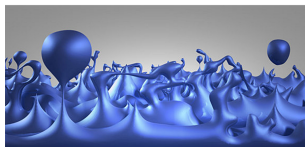


# The conformal bootstrap

Silviu S. Pufu

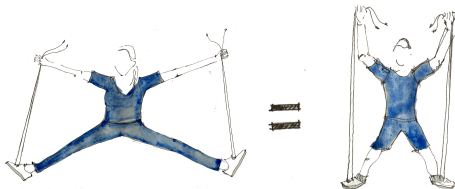
Princeton University

Seattle, July 23, 2022



# The bootstrap philosophy

**My task:** Talk about bootstrap, with emphasis on bootstrapping string theory.



- **Bootstrap** = the use of **symmetry** and other principles (e.g. **unitarity/positivity, analyticity, crossing symmetry**) to constrain (or even determine) some physical quantity

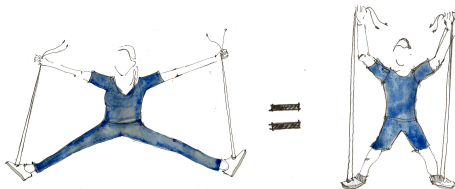
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(most well developed)

- Has been applied to:

→ scattering amplitudes  
→ matrix models  
→ lattice systems  
→ etc.

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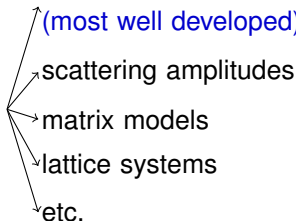
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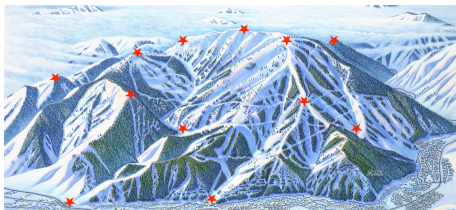
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# Conformal Field Theory

- **This talk:** Focus on bootstrapping CFT.
- CFTs are QFTs with no intrinsic length/energy scale for which any angle-preserving transformation of spacetime points is a symmetry.
- May not be directly relevant for particle physics b/c CFTs have only massless excitations  $E \propto |\vec{p}|$ .
- **BUT:** CFTs are landmarks in the space of QFTs. They are endpoints of Renormalization Group (RG) flow, with RG flows connecting them



⇒ can deform CFTs to learn about more general QFTs [Hogervorst, Rychkov, van Rees, Katz, Fitzpatrick, Anand, Genest, Khandker, Walters, Xin, Chen, ...]

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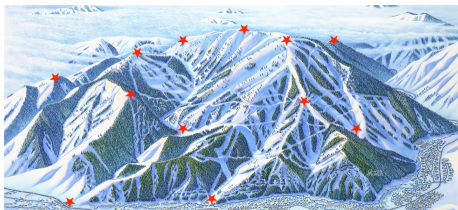
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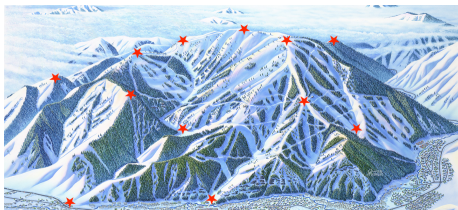
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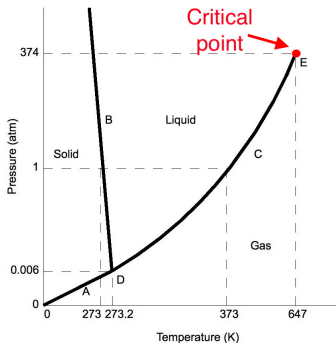


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# Conformal Field Theory

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- **statistical physics:** Euclidean CFTs describe 2nd order thermal phase transitions (e.g. water/vapor critical pt  $T_c = 647$  K and  $P_c = 374$  atm.)



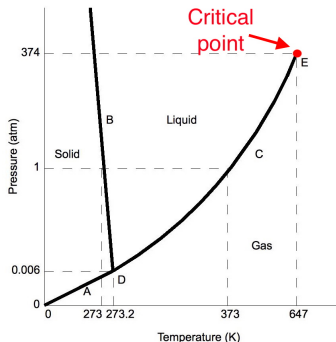
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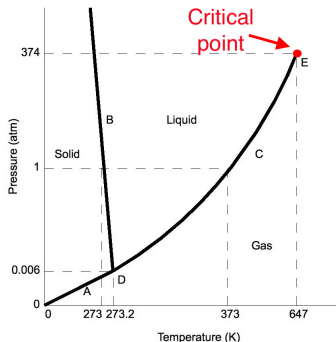


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- Conformal symmetry  $\implies$

$$\langle \phi_i(\vec{x}) \phi_j(\vec{y}) \rangle = \frac{\delta_{ij}}{|\vec{x} - \vec{y}|^{2\Delta_i}}$$

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where  $\Delta_{ijk} \equiv \Delta_i + \Delta_j - \Delta_k$ .

Higher pt correlators can be computed from these.

- In the context of phase transitions, the scaling dimensions  $\Delta_i$  are related to “critical exponents”.
- For example,

$$\rho_\ell - \rho_v \propto (T_c - T)^\beta, \quad \beta = \frac{\Delta_\sigma}{3 - \Delta_\epsilon}$$

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- **CFT data:** list of observables  $\phi_i$  w/ their properties:  $\Delta_i$  (the scaling dimensions), spin, other symmetry charges; and 3-pt function coeffs  $f_{ijk}$ .
- **Conformal Bootstrap** = an approach that relies on **general principles**. (No Lagrangian / action / partition function needed.)
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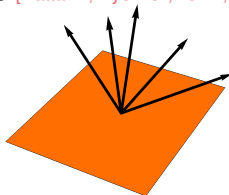


# Conformal bootstrap

- One can compute a 4-pt function  $\langle \phi_1(\vec{x}_1) \phi_2(\vec{x}_2) \phi_3(\vec{x}_3) \phi_4(\vec{x}_4) \rangle$  in two different ways, and they must agree:

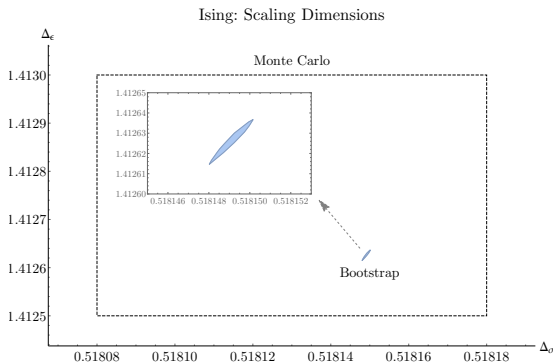
$$\langle \phi_1 \phi_2 \phi_3 \phi_4 \rangle = \sum_k f_{12k} \phi_k \begin{array}{c} \phi_1 \quad \phi_4 \\ \diagdown \quad \diagup \\ \text{---} \phi_k \text{---} \\ \diagup \quad \diagdown \\ \phi_2 \quad \phi_3 \end{array} f_{34k} = \sum_k \begin{array}{c} \phi_1 \quad \phi_4 \\ \diagdown \quad \diagup \\ \text{---} f_{14k} \text{---} \\ \diagup \quad \diagdown \\ \phi_2 \quad \phi_3 \end{array} \phi_k f_{23k}$$

- In general, unknown set of  $\phi_k$  and unknown  $f_{ijk}$ .
- Looking for situations that make this impossible can be rephrased as a semi-definite programming problem (numerical optimization)  $\Rightarrow$  exclude regions in CFT data space [Rattazzi, Rychkov, Tonni, Vichi, El-Showk, Kos, Paulos, Poland, Simmons-Duffin, ...] .



# Numerical results

- “Precision islands”: 3d Ising,  $O(N)$ , Gross-Neveu-Yukawa models.
- Ising: IR fixed pt of  $L = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}m^2\phi^2 + \lambda\phi^4$  with  $m$  tuned to  $m_{\text{cr}}$ .

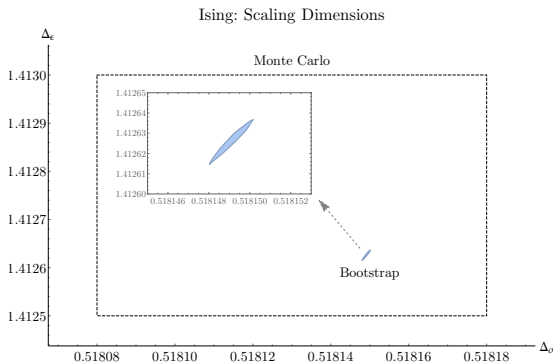


[Kos, Poland, Simmons-Duffin, Vichi '16]

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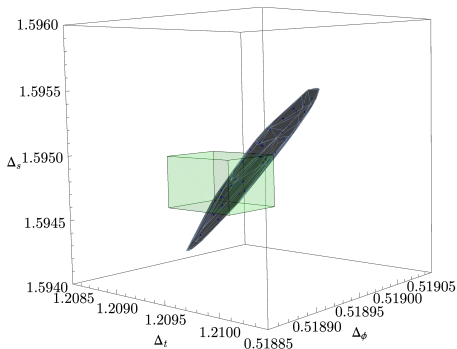
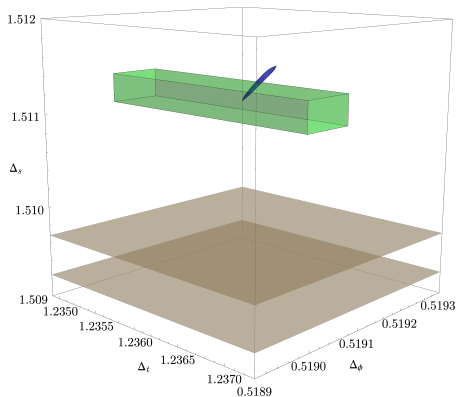
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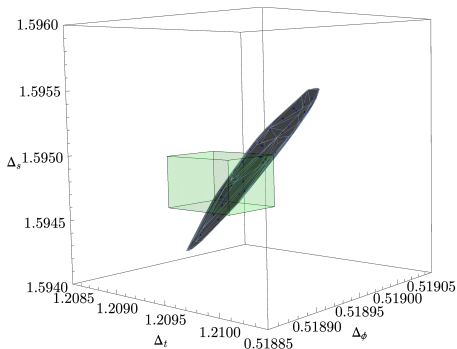
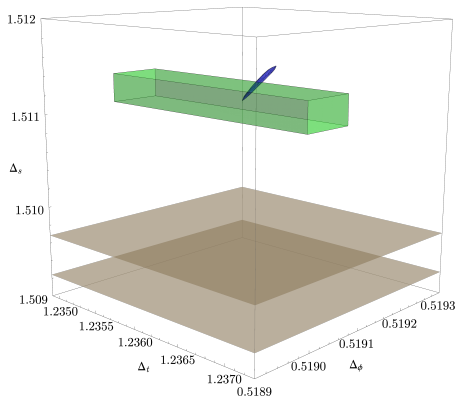
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[Chester, Landry, Liu, Poland, Simmons-Duffin, Su, Vichi '19, 20']

- Next frontier: gauge theories in 3 and 4 dimensions.
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# Bootstrapping Quantum Gravity

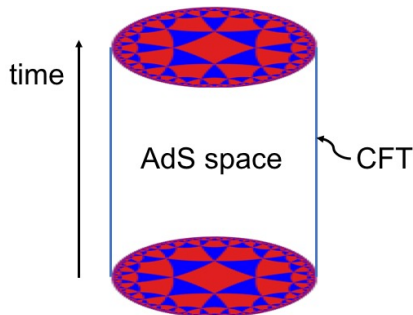
- Rest of the talk: what bootstrap can teach us about Quantum Gravity and string theory.
- Connection: **AdS / CFT correspondence** (gauge / gravity duality; holographic duality) [Maldacena '97; Gubser, Klebanov, Polyakov '98; Witten '98] :  
  
certain CFTs that are generalizations of QCD with  $N$  colors have dual descriptions in terms of gravity w/ negative cosmological constant (as a limit of string theory or M-theory)
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# AdS/CFT

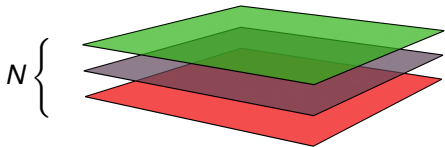
- Precise correspondence:  
CFT quantities (e.g. correlation functions) can be computed from the bulk.



- $L/\ell_p \propto N^\#$  where  $L$  is the curvature radius of AdS, and  $\ell_p$  is the Planck length.
- (Semi-classical) gravity is a good approximation when the number of colors  $N$  is large ( $\implies L/\ell_p$  is large, i.e. the curvature of spacetime is small)



# Examples

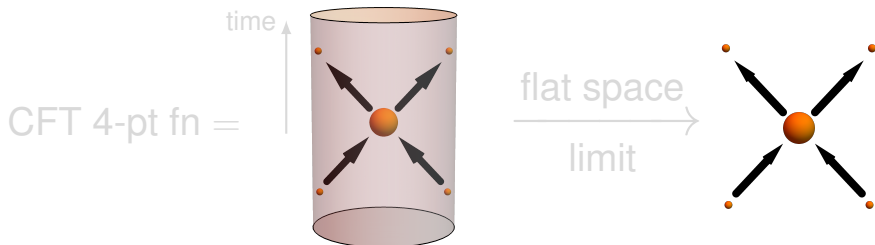


Examples of CFTs with AdS duals (both with maximal SUSY):

- $\mathcal{N} = 4$  supersymmetric Yang-Mills theory in  $3 + 1$  dimensions.
  - $L/\ell_p \propto N^{1/4}$ .
  - Dual to  $AdS_5 \times S^5$ .
  - Describes physics of  $N$  membranes in 10-dimensional string theory.
- ABJM theory in  $2 + 1$  dimensions [Aharony, Bergman, Jafferis, Maldacena '09].
  - $L/\ell_p \propto N^{1/6}$ .
  - Dual to  $AdS_4 \times S^7$ .
  - Describes physics of  $N$  membranes in 11-dimensional M-theory.

# Study Quantum Gravity through CFT

- What can one learn about string theory / Quantum Gravity by studying these CFTs?
- Example: **graviton scattering** [Polchinski, Giddings, Penedones, Fitzpatrick, Kaplan, Goncalves, ...]



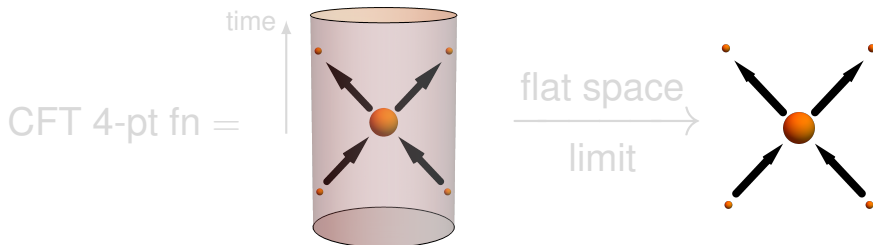
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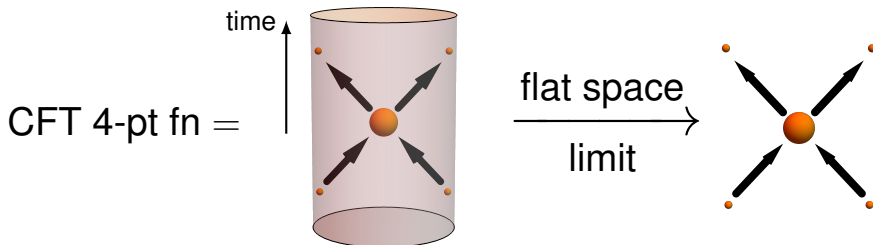
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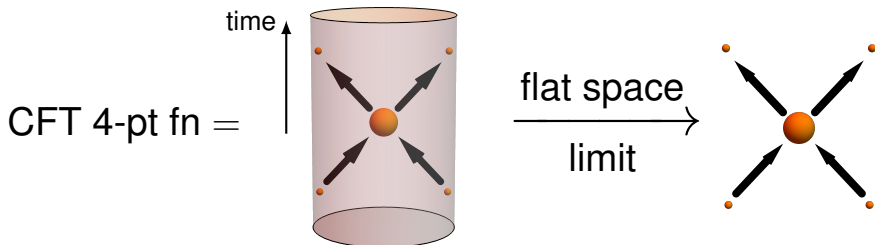
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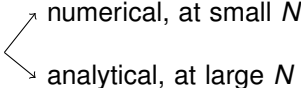
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# Study Quantum Gravity through CFT

Of all CFTs w/ AdS duals, the most constrained are those with **supersymmetry** (such as  $\mathcal{N} = 4$  SYM or ABJM theory), for which one can compute “**protected**” quantities.

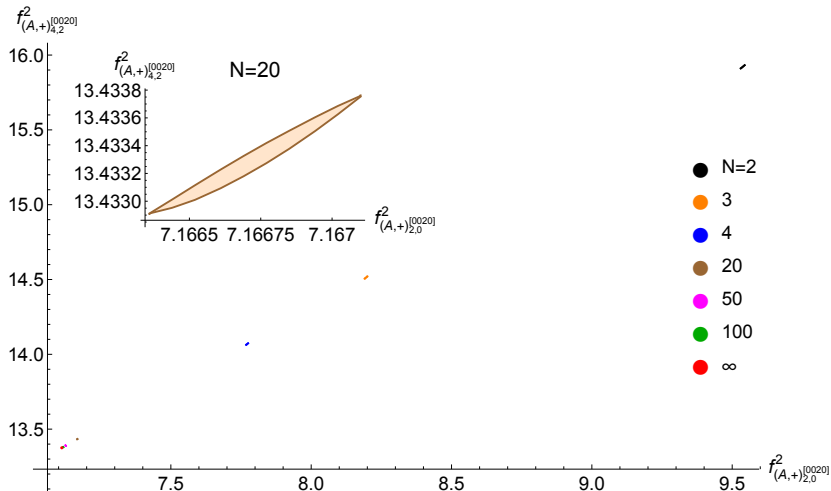
- Example: integrals of correlators of certain operators.

Ongoing efforts 

- numerical, at small  $N$
- analytical, at large  $N$

# Numerical study in ABJM theory (in $2 + 1$ dimensions)

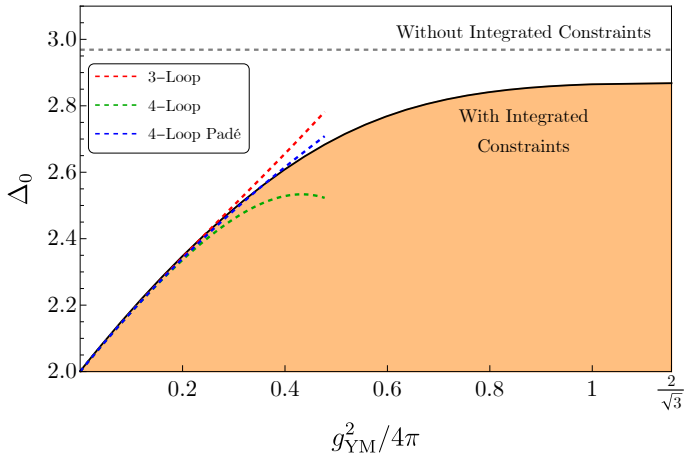
Numerical bootstrap **islands** for ABJM theory [Chester, Lee, SSP, Yacoby '14; Agmon, Chester, SSP '19] (uses SUSY [Dedushenko, Fan, SSP, Yacoby '16, '17, '18]).



# Numerical study in $\mathcal{N} = 4$ SYM (in $3 + 1$ dimensions)

Numerical bootstrap **bounds** on lowest unprotected op in  $SU(2)$   $\mathcal{N} = 4$  SYM

[Chester, Dempsey, SSP '19]



Bounds can be improved by including more SUSY info, mixed correlators, etc.



# Analytic bootstrap: a 4d example

- In  $\mathcal{N} = 4$  SYM, schematically at **fixed**  $g_{\text{YM}}$

$$\langle TTTT \rangle = 1 + N^{-2} + \textcolor{red}{N}^{-\frac{7}{2}} + N^{-4} + \textcolor{blue}{N}^{-\frac{9}{2}} + \dots$$

and each coefficient can be determined from Witten diagrams.

- Analytic properties of Witten diagrams + symmetry + exact results from SUSY completely determine  $\langle TTTT \rangle$  at the first few orders in  $1/N$ .
- Flat space limit of  $\langle TTTT \rangle \implies$  the first few corrections to the Einstein-Hilbert action (as seen by graviton scattering). Schematically:

$$S_{10d} = \int \sqrt{-g} \left[ R + \textcolor{red}{Riemann}^4 + \textcolor{blue}{D^4 Riemann}^4 + \dots \right]$$

[Chester, SSP, Yin '18; Binder, Chester, SSP, Wang '19; Binder, Chester, SSP '19; Chester, Green, SSP, Wang, Wen '19, '20] (See also [Alday, Bissi, Perlmutter, Heslop, Paul, ...] ).

- Can derive the  $\textcolor{red}{Riemann}^4$ ,  $\textcolor{blue}{D^4 Riemann}^4$  terms in 10-dimensional string theory (and also in 11-dimensional M-theory from ABJM theory) **from CFT bootstrap + SUSY!**

# Comments

- Bootstrap is **crucial**, b/c the standard technique (Witten diagrams in AdS) cannot be used due to the fact that the interaction vertices **are not fully known**.
- In  $\mathcal{N} = 4$  SYM,  $\langle TTTT \rangle$  is a fn of two params:  $N$  and  $g_{\text{YM}}$ .
  - $g_{\text{YM}}^2 \leftrightarrow g_s$  (string coupling)
  - $1/N \rightarrow$  (small momentum expansion of scattering amplitudes)
- In flat space, graviton S-matrix is known:
  - At leading order in  $g_{\text{YM}}$  exactly in momentum (Virasoro-Shapiro)
  - At low orders in momentum ( $R^4, D^4 R^4, D^6 R^4$ ) exactly in  $g_{\text{YM}}$ .
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# Comments

- Bootstrap is **crucial**, b/c the standard technique (Witten diagrams in AdS) cannot be used due to the fact that the interaction vertices **are not fully known**.
- In  $\mathcal{N} = 4$  SYM,  $\langle TTTT \rangle$  is a fn of two params:  $N$  and  $g_{\text{YM}}$ .
  - $g_{\text{YM}}^2 \leftrightarrow g_s$  (string coupling)
  - $1/N \rightarrow$  (small momentum expansion of scattering amplitudes)
- In flat space, graviton S-matrix is known:
  - At leading order in  $g_{\text{YM}}$  exactly in momentum (Virasoro-Shapiro)
  - At low orders in momentum ( $R^4, D^4 R^4, D^6 R^4$ ) exactly in  $g_{\text{YM}}$ .
- Two concrete goals:
  - Determine AdS analog of Virasoro-Shapiro
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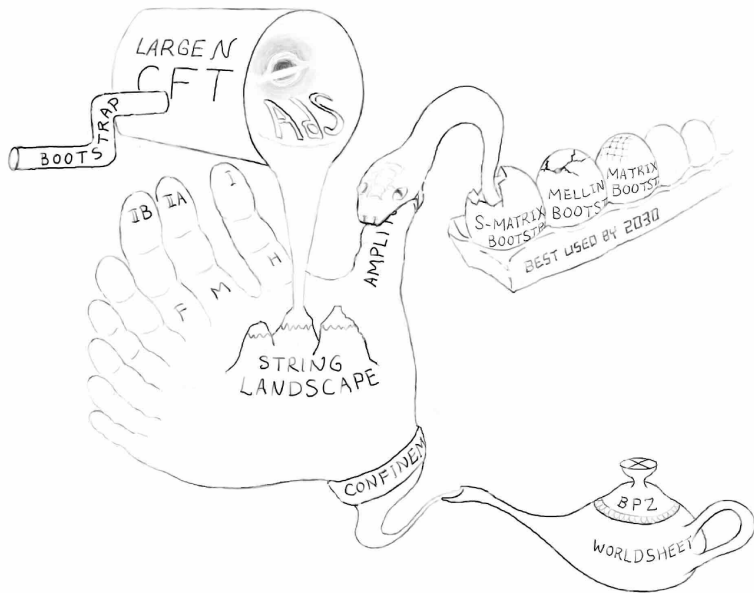
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# Conclusion and other directions

- CFT gives a universal language that applies to a variety of systems that exhibit 2nd order phase transitions and even to **Quantum Gravity**.

## Other ways to bootstrap string theory:

- Bootstrap S-matrices directly. Lots of progress recently. [Vieira, Penedones, Guerrieri, Caron-Huot, Rastelli, Simmons-Duffin, Mazac, ...]
- Bootstrap worldsheet CFT (perhaps together with dual CFT).
- Instead of OPE decomposition, use a different expansion of the correlation function. (Mellin bootstrap?) [Gopakumar, Kaviraj, Sen, Sinha, ...]
- Finite temperature bootstrap [Iliesiu, Kologlu, Mahajan, Perlmutter, Simmons-Duffin, ...]
- Swampland questions: is it possible to rule in/out scale separated AdS vacua? String universality?
- Bootstrapping de Sitter Quantum Gravity [Arkani-Hamed, Baumann, Lee, Pimentel, Pajer, Stefanyszyn, Supel, Goodhew, Jazayeri, Sleight, Taronna, Duaso Pueyo, Joyce, Meltzer, Di Pietro, Gorbenko, Komatsu, Hogervorst, penedones, Vaziri, ...]



[Illustration by Xi Yin, from *[Gopakumar, Perlmutter, Pufu, Yin '22]*]