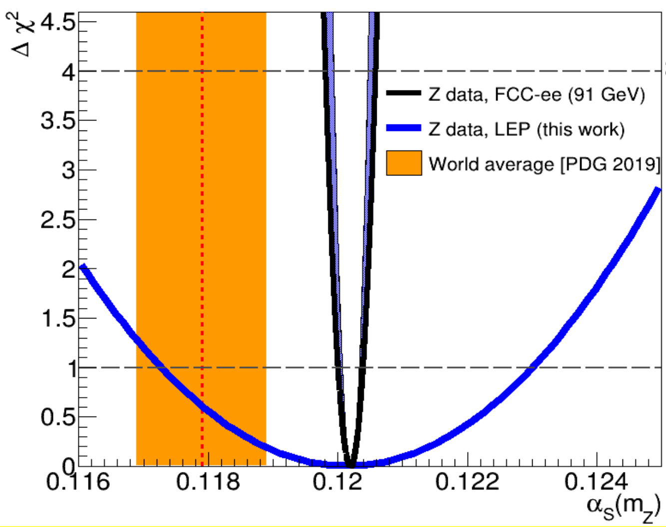


$\alpha_s(m_z) = 0.1203 \pm 0.0028$ (±2.3%)

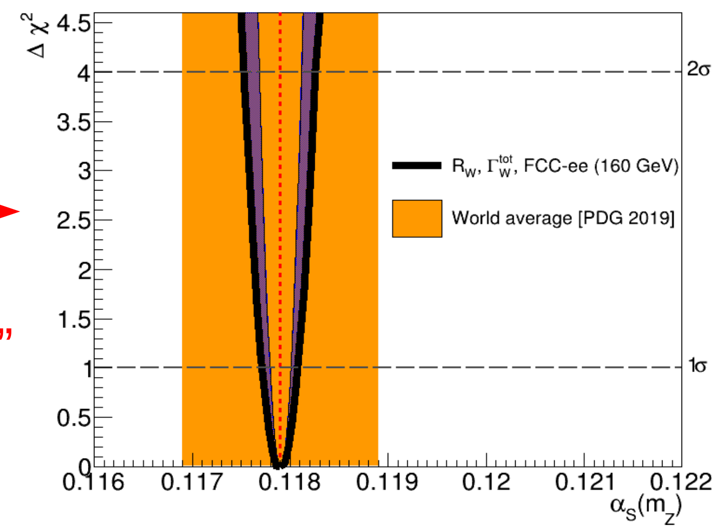


$\alpha_s(m_z) = 0.101 \pm 0.027$ (±27%)

Future: Permil uncertainty possible only with a machine like FCC-e^+e^-



Strong SM "stress test"



$\alpha_s(m_z) = 0.12030 \pm 0.00028$ ±0.2% (tot), ±0.1% (exp)

$\alpha_s(m_z) = 0.11790 \pm 0.00023$ ±0.2% (tot), ±0.1% (exp)

Summary (I): Current & future α_s precision

Method	Relative $\alpha_S(m_Z^2)$ uncertainty: Current — theory & exp. uncertainties sources	Relative $\alpha_S(m_Z^2)$ uncertainty: Near (long-term) future — theory & experimental progress
(1) Lattice	0.7%	$\approx$ 0.3% (0.1%)
	Finite lattice spacing & stats.	Reduced latt. spacing. Add more observables
	$N^{2,3}$LO pQCD truncation	Add $N^{3,4}$LO, active charm (QED effects)
		Higher renorm. scale via step-scaling to more observ.
(2) τ decays	1.6%	< 1.%
	N^3LO CIPT vs. FOPT diffs.	Add N^4LO terms. Solve CIPT–FOPT diffs.
	Limited τ spectral data	Improved τ spectral functions at Belle II
(3) $Q\overline{Q}$ bound states	3.3%	$\approx$ 1.5%
	$N^{2,3}$LO pQCD truncation	Add $N^{3,4}$LO & more ($c\bar{c}$), ($b\bar{b}$) bound states
	$m_{c,b}$ uncertainties	Combined $m_{c,b} + \alpha_S$ fits
(4) DIS & PDF fits	1.7%	$\approx$ 1% (0.2%)
	$N^{2,(3)}$LO PDF (SF) fits	N^3LO fits. Add new SF fits: $F_2^{p,d}$, g_i (EIC)
	Span of PDF-based results	Better corr. matrices. More PDF data (LHeC/FCC-eh)
(5) e^+e^- jets & evt shapes	2.6%	$\approx$ 1.5% (< 1%)
	NNLO+$N^{(1,2,3)}$LL truncation	Add $N^{2,3}$LO+N^3LL, power corrections
	Different NP analytical & PS corrs.	Improved NP corrs. via: NNLL PS, grooming
	Limited datasets w/ old detectors	New improved data at B factories (FCC-ee)
(6) Electroweak fits	2.3%	($\approx$ 0.1%)
	N^3LO truncation	N^4LO, reduced param. uncerts. ($m_{W,Z}$, α, CKM)
	Small LEP+SLD datasets	Add W boson. Tera-Z, Oku-W datasets (FCC-ee)
(7) Hadron colliders	2.4%	$\approx$ 1.5%
	NNLO(+NNLL) truncation, PDF uncerts.	N^3LO+NNLL (for color-singlets), improved PDFs
	Limited data sets ($t\bar{t}$, W, Z, e-p jets)	Add more datasets: Z p_T, p-p jets, σ_i/σ_j ratios,...
World average	0.8%	$\approx$ 0.4% (0.1%)

Summary (II): α_s wish-list

■ Experimental/Theoretical needs to reach $\mathcal{O}$(0.1%) precision:

(1) Lattice QCD. Sufficient dedicated computing resources & person-power to:

- Develop pQCD $N^{3,4}$LO theory for observables in a finite space-time volume
- Extend higher renormalization scales via step-scaling to more observables

(2) Other theory efforts:

- Completion of hadronic τ decay renormalon analysis
- Advanced power corrections for e^+e^- event shapes and resummation for jet rates
- NNLL accuracy parton showers matched to NNLO
- NNLO(+NNLL) MCs for complex final states in e^+e^-, e-p, p-p
- Differential NNLO predictions for LHC & HERA multi-jet observables,…

(3) Extension of NNLO hadron-collider- and/or PDF-based extractions via:

- Incorporation of multiple new LHC precision observables and datasets
- Improved treatment of exp. correlation matrices uncertainties among measurements.
- New DIS measurements at high-energy facilities: EIC first, LHeC/FCC-eh longer future (approx. ±0.2%).

(4) Hadronic Z (and W) decays is the only non-lattice method known that can reach permil precision: Tera-Z (FCC-ee) machine needed.

Backup slides

α_s from hadronic EW decays (Update)

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]

- Incorporated new α^2,α^3 EW corrections(*) to Z pseudoobserv:

(*)Dubovyk/Chen/Freitas et al. arXiv:1906.08815, arXiv:2002.05845

- The W and Z hadronic widths :

$$\Gamma^{\rm had}_{\rm W,Z}(Q) = \Gamma^{\rm Born}_{\rm W,Z}\left(1 + \sum_{i=1}^{4} a_i(Q)\left(\frac{\alpha_S(Q)}{\pi}\right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\rm EW} + \delta_{\rm mix} + \delta_{\rm np}\right)$$

- The ratio of W, Z hadronic-to-leptonic widths :

$$\mathrm{R}_{\rm W,Z}(Q) = \frac{\Gamma^{\rm had}_{\rm W,Z}(Q)}{\Gamma^{\rm lep}_{\rm W,Z}(Q)} = \mathrm{R}^{\rm EW}_{\rm W,Z}\left(1 + \sum_{i=1}^{4} a_i(Q)\left(\frac{\alpha_S(Q)}{\pi}\right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\rm mix} + \delta_{\rm np}\right)$$

- In the Z boson case, the hadronic cross section at the resonance peak in e^+e^- :

$$\sigma^{\rm had}_{\rm Z} = \frac{12\pi}{m_{\rm Z}} \cdot \frac{\Gamma^e_{\rm Z}\Gamma^{\rm had}_{\rm Z}}{(\Gamma^{\rm tot}_{\rm Z})^2}$$

TH uncertainties:
±0.01% (Z)
±0.02% (W)

Parametric uncerts.: (α_s,$m_{Z,W}$; $V_{cs,ud}$):
±0.03% (Z)
±1.7% (W)
±0.03% (W, CKM unit)

- Incorporated modified LEP data due to luminosity bias correction(*):

	theory			experiment		
	previous	new (this work)	change	previous [6]	new [20, 21]	change
$\Gamma^{\rm tot}_{\rm Z}$ (MeV)	$2494.2 \pm 0.8_{\rm th}$	$2495.2 \pm 0.6_{\rm par} \pm 0.4_{\rm th}$	+0.04%	2495.2 ± 2.3	2495.5 ± 2.3	+0.012%
$\mathrm{R_Z}$	$20.733 \pm 0.007_{\rm th}$	$20.750 \pm 0.006_{\rm par} \pm 0.006_{\rm th}$	+0.08%	20.767 ± 0.025	20.7666 ± 0.0247	−0.040%
$\sigma^{\rm had}_{\rm Z}$ (pb)	$41\,490 \pm 6_{\rm th}$	$41\,494 \pm 5_{\rm par} \pm 6_{\rm th}$	+0.01%	$41\,540 \pm 37$	$41\,480.2 \pm 32.5$	−0.144%

W boson observables	GFITTER 2.2 (NNLO)	this work (N³LO) (exp. CKM)	this work (N³LO) (CKM unit.)	experiment
$\Gamma^{\rm had}_{\rm W}$ (MeV)	–	$1440.3 \pm 23.9_{\rm par} \pm 0.2_{\rm th}$	$1410.2 \pm 0.8_{\rm par} \pm 0.2_{\rm th}$	1405 ± 29
$\Gamma^{\rm tot}_{\rm W}$ (MeV)	$2091.8 \pm 1.0_{\rm par}$	$2117.9 \pm 23.9_{\rm par} \pm 0.7_{\rm th}$	$2087.9 \pm 1.0_{\rm par} \pm 0.7_{\rm th}$	2085 ± 42
$\mathrm{R_W}$	–	$2.1256 \pm 0.0353_{\rm par} \pm 0.0008_{\rm th}$	$2.0812 \pm 0.0007_{\rm par} \pm 0.0008_{\rm th}$	2.069 ± 0.019

Recent update of LEP luminosity bias change the Z values by few permil

(*) Voutsinas et al. arXiv:1908.01704, Janot et al. arXiv:1912.02067

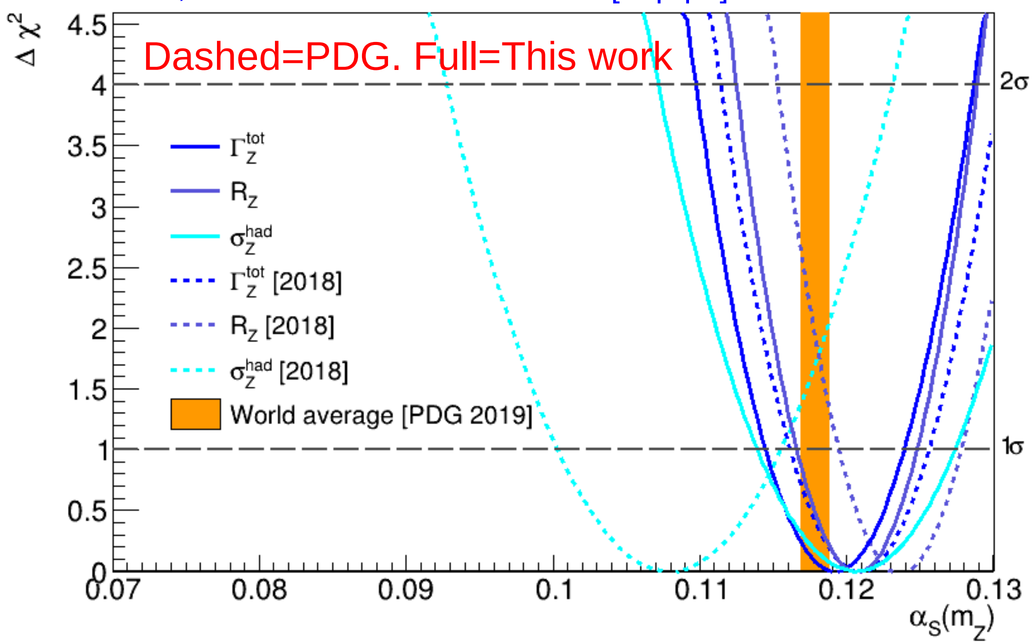
α_s from hadronic Z decays (Update)

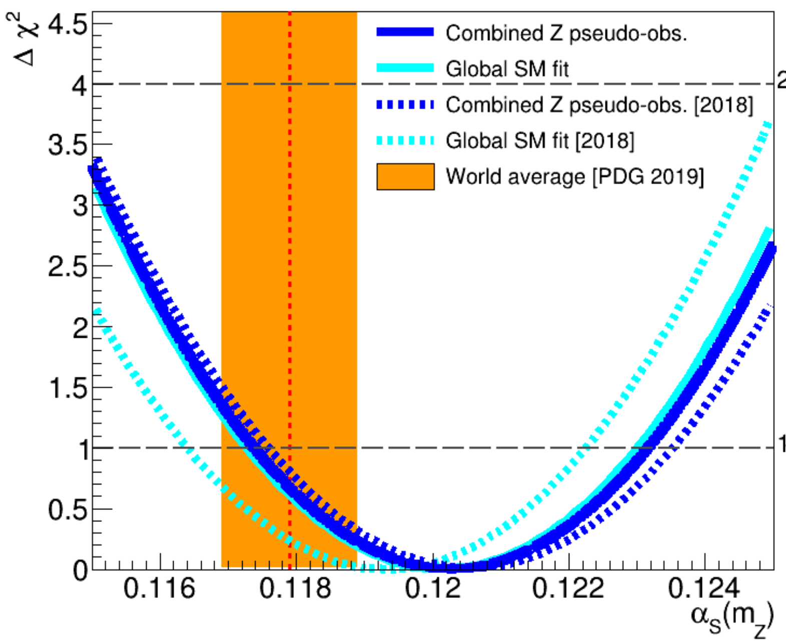
- QCD coupling extracted from:

(i) Combined fit of 3 Z pseudo-observ:
(ii) Full SM fit (with α_s free parameter)

Z boson observable	$\alpha_S(m_Z)$ extraction	uncertainties exp.	param.	theor.
Γ_Z^{tot}	0.1192 ± 0.0047	± 0.0046	± 0.0005	± 0.0008
R_Z	0.1207 ± 0.0041	± 0.0041	± 0.0001	± 0.0009
σ_Z^{had}	0.1206 ± 0.0068	± 0.0067	± 0.0004	± 0.0012
All combined	0.1203 ± 0.0029	± 0.0029	± 0.0002	± 0.0008
Global SM fit	0.1202 ± 0.0028	± 0.0028	± 0.0002	± 0.0008

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]





- LEP lumi-bias updates lead to much better agreement among Γ_Z, R_Z, σ_0 extractions:

$\alpha_s(m_z) = 0.1203 \pm 0.0028$ (±2.3%)

PDG'21: $\alpha_s(m_z) = 0.1221 \pm 0.0027$ (±2.3%)

- EXP/TH updates lead to better agreement with full SM fit:

$\alpha_s(m_z) = 0.1202 \pm 0.0028$

PDG'21: $\alpha_s(m_z) = 0.1194 \pm 0.0029$

α_s from hadronic W decays (Update)

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]

- Parametrized W boson Γ_W^{lep}, Γ_W^{had}, Γ_W^{tot}, R_W (with α_s^4,α,$\alpha_s\alpha$ corrections):

$$\Gamma_W^{\rm lep} = \Gamma_0 + c_1\,\Delta_W + c_4\,\Delta_H + c_5\,\Delta_t + c_7\,\Delta_\tau\,,\qquad \Gamma_W^{\rm had} = \Gamma_0 + c_1\,\Delta_W + c_2\,\Delta_{\rm CKM} + c_3\,\Delta_{\alpha_S} + c_6\,\Delta_{\alpha_S}^2\,,$$

$$\Delta_W = \left(\frac{m_W}{80.379}\right)^3 - 1,\quad \Delta_H = \log\left(\frac{m_H}{125.10}\right),\quad \Delta_t = \left(\frac{m_t}{172.9}\right) - 1,\quad \Delta_\tau = \left(\frac{m_\tau}{1.777}\right) - 1\quad \Delta_{\alpha_S} = \frac{\alpha_S(m_Z)}{0.1179} - 1,\quad \Delta_{\rm CKM} = \frac{|V_{cd}|^2 + |V_{cs}|^2}{0.218^2 + 0.997^2} - 1$$

W widths (GeV)	Γ_0	c_1	c_2	c_3	c_4	c_5	c_6	c_7	Max dev.
$\Gamma_W^{\rm lep}$	679.35	676.78	–	–	0.04674	0.47745	–	−0.347428	< 0.00002
$\Gamma_W^{\rm had}$ (exp. CKM)	1440.28	1446.61	734.557	53.76	–	–	−1.24411	–	< 0.0002
$\Gamma_W^{\rm had}$ (CKM unit.)	1410.21	1409.59	–	52.34	–	–	−1.15932	–	< 0.0002
$\Gamma_W^{\rm tot}$ (exp. CKM)	2119.58	2044.8	732.55	50.67	0.03980	0.46258	−1.0723	−0.36408	< 0.0002
$\Gamma_W^{\rm tot}$ (CKM unit.)	2089.51	2088.26	–	52.28	0.04790	0.47842	−1.2683	−0.32942	< 0.0002

- Numerical evaluation of W boson (N^{3}LO + EW corrs.) pseudo-observables :

W boson observables	GFITTER 2.2 (NNLO)	this work (N^{3}LO) (exp. CKM)	this work (N^{3}LO) (CKM unit.)	experiment
$\Gamma_W^{\rm lep}$ (MeV)	–	$679.4 \pm 0.3_{\rm par} \pm 0.5_{\rm th}$	$679.4 \pm 0.3_{\rm par} \pm 0.5_{\rm th}$	682.2 ± 10.2
$\Gamma_W^{\rm had}$ (MeV)	–	$1440.3 \pm 23.9_{\rm par} \pm 0.2_{\rm th}$	$1410.2 \pm 0.8_{\rm par} \pm 0.2_{\rm th}$	1405 ± 29
$\Gamma_W^{\rm tot}$ (MeV)	$2091.8 \pm 1.0_{\rm par}$	$2119.6 \pm 23.9_{\rm par} \pm 0.7_{\rm th}$	$2089.5 \pm 1.1_{\rm par} \pm 0.7_{\rm th}$	2085 ± 42
R_W	–	$2.1200 \pm 0.0352_{\rm par} \pm 0.0016_{\rm th}$	$2.0757 \pm 0.0014_{\rm par} \pm 0.0015_{\rm th}$	2.0684 ± 0.0254

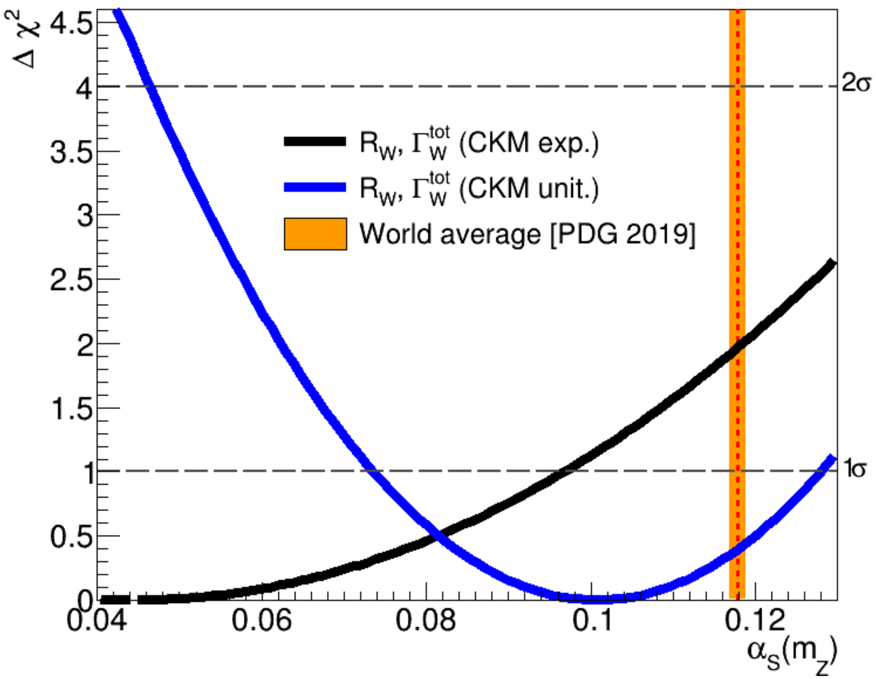
α_s from hadronic W decays (Update)

- QCD coupling extracted from new N³LO fit of combined Γ_W, R_W pseudo-observ.:

W boson observables	$\alpha_S(m_Z)$ extraction	uncertainties exp.	param.	theor.
Γ_W^{tot}, R_W (exp. CKM)	0.044 ± 0.052	± 0.024	± 0.047	(± 0.0014)
Γ_W^{tot}, R_W (CKM unit.)	0.101 ± 0.027	± 0.027	(± 0.0002)	(± 0.0016)
Γ_W^{tot}, R_W (FCC-ee, CKM unit.)	0.11790 ± 0.00023	± 0.00012	± 0.00004	± 0.00019

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]

- Still very imprecise extraction:
  - Large propagated parametric uncert. from poor V_{cs} exp. precision (±2%): QCD coupling unconstrained: 0.04±0.05
  - Imposing CKM unitarity: large exp. uncertainties from Γ_W, R_W (±0.9–2%): QCD coupling with ~27% precision
  - Propagated TH uncertainty much smaller today than exp. ones: ~1%



$\alpha_s(m_Z)$ = 0.101 ± 0.027 (±27%)

α_s from hadronic Z decays (future)

- QCD coupling extracted from:

(i) Combined fit of 3 Z pseudo-observ:
(ii) Full SM fit (with α_s free parameter)

Z boson observable	$\alpha_S(m_Z)$ extraction	uncertainties exp.	param.	theor.
All combined	0.1203 ± 0.0029	±0.0029	±0.0002	±0.0008
Global SM fit	0.1202 ± 0.0028	±0.0028	±0.0002	±0.0008
All combined (FCC-ee)	0.12030 ± 0.00026	±0.00013	±0.00005	±0.00022
Global SM fit (FCC-ee)	0.12020 ± 0.00026	±0.00013	±0.00005	±0.00022

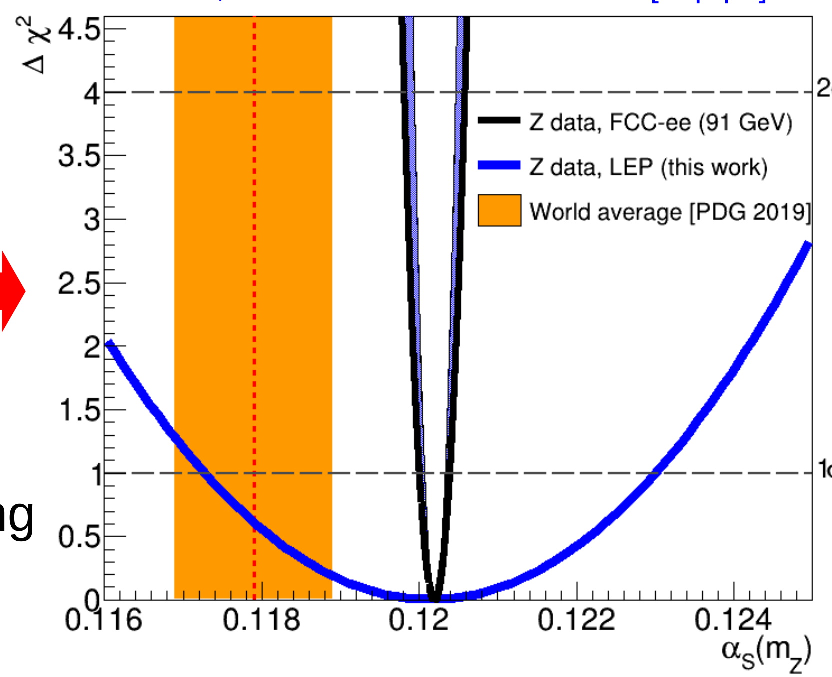
- FCC-ee:

– Huge Z pole stats. ($\times 10^5$ LEP):
– Exquisite systematic/parametric precision (stat. uncert. negligible):

$$\Delta R_Z = 10^{-3}, \quad R_Z = 20.7500 \pm 0.0010$$
$$\Delta\Gamma_Z^{tot} = 0.1\ \text{MeV}, \quad \Gamma_Z^{tot} = 2495.2 \pm 0.1\ \text{MeV}$$
$$\Delta\sigma_Z^{had} = 4.0\ \text{pb}, \quad \sigma_Z^{had} = 41\,494 \pm 4\ \text{pb}$$
$$\Delta m_Z = 0.1\ \text{MeV}, \quad m_Z = 91.18760 \pm 0.00001\ \text{GeV}$$
$$\Delta\alpha = 3 \cdot 10^{-5}, \quad \Delta\alpha_{had}^{(5)}(m_Z) = 0.0275300 \pm 0.0000009$$

– TH uncert. to be reduced by $\times 4$ computing missing α_s^5, α^3, $\alpha\alpha_s^2$, $\alpha\alpha_s^2$, $\alpha^2\alpha_s$ terms

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]



- 10 times better precision than today: $\delta\alpha_s/\alpha_s$ ~ ±0.2% (exp+th), ±0.1% (exp)
Strong (B)SM consistency test.

$\alpha_s(m_Z) = 0.12030 \pm 0.00028$ (±0.2%)

α_s from hadronic W decays (future)

- QCD coupling extracted from new N³LO fit of combined Γ_W, R_W pseudo-observ.:

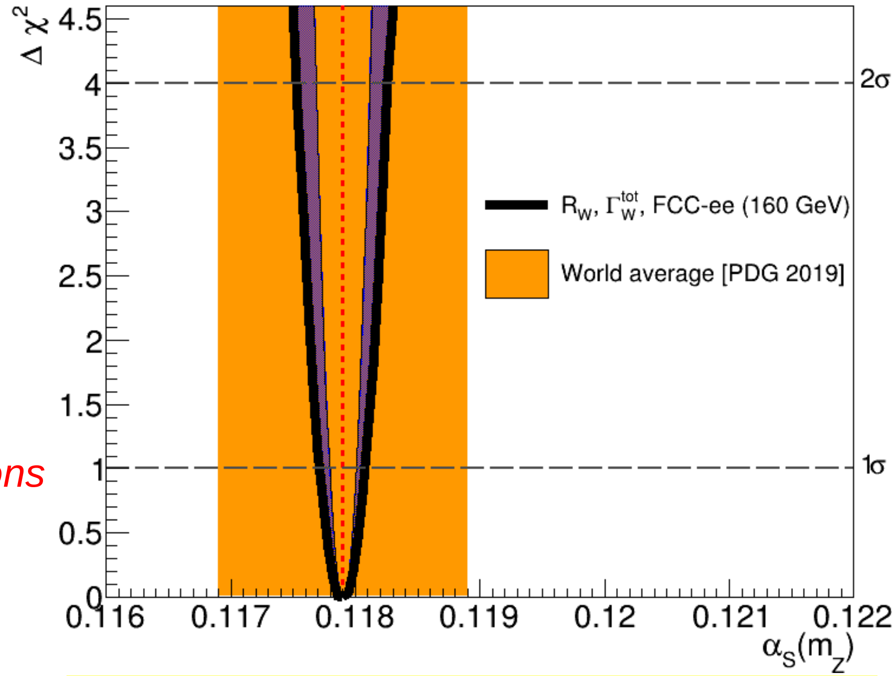
W boson observables	$\alpha_S(m_Z)$ extraction	uncertainties exp.	param.	theor.
Γ_W^{tot}, R_W (exp. CKM)	0.044 ± 0.052	±0.024	±0.047	(±0.0014)
Γ_W^{tot}, R_W (CKM unit.)	0.101 ± 0.027	±0.027	(±0.0002)	(±0.0016)
Γ_W^{tot}, R_W (FCC-ee, CKM unit.)	0.11790 ± 0.00023	±0.00012	±0.00004	±0.00019

- FCC-ee extraction:
  - Huge W pole stats. ($\times10^4$ LEP-2).
  - Exquisite syst./parametric precision:

$$\Gamma_W^{tot} = 2088.0 \pm 1.2 \text{ MeV}$$
$$R_W = 2.08000 \pm 0.00008$$
$$m_W = 80.3800 \pm 0.0005 \text{ GeV}$$
$$|V_{cs}| = 0.97359 \pm 0.00010$$ ← *O(10^{12}) D mesons*

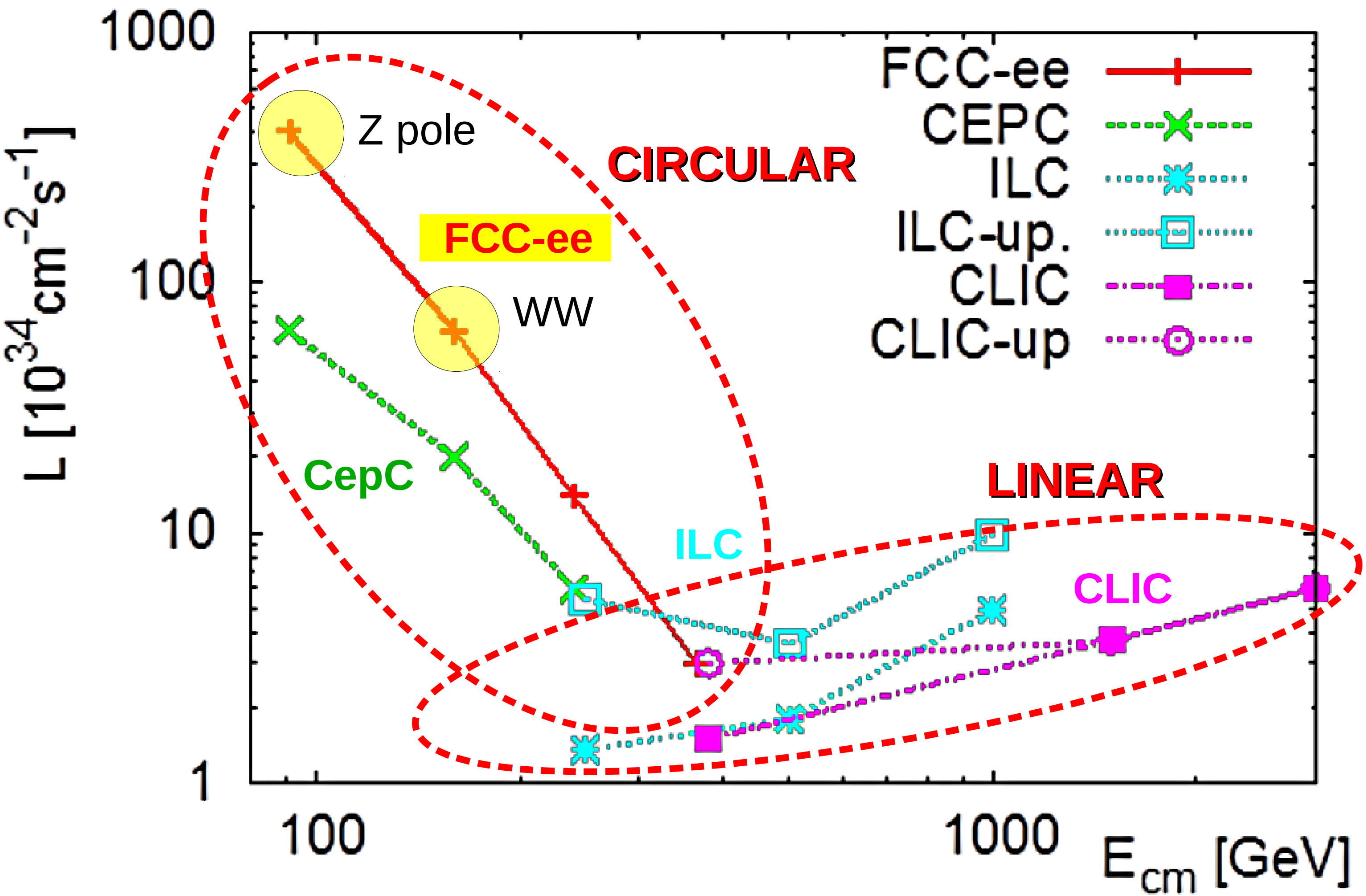
  - TH uncertainty to be reduced by $\times10$ after computing missing α_s^5, α^2, α^3, $\alpha\alpha_s^2$, $\alpha\alpha_s^2$, $\alpha^2\alpha_s$ terms

DdE, Jacobsen: arXiv:2005.04545 [hep-ph]



$\alpha_s(m_z) = 0.11790 \pm 0.00023$ (±0.2%)

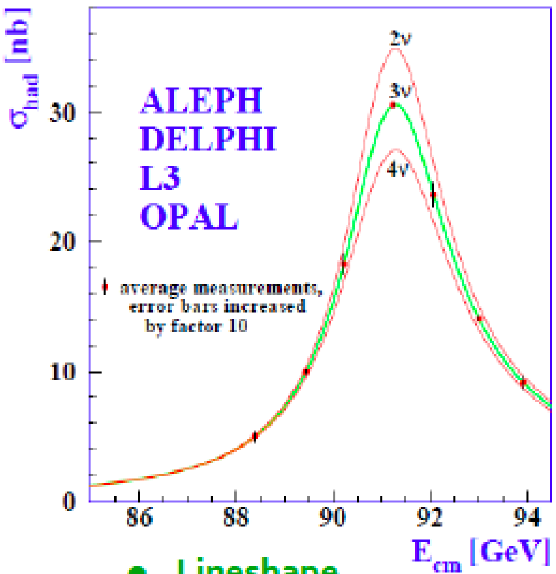
Future e^+e^- colliders under discussion



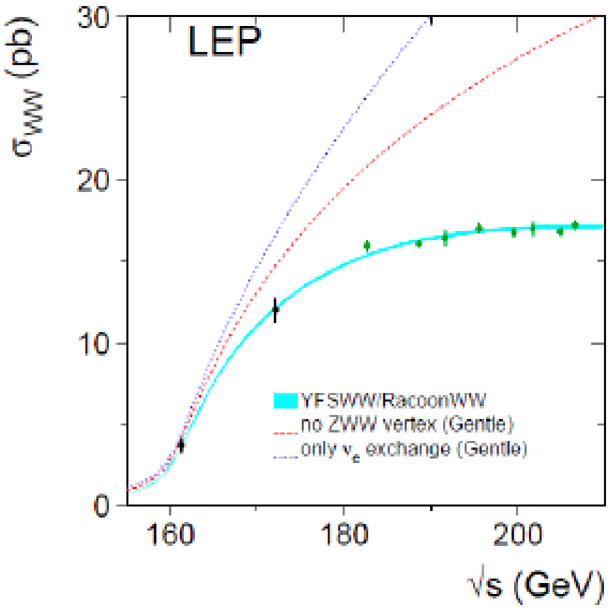
- FCC-ee features lumis a few times larger than other machines over 90–240 GeV
- Unparalleled Z, W, jets, τ,… data sets: Negligible α_s stat. uncertainties

Ultra-precise W, Z, top physics at FCC-ee

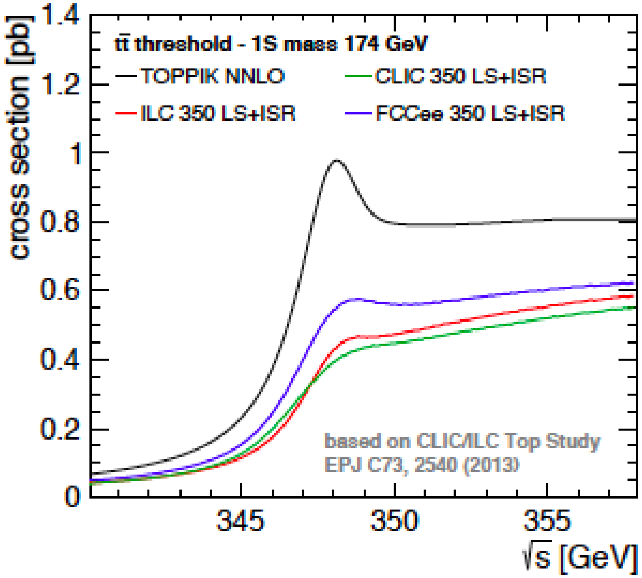
$\sqrt{s}$=91 GeV, 10^{12} Z's | **$\sqrt{s}$=161 GeV, 10^{8} W's** | **$\sqrt{s}$=350 GeV, 10^{6} tops**



- Lineshape
  - Exquisite E_{beam} (unique!)
  - m_Z, Γ_Z to 10 keV (stat.) 100 keV (syst.)
- Asymmetries
  - $\sin^2\theta_W$ to 5×10^{-6}
- Branching ratios, R_l, R_b
  - $\alpha_s(m_Z)$ to 0.0002
- Predict m_{top}, m_W in SM



- Threshold scan
  - m_W to 500 keV
- Branching ratios R_l, R_{had}
  - $\alpha_s(m_W)$ to 0.0002
- Radiative returns $e^+e^-\to\gamma Z$ ($Z\to\nu\nu$, $\mu^+\mu^-$)
  - N_ν to 0.001



- Threshold scan + 4D fit
  - m_{top} to 10 MeV (stat.) 40 MeV (th.)
  - λ_{top} to 13%
  - EWK couplings to 1–10%

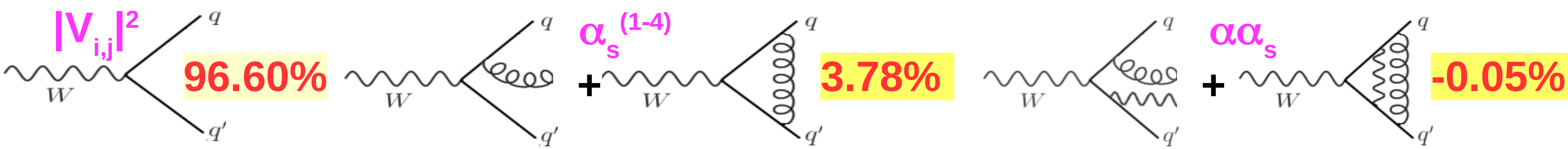
- Unparalleled Z, W, jets, τ,… data sets: Negligible α_s stat. uncertainties
- Unparalleled syst. uncert.: δE_{cm}(Z,W) ~ 0.1, 0.3 MeV → Very precise $\Gamma_{W,Z}$

α_s from hadronic W decays (today)

- Width known at N³LO. As for Z boson, small sensitivity to α_s (only beyond Born):

$$\Gamma_{\rm W,had} = \frac{\sqrt{2}}{4\pi} G_F m_W^3 \sum_{\rm quarks\ i,j} |V_{\rm i,j}|^2 \left[1 + \sum_{k=1}^{4} \left(\frac{\alpha_s}{\pi}\right)^k + \delta_{\rm electroweak}(\alpha) + \delta_{\rm mixed}(\alpha\alpha_s)\right]$$

[EWK: -0.35%]



- Recalculation of partial and total hadronic W widths:

DdE, Srebre:arXiv:1603.06501

Partial widths (MeV)	$\Gamma^{(0)}$	$\Gamma^{(1)}_{\rm QCD}$	$\Gamma^{(2)}_{\rm QCD}$	$\Gamma^{(3)}_{\rm QCD}$	$\Gamma^{(4)}_{\rm QCD}$	$\Gamma_{\rm ewk}$	$\Gamma_{\rm mixed}$	$\Gamma^{\rm W}_{\rm had}$
$W \to qq'$ (exp. $V_{\rm ij}$)	1379.851	52.931	2.857	−0.992	−0.238	−5.002	−0.755	$1428.65 \pm 22.40_{\rm par} \pm 0.04_{\rm th}$
$W \to qq'$ ($V_{\rm ij}V_{\rm jk} = \delta_{\rm ik}$)	1363.197	52.291	2.822	−0.980	−0.235	−4.942	−0.746	$1411.40 \pm 0.96_{\rm par} \pm 0.04_{\rm th}$
$W \to qq'$ (exp. $V_{\rm ij}$) [5]	1408.980	54.087	2.927	−1.018	−0.245	−5.132	−0.779	$1458.820 \pm 0.006_{\rm th}$
$W \to qq'$ ($V_{\rm ij}V_{\rm jk} = \delta_{\rm ik}$) [5]	1363.640	52.346	2.833	−0.985	−0.237	−4.940	−0.748	$1411.910 \pm 0.006_{\rm th}$

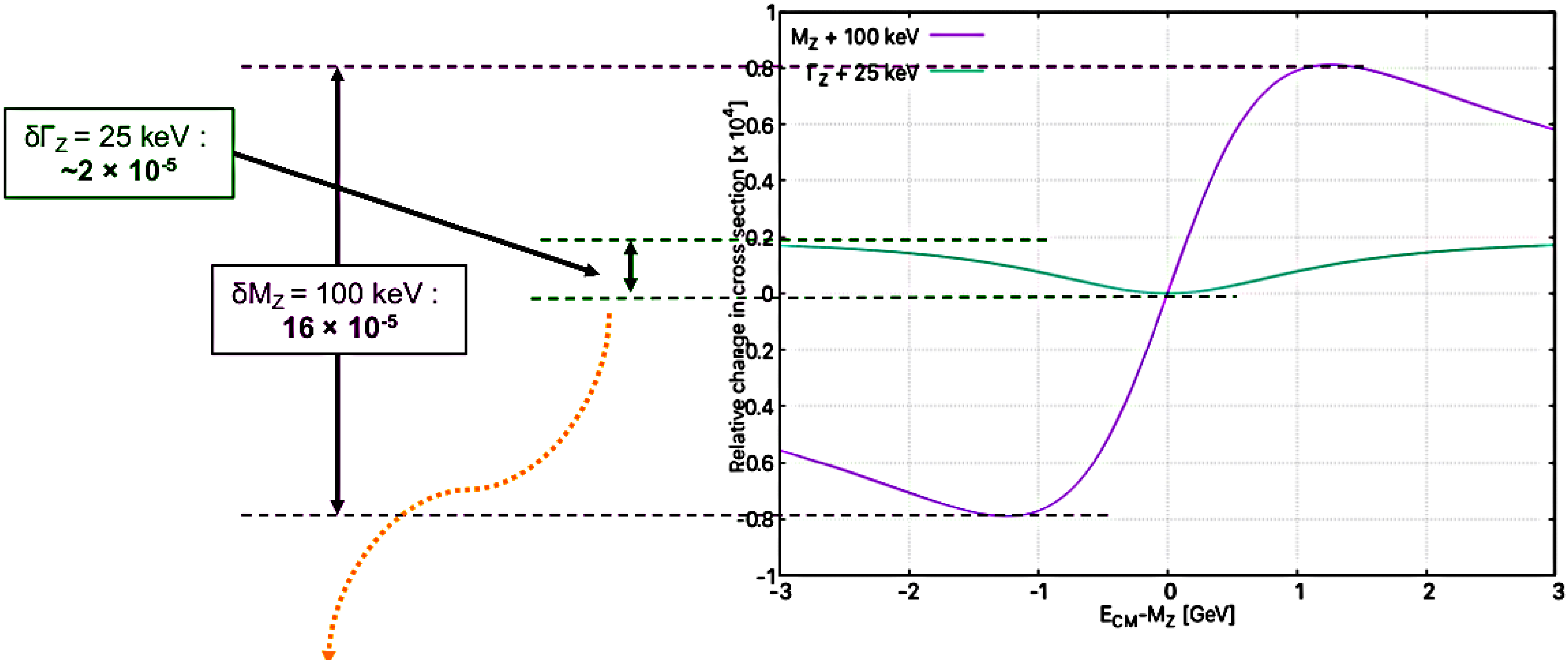
- Careful evaluation of parametric ($V_{i,j}$, m_W) & theoretical uncertainties:
  - Parametric uncertainty: ±22.40 MeV (dominated by V_{cs})
    ±0.96 MeV (CKM unitarity, dominated by m_W)
  - Higher-order QCD corrections: ±0.2 MeV (diffs. N³LO vs N⁴LO for Γ_Z)
  - Higher-order EWK, mixed corrections: ±0.035 MeV (from D.Kara NPB877(2013)683)
  - Others (non-pQCD $(\Lambda_{QCD}/m_W)^4$, finite q masses above LO, ren. scheme,…): negligible

FCC-ee (91 GeV) syst. uncertainties

- FCC-ee goal: Via Z line-shape scan, determine Z parameters to precisions:

  $\delta M_Z = 100$ keV ; $\delta\Gamma_Z = 25$ keV

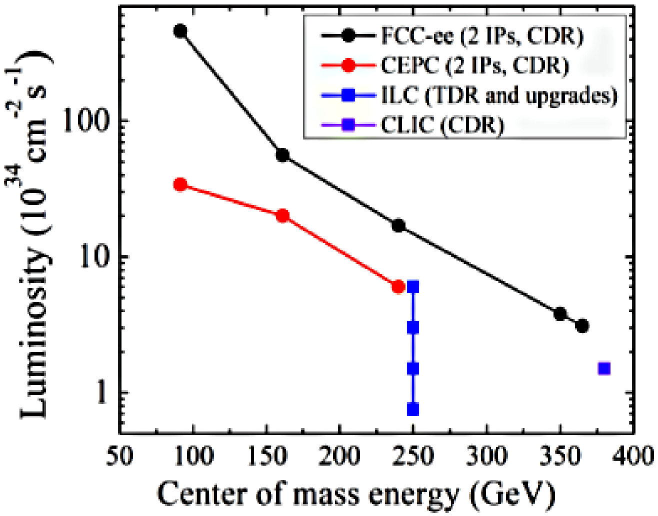
  - Plot shows relative change in cross section across Z resonance for parameter variation of this size

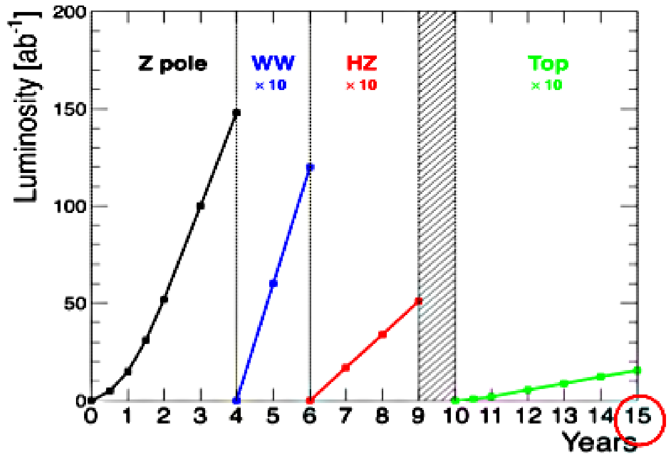


- Z width measurement most demanding: Need **relative** normalisation to about 10^{-5}
  - Need statistics of order 10^{10}
  - Need careful control of energy dependent effects

FCC-ee Luminosity, Operation, Data samples

Largest luminosities in the 88 – 365 GeV energy range





Event statistics	√s precision
5×10^{12} $e^+e^- \rightarrow Z$	100 keV
10^8 $e^+e^- \rightarrow W^+W^-$	300 keV
10^6 $e^+e^- \rightarrow HZ$	1 MeV
10^6 $e^+e^- \rightarrow t\bar{t}$	2 MeV

Working point	Z, years 1-2	Z, later	WW	HZ	tt threshold...	... and above
√s (GeV)	88, 91, 94		157, 163	240	340 – 350	365
Lumi/IP (10^{34} cm^{-2}s^{-1})	100	200	25	7	0.8	1.4
Lumi/year (2 IP)	24 ab^{-1}	48 ab^{-1}	6 ab^{-1}	1.7 ab^{-1}	0.2 ab^{-1}	0.34 ab^{-1}
Physics goal	150 ab^{-1}		10 ab^{-1}	5 ab^{-1}	0.2 ab^{-1}	1.5 ab^{-1}
Run time (year)	2	2	2	3	1	4