What is QFT?

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Plan

Introduction

Approaches to axiomatic QFT

Conclusion: need to update axioms?
Hundreds of references in the white paper 2203.08053

This is a review of the 70 years old subject
“Theoretical physics of today is technology of tomorrow.”

-Source???
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Experimental

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“Experimental physics of today is experimental physics of tomorrow.”

“Mathematical physics of today is theoretical physics of tomorrow.”
“Theoretical physics of today is technology of tomorrow.”

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“Mathematical physics of today is experimental physics of years to come.”

“Mathematical physics of today is theoretical physics of decades to come.”
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decades to come

... Mathematics
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Right approach - develop all at once.
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My interest: mathematical content of theories.
What is a “theory”? 
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- (Logically self-consistent set of statements) \( \subset \) (Math)
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- Sometimes lands onto a known subfield.
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- (Logically self-consistent set of statements) ∈ (Math)

- Sometimes lands onto a known subfield.

- Sometimes leads to new math.
Examples
Examples

Classical mechanics
Examples

Classical mechanics

Symplectic geometry
Examples

- Classical mechanics
- Symplectic geometry
- Electrodynamics
Examples

- Classical mechanics
- Symplectic geometry
- Electrodynamics
  - Analysis of PDE
  - Differential geometry
Examples

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General relativity
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Quantum mechanics
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Quantum mechanics
Linear algebra
Functional analysis
QFT

???
QFT

More precisely:
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- Particle theory
- Condensed matter
- String Theory
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- Particle theory
- Condensed matter
- String Theory

But what is QFT?  
(No such subfield in math)

New Math?
Worse: the complete consistent set of rules is not even known.
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Currently investigating and expanding.

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A number of definitions (systems of axioms) have been proposed over the years...
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QFT is almost a century old subject! A number of definitions (systems of axioms) have been proposed over the years...

Let us briefly review them.
Wightman's approach (50's)
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Main objects are Wightman functions:
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$$\langle 0 | \varphi_1(x_1) \varphi_2(x_2) \ldots \varphi_n(x_n) | 0 \rangle \equiv W(x_1, \ldots, x_n)$$

Points in Lorentzian spacetime
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Obey axioms:

(W0) Representation th' of Poincare group, spectral condition, uniqueness of vacuum;

(W1) Fields are operator-valued tempered distributions;

(W2) Poincare-covariance of fields;

(W3) Locality (microcausality).
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\text{Span of } \varphi_1(f_1) \ldots \varphi_n(f_n) | 0 \rangle \text{ approximates all states}
\]
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Ed Nelson’s version: also probabilistic, requires Markov property.
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\[ (N) \Rightarrow (W). \quad (\text{Nelson's reconstruction thm}) \]
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Can prove:

- Cluster decomposition
- CPT
- Analytic continuation
- Spin-statistics

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(A) “Solve axioms” — Bootstrap!
(B) Constructive field theory.
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Especially powerful with conformal invariance — conformal bootstrap.
(see Silviu’s talk)
Constructive QFT
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Basic philosophy: Construct path integral rigorously → Check the axioms (OS or GJ or W)
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Main achievements:
Scalar QFT in 2d with arbitrary potentials;
$\phi^4$ in three dimensions;
$\phi^4$ in $>4$ dimensions is free; ($D=4$ is recent)
Yukawa models in 2D and 3D;
Gross-Neveu;
Thirring model;
Gauge theories in 2D and 3D (via lattice);
Partial success in 4D YM.
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Very technical field

Biggest question: Gap in 4D YM (Millenium problem)
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\[ \mathcal{A}(U_1) \hookrightarrow \mathcal{A}(U_2) \]
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  \[ \mathcal{A}(U_1) \hookrightarrow \mathcal{A}(U_2) \]  (Isotony)

- \( [\mathcal{A}(U_1), \mathcal{A}(U_2)] = 0 \) inside \( \mathcal{A}(U) \)  (Locality)

- \( U_1 \) and \( U_2 \) spacelike separated

- \( U_1 \) and \( U_2 \) Cauchy slice
  \[ \mathcal{A}(U_1) \cong \mathcal{A}(U_2) \]  (Time slice axiom)
Different approaches
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(Araki-Haag-Kastler)
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(Benini-Schenkel-...)  Homotopy AQFT (fixes problems of AQFT in gauge theories)
One more AQFT approach

(Costello-Gwilliam) Factorization algebras.

(Became popular among mathematicians recently)
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- Allows to prove general facts. Entanglement properties.
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$\sigma \mapsto \langle \psi | O | \psi \rangle$ or $\text{Tr}(O \rho_\psi)$.
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Key property is the cutting/gluing law:
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Computes correlation functions, partition functions, states...

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(Hard in all approaches.)
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Conformal invariance + AQFT = conformal nets (In 2d, conformal net $\rightarrow$ VOA)
Many other ideas and directions in the literature...

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Hu and Losev proposal: QFT should be defined on “Feynman geometries” (spacetimes with UV cutoff)
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We have an amazing intuition for what QFT encompasses. Making it precise requires some work.

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Thank you!