What is QFT?

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Plan

Introduction

Approaches to axiomatic QFT

Conclusion: need to update axioms?

Hundreds of references in the white paper
2203.08053

This is a review of the 70 years old subject

"Theoretical physics of today is technology of tomorrow."

-Source???

"~~Theoretical~~ Experimental physics of today is technology of tomorrow."

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Mathematics

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Practicality

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- My interest: mathematical content of theories.

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- Sometimes leads to new math.

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Linear algebra
Functional analysis

QFT

???

QFT

???

More precisely:

QFT

???

More precisely:

Particle theory

QFT

Condensed matter

QFT

String Theory

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But what is QFT?
(No such subfield in math)

New Math?

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Let us briefly review them.

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Span of $\varphi_1[f_1] \ldots \varphi_n[f_n] |0\rangle$ approximates all states

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(N) => (W). (Nelson's reconstruction thm)

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Especially powerful with conformal invariance – conformal bootstrap. (see Silviu's talk)

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Scalar QFT in 2d with arbitrary potentials;
φ^4 in three dimensions;
φ^4 in ≥ 4 dimensions is free; (D=4 is recent)
Yukawa models in 2D and 3D;
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Biggest question: Gap in 4D YM (Millenium problem)

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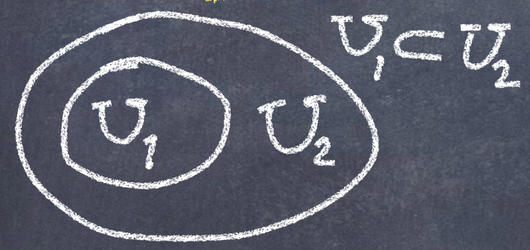

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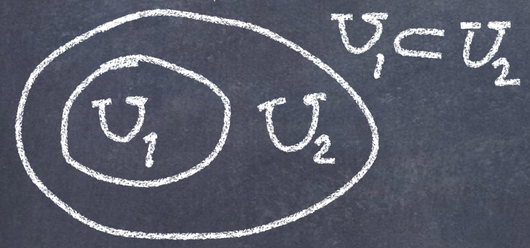

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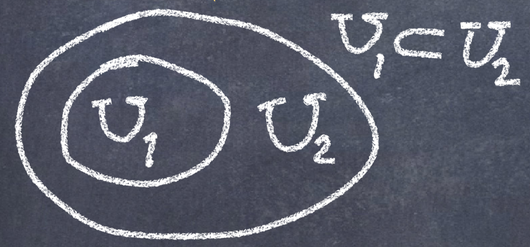

$\mathcal{A}(U_1) \hookrightarrow \mathcal{A}(U_2)$ $\quad$ (Isotony)

$[\mathcal{A}(U_1), \mathcal{A}(U_2)] = 0$ inside $\mathcal{A}(U)$ $\quad$ (Locality)

Spacelike separated

$\Longrightarrow \mathcal{A}(U_1) \simeq \mathcal{A}(U_2)$ $\quad$ (Time slice axiom)

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(Benini-Schenkel-...) Homotopy AQFT (fixes problems of AQFT in gauge theories)

One more AQFT approach

(Costello-Gwilliam) Factorization algebras.

(Became popular among mathematicians recently)

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$$\mathcal{O} \longmapsto \langle \mathcal{O} \rangle_\psi = \langle \psi | \mathcal{O} | \psi \rangle \quad \text{or} \quad \mathrm{Tr}(\mathcal{O} \rho_\psi).$$

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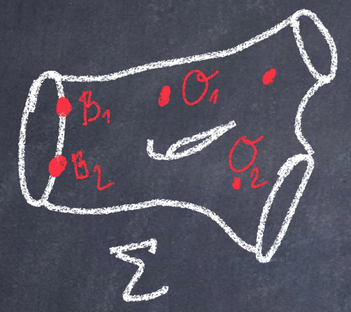

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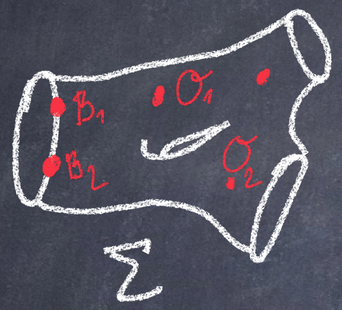

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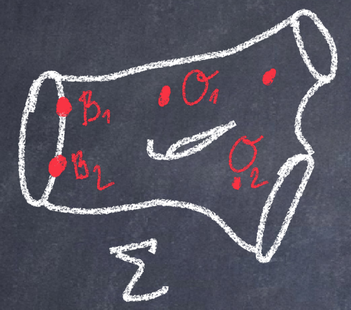

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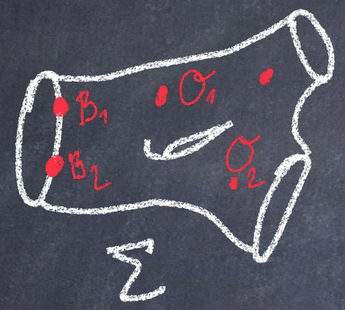

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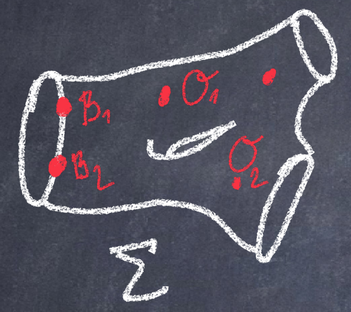

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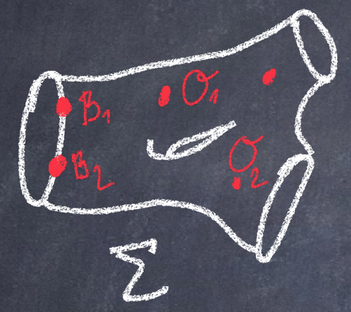

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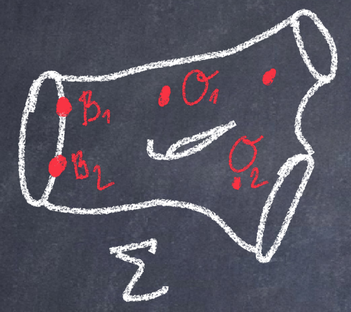

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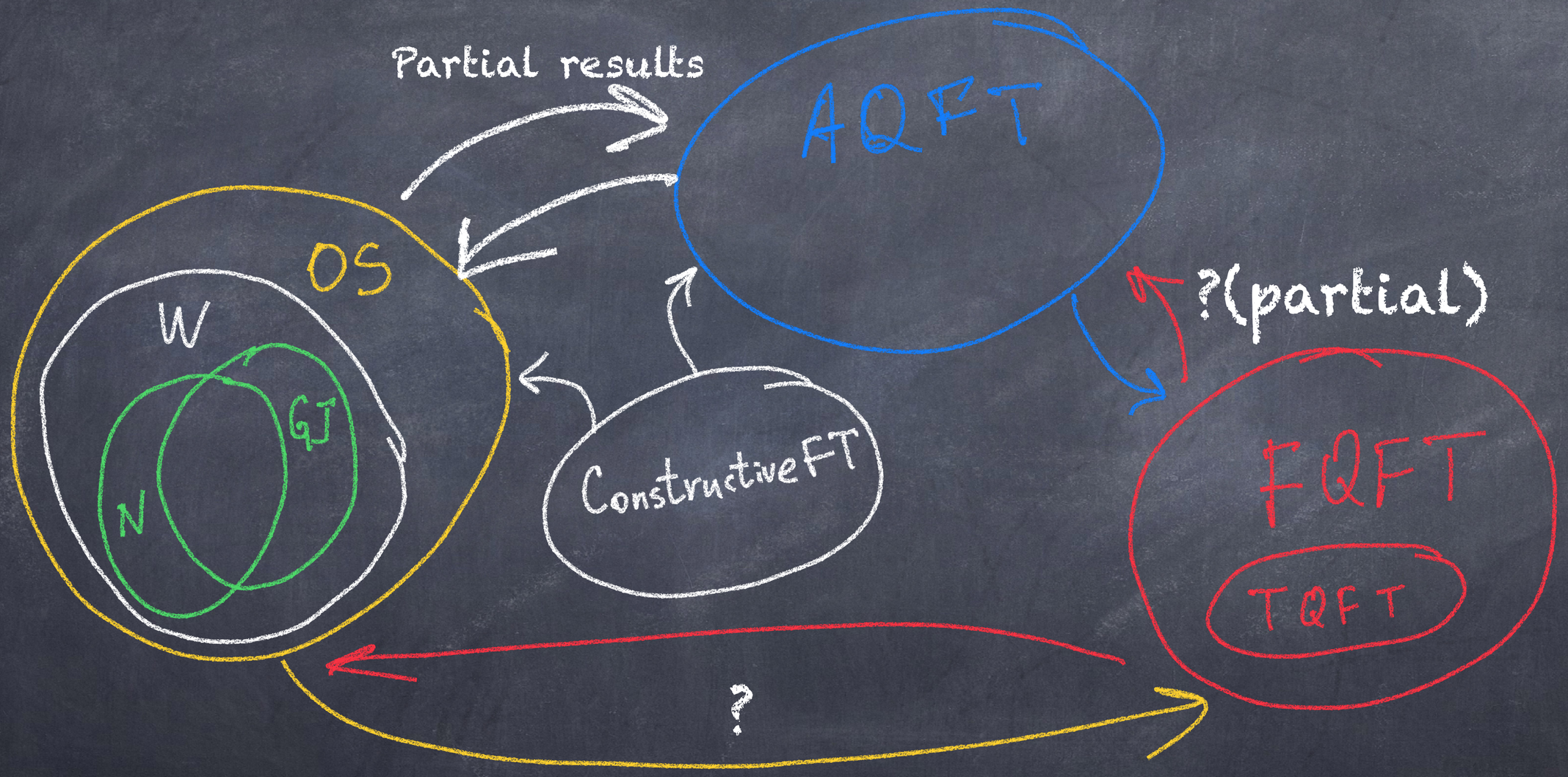
Conformal invariance + AQFT = conformal nets (In 2d, conformal net → VOA)


Partial results
AQFT
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GJ
N
Constructive FT
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