Top Mass Measurements at $e^+e^-$ Colliders

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Outline

• Different experimental routes
• A detailed look at the threshold
• Snowmass ’21 Systematics summary
• A further look at key uncertainties
Top Quark Mass: Measurement Strategies

At and above threshold

- The accelerator side: Requires sufficient collision energy for top pair production
- So far thoroughly studied for ILC, CLIC, threshold studies common for CLIC, FCC-ee, ILC

Three approaches to the top mass

- The threshold scan around 350 GeV
- The top mass from radiative events
- Direct kinematic reconstruction

threshold - QQbar_Threshold NNNLO
ISR + ILC Luminosity Spectrum
- default - m_t = 171.5 GeV, τ = 1.37 GeV
- variations +0.1 GeV
- variations -0.15 GeV

efficiencies and signal yields from EPJC73, 2530 (2013)
Top Quark Mass: Measurement Strategies

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Three approaches to the top mass

The threshold scan around 350 GeV

The top mass from radiative events

Direct kinematic reconstruction

Key references:
- EPJ C73, 2530 (2013) (CLIC, (ILC): Threshold, direct)
- JHEP 11, 003 (2019) (CLIC: Threshold, radiative, direct)
- PLB 804,135353 (2020) (ILC, CLIC: radiative)

+ a rich set of reports and conference proceedings on arXiv
Direct Kinematic Reconstruction above Threshold

Interpretation challenges

• 80 MeV statistical precision for 100 fb$^{-1}$ at 500 GeV (extracted via template fits) in invariant mass of $W_b$ final state.
  Newer study: 30 MeV at 380 GeV, 1 ab$^{-1}$ at CLIC
• Experimental systematics, in particular $b$-JES a clear challenge
• On the theory side: interpretation of the measured mass, other corrections

[EPJ C73, 2530 (2013)]

[JHEP 11, 03 (2019)]
Top Mass in Radiative Events

A threshold scan at higher collision energies

- A new(er) idea to measure the top mass in a theoretically well-defined scheme in high-energy running above the threshold
Top Mass in Radiative Events

A threshold scan at higher collision energies

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<th>ILC, $\sqrt{s} = 500$ GeV</th>
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<tbody>
<tr>
<td>luminosity [fb$^{-1}$]</td>
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<td>1000</td>
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<tr>
<td>statistical</td>
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<td>90 MeV</td>
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<td>theory</td>
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<td>55 MeV</td>
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<tr>
<td>lum. spectrum</td>
<td>20 MeV</td>
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</tr>
<tr>
<td>photon response</td>
<td>16 MeV</td>
<td>85 MeV</td>
</tr>
<tr>
<td>total</td>
<td>150 MeV</td>
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matched NNLO + NNLL calculation, luminosity spectrum folded in explicitly; Extraction of short distance MSR mass
A threshold scan at higher collision energies

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<td>luminosity [fb$^{-1}$]</td>
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<td></td>
</tr>
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<td>140 MeV 90 MeV 350 MeV 110 MeV</td>
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matched NNLO + NNLL calculation, luminosity spectrum folded in explicitly; Extraction of short distance MSR mass

can provide 5σ evidence for scale evolution (“running”) of the top quark MSR mass from ILC500 data alone
The Top Quark Mass

_Ultimate precision at the threshold_

- Exploit precise theoretical calculations of cross section in the threshold region, in well-defined mass schemes ($m_t^{PS}$, $m_t^{1S}$...) -> Can be converted directly into MSbar mass.
The Top Quark Mass

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Differences between Colliders

The Luminosity Spectrum

- Linear collider luminosity spectra are characterized by a beamstrahlung tail, FCC-ee is close to Gaussian
Differences between Colliders

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![Graph showing differences between FCC and ILC luminosity spectra.](image)

- The Luminosity Spectrum

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Differences between Colliders

The Luminosity Spectrum

- Linear collider luminosity spectra are characterized by a beamstrahlung tail, FCC-ee is close to Gaussian

The Luminosity Spectrum

\[
\begin{align*}
\text{tt threshold} - m_T^{171.5 \text{ GeV}} \\
\text{QQbar Threshold NNNLO} & \quad \text{FCC-ee 350 LS only} \\
\text{ISR only} & \quad \text{FCC-ee 350 LS+ISR}
\end{align*}
\]

\[
\begin{align*}
\text{ILC} & \quad 350 \text{ LS only} \\
\text{ILC 350 LS+ISR}
\end{align*}
\]

\[
\begin{align*}
\text{91 km ring circumference} \\
\text{based on CLIC/ILC Top Study EPJ C73, 2530 (2013)}
\end{align*}
\]

Requires: Precise understanding and measurement of spectrum.
In this case:
~ 30% reduction of cross section -> 15% hit on statistical uncertainty
The Standard Threshold Scan

Experimental Assumptions

- The standard assumptions:
  Efficiency, signal and background yields taken from EPJ C73, 2530 (2013):
  70.2% signal efficiency, 73 fb effective background cross section after selection

- A 10-point threshold scan, with equal luminosity sharing, spacing by 1 GeV, from 340 - 349 GeV

- ILC, FCC-ee assume 200 fb$^{-1}$ total, CLIC 100 fb$^{-1}$
  (for easier comparisons, 200 fb$^{-1}$ numbers are often also quoted for CLIC)

- Top mass (and other parameters, such as $\Gamma_t$, $y_t$, $\alpha_s$) extracted via template fits of predicted cross sections with different input parameters.
  **Theory essential** - here NNNLO QCD [Beneke et.al.]
Theory Uncertainties

A key factor

- QCD scale uncertainties highly relevant.
Theory Uncertainties

A key factor

- QCD scale uncertainties highly relevant.
- Also need to calculate other effects, such as ISR, to the required precision!

The step from black to green

[only approximate in current experimental studies]
Choosing the Scan Range

Parameter Sensitivity

- Plot shows the derivative of the cross section for various parameters - to make this understandable this is normalised to typical changes of these parameters.
- Full use to optimize scan range requires knowledge of mass to ~ 200 MeV in PS scheme. Can be achieved with 2 x 5 fb\(^{-1}\):
  - point 1: \(\sqrt{s} = 2 \times m_{t,\text{PS},LHC} - 1.5\) GeV
  - point 2: \(\sqrt{s} = 2 \times m_{t,\text{PS},LHC} + 0.5\) GeV  [arXiv:1902.07246]
  (N.B.: This is safe also when taking theory uncertainties into account)
- Optimizing for particular parameters can reduce the statistical uncertainty by ~ 25% [JHEP 7, 70 (2021)]
Choosing the Scan Range

Enter theory uncertainty

- QCD scale uncertainties dominate over point-by-point statistical uncertainties for typical threshold scans: At this point optimising scan strategies to reduce statistical uncertainties does not improve the total uncertainty - in fact concentrating on a very small range may make systematic control more difficult.

- In general: Also to separate contributions from different parameters, the most relevant range is 340 - 346 GeV. Higher energy points would primarily benefit a $y_t$ measurement.
Choosing the Scan Range

**Bottom line for FCC-ee studies**

- Mildly optimized scan (mass & width) for FCC-ee as a balance between different sensitivities:
  - 8 points in the range of 340 - 346 GeV

assumed for most results in the following
Fitting Multiple Parameters

Mass, Width, Yukawa Coupling

- ~ 45 MeV on width

- ~ 11.5% on Yukawa coupling
Uncertainties Overview

**ILC as starting point**

- Relatively thorough evaluation for ILC leading up to European Strategy, and adjusted also for CLIC in the framework of the CLIC top physics paper.

<table>
<thead>
<tr>
<th>error source</th>
<th>$\Delta m_t^{\text{PS}}$ [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>stat. error (200 fb$^{-1}$)</td>
<td>13</td>
</tr>
<tr>
<td>theory (NNNLO scale variations, PS scheme)</td>
<td>40</td>
</tr>
<tr>
<td>parametric ($\alpha_s$, current WA: 9 x 10$^{-4}$)</td>
<td>26</td>
</tr>
<tr>
<td>non-resonant contributions (such as single top)</td>
<td>$&lt; 40$</td>
</tr>
<tr>
<td>residual background / selection efficiency</td>
<td>10 – 20</td>
</tr>
<tr>
<td>luminosity spectrum uncertainty</td>
<td>$&lt; 10$</td>
</tr>
<tr>
<td>beam energy uncertainty</td>
<td>$&lt; 17$</td>
</tr>
<tr>
<td>combined theory &amp; parametric</td>
<td>30 – 50</td>
</tr>
<tr>
<td>combined experimental &amp; backgrounds</td>
<td>25 – 50</td>
</tr>
<tr>
<td>total (stat. + syst.)</td>
<td>40 – 75</td>
</tr>
</tbody>
</table>

assuming standard 10 point scan
40 - 45, depending on scan range
evaluated with current WA

2015 study, reduce with theory work
2012 study, basis for all newer threshold studies
evaluated for CLIC
very conservative estimate - expect to be better
Uncertainties Overview

My Snowmass’21 Bottom Line

<table>
<thead>
<tr>
<th>$\delta m_t^{PS}$ [MeV]</th>
<th>ILC</th>
<th>CLIC</th>
<th>FCC-ee</th>
</tr>
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<tbody>
<tr>
<td>$\mathcal{L}$ [fb$^{-1}$]</td>
<td>200</td>
<td>100 [200]</td>
<td>200</td>
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Theoretical uncertainty (QCD): 40 – 45 |
Parametric uncertainty $\alpha_s$: 26 | 26 | 3.2 |
Parametric uncertainty $y_t$ (HL-LHC): 5 |
Non-resonant contributions: < 40 |
Experimental systematic uncertainty: 15 – 30 | 11 – 20 |
Total uncertainty: 40 – 75 |

- Assumptions on experimental systematics:
  - residual background and selection similar for all
  - very slight advantage for FCC-ee due to absence of luminosity spectrum uncertainty
  - Beam energy uncertainty for FCC-ee 3 MeV (for 5 MeV energy uncertainty) - original conservative estimate for ILC and CLIC can likely be improved to similar range (same energy measurement techniques for linear and circular)

slightly compressed scans for ILC, FCC-ee

latest evaluation with 8-point scans

ultimate $\alpha_s$ ($1.2 \times 10^{-4}$) assumed for FCC-ee, current WA for ILC, CLIC
(Do we have something better?)
Uncertainties: Luminosity Spectrum

A few more details

• Beamstrahlung tail at linear colliders smears the threshold curve: Requires accurate modeling in analysis

• Studied with the CLIC spectrum, with full detector simulations for spectrum reconstruction. Effect on top mass controlled to better than 10 MeV
Uncertainties - Parametric

A few more details

Correlation of mass with $\alpha_s$, $y_t$

Uncertainty scales with input precision:

- $\Delta m \sim 2.6 \text{ MeV per } 10^{-4} \text{ in } \alpha_s$
- $\Delta m \sim 1.6 \text{ MeV per } 1\% \text{ in } y_t: \sim 5 \text{ MeV for } 3.4\%$
- from HL-LHC
Uncertainties - Non-resonant contributions

A few more details

- Studied in EPJ C75, 223 (2015) (Fuster et al)

- Non-resonant contributions in the threshold region are non-negligible.
- Contribution to yield depends strongly on cuts.
- Cuts can influence shape

- Precise understanding and control important: need to limit the effect to well below 1% of cross section to make uncertainty smaller than statistics!
Uncertainties - Scale

A few more details

- Impact of QCD scale uncertainties on mass, width, Yukawa extraction

![Graph showing the impact of QCD scale uncertainties on mass, width, and Yukawa extraction.](image-url)
Uncertainties - Scale

A few more details

- Impact of QCD scale uncertainties on mass, width, Yukawa extraction

February 2022

FCC-ee top threshold scan 340 - 349 GeV, flexible
PS scheme, input $m_{tPS} = 171.5$ GeV
standard template fit

efficiencies and signal yields taken from EPJ C73, 2530 (2013)

February 2022

FCC-ee flexible top threshold scan , 200.0 fb$^{-1}$
PS scheme, input $m_{tPS} = 171.5$ GeV, $\Gamma_t = 1.37$ GeV

- $\Delta m$ (2D template fit)
- $\Delta \Gamma$ (2D template fit)
- $\Delta \Gamma$ (1D template fit, fixed $m_t$)

efficiencies and signal yields taken from EPJ C73, 2530 (2013)
Uncertainties - Scale

A few more details

- Impact of QCD scale uncertainties on mass, width, Yukawa extraction
Uncertainties - Scale

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The leading systematic:
Improvements directly propagate to total precision
Bottom Line

• $e^+e^-$ Colliders running at the top quark pair production threshold will provide the ultimate precision on the top quark mass

• A challenge for theory: Understanding parameters on a level comparable to expected experimental precision. Theory is a / the leading systematic for many measurements - for the mass it is the leading uncertainty overall.

⇒ Advances in theory directly translate into improvements of overall precision.